



Supply chain coordination with a single supplier and multiple retailers considering customer arrival times and route selection



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ABSTRACT

We address a novel decentralized supply chain with one supplier and multiple independent retailers based on the practice of several supply chains in the real world. Coordination of such a supply chain has rarely been studied. Despite overcoming the well-known double marginalization, the supplier's route selection can obstruct supply chain coordination. We present a wholesale-price-and-carpooling contract to coordinate such a supply chain. We demonstrate supply chain coordination under such a contract and show that the profit along the supply chain can be arbitrarily allocated. We show that the popular revenue-sharing contract may lose flexibility in profit allocations.

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1. Introduction

Managers of many supply chains with a single supplier and multiple retailers face a pricing-and-routing problem. With one vehicle, the supplier distributes newsvendor-type products to different retailers located within different geographic regions. The retailers do not compete with each other and make ordering decisions on the basis of the wholesale prices charged by the supplier. These newsvendor-type products require long lead times and are sold in short selling seasons. The arrival time of products at each retailer depends on the delivery route selected by the supplier, but the arrival times of the customers to purchase the products vary. Suppliers in these supply chains cannot start production too early because of specific constraints. Therefore, some customers visit local retailers before product arrive, and because these customers typically do not look for the products at other retailers, the orders are lost. This problem can be widely found in many supply chains in the real world and our study was motivated by the following real examples.

In China, customers use many traditional food supply chains. In each supply chain, a single supplier distributes food to multiple retailers every day. Each retailer only places one order every day. Because the food must be used shortly after harvested or produced, customers will not purchase food after specific, fixed, time points. A traditional food supply chain, as described, has several unique characteristics. On the supply side, to guarantee product freshness or appropriateness, suppliers cannot start production too early. Because suppliers usually deliver food to retailers with a single vehicle, the route selection influences when retailers can sell the food. On the demand side, consumers of traditional foods in rural China, primarily housewives, visit retailers in the morning to purchase traditional foods and other goods for use at lunch or dinner. The housewife consumers in our example visit at different time points as determined by their own schedules. Those who visit local retailers before product arrival fail to purchase the desired product, but they typically do not search for the product

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at other retailers. In addition, although some retailers have predictably early-arriving clientele, the supplier may not serve them first because of the increased transportation costs, which might exceed the increased revenue gained by delivering early. Some retailers who receive late deliveries complain to the supplier or even quit the supply chain. Their pricing and ordering decisions are more complicated by customer arrival times and supplier route selections. A similar situation is found in the traditional bread supply chains of Argentina.

The second example involves newspaper supply chains. One traveling salesperson, who determines the route, delivers newspapers to several newsstands. In the morning of every business day, workers pass by newsstands near mass transit areas and buy available newspapers. Those who arrive at their local newsstands before the newspapers are delivered typically neither wait for the supply to arrive nor look for the newspapers at other newsstands. This purchasing pattern also characterizes many university campuses where instructors and students try to buy newspapers from local vendors during break times. In these cases, customer arrivals fall within a wide range of time points at different newsstands. However, because of the nature of the news cycle, publishers cannot start production too early.

On the basis of the practice, we developed our model that differs from others in the literature in two aspects. First, the demands at all retailers are generated by customers arriving at different times; in most real cases, customers enter the system at different time points. With the assumption that the products can be delivered to retailers before the selling time (season) starts, many studies on supply chain coordination describe the total demand at each retailer as a probability distribution. In some supply chains, however, the arrival time of each customer needs to be considered. For example, Grubbström (2010) studied the newsvendor problem to show ways to maximize the net present value of the payments involved, and customer demand was considered a compound renewal process. In our supply chain model, we account for the arrival time of customers because the product arrival times are decision variables that affect the overall supply chain profit. Second, the route decided by the supplier influences the profits of all supply chain members. Many studies show that supply chain coordination cannot be achieved because of double marginalization (Spengler, 1950) and the competition between retailers. A well-known phenomenon, double marginalization is found when the retailer's optimal order quantity is smaller than the channel-wide optimal inventory level. A fairly large number of contracts have been developed to align retailers' pricing and ordering decisions. In our supply chain model, the supplier uses a new decision framework and plays a more important role than in existing supply chain models. We show that the supplier's route selection can create a new barrier to supply chain coordination. To the best of our knowledge, this type of supply chain has rarely been studied. Fig. 1 illustrates such a supply chain with n retailers.

Managers of one food supply chain in China realized supply chain profit losses and raised the following questions: Are the profits of members maximized in a decentralized supply chain? If not, how do we increase the profits of supply chain members? Can existing supply chain contracts improve the supply chain profit? On the basis of these questions, we study the coordination of supply chains. To the best of our knowledge, coordination of such supply chains has been unexplored, and this article is the first to answer these managerial questions. Therefore, our study fills the research gap by coordinating the decentralized supply chain. The specific contributions of this study are as follows:

- (1) We uncover barriers to the coordination of the supply chain, in which the supplier charges each retailer a price for each unit purchased and decides the route. On the basis of prices and route, the retailers decide their own selling prices and order quantities. Double marginalization exists in all of the subsystems, each of which involves a retailer and the supplier. In addition, the supplier may prefer a local optimal route that cannot maximize supply chain profit (this phenomenon is described in detail in Section 3).
- (2) In contrast to combining or extending existing contracts, we developed a wholesale-price-and-carpooling (WPC) contract based on practice in the taxi industry in China to address the unique characteristics of the supply chain, which enhances the feasibility of our contract. The contract suggests use of a carpooling strategy through which retailers pay the supplier wholesale prices for each unit purchased and share the transportation costs of the delivery routes, which they share in part. We demonstrate that the WPC contract can achieve supply chain coordination and arbitrarily allocate supply chain profit. Principles of carpooling have not been used to design a supply chain contract. In traditional carpooling, passengers share a transportation service, usually one vehicle in which several people ride, sacrificing some individual benefits, such as streamlined travel time and privacy, while saving transportation fees, easing traffic

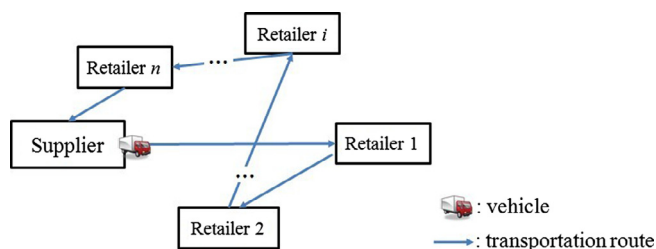


Fig. 1. Supply chain model.

congestion, and emitting less carbon dioxide. In Beijing, the government encourages carpooling contracts. When two independent passengers share one taxi, for example, they pay 60% of the taxi fee for the common route. The passenger with a farther destination pays the remaining costs of the taxi ride. In this case, both passengers spend less money than if they each employed a separate taxi, and the taxi driver also earns more through carpooling than when serving one passenger. Several types of carpooling problems and relevant studies have focused on developing algorithms to obtain optimal routes for vehicles (e.g., Baldacci et al., 2004; Yan and Chen, 2011). See Agatz et al. (2012) for a detailed discussion of existing studies on optimization for carpooling problems.

- (3) We analyze the way the popular revenue-sharing contract and the buy-back contract perform in newsvendor supply chains. We found that these two contracts lose flexibility in terms of profit allocations. We also discuss the way the decision variables under the contracts influence supply chain profit.

This paper is organized as follows: In Section 2, we review related studies to reveal our contribution to the literature. In Section 3, we analyze the decision frameworks of members in centralized and decentralized supply chains. Section 4 presents the coordinating WPC contract. Section 5 offers discussion of the revenue-sharing contract under a newsvendor supply chain. In Section 6, we summarize the managerial implications of this study, and we give concluding remarks, including suggestions for several directions of future research, in Section 7.

2. Literature review

The price-setting newsvendor problem is a classic inventory problem that has been widely studied. The retailer decides order quantity and selling price under different conditions to achieve sales targets (Xu et al., 2010; Murray et al., 2012; Abad, 2014; Wang et al., 2014; Rubio-Herrero et al., 2015). For example, Wang and Chen (2015) studied the retailer's optimal ordering policy with option contracts under which the retailer can purchase products from a supplier or purchase options. Luo et al. (2016) discussed a price-setting newsvendor problem under a demand function in which the stochastic part also depends on the selling price. Ye and Sun (2016) extended the basic price-setting newsvendor problem by considering strategic customers, some of whom may purchase the products at the end of the selling season at the salvage value. The above studies focused on the optimal decision of the retailer.

When the optimization problem is extended to the supply chain, contracts which have been widely studied and used in the real world are essential for supply chain coordination, and this paper is mostly related to this topic. See Cachon (2003) for a review of contracts for supply chains with a single supplier and a single retailer. Many studies extended the contracts to coordinate supply chains consisting of one supplier and multiple retailers. Ingene and Parry (1995) demonstrated that a two-part tariff wholesale pricing policy can be used to coordinate the supply chain with multiple independent retailers. Revenue-sharing contracts are reportedly used widely in the video rental supply chain with multiple retailers (e.g., see Dana and Spier, 2001). Cachon and Lariviere (2005) proposed the revenue-sharing contracts under which the supply chain with multiple competing retailers can be coordinated. Bernstein and Federgruen (2005) proposed a buy-back contract (also called price-discount sharing contract) that can achieve coordination for supply chains with competing retailers under demand uncertainty. Zhao (2008) showed that a buy-back contract can be employed to coordinate a supply chain with retailers under both price and inventory competition. Chen and Xiao (2009) analyzed the supply chain model with a dominant retailer and several fringe retailers. That study also compared the performances of quantity discount and wholesale price contracts with the consideration of demand disruption. Zhang et al. (2012) developed the revenue-sharing contract to coordinate supply chains with competing retailers and demand disruption. Cao et al. (2013) extended the revenue-sharing contract for supply chains with competing retailers by considering both demand and production cost disruptions. Chiu et al. (2015) proposed menus of contracts on the basis of target sales rebate, fixed quantity, and quantity discount to coordinate supply chains with mean-variance retailers. David and Adida (2015) studied the coordination of a supply chain in which a single supplier sells the products through a direct channel and multiple retailers. In that paper, the quantity discount contract is extended to coordinate the supply chain. The above studies focused on the supply chain operating newsvendor-type products.

Several researchers also investigated the coordination of supply chains in which multiple retailers use continuous review inventory policy and have unlimited opportunities of sending orders to a single supplier. For example, Chen et al. (2001) proposed a composite contract with fixed payment and quantity discount mechanisms to coordinate such supply chains with independent retailers. Boyaci and Gallego (2002) developed a consignment mechanism for the coordination of the supply chain. Chiou et al. (2007) investigated a supply chain in which multiple retailers have budget constraints and developed a price-discount-and-savings-sharing contract to coordinate the supply chain. Chen and Xiao (2017) extended the model by considering competing retailers and proposed a Grove wholesale price contract for coordination.

Table 1 summarizes the relevant studies on the supply chain coordination with consideration of multiple retailers. As can be seen from the literature described, most existing studies on the supply chain coordination with newsvendor-type products consideration were based on considerations of total demand of each retailer. Customer arrival and route selection of the supplier were not included in those studies. The related research in the continuous review inventory model did not consider how the supplier's route selection affects the supply chain performance. Moreover, the contracts in these studies are

Table 1

Comparison of this study with some relevant studies.

	Inventory model	Problem characteristic	Customer arrival	Supplier's route selection	Coordination mechanism
Ingene and Parry (1995)	N	Independent retailers	/	/	Two-part tariff
Chen et al. (2001)	C	Independent retailers	✓	/	Quantity discount and fixed payment
Dana and Spier (2001)	N	Inventory and price competing retailers	/	/	Revenue-sharing
Boyaci and Gallego (2002)	C	Independent retailers	✓	/	Consignment
Cachon and Lariviere (2005)	N	Price competing retailers	/	/	Revenue-sharing
Bernstein and Federgruen (2005)	N	Price competing retailers	/	/	Buy-back
Chiou et al. (2007)	C	Budget-constrained retailers	✓	/	Price discount and savings sharing
Zhao (2008)	N	Inventory and price competing retailers	/	/	Buy-back
Chen and Xiao (2009)	N	A dominant retailer	/	/	Quantity discount; Wholesale price
Zhang et al. (2012)	N	Demand disruption	/	/	Revenue-sharing
Cao et al. (2013)	N	Cost and demand disruption	/	/	Revenue sharing
Chiu et al. (2015)	N	Risk consideration	/	/	Target sales rebate
David and Adida (2015)	N	Dual channel	/	/	Quantity discount
Chen and Xiao (2017)	C	Price competing retailers	✓	/	Wholesale price
This study	N	Timing issue of demand and supply	✓	✓	Wholesale-price- and carpooling

"N" represents *Newsvendor-type product model* and "C" represents *continuous review model*

"✓" represents *covered* and "/" represents *not covered*

developed with the combination or extension of existing classic contracts. In contrast to the existing studies, our study is based on consideration of the customer arrival times and the route, as decided by the supplier, that influence the profits of all supply chain members. Moreover, the idea of carpooling is unexplored in the supply chain literature, despite the use of it in some industries. We develop a novel WPC contract that can bring new methodology and insights applicable to the supply chain contract.

In addition to supply chain coordination, there are other three streams of literature on the supply chain with one supplier and multiple retailers. The first direction focuses on transshipment under which a retailer can move excess inventories or capacity to another with excess demand (Anupindi et al., 2001; Shao et al., 2011; Li and Zhang, 2015; Feng et al., 2017). The second type of literature features inventory competition under which customers of one retailer can switch to other retailers when the demand is not fully satisfied. Lippman and McCardle (1997) studied multiple retailers whose demand is allocated by using a splitting rule based on the realization of demand. Anupindi and Bassok (1999) analyzed how the level of market search influences the profit of the manufacturer. Netessine and Rudi (2003) studied the system with n different types of products. In that system, a deterministic fraction of the excess demand of each product shows the ratio of consumers who will switch to another product as a substitute. Zhao and Atkins (2008) studied multiple competitive newsvendors in which a proportion of customers lost by one vendor switch to another one. Jiang and Anupindi (2010) compared the customer-driven search with transshipment between two retailers. However, these studies did not consider the supplier's decision making or vertical coordination of the supply chain.

The third direction is known as the inventory routing problem (IRP). The IRP involves a single supplier who is responsible for monitoring and replenishing inventories for multiple geographically dispersed retailers (customers). The decision includes when to replenish, how much to replenish, and which route to use (Song and Savelsbergh, 2007). There exists a rich body of literature on the IRP (e.g., Bassok and Ernst, 1995; Xu et al., 2007; Kleywegt et al., 2002; Campbell and Savelsbergh, 2004; Gaur and Fisher, 2004; Song and Savelsbergh, 2007; Zhao et al., 2007; Yu et al., 2008; Li et al., 2011; Niakan and Rahimi, 2015; Cheng et al., 2017). These studies were based on the vendor managed inventory systems in which the retailer's role was ignored. In these models, the supplier made all of the decisions with the purpose of optimizing its own or the supply chain's target value. The trade-off is between inventory cost and utilization of vehicles (transportation cost). Liu and Chen (2011) considered the deterministic demand that is a linear function of the sales prices. However, they also assumed that the supplier and retailers act as one company to maximize the total supply chain profit. Existing studies on the IRP focused on developing efficient policies or algorithms to obtain a channel-wide optimal solution. In our supply chain model, the supplier cannot control retailers' ordering decisions, and our study focuses on design of contracts under which supply chain coordination can be achieved. In particular, Alaei and Setak (2015) studied a multi-objective coordination problem of a supply chain with route selection by using the revenue-sharing contract. Their model took into account total demand at each retailer but did not explore the impact of supplier's route selection on supply chain coordination.

3. Model analysis

Our model presents a single-period supply chain model with one supplier and n independent retailers, all of whom are risk neutral. Each retailer has an exclusive territory, which means that there is no pricing and inventory competition among these retailers. In each selling season, with one vehicle and one trip, the supplier distributes a common product to the retailers in the chain. Customers leave the system if they find the products unavailable when they arrive at their local retailers. In a set $SC = \{0, 1, \dots, n\}$ of supply chain members, member 0 represents the supplier, and member i represents retailer i , where $i = 1, 2, \dots, n$. The supplier decides wholesale prices to charge retailers and the delivery route that determines the product arrival time at each retailer. Based on the product arrival time and wholesale price, retailer i makes two decisions: the order quantity of the product, q_i , and its selling price, p_i . The time when the products arrive at retailer i is denoted by t_i . In this model, the revenue of retailer i generated from sales is influenced by t_i . $R_i(p_i, q_i|t_i)$ is the total revenues of retailer i under t_i , including the revenue generated from sales and salvage activity. We assume that, for any pair of $\{p_i, q_i\}$, $R_i(p_i, q_i|t_i)$ is non-increasing with t_i . The supplier's unit production cost is c , and $p_i > c > 0, i = 1, 2, \dots, n$. $R_i(p_i, q_i|t_i)$ can present the revenue under either a deterministic or stochastic demand. $R_i(p_i, q_i|t_i)$ represents the expected revenue of retailer i who faces a stochastic demand. If we set p_i as exogenously specified and $p_1 = p_2 = \dots = p_n$, the demand function is close to the case in newspaper supply chains. Without loss of generality, we assume that the production costs of the retailers are zero and the speed of the vehicle is a constant. We assume that the vehicle capacity is unlimited. This assumption is based on the observation on the food supply chain in China that the utilization rate of the vehicle capacity is around 50% in each trip. The goodwill penalty for lost sales is not considered in this model.

3.1. Centralized supply chain

In a centralized supply chain, the supplier and all of the retailers act to maximize the supply chain profit. Let x_{ij} be equal to 1 if and only if member j is visited immediately after member i and it is 0 otherwise. The travel time and transportation cost from member i to member j are respectively represented by g_{ij} and a_{ij} , where $i, j = 0, 1, \dots, n$. For simplicity, we assume that the service time of member i is negligible. Without loss of generality, we let the time when the product leaves the supplier, t_0 be 0 and set M as a big positive number. We denote the vectors of selling prices, order quantities, product arrival times, and sub-routes (arcs) as $\bar{p} = (p_1, \dots, p_n), \bar{q} = (q_1, \dots, q_n), \bar{t} = (t_1, \dots, t_n)$, and $\bar{x} = (x_{01}, \dots, x_{0n}, \dots, x_{i0}, \dots, x_{in}, \dots, x_{n0}, \dots, x_{n,n-1})$. $\pi_{sc}(\bar{p}, \bar{q}, \bar{t}, \bar{x})$ is the supply chain profit. With the approach of modeling the traveling salesman problem (TSP), we obtain the decision framework of the supply chain as follows.

(P1)

$$\text{Max } \pi_{sc}(\bar{p}, \bar{q}, \bar{t}, \bar{x}) = \sum_{i=1}^n R_i(p_i, q_i|t_i) - c \sum_{i=1}^n q_i - \sum_{i,j \in SC, i \neq j} a_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j \in SC, j \neq i} x_{ij} = 1 \quad \forall i \in SC \tag{2}$$

$$\sum_{i \in SC, i \neq j} x_{ij} = 1 \quad \forall j \in SC \tag{3}$$

$$t_j \geq t_i + g_{ij} - M(1 - x_{ij}) \quad \forall i, j \in SC, j \neq 0 \tag{4}$$

$$x_{ij} = 0, 1 \quad \forall i, j \in SC \tag{5}$$

$$t_j \geq 0 \quad \forall j \in SC \tag{6}$$

In (P1), the objective function is to maximize the supply chain profit. Constraints (2) and (3) ensure that the supplier and each retailer are visited exactly once. Constraint (4) guarantees that the product arrival time to a retailer is longer than or equal to the product arrival time to the retailer visited immediately before (or t_0 in case $i = 0$) plus the travel time between the two members. Because g_{ij} is positive for all $i, j \in SC$, sub-tour elimination can also be achieved by this constraint. Any \bar{x} satisfying these constraints presents a feasible route. As we can see, (P1) is an extension of the TSP problem that is NP-hard. Therefore, small problems of centralized supply chains can be solved by using optimization tools like LINDO and CPLEX. We can obtain the channel-wide optimal solution for large problems by using existing algorithms. We set $\bar{p}^o, \bar{q}^o, \bar{t}^o$, and \bar{x}^o as the vectors of the optimal prices, order quantities, product arrival times, and sub-routes that maximize the supply chain profit, i.e., $\bar{p}^o = (p_1^o, \dots, p_n^o), \bar{q}^o = (q_1^o, \dots, q_n^o), \bar{t}^o = (t_1^o, \dots, t_n^o)$, and $\bar{x}^o = (x_{01}^o, \dots, x_{0n}^o, \dots, x_{i0}^o, \dots, x_{in}^o, \dots, x_{n0}^o, \dots, x_{n,n-1}^o)$, respectively. We assume that $R_i(p_i, q_i|t_i)$ is differentiable. For the case with stochastic demand, the marginal revenue of retailer i is decreasing in q_i , i.e., $\partial^2 R_i(p_i, q_i|t_i) / \partial (q_i)^2 < 0$ (e.g., see [Cachon and Lariviere, 2005](#)). Because \bar{t}^o and \bar{x}^o satisfy Constraints (2)–(6), for all $i = 1, 2, \dots, n, p_i^o$ and q_i^o should satisfy

$$\partial R_i(p_i^o, q_i^o|t_i^o) / \partial p_i = 0, \tag{7}$$

$$\partial R_i(p_i^o, q_i^o|t_i^o) / \partial q_i = c. \tag{8}$$

Eqs. (7) and (8) present the necessary conditions for p_i^0 and q_i^0 , respectively. Considering that $R_i(p_i, q_i|t_i)$ is non-increasing with t_i , \bar{t} can be determined by \bar{x} based on Constraint (4) and t_0 is always equal to 0. Consequently, we can use \bar{t} to present the route and write the objective function as

$$\text{Max } \pi_{sc}(\bar{p}, \bar{q}, \bar{t}) = \sum_{i=1}^n R_i(p_i, q_i|t_i) - c \sum_{i=1}^n q_i - TC(\bar{t}) \tag{9}$$

where $TC(\bar{t}) = \sum_{i,j \in SC, i \neq j} a_{ij} x_{ij}$.

3.2. Decentralized supply chain

In a decentralized supply chain, members make decisions in three stages. As shown in Fig. 2, the supplier makes two decisions: the route and the wholesale price, w_i , that retailer i pays per unit purchased. Based on t_i and w_i , retailer i decides p_i and q_i to maximize its own profit, $i = 1, 2, \dots, n$.

With a specific route and wholesale price, the profit function of retailer i is

$$\pi_i(p_i, q_i|t_i, w_i) = R_i(p_i, q_i|t_i) - w_i q_i. \tag{10}$$

We set p_i^* and q_i^* as the optimal selling price and order quantity for retailer i . For any t_i and w_i , p_i^* and q_i^* should respectively satisfy

$$\partial R_i(p_i^*, q_i^*|t_i) / \partial p_i = 0, \tag{11}$$

$$\partial R_i(p_i^*, q_i^*|t_i) / \partial q_i = w_i. \tag{12}$$

We can see that p_i^* and q_i^* are functions of t_i and w_i , where $i = 1, 2, \dots, n$. We denote the vector of wholesale prices as $\bar{w} = (w_1, \dots, w_n)$ and the decision framework of the supplier in the decentralized supply chain (P2) can be written as follows:

$$\text{Max } \pi_0(\bar{w}, \bar{t}) = \sum_{i=1}^n (w_i - c) q_i^*(w_i, t_i) - \sum_{i,j \in SC, i \neq j} a_{ij} x_{ij} \tag{13}$$

subject to

Constraints (2)–(6).

In (P2), the objective function is to maximize the supplier's profit and the same constraints as in (P1) are used. It is easy to show that the optimal w_i in (P2) should satisfy $w_i > c$, $i = 1, 2, \dots, n$.

Proposition 1. *If $(\bar{p}^0, \bar{q}^0, \bar{p}^*, \bar{q}^*)$ are determined by Eqs. (7), (8), (11) and (12), respectively, then the channel-wide optimal prices and order quantities, \bar{p}^0 and \bar{q}^0 , cannot be achieved in decentralized supply chains.*

Proposition 1 still holds under the case with deterministic demand. On the basis of Proposition 1, we can infer that for any \bar{t} satisfying Constraints (2)–(6) and \bar{p} , the retailers' optimal order quantities cannot maximize supply chain profit. This finding means that a retailers' optimal pricing-and-ordering decision differs from the channel-wide optimal decision. When the selling prices are fixed, retailer order quantities are lower than the channel-wide optimal order quantities. The well-known *double marginalization* exists in all of the subsystems in the newsvendor supply chain. Therefore, managers need to implement contracts to coordinate the supply chain and improve total profits. Moreover, they should pay attention to the retailer decisions during contract development.

3.3. Supplier's problem

In many supply chain models, double marginalization and competition among retailers are considered barriers for supply chain coordination. In our model, however, the supplier's route decision creates a potential barrier for improving the supply chain profit. In this subsection, we discuss how the route selection influences the total profit in a decentralized supply chain. Because p_i^* and q_i^* are functions of t_i and w_i , we can obtain the decentralized supply chain's profit as a function of \bar{w} and \bar{t} , i.e., $\pi_{sc}(\bar{w}, \bar{t})$. Let \bar{w}^* and \bar{t}^* be the vectors of (w_1^*, \dots, w_n^*) and (t_1^*, \dots, t_n^*) that maximize $\pi_{sc}(\bar{w}, \bar{t})$ and that satisfy Constraints



Fig. 2. Time line of decisions.

(2)–(6). The optimal supply chain profit is $\pi_{sc}(\bar{w}^*, \bar{t}^*) = \sum_{i=1}^n R_i(p_i^*, q_i^* | t_i^*) - c \sum_{i=1}^n q_i^* - TC(\bar{t}^*)$. From (P2) we can see that, for any \bar{t} and \bar{x} that satisfy Constraints (2)–(6), the supplier has an optimal \bar{w} to maximize $\pi_0(\bar{w}, \bar{t})$. Suppose that route A is the optimal route determined by (P2), and the transportation cost under route A is smaller than under the channel-wide optimal route. Under route A with \bar{t}^A , the supplier's profit is $\pi_0(\bar{w}^A, \bar{t}^A) = \sum_{i=1}^n (w_i^A - c)q_i^A - TC(\bar{t}^A)$, in which \bar{w}^A and \bar{q}^A are the supplier's optimal \bar{w} and retailer i 's optimal order quantity under route A. We obtain the optimal profit of the supply chain under route A as $\pi_{sc}(\bar{w}^A, \bar{t}^A) = \sum_{i=1}^n R_i(p_i^A, q_i^A | t_i^A) - c \sum_{i=1}^n q_i^A - TC(\bar{t}^A)$ and $\pi_{sc}(\bar{w}^*, \bar{t}^*) \geq \pi_{sc}(\bar{w}^A, \bar{t}^A)$. Let the gross profits of the supply chain and the supplier be $\sum_{i=1}^n R_i(p_i, q_i | t_i) - c \sum_{i=1}^n q_i$ and $\sum_{i=1}^n (w_i - c)q_i$, respectively. We can see that when switching from route A to the channel-wide optimal route, the supplier may find that its increased gross profit is smaller than the increased channel gross profit. It means that the increased channel gross profit is shared by the supplier and retailers. Meanwhile, the transportation cost is undertaken by the supplier. This difference may distort the supplier's route decision. If the increased transportation cost is higher than the increased gross profit of the supplier, then the supplier will select a suboptimal route. Mathematically speaking, if $\pi_0(\bar{w}^*, \bar{t}^*) - \pi_0(\bar{w}^A, \bar{t}^A) = \sum_{i=1}^n (w_i^* - c)q_i^* - \sum_{i=1}^n (w_i^A - c)q_i^A - TC(\bar{t}^*) + TC(\bar{t}^A) < 0$, then the supplier prefers route A and the optimal supply chain profit cannot be achieved.

The above discussion shows that the supplier's route selection can set up a barrier to improved supply chain profit in the decentralized setting. Because the proof under a general demand function is complicated, we take the linear demand function as an example to prove that there exist cases in which the supplier may prefer a suboptimal route. We select this demand function for the following two reasons. First, this linear demand form is commonly used in the fields of supply chain management, inventory management, and marketing. Second, it is close to the demands observed from the food and newspaper supply chains in China. As discussed in Section 1, each customer's arrival at the local retailer is fairly certain on each working day. Note that this linear demand function is used only for the proof of supplier's route selection problem. All of other parts in this article are based on the general demand function. Assume that the customers arrive at retailer i at a deterministic rate per time unit determined by p_i ; that is $D_i(p_i) = e_i - p_i b_i$ where e_i and b_i are positive constants. We assume that $e_i - cb_i > 0$ and each customer orders one unit of product. The selling period of retailer i is limited within the range of $[f_i, l_i]$ and we assume that l_i is always greater than t_i . In the real world, f_i and l_i can be uncertain. However, based on our observation that the uncertainties are relatively low compared to the selling periods, we take f_i and l_i as exogenously specified, $i = 1, 2, \dots, n$. The profit function of retailer i in the decentralized supply chain is

$$\pi_i(p_i, q_i | t_i, w_i) = (p_i - w_i)(e_i - p_i b_i)T_i \quad (14)$$

In Eq. (14), $q_i = (e_i - p_i b_i)T_i$ and $T_i = l_i - \max(f_i, t_i)$. Therefore, $p_i^* = (e_i + b_i w_i)/(2b_i)$ and $q_i^* = (e_i - b_i w_i)T_i/2$, and the supplier's optimal wholesale price to retailer i is $w_i^* = (e_i + b_i c)/(2b_i)$. Therefore, $e_i - b_i w_i^* > 0$ and q_i^* is non-increasing with t_i . For any route, \bar{x}, \bar{t} can be obtained based on Constraint (4). In addition, we can obtain $p_i^* = (3e_i + b_i c)/(4b_i)$ and $q_i^* = (e_i - b_i c)T_i/4$. In this case, the decision framework of the supplier (P3) can be represented as

$$\text{Max } \pi_0(\bar{w}^*, \bar{t}) = \sum_{i=1}^n \frac{(e_i - b_i c)^2 T_i}{8b_i} - TC(\bar{t}) \quad (15)$$

subject to

Constraints(2)–(6).

Moreover, the decision framework of the supply chain (P4) can be represented as

$$\text{Max } \pi_{sc}(\bar{w}^*, \bar{t}) = \sum_{i=1}^n \frac{3(e_i - b_i c)^2 T_i}{16b_i} - TC(\bar{t}) \quad (16)$$

subject to

Constraints (2)–(6).

Let \bar{t}^* be the vector of $(t_1^*, t_2^*, \dots, t_n^*)$ determined by a route that maximizes $\pi_{sc}(\bar{w}^*, \bar{t})$ and that satisfies Constraints (2)–(6).

Proposition 2. In a decentralized supply chain with the linear demand functions, (i) if $TC(\bar{t}^*)$ is smaller than or equal to all other $TC(\bar{t})$ values in which \bar{t} satisfies Constraints (2)–(6), \bar{t}^* represents the optimal route of the supplier; (ii) otherwise, the supplier may prefer another route that cannot maximize the supply chain profit.

Proposition 2 indicates that, in the decentralized supply chain under linear demand functions, the supplier is incentivized to select the channel-wide optimal route only when it generates the smallest transportation cost. In other cases, the supplier may prefer a route that is not optimal for the supply chain. When switching from one route to another, the supplier may find that the gross profit increment for the channel is larger than its gross profit increment generated from supplying the chain. Furthermore, the supplier shoulders any increased transportation cost. This imbalance may distort the supplier's route selection, which should also be considered in the contract design. This phenomenon is illustrated in Example 1. The optimal routes for the supplier and supply chain are denoted as routes S and O, respectively.

Example 1. For a supply chain with one supplier and four retailers, we let $c = \$1$ and $a_{ij} = g_{ij} \times 1$ (\$/min). The demand rates and selling period ranges are shown in Table 2. Table 3 shows the travel time between different members. We solved (P3) and (P4) by using CPLEX, and the computational time is negligible.

In this example, the optimal route that maximizes the supplier's profit is 0-1-2-3-4-0 (route S); while the optimal route that maximizes the supply chain profit is 0-2-1-3-4-0 (route O). Product arrival times of the four retailers under routes S and O are shown in Table 4. From Eqs. (15) and (16), we calculate the profits of the supplier and the supply chain. Let T_i^S and T_i^O be the product arrival times at retailer i under routes S and O, respectively. Using route S in Example 1, the supplier's gross profit is $\sum_{i=1}^4 (w_i - c)q_i = \sum_{i=1}^4 \frac{(e_i - b_i c)^2 T_i^S}{8b_i} = \414.9 , and the transportation cost is \$112.0. The gross profit of the supply chain is $\sum_{i=1}^4 R_i(p_i, q_i | t_i) - c \sum_{i=1}^4 q_i = \sum_{i=1}^4 \frac{3(e_i - b_i c)^2 T_i^S}{16b_i} = \622.4 . In this case, the profits, after expenses, of the supplier and the supply chain are \$302.9 and \$510.4, respectively. Under route O, the supplier's gross profit is $\sum_{i=1}^4 \frac{(e_i - b_i c)^2 T_i^O}{8b_i} = \430.3 and the transportation cost is \$129.0. The supply chain's gross profit is $\sum_{i=1}^4 \frac{3(e_i - b_i c)^2 T_i^O}{16b_i} = \645.5 . The final profits of the supplier and the supply chain are \$301.3 and \$516.5, respectively. We can see that when switching from route S to route O, the supplier's transportation cost increases by \$17.0 and the gross profit of the supply chain increases by \$23.1. This finding means that the supply chain benefits from the selection of route O because the profit of the channel is maximized. However, in this case, the supplier's gross profit increases by \$15.4, which is smaller than the increased transportation cost. Therefore, the supplier is incentivized to choose route S to maximize its own profit, which leads to a lower total profit for the supply chain.

As Example 1 shows, if the supplier switches from route S to route O, then Retailer 2 obtains the products earlier and the supply chain's gross profit can be increased. However, the gap between the gross profits of the supplier and the supply chain implies that the supplier's gross profit increment is smaller than the one of the supply chain. Because the transportation cost is undertaken by the supplier, this imbalance between gross profit and transportation cost may result in a suboptimal route selected by the supplier. When managers seek to improve the performance of the supply chain, much attention should be paid to encouraging the supplier to select the channel-wide optimal route. In such supply chains, supply chain coordination is achieved when the retailers and the supplier select \bar{p}^o , \bar{q}^o , and \bar{t}^o simultaneously. However, coordinating such supply chains is more difficult than coordinating the classic ones, because the supplier's route decision is involved.

When the vehicle has a capacity constraint, the constraints in (P1) and (P2) can be extended to a multi-vehicle routing problem in which one vehicle delivers products to retailers in one cluster. We can obtain the objective functions of the supply chain and the supplier by considering gross profits and transportation costs. In this case, a gap is found between the gross profits of the supplier and channel. On the basis of the analysis in Sub Section 3.3, we can find cases in which the supplier may prefer a local optimal transportation solution when multiple capacitated vehicles are used.

4. Coordinating wholesale-price-and-carpooling contracts

Carpooling contracts in the taxi industries have been encouraged by the government of cities in some cities with a large population and relatively limited transportation supply capacities, such as Beijing, China.

For any route, we rearrange the retailers in ascending order according to the product arrival time:

$$t_{(1)} < t_{(2)} < \dots < t_{(n)} \tag{17}$$

The supplier is Member (0), and let the arc between members $(i - 1)$ and (i) be sub-route (i) with transportation cost $a_{(i)}$, $i = 1, \dots, n$. Because the supplier must be the last visited member, the last sub-route is sub-route (0) with transportation $a_{(0)}$. For example, in one route with three retailers, the sequence of visited retailers is 0-2-3-1-0. Because $t_2 < t_3 < t_1$, Retailer (1) represents Retailer 2, Retailer (2) represents Retailer 3, and Retailer (3) represents Retailer 1. We can also obtain the indexes of sub-routes, as Fig. 3 shows.

Definition 1 (Partly common route). Sub-route (i) is the partly common route of retailers $(i), (i + 1), \dots, (n)$. In particular, the sub-route between retailer (n) and the supplier is the partly common route of all of the retailers.

Under the WPC contract, each retailer pays the supplier a wholesale price for each unit purchased. In addition, the supplier obtains a payment from the retailers for each sub-route. Let $Z_{ij}a_{ij}$ be the payment for the sub-route between member i and member j , in which Z_{ij} is a contract parameter. When the supplier selects a route, we can obtain sub-route (i) based on (17) and arrange the retailers, $i = 0, 1, \dots, n$. For simplicity, let $Z_{(i)}$ represent Z_{jk} if the arc between member j and member k is

Table 2
Demand rates and selling periods (minutes).

Retailer	e_i	b_i	f_i	l_i
1	8	2	10	100
2	6	1	10	80
3	9	3	10	120
4	5	2	10	180

Table 3
Travel time between different members (minutes).

Member	0	1	2	3	4
0	0	20	15	25	25
1	20	0	27	42	40
2	15	27	0	20	31
3	25	42	20	0	20
4	25	40	31	20	0

Table 4
Product arrival times (minutes) under different routes.

	t_1	t_2	t_3	t_4
route S	20	47	67	87
route O	42	15	84	104

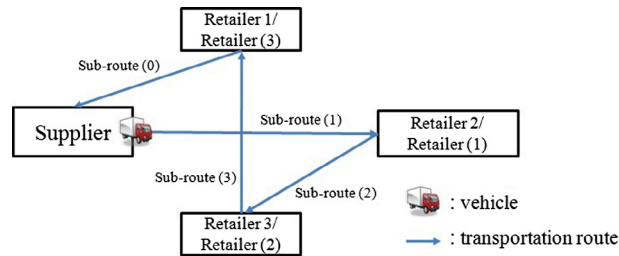


Fig. 3. An example of indexes of retailers and sub-routes.

sub-route $(i), i, j, k = 0, 1, \dots, n$ and $i \neq j \neq k$. When the vehicle starts from the supplier and goes to Retailer (1), it is loaded with the products of all of the retailers; this sub-route is the partly common route of Retailers (1)– (n) . The sub-route between retailers $(n - 1)$ and (n) is the part in which the products of only retailer (n) are carried. The number of partly common routes of retailer (i) is $i + 1, i = 1, 2, \dots, n$. Retailers share the payment for their partly common route. The WPC contract works as follows.

- Prior to the selling season, the supplier and retailers agree to the wholesale prices and the payment of $Z_{ij}a_{ij}$ to the supplier for using the sub-route between member i and member j .
- The supplier selects the route based on $Z_{ij}a_{ij}$.
- Retailer i places an order for q_i units and pays the supplier w_i for each unit purchased.
- The supplier delivers the products to the retailers. The retailers are rearranged based on the route and share $Z_{(i)}a_{(i)}, i, j = 0, 1, \dots, n$ and $i \neq j$.

Under the WPC contract, the supplier's profit function is

$$\pi_0(\bar{w}, \bar{t}) = \sum_{i=1}^n (w_i - c)q_i^*(w_i, t_i) + \sum_{i,j \in SC, i \neq j} (Z_{ij} - 1)a_{ij}x_{ij} \tag{18}$$

Lemma 1. By setting $Z_{ij} < 1$ if $x_{ij}^0 = 0$ and $Z_{ij} > 1$ otherwise, the supplier's optimal route is \bar{x}^0 when $w_i = c, i, j = 0, 1, \dots, n$ and $i \neq j$.

By setting w_i and $\{Z_{ij}|i, j \in SC \text{ and } i \neq j\}$ based on the Lemma, the supplier's route decision can be optimized. Then, we can rearrange the retailers based on (17) to obtain $Z_{(i)}, i = 0, 1, \dots, n$. Let $\pi_{(i)}$ be the profit of member (i) under the WPC contract, $i = 0, 1, \dots, n$, and π_{sc}^0 be the maximum profit of the supply chain that can be achieved by $\{\bar{p}^0, \bar{q}^0, \bar{t}^0\}$. For a set of $\{k_i|i \in SC\}$ and a selected route, let $k_{(i)}$ represent k_j if member (i) is member $j, i, j = 0, 1, \dots, n$.

Theorem 1. For any set of $\{k_i|i \in SC\}$ wherein $\sum_{i=0}^n k_i = \pi_{sc}^0$ and $k_i \in (0, \pi_{sc}^0)$, the WPC contract with $w_i = c$ and $\{Z_{ij}|i, j \in SC \text{ and } i \neq j\}$ can be obtained from the Lemma where $\sum_{i=0}^n (Z_{(i)} - 1)a_{(i)} = k_{(0)}$ under the channel-wide optimal route. In addition, retailer (i) pays the supplier $\phi_{(i)(j)}$ of $a_{(j)}$ if sub-route (j) is its partly common route, satisfying $\sum_{i=m}^n \phi_{(i)(m)} = Z_{(m)}$ and $\sum_{i=1}^n \phi_{(i)(0)} = Z_{(0)}$

where $j = 0, 1, \dots, n$ and $m = 1, 2, \dots, n$. We can always find a set of $\{Z_{(j)}|j \in SC\}$ and $\{\phi_{(i)(j)}|i, j \in SC\}$ for the WPC contract under which supply chain coordination is achieved and $\pi_{(i)} = k_{(i)}$, $i = 0, 1, \dots, n$.

The theorem indicates that supply chain coordination can be achieved and the total profit can be arbitrarily allocated under the WPC contracts. When $w_i = c$, the retailers will select the channel-wide optimal pricing and ordering decisions for any route; so the double marginalization can be solved under the WPC contract. Because the wholesale prices are the same with the supplier's unit production cost, the supplier needs to receive some compensation for the decreased wholesale prices. Meanwhile, it should be guaranteed that the channel-wide route will be selected by the supplier. The idea of carpooling takes good care of these two considerations. By using the carpooling, if the arc between retailers i and j is selected, the supplier can obtain $Z_{ij}a_{ij}$. The lemma and [Theorem 1](#) show that there exist settings of WPC contract under which the supplier will select the channel-wide optimal route. Therefore, the double marginalization and route selection are solved simultaneously under the WPC contract. When the channel-wide optimal route is selected, the supplier can obtain $Z_{(i)}a_{(i)}$ for sub-route (i) that is greater than the related transportation cost, $a_{(i)}$, and this is the compensation for the low wholesale prices.

Moreover, under the WPC contract, the total profit can be arbitrarily allocated among all supply chain members. At first glance, it seems that the contract might be unfair to the retailers who receive their products last by considering that they need to pay for most of the sub-routes. However, this intuition does not hold under the WPC contract, because the payment can be adjusted by setting Φ . For example, even though retailer (n) needs to pay for all of the sub-routes, the total payment can be smaller than other retailers' if $\Phi_{(n)(j)}$ is sufficiently small, $j = 0, 1, \dots, n$. Note that the total supply chain profit can be arbitrarily allocated under the WPC contract. The profits of supply chain members are determined by $\{k_i|i \in SC\}$ that depends on the members' relative bargaining power. The supplier's profit is increased with $\sum_{i=0}^n (Z_{(i)} - 1)a_{(i)}$ and retailer (i)'s profit is decreased with $\sum_{j=0}^i \phi_{(i)(j)}a_{(j)}$, $i = 1, 2, \dots, n$. The Lemma shows an easy way of setting $\{Z_{ij}|i, j \in SC \text{ and } i \neq j\}$ that can be easily understood and used by managers. Let member i 's outside opportunity profit be k_i where $\sum_{i=0}^n k_i \leq \pi_{sc}^0$ and $k_i > 0$, $i = 0, 1, \dots, n$ (see [Cachon and Lariviere, 2005](#)). We assume a profit split $(k_0 + \eta_0, k_1 + \eta_1, \dots, k_n + \eta_n)$ with $\sum_{i=0}^n (k_i + \eta_i) = \pi_{sc}^0$ and $\eta_i \geq 0$ for all $i = 0, 1, \dots, n$. For a selected route, let $\eta_{(i)}$ represent η_j if member (i) is member j , $i, j = 0, 1, \dots, n$. Based on the proof of [Theorem 1](#), we can always find a set of $\{Z_{(j)}|j \in SC\}$ and $\{\phi_{(i)(j)}|i, j \in SC \text{ and } i \neq j\}$ wherein $\sum_{i=0}^n (Z_{(i)} - 1)a_{(i)} = k_{(0)} + \eta_{(0)}$ for the WPC contract under which $\pi_{(i)} = k_{(i)} + \eta_{(i)}$, $i = 0, 1, \dots, n$. Consequently, by using the $\{Z_{ij}|i, j \in SC \text{ and } i \neq j\}$ obtained from the Lemma, all of the opportunity profits are satisfied.

The above theorem presents the WPC contract in a general form. In the real world, the WPC contract can be used in some simplified ways by adding some constraints on the decision variables.

Theorem 2. For any set of $\{k_i|i \in SC\}$ where $\sum_{i=0}^n k_i = \pi_{sc}^0$ and $k_i \in (0, \pi_{sc}^0)$, we can look at the WPC contract with $w_i = c$ and $\phi_{(j)(j)} = \phi_{(j+1)(j)} = \dots = \phi_{(n)(j)}$, $j = 1, 2, \dots, n$. For any $\{Z_{ij}|i, j \in SC \text{ and } i \neq j\}$ satisfying the Lemma, there exists a unique $\{\phi_{(i)(j)}|i, j \in SC\}$ for the WPC contract under which supply chain coordination is achieved and $\pi_{(i)} = k_{(i)}$, $i = 0, 1, \dots, n$.

As [Theorem 2](#) shows, the involved retailers pay the same percentage of the transportation cost for each sub-route. The number of decision variables under this contract is fewer than under the general contract shown in [Theorem 1](#). It implies that this simplified version of WPC contract has the advantage of easy implementation.

The information required to implement the WPC contract is not difficult to manage. For the customer arriving process of each retailer, the channel-wide optimal route can be consistent in different selling seasons and the optimal route is independent of profit allocation. Retailer does not need to monitor each other's revenues. In addition, retailers can easily monitor the delivery route. From (P1), we can see that the product arrival times to retailers create discrete variables. Retailers can conjecture about the sequence of retailers visited first based on their own product arrival time. This characteristic also supports that the WPC contract cannot be unfair to the retailers visited last. When all of the retailers can monitor their partly common routes (except the last sub-route), the supplier should follow sub-routes from (1) to (n). Because each route consists of a Hamiltonian tour, the number of sub-routes is $n + 1$. Then, the last sub-route is also followed. It means that when the retailers who are served relatively late monitor their partly common routes, they also guarantee the final sub-route agreed upon by all members.

5. Supply chain coordination under revenue-sharing contracts

The revenue-sharing and buy-back contracts are the most widely studied contracts under supply chains with a single supplier and multiple retailers. These two contracts are also shown to be more attractive than some other contracts for these supply chains (e.g., see [Dana and Spier, 2001](#); [Cachon and Lariviere, 2005](#)). Buy-back contracts have been studied for coordinating supply chains with multiple retailers (e.g., [Cachon, 2003](#); [Bernstein and Federgruen, 2005](#); [Zhao, 2008](#)). Supply chain coordination is achieved under the buy-back contracts by making the retailer's profit linearly increasing with the supply chain profit. Based on this property, we infer that buy-back contracts share the same limitations as the revenue-sharing contract. Therefore, we discuss only the revenue-sharing contract under our supply chain model. Under the revenue-sharing contract, each retailer pays the supplier a wholesale price for each unit purchased, and shares its total revenue with the supplier.

Proposition 3. Under the revenue-sharing contract, retailer i keeps γ_i of its revenue and $w_i = \gamma_i c$, $\gamma_i \in (0, 1)$. For any route, retailer i 's optimal p_i and q_i maximize subsystem i 's profit, $i = 1, 2, \dots, n$. The profit function of the supplier is

$$\pi_0(\bar{\gamma}, \bar{t}) = \sum_{i=1}^n (1 - \gamma_i) \Pi_i - TC(\bar{t}). \quad (19)$$

Proposition 3 tells that the revenue-sharing contract can address the double marginalization for any route. Hence, when the supplier selects the channel-wide optimal route, subsystem i 's profit is Π_i^0 and supply chain coordination is achieved under the revenue-sharing contract. In addition, profit split is determined by $(\gamma_1, \gamma_2, \dots, \gamma_n)$. However, the supplier's route selection problem is not considered in this contract. We can obtain the supplier's decision framework under the revenue-sharing contract including Constraints (2)–(6) and the objective function to maximize $\pi_0(\bar{\gamma}, \bar{t})$.

Proposition 4. Under the revenue-sharing contract, if $TC(\bar{t}^0)$ is not the smallest among all feasible values of $TC(\bar{t})$, the supply chain profit cannot be arbitrarily allocated.

Proposition 4 shows that, in some cases, supply chain coordination cannot be achieved under certain profit-split scenarios. Because the revenue-sharing contract focuses on coordinating each retailer's pricing-and-ordering decision, the incentives to align retailers' decisions may distort the supplier's route selection. If the available profit-split scenarios cannot be accepted by some members, then those contracts are proven infeasible and the supply chain coordination cannot be achieved. Therefore, contracts with a limited range of available profit split may not be widely implemented. In addition, in the real world, the profit split usually depends on each supply chain member's relative bargaining powers or outside dynamic opportunity to earn higher profits. For instance, if retailer i has a higher opportunity profit outside the supply chain, it may require a higher γ_i under which the supplier is not willing to select the channel-wide optimal route. Then, the revenue-sharing contract may fail to coordinate the supply chain. This limitation can result in a negative impact on the robustness of long-term implementation of the contract. Hence, the flexibility of profit allocation is an important characteristic of supply chain contracts (e.g., [Cachon and Lariviere, 2005](#); [Feng et al., 2015](#)).

Example 2 illustrates the limitation of the revenue-sharing contract in terms of profit split.

Example 2. Under the revenue-sharing contract, for any profit split and route, retailer i 's optimal p_i and q_i satisfy $p_i^0 = (e_i + b_i c)/(2b_i)$ and $q_i^0 = (e_i - b_i c)T_i/2$, $i = 1, 2, 3, 4$. For a profit split of $\gamma_1 = 30\%$, $\gamma_2 = 50\%$, $\gamma_3 = 60\%$, and $\gamma_4 = 30\%$, the supplier's decision framework is

$$\pi_0(\bar{t}) = \sum_{i=1}^4 \frac{(1 - \gamma_i)(e_i - b_i c)^2 T_i}{4b_i} - TC(\bar{t}) \quad (20)$$

subject to

Constraints (2)–(6).

The supply chain's decision framework is

$$\pi_{sc}(\bar{t}) = \sum_{i=1}^4 \frac{(e_i - b_i c)^2 T_i}{4b_i} - TC(\bar{t}) \quad (21)$$

subject to

Constraints (2)–(6).

We solved this problem by using CPLEX, and Table 5 shows the profits under routes S and O in Example 2. In this example, the route that maximizes the supplier's profit is 0-1-2-3-4-0 (route S); while the channel-wide optimal route is 0-2-1-3-4-0 (route O). Under route S, the supplier's profit, exclusive of transportation cost, is \$491.90 and the transportation cost is \$112.00. In this case, the final profits of the supplier and the supply chain are \$379.90 and \$717.80, respectively. Under route O, the profits of the supplier and the supply chain are \$359.80 and \$731.70, respectively. As a consequence, the supplier prefers route S, which leads to a lower total profit for the supply chain.

If a contract cannot allow for arbitrarily allocated supply chain profit, some supply chain members cannot reap opportunity profits. In this case, the members cannot agree upon a revenue-sharing contract and supply chain coordination cannot be achieved. Because the profits of the subsystems are fixed under supply chain coordination, profit split is also fixed

Table 5
Profit comparison under different routes.

Route	Π_i				π_{sc}	$(1 - \gamma_i)\Pi_i$				π_0
	1	2	3	4		1	2	3	4	
S	360.0	206.2	159.0	104.6	717.8	252.0	103.1	63.6	73.2	379.9
O	261.0	406.2	108.0	85.5	731.7	182.7	203.1	43.2	59.8	359.8

under a given $(\gamma_1, \gamma_2, \dots, \gamma_n)$. For example, if the opportunity profits of supply chain members require a profit split of $\gamma_1 = 30\%, \gamma_2 = 50\%, \gamma_3 = 60\%, \gamma_4 = 30\%$, supply chain coordination cannot be achieved, as [Example 2](#) shows. To gain more insight into how the revenue-sharing contract performs under such supply chains, we analyzed the profits under different profit splits.

Proposition 5. For any $(\gamma_1, \gamma_2, \dots, \gamma_n)$, we assume retailer f is the first visited retailer under route S (i.e., $t_f^S < t_i^S, i = 1, 2, \dots, n$ and $i \neq f$). For any ε , the supplier's optimal \bar{t} is also \bar{t}^S under $(\gamma_1, \dots, \gamma_f - \varepsilon, \dots, \gamma_n)$ wherein $\gamma_f - \varepsilon > 0$ and $\varepsilon > 0$.

[Proposition 5](#) discusses how profit split of the subsystem containing the first visited retailer (retailer f) influences the supply chain profit under the revenue-sharing contract. There may exist profit split scenarios under which route S is the same as route O and supply chain coordination is achieved. However, there is an upper bound on γ_f . If γ_f is beyond the upper bound, the supplier cannot obtain its maximum profit by serving retailer f first and will select another route that cannot maximize the supply chain profit. [Fig. 4](#) illustrates this phenomenon by using the data in [Example 2](#) with $\gamma_1 = 30\%, \gamma_3 = 60\%$, and $\gamma_4 = 30\%$. In this case, to achieve supply chain coordination, the upper bound on γ_2 is 39.9%. Moreover, the supplier's profit is linearly decreasing and Retailer 2's profit is linearly increasing with γ_2 . When γ_2 is greater than 39.9%, however, the proportion of the subsystem's profit obtained by the supplier is relatively small. Therefore, the supplier prefers another route under which Retailer 2 is not the first visited retailer and supply chain coordination is not achieved, as [Example 2](#) shows. Because the subsystem's profit is decreased, the decreasing rate of the supplier's profit and the increasing rate of Retailer 2's profit are respectively smaller than the rates when $0 < \gamma_2 \leq 39.9\%$. In addition, Retailer 2 can obtain a higher profit by selecting a sufficiently high γ_2 than its maximum profit under supply chain coordination (when $\gamma_2 = 39.9\%$). Consequently, if Retailer 2 has sufficient bargaining power, it can obtain a large γ_2 and supply chain coordination cannot be achieved.

The cases for other retailers are more complex. Consider the retailers who are not visited first under route O . Under other routes, the product arrival times at these retailers can be either increased or decreased. Suppose retailer j is not visited first under route O . Intuition suggests that under a coordinating profit split there could be a lower bound and an upper bound on γ_j for these retailers. If γ_j is sufficiently small or large, the supplier may prefer another route under which t_j is smaller or greater than under route O . [Fig. 5](#) shows the profits under different values of γ_1 by using the data in [Example 2](#) with $\gamma_2 = 39.9\%, \gamma_3 = 60.0\%$, and $\gamma_4 = 30.0\%$. The supply chain coordination can only be achieved when $29.9\% \leq \gamma_1 \leq 53.2\%$. When γ_1 is not within this range, the supplier selects two different routes under which Retailer 1 receives the product at varying times. When $\gamma_1 \leq 53.2\%$, for example, the product arrival time of Retailer 1 is sooner than the one in which $\gamma_1 > 53.2\%$.

In some cases $\gamma_j (j \neq f)$ has only a lower bound to guarantee supply chain coordination. The possibility of these cases for the last retailer visited under route O might be greater than the possibilities of other retailers. If retailer l is the last visited retailer under route O and γ_l is increased, the supplier can obtain a smaller proportion of this subsystem's profit. Consequently, the supplier has lower incentive to decrease t_l . If there is no other route under which t_l is greater than under route O , the supplier will keep route O for any γ_l between the lower bound and 1. [Fig. 6](#) shows the profits under different γ_4 by using the data in [Example 2](#) with $\gamma_1 = 30.0\%, \gamma_2 = 39.9\%$, and $\gamma_3 = 60.0\%$. The results show that supply chain coordination can be achieved when $\gamma_4 \geq 29.5\%$. When $\gamma_4 < 29.5\%$, the product arrival time at Retailer 4 is sooner and this subsystem's profit is increased. Consequently, as [Fig. 6](#) shows, the profits of the supplier and Retailer 4 have greater decreasing and increasing rates than the rates when $\gamma_4 \geq 29.5\%$.

It can be proven that, given a $(\gamma_1, \dots, \gamma_{i-1}, \gamma_{i+1}, \dots, \gamma_n)$, the supplier's profit is decreasing with γ_i , for all $i = 1, 2, \dots, n$. If the supplier has sufficient bargaining power to keep γ_i low, the channel-wide optimal route may not be achieved, as [Figs. 5 and 6](#) illustrate. In [Section 3](#), we show that the supplier's route selection can set up a barrier to improving the supply chain profit when the double marginalization exists. The results in this section suggest that even though retailers' pricing-and-ordering

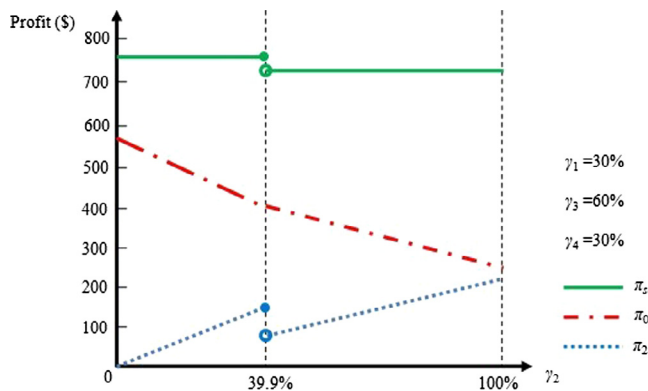


Fig. 4. Profits under different γ_2 .

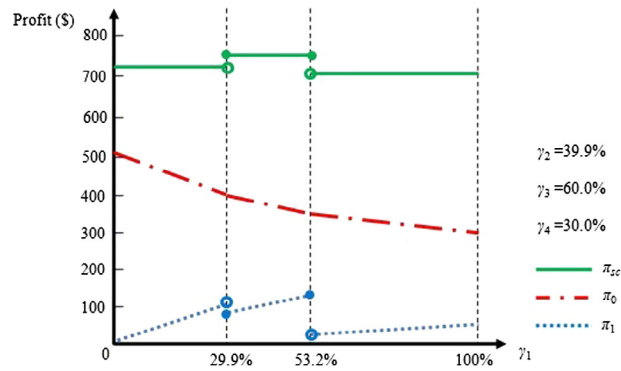


Fig. 5. Profits under different γ_1 .

decisions are coordinated, route selection could still create an obstacle to supply chain coordination under the revenue-sharing contract. Some extended revenue-sharing contracts have been developed for supply chains with multiple retailers. For example, Cachon and Lariviere (2005) extended the classic revenue-sharing contract to coordinate supply chains with competing retailers. Under such a contract, the wholesale price of each retailer depends on the revenue functions and optimal order quantities of other retailers. It means that the retailers must monitor other retailers' revenue, an often expensive or infeasible strategy. Therefore, we only discuss the classic revenue-sharing contract.

Many contracts have been developed for the coordination of supply chains. Therefore, comparison of our WPC contract with all of them creates challenges. However, many contracts share the principle that supply chain members' profit functions should be some fixed linear functions of supply chain profit. Under this typical stipulation, channel members' profit-maximizing incentives are consistent with overall profit maximization for the channel (Qin et al., 2011). This finding implies that under those fixed-linear function requirements in similar contracts, the profit function of each member includes other members' profit functions. In this situation, each member must monitor other members' revenues and costs, which can be very costly or infeasible in practice. Hence, the contracts that make each retailer's profit function linear and incorporated into the corresponding subsystem profit are difficult to implement. From the analysis of revenue-sharing contracts under this type of supply chain, we conjecture that those contracts can only achieve limited profit-split scenarios. The WPC contract, however, addresses double marginalization and supplier's route selection simultaneously. By using the wholesale-price mechanism, we coordinate the retailers' pricing and ordering decisions. To compensate for the low wholesale price, the supplier can charge a transportation service fee for each route. With appropriate values of contract parameters, the supplier is incentivized to select the channel-wide optimal route. Because the WPC contract can coordinate the supply chain and arbitrarily split the total profit, it can be easily implemented in many supply chains in the real world.

6. Summary of managerial implications

In the decentralized supply chain, the retailers make pricing and ordering decisions on the basis of the time when the products arrive. In this case, the delivery route not only determines the supplier's transportation cost but also influences retailers' order quantities; thus, supplier's profit is determined. Hence, the supplier's route selection influences all of the

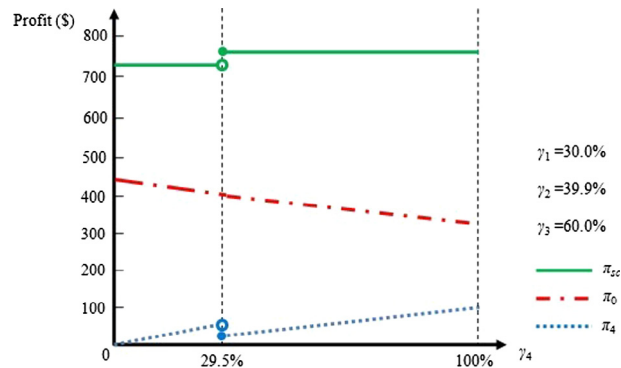


Fig. 6. Profits under different γ_4 .

members' profits by determining when each retailer can obtain products. Moreover, under the WPC, retailers can be more incentivized to promote their sales than under conventional supply chain models. By making an effort to increase demand, a retailer may attract more customers to visit and purchase in a specific period. Under this condition, the retailer benefits the supplier delivering products relatively early. The supplier needs to consider the retailers' demands and order quantities when deciding the delivery route. On the basis of our observation from food and newspaper supply chains, the suppliers select the route mainly on the basis of transportation cost without comprehensively considering retailers' decisions. Our results suggest that suppliers notice the trade-off between gross profit and transportation cost. The supplier may benefit from making the effort to deliver early to the retailer with a large demand rate and early opening time.

The maximized total profit means that all members of the supply chain can obtain higher profits than when the channel is under decentralized control (Feng et al., 2014). Hence, it is beneficial for managers to coordinate supplier and retailers' decisions. When designing a contract to coordinate such supply chains, all parties need to pay attention to both route selection and ordering decisions. Under the WPC contract, the supplier can charge a low wholesale price to encourage the retailers to order more products. Meanwhile, retailers pay for the delivery service that is a main part of the supplier's revenue. By using our WPC contract, supply chain managers can coordinate their decisions to benefit from a high performance supply chain, and each member can obtain a higher profit than when the channel is under decentralized control. Therefore, the WPC contract is likely to appeal to all supply chain members. However, the WPC contract would not be optimal in all supply chains. For example, if the transportation cost is dominated by the revenue earned by the supplier, no trade-off exists between transportation costs and product arrival times. In the studied food supply chains in China, transportation costs of different delivery routes play an important part in supplier profit. This profit structure could explain the reason that managers in those supply chains attempt to coordinate the route selection of the supplier.

Under a revenue-sharing contract, the retailers' pricing and ordering decisions can be coordinated for the given route. Our numerical results indicate that profit-split scenarios are possible under which the revenue-sharing contract can coordinate the supply chain. However, one retailer can encourage the supplier to deliver the products earlier by sharing a smaller proportion of the subsystem's profit. Because gross profit on a proportion basis of the retailer's revenue is relative high, the supplier may prefer a route under which a retailer with more orders than others is visited early enough to maximize retail sales. This optimal route cannot be achieved because, in this type of situation, the revenue-sharing contract cannot arbitrarily allocate the supply chain profit. If the opportunity profits of supply chain members are not feasible under the revenue-sharing contract, then it cannot be used to coordinate the members. Although we resolved double marginalization, the route selection can still present a barrier to supply chain coordination. However, the supplier can implement the contract with some of the retailers, which can improve supply chain performance to some extent. The flexibility of partner selection may make the contract easy to implement in practice. In summary, the revenue-sharing contract can be helpful in limited situations.

7. Conclusions and future research

In our supply chain model, the arrival time of customers influences supply chain profit because product arrival times are decision variables. For any route selected by the supplier, retailers' local optimal ordering and pricing decisions differ from the channel-wide optimal ones because of double marginalization. In addition, we showed that the supplier's route selection creates a new obstacle to supply chain coordination under decentralized control. Because of the gap between the gross profits of supplier and the channel, in some cases, the supplier may prefer a route that cannot maximize supply chain profit. Under linear demand functions, if the channel-wide optimal route generates the smallest transportation costs among all feasible routes, this route can also maximize the supplier's profit; otherwise, the supplier prefers another route that may not maximize supply chain profit.

We developed a novel WPC contract under which the supplier charges low wholesale prices and retailers share the transportation costs for parts of the routes they share. Our results demonstrated that the WPC contract can be used to coordinate a supply chain and arbitrarily allocate supply chain profit. The general WPC contract can be rendered in several simplified versions. We analyzed one case in which retailers share the same proportion of their partly common routes (except the last sub-route) and showed that supply chain coordination can be achieved under this simplified contract. Moreover, the information required to implement this contract is also easy to manage. Supply chain members need not negotiate the delivery route for every selling season. The supply chain also generates an easy information flow that decreases the administrative cost of the WPC contract.

To explore the coordination of a large supply chain further, we studied the popular revenue-sharing contract. We showed that revenue sharing can resolve double marginalization, but it also distorts the supplier's route selection. By using the revenue-sharing contract, supply chain coordination can be achieved, but the supply chain profit cannot be arbitrarily allocated. As a consequence, the deal made between supply chain members can be relatively complex, creating an infeasible contract if the opportunity profits of supply chain members cannot be satisfied. On the basis of the profit allocation to all retailers, for the first retailer on channel-wide optimal route, an upper bound on γ_j guarantees supply chain coordination. However, there could be both upper and lower bounds on γ_j for all other retailers. In particular, in some cases, for the last retailer visited under the channel-wide optimal route, γ_l has only a lower bound. Under the revenue-sharing contract, the

supplier must monitor the retailers’ revenues at every selling season. Therefore, compared with the WPC contract, the information flow could create an obstacle to the implementation of revenue-sharing contracts.

This study has several limitations that can be addressed in future research. First, on the basis of the observations of food supply chains in China, we proposed a model using the assumption that retailers do not compete in pricing or inventory. However, in some other supply chains, retailers compete on the basis of price and customers search for the products in several retailers. This type of supply chains is much more complicated than the one we studied. Second, we assumed that the supplier delivers products to retailers with a single vehicle without capacity limitation. However, multiple vehicles or vehicles that make multiple trips can be essential to meet a high total demand. In this case, considerations of vehicle capacity can make developing a WPC contract much more difficult. Under the WPC contract, retailers undertake the transportation cost on the basis of their partly common route. If multiple vehicles deliver products or one vehicle delivers products in multiple trips, then retailers are assigned to different clusters and the partly common routes of retailers from different clusters would be difficult to define. Third, we proved the supplier’s problem under a linear demand assumption. Extending the proof by assuming that customer arrivals follow a Poisson or renewal process might yield interesting findings. Fourth, an alternative version that encourages lower administrative costs would be worthy of study. A simple contract that shares the advantages of the WPC contract might present interesting possibilities for supply chain management.

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Appendix A

Proof of Proposition 1. As seen from Eq. (9), p_i^o and q_i^o should satisfy $\partial R_i(p_i^o, q_i^o | t_i^o) / \partial q_i = c$. In the decentralized supply chain, with p_i^o and t_i^o , retailer i ’s optimal order quantity, q_i^* , satisfies $\partial R_i(p_i^o, q_i^* | t_i^o) / \partial q_i = w_i$. Because $\partial^2 R_i(p_i, q_i | t_i) / \partial (q_i)^2 < 0$ and $w_i > c$, we can infer that retailer i ’s optimal order quantity is less than q_i^o . Therefore, \bar{p}^o and \bar{q}^o cannot be achieved. □

Proof of Proposition 2. (i) Because \bar{t}^* maximizes $\pi_{sc}(\bar{w}^*, \bar{t})$, for any route, route A with \bar{t}^A , we obtain

$$\sum_{i=1}^n \frac{3(e_i - b_i c)^2 T_i^*}{16b_i} - \sum_{i=1}^n \frac{3(e_i - b_i c)^2 T_i^A}{16b_i} - TC(\bar{t}^*) + TC(\bar{t}^A) \geq 0 \tag{A.1}$$

where $T_i^* = l_i - \max(f_i, t_i^*)$ and $T_i^A = l_i - \max(f_i, t_i^A)$, $i = 1, 2, \dots, n$. If $\sum_{i=1}^n \frac{3(e_i - b_i c)^2 (T_i^* - T_i^A)}{16b_i} \leq 0$, then

$$\sum_{i=1}^n \frac{3(e_i - b_i c)^2 (T_i^* - T_i^A)}{16b_i} \leq \sum_{i=1}^n \frac{(e_i - b_i c)^2 (T_i^* - T_i^A)}{8b_i} \tag{A.2}$$

and

$$\sum_{i=1}^n \frac{(e_i - b_i c)^2 (T_i^* - T_i^A)}{8b_i} - TC(\bar{t}^*) + TC(\bar{t}^A) \geq 0 \tag{A.3}$$

If $\sum_{i=1}^n \frac{3(e_i - b_i c)^2 (T_i^* - T_i^A)}{16b_i} > 0$, it follows that

$$\sum_{i=1}^n \frac{(e_i - b_i c)^2 (T_i^* - T_i^A)}{8b_i} > 0 \tag{A.4}$$

Considering that $TC(\bar{t}^*) \leq TC(\bar{t}^A)$, we get

$$\sum_{i=1}^n \frac{(e_i - b_i c)^2 (T_i^* - T_i^A)}{8b_i} - TC(\bar{t}^*) + TC(\bar{t}^A) > 0 \tag{A.5}$$

Consequently, \bar{t}^* represents the optimal route of the supplier. (ii) If route B is characterized with \bar{t}^B and $TC(\bar{t}^*) > TC(\bar{t}^B)$, the difference between the supply chain profits under two routes is

$$\sum_{i=1}^n \frac{3(e_i - b_i c)^2 (T_i^* - T_i^B)}{16b_i} - TC(\bar{t}^*) + TC(\bar{t}^B) \geq 0 \tag{A.6}$$

Based on Inequality (22) and $TC(\bar{t}^*) > TC(\bar{t}^B)$,

$$\sum_{i=1}^n \frac{3(e_i - b_i c)^2 (T_i^* - T_i^B)}{16b_i} > \sum_{i=1}^n \frac{(e_i - b_i c)^2 (T_i^* - T_i^B)}{8b_i} > 0 \tag{A.7}$$

The supplier prefers route B when

$$\sum_{i=1}^n \frac{(e_i - b_i c)^2 (T_i^* - T_i^B)}{8b_i} < TC(\bar{t}^*) - TC(\bar{t}^B) \tag{A.8}$$

In this case, the supplier selects the route that cannot maximize $\pi_{sc}(\bar{w}^*, \bar{t})$. \square

Proof of Lemma. When $w_i = c$, we can write the decision framework of the supplier (P3) as follows.

$$\text{Max } \pi_0(\bar{w}, \bar{t}) = \sum_{i,j \in SC, i \neq j} (Z_{ij} - 1) a_{ij} x_{ij} \tag{A.9}$$

subject to

Constraints (2)–(6).

Each feasible solution of (P3) is a Hamiltonian tour in which the number of x_{ij} that is not equal to 0, is $n + 1$. For a feasible route, \bar{x}^c that is different from \bar{x}^0 , and where $Z_{ij} < 1$ if $x_{ij}^0 = 0$ and $Z_{ij} > 1$ otherwise, we can infer that $\sum_{i,j \in SC, i \neq j} (Z_{ij} - 1) a_{ij} x_{ij}^0 > \sum_{i,j \in SC, i \neq j} (Z_{ij} - 1) a_{ij} x_{ij}^c$. Consequently, the supplier’s optimal route is the channel-wide optimal route, \bar{x}^0 . \square

Proof of Theorem 1. By setting w_i and $\{Z_{ij} | i, j \in SC \text{ and } i \neq j\}$ based on the Lemma, the supplier will select \bar{x}^0 . Under the WPC contract, the profit functions of the supplier and retailer (i) can be shown as

$$\pi_{(0)} = \sum_{i=0}^n (Z_{(i)} - 1) a_{(i)}, \tag{A.10}$$

$$\pi_{(i)}(p_{(i)}, q_{(i)} | t_{(i)}^0, w_{(i)}) = R_{(i)}(p_{(i)}, q_{(i)} | t_{(i)}^0) - cq_{(i)} - \sum_{j=0}^i \phi_{(i)(j)} a_{(j)}. \tag{A.11}$$

Consequently, retailer (i)’s optimal $p_{(i)}$ and $q_{(i)}$ satisfy Eqs. (7) and (8) and supply chain coordination is achieved. Let $\Pi_{(i)}^0$ be the maximum profit of subsystem (i) under \bar{t}^0 such that $\Pi_{(i)}^0 = R_{(i)}(p_{(i)}^0, q_{(i)}^0 | t_{(i)}^0) - cq_{(i)}^0$, then we have

$$\sum_{i=1}^n \Pi_{(i)}^0 - TC(\bar{t}^0) = \sum_{i=1}^n \Pi_{(i)}^0 - \sum_{i=0}^n a_{(i)} = \pi_{sc}^0. \tag{A.12}$$

For any $k_{(0)}$, we can always find a set of $\{Z_{ij} | i, j \in SC \text{ and } i \neq j\}$ obtained from the Lemma that satisfies $\sum_{i=0}^n (Z_{(i)} - 1) a_{(i)} = k_{(0)}$. Then, we can obtain the following linear equation set.

$$\left\{ \begin{array}{l} \Pi_{(1)}^0 - (\phi_{(1)(1)} a_{(1)} + \phi_{(1)(0)} a_{(0)}) = k_{(1)} \\ \Pi_{(2)}^0 - (\phi_{(2)(1)} a_{(1)} + \phi_{(2)(2)} a_{(2)} + \phi_{(2)(0)} a_{(0)}) = k_{(2)} \\ \vdots \\ \Pi_{(n)}^0 - (\phi_{(n)(1)} a_{(1)} + \phi_{(n)(2)} a_{(2)} + \dots + \phi_{(n)(n)} a_{(n)} + \phi_{(n)(0)} a_{(0)}) = k_{(n)} \\ (\phi_{(1)(1)} + \phi_{(2)(1)} + \dots + \phi_{(n)(1)}) a_{(1)} = Z_{(1)} a_{(1)} \\ (\phi_{(2)(2)} + \phi_{(3)(2)} + \dots + \phi_{(n)(2)}) a_{(2)} = Z_{(2)} a_{(2)} \\ \vdots \\ \phi_{(n)(n)} a_{(n)} = Z_{(n)} a_{(n)} \\ (\phi_{(1)(0)} + \phi_{(2)(0)} + \dots + \phi_{(n)(0)}) a_{(0)} = Z_{(0)} a_{(0)} \end{array} \right. \tag{A.12}$$

Equation set (A.13) has $n(n+3)/2$ variables and $2n+1$ equations. The difference is $(n+1)(n-2)/2$. Because $n \geq 2$, the number of equations is no more than the number of variables. Let Φ and v be

$$(\phi_{(1)(1)}, \dots, \phi_{(n)(1)}, \phi_{(2)(2)}, \dots, \phi_{(n)(2)}, \dots, \phi_{(n)(n)}, \phi_{(1)(0)}, \dots, \phi_{(n)(0)})^T$$

and

$$(\Pi_{(1)}^o - k_{(1)}, \dots, \Pi_{(n)}^o - k_{(n)}, Z_{(1)}a_{(1)}, \dots, Z_{(n)}a_{(n)}, Z_{(0)}a_{(0)})^T.$$

The above equation set can be presented as $G\Phi = v$ in which G is the coefficient matrix, given by the following augmented matrix:

$$\left(\begin{array}{ccc|ccc|ccc|c} a_{(1)} & & & 0 & & & 0 & a_{(0)} & & \Pi_{(1)}^o - k_{(1)} \\ & a_{(1)} & & a_{(2)} & & & \vdots & & & \Pi_{(2)}^o - k_{(2)} \\ & & \ddots & & \ddots & & 0 & & & \vdots \\ & & & & & \dots & & & & \Pi_{(n)}^o - k_{(n)} \\ \hline a_{(1)} & a_{(1)} & \dots & a_{(1)} & & & 0 & & & Z_{(1)}a_{(1)} \\ & & & a_{(2)} & \dots & a_{(2)} & \dots & & & Z_{(2)}a_{(2)} \\ & & & & & \dots & 0 & & & \vdots \\ & & & & & & a_{(n)} & & & Z_{(n)}a_{(n)} \\ & & & & & & 0 & a_{(0)} & a_{(0)} & \dots & a_{(0)} \\ & & & & & & & & & & Z_{(0)}a_{(0)} \end{array} \right) \begin{array}{l} \left. \vphantom{\begin{array}{c} a_{(1)} \\ \vdots \\ a_{(n)} \end{array}} \right\} n \\ \left. \vphantom{\begin{array}{c} Z_{(1)}a_{(1)} \\ \vdots \\ Z_{(0)}a_{(0)} \end{array}} \right\} n+1 \end{array}$$

Add the $(n+1)$ th, $(n+2)$ th, ..., and $2n$ th rows to the last row, and subtract the sum of the first n rows to obtain

$$\underbrace{(0 \ 0 \ \dots \ 0)}_{n(n+3)/2} \left| \sum_{i=0}^n Z_{(i)}a_{(i)} - \sum_{i=1}^n \Pi_{(i)}^o + \sum_{i=1}^n k_{(i)} \right.$$

Because $\sum_{i=0}^n (Z_{(i)} - 1)a_{(i)} = k_{(0)}$ and $\sum_{i=1}^n \Pi_{(i)}^o - \sum_{i=0}^n a_{(i)} = \sum_{i=1}^n k_{(i)} + k_{(0)}$, the last element in the last row satisfies

$$\sum_{i=0}^n Z_{(i)}a_{(i)} - \sum_{i=1}^n \Pi_{(i)}^o + \sum_{i=1}^n k_{(i)} = 0. \tag{A.14}$$

By moving the last n columns of the coefficient matrix to the left side of the matrix, we can obtain an augmented matrix U that is an echelon matrix with $2n$ non-zero rows.

$$\left(\begin{array}{ccc|ccc|ccc|c} a_{(0)} & & & a_{(1)} & & & 0 & & & 0 & \Pi_{(1)}^o - k_{(1)} \\ & a_{(0)} & & & a_{(1)} & & a_{(2)} & & & \vdots & \Pi_{(2)}^o - k_{(2)} \\ & & \ddots & & & \ddots & & & & 0 & \vdots \\ & & & & & & & & & a_{(n)} & \Pi_{(n)}^o - k_{(n)} \\ \hline & & & a_{(1)} & a_{(1)} & \dots & a_{(1)} & & & 0 & Z_{(1)}a_{(1)} \\ & & & & & & a_{(2)} & \dots & a_{(2)} & \vdots & Z_{(2)}a_{(2)} \\ & & & & & & & & & 0 & \vdots \\ & & & & & & & & & a_{(n)} & Z_{(n)}a_{(n)} \\ & 0 & 0 & \dots & 0 & 0 & \dots & 0 & & 0 & 0 \end{array} \right) \begin{array}{l} \left. \vphantom{\begin{array}{c} a_{(0)} \\ \vdots \\ a_{(n)} \end{array}} \right\} n \\ \left. \vphantom{\begin{array}{c} Z_{(1)}a_{(1)} \\ \vdots \\ Z_{(0)}a_{(0)} \end{array}} \right\} n+1 \end{array}$$

The rank of G is equal to the rank of U and is lower than $n(n+3)/2$. Consequently, there always exist Φ satisfying $G\Phi = v$.
□

Proof of Theorem 2. If $\phi_{(j)} = \phi_{(j)(j)}, j = 1, 2, \dots, n$, we can obtain the following linear equation set.

$$\pi_0(\bar{\gamma}, \bar{t}) = \sum_{i=1}^n (1 - \gamma_i)(R_i(p_i, q_i | t_i) - cq_i) - TC(\bar{t}) = \sum_{i=1}^n (1 - \gamma_i)\Pi_i - TC(\bar{t}). \quad \square \quad (\text{A.17})$$

Proof of Proposition 4. Consider route S , that is not channel-wide optimal with $TC(\bar{t}^S) < TC(\bar{t}^0)$. Let Π_i^S be subsystem i 's maximum profit under route S . Because \bar{p}^0, \bar{q}^0 , and \bar{t}^0 maximize the supply chain profit, we can obtain

$$\sum_{i=1}^n \Pi_i^0 - \sum_{i=1}^n \Pi_i^S - TC(\bar{t}^0) + TC(\bar{t}^S) > 0. \quad (\text{A.18})$$

H is the set containing all of subsystem i wherein $\Pi_i^0 - \Pi_i^S \geq 0$. If L contains all of subsystem i wherein $\Pi_i^0 - \Pi_i^S < 0$ and $\Delta\Pi_i = \Pi_i^0 - \Pi_i^S$, we can write Inequality (A.18) as

$$\sum_{h \in H} \Delta\Pi_h + \sum_{l \in L} \Delta\Pi_l - TC(\bar{t}^0) + TC(\bar{t}^S) > 0. \quad (\text{A.19})$$

When switching from \bar{t}^S to \bar{t}^0 , the supplier's profit increment is

$$\Delta\pi_0(\bar{\gamma}) = \sum_{h \in H} (1 - \gamma_h)\Delta\Pi_h + \sum_{l \in L} (1 - \gamma_l)\Delta\Pi_l - TC(\bar{t}^0) + TC(\bar{t}^S) \quad (\text{A.20})$$

When $\gamma_h = 1$ for all $h \in H$, we can infer that

$$\Delta\pi_0(\bar{\gamma}) = \sum_{l \in L} (1 - \gamma_l)\Delta\Pi_l - TC(\bar{t}^0) + TC(\bar{t}^S) < 0 \quad (\text{A.21})$$

Considering that $\pi_0(\bar{\gamma})$ is continuous in $\bar{\gamma}$, we can always find a $\bar{\gamma}$ with $\gamma_i \in (0, 1)$ (for all $i = 1, 2, \dots, n$), under which $\Delta\pi_0(\bar{\gamma}) < 0$. In this case, \bar{t}^0 cannot maximize the supplier's profit and the supplier prefers route S under which supply chain coordination cannot be achieved. Therefore, the revenue-sharing contract cannot arbitrarily allocate the supply chain profit. \square

Proof of Proposition 5. Without loss of generality, we let Retailer 1 be the first visited retailer under route S and $(\gamma_1, \gamma_2, \dots, \gamma_n)$. Consider any other route W under which \bar{t}^W is different from \bar{t}^S . Because route S is the supplier's optimal route under $(\gamma_1, \gamma_2, \dots, \gamma_n)$, we have

$$(1 - \gamma_1)(\Pi_1^S - \Pi_1^W) + \sum_{j=2}^n (1 - \gamma_j)(\Pi_j^S - \Pi_j^W) + TC(\bar{t}^S) - TC(\bar{t}^W) > 0. \quad (\text{A.22})$$

Because Retailer 1 is the first visited retailer under route S , we have $t_1^S \leq t_1^W$ and $\Pi_1^S \geq \Pi_1^W$. Considering that $1 - \gamma_1 + \varepsilon > 1 - \gamma_1 > 0$, we have $(1 - \gamma_1 + \varepsilon)(\Pi_1^S - \Pi_1^W) \geq (1 - \gamma_1)(\Pi_1^S - \Pi_1^W)$. Therefore,

$$(1 - \gamma_1 + \varepsilon)(\Pi_1^S - \Pi_1^W) + \sum_{j=2}^n (1 - \gamma_j)(\Pi_j^S - \Pi_j^W) + TC(\bar{t}^S) - TC(\bar{t}^W) > 0. \quad (\text{A.23})$$

It means that route S is still the supplier's optimal route under $(\gamma_1 - \varepsilon, \gamma_2, \dots, \gamma_n)$. \square

References

- Abad, P., 2014. Determine optimal price and order size for a price setting newsvendor under cycle service level. *Int. J. Prod. Econ.* 158, 106–113.
- Agatz, N., Erera, A., Savelsbergh, M., Wang, X., 2012. Optimization for dynamic ride-sharing: a review. *Eur. J. Oper. Res.* 223, 295–303.
- Alaei, S., Setak, M., 2015. Multi objective coordination of a supply chain with routing and service level consideration. *Int. J. Prod. Econ.* 167, 271–281.
- Anupindi, R., Bassok, Y., 1999. Centralization of stocks: retailers vs. manufacturer. *Manage. Sci.* 45 (2), 178–191.
- Anupindi, R., Bassok, Y., Zemel, E., 2001. A general framework for the study of decentralized distribution systems. *Manuf. Service Oper. Manage.* 3 (4), 349–368.
- Baldacci, R., Maniezzo, V., Mingozzi, A., 2004. An exact method for the car pooling problem based on lagrangean column generation. *Oper. Res.* 52 (3), 422–439.
- Bassok, Y., Ernst, R., 1995. Dynamic allocations for multi-product distribution. *Transport. Sci.* 29 (3), 256–266.
- Bernstein, F., Federgruen, A., 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Manage. Sci.* 51 (1), 18–29.
- Boyaci, T., Gallego, G., 2002. Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. *Int. J. Prod. Econ.* 77 (2), 95–111.
- Cachon, G.P., 2003. Supply chain coordination with contracts. In: de Kok, A.G., Graves, S.C. (Eds.), *Handbook in Operations Research and Management Science, Volume on Supply Chain Management: Design, Coordination and Operation*. North Holland, Amsterdam, pp. 229–339.
- Cachon, G.P., Larivière, M.A., 2005. Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Manage. Sci.* 51 (1), 30–44.
- Campbell, A.M., Savelsbergh, M.W.P., 2004. A decomposition approach for the inventory-routing problem. *Transport. Sci.* 38 (4), 488–502.
- Cao, E.B., Wan, C., Lai, M.Y., 2013. Coordination of a supply chain with one manufacturer and multiple competing retailers under simultaneous demand and cost disruptions. *Int. J. Prod. Econ.* 141, 245–433.

- Chen, F.R., Federgruen, A., Zheng, Y.S., 2001. Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Manage. Sci.* 47 (5), 693–708.
- Chen, K.B., Xiao, T.J., 2009. Demand disruption and coordination of the supply chain with a dominant retailer. *Eur. J. Oper. Res.* 197, 225–234.
- Chen, K.B., Xiao, T.J., 2017. Pricing and replenishment policies in a supply chain with competing retailers under different retail behaviors. *Comput. Indust. Eng.* 103, 145–157.
- Cheng, C., Yang, P., Qi, M.Y., Rousseau, L.M., 2017. Modeling a green inventory routing problem with a heterogeneous fleet. *Transport. Res. Part E* 97, 97–112.
- Chiou, C.C., Yao, M.J., Tsai, J., 2007. A mutually beneficial coordination mechanism for a one-supplier multi-retailers supply chain. *Int. J. Prod. Econ.* 108, 314–328.
- Chiu, C.H., Choi, T.M., Hao, G., Li, X., 2015. Innovative menu of contracts for coordinating a supply chain with multiple mean-variance retailers. *Eur. J. Oper. Res.* 246, 815–826.
- Dana, J.D., Spier, K.E., 2001. Revenue sharing and vertical control in the video rental industry. *J. Indust. Econ.* 59 (3), 223–245.
- David, A., Adida, E., 2015. Competition and coordination in a two-channel supply chain. *Prod. Oper. Manage.* 24 (8), 1358–1370.
- Feng, X.H., Moon, I.K., Ryu, K.Y., 2014. Revenue-sharing contracts in an N-stage supply chain with reliability considerations. *Int. J. Prod. Econ.* 147, 20–29.
- Feng, X.H., Moon, I.K., Ryu, K.Y., 2015. Supply chain coordination under budget constraints. *Comput. Indust. Eng.* 88, 487–500.
- Feng, X.H., Moon, I.K., Ryu, K.Y., 2017. Warehouse capacity sharing via transshipment for an integrated two-echelon supply chain. *Transport. Res. Part E* 104, 17–35.
- Gaur, V., Fisher, M.L., 2004. A periodic inventory routing problem at a supermarket chain. *Oper. Res.* 52 (6), 813–822.
- Grubbström, R.W., 2010. The newsboy problem when customer demand is a compound renewal process. *Eur. J. Oper. Res.* 203, 134–142.
- Ingene, C.A., Parry, M.E., 1995. Coordination and manufacturer profit maximization: the multiple retailer channel. *J. Retail.* 71 (2), 129–151.
- Jiang, L., Anupindi, R., 2010. Customer-driven vs. retailer-driven search: channel performance and implications. *Manuf. Service Oper. Manage.* 12 (1), 102–119.
- Kleywegt, A.J., Nori, V.S., Savelsbergh, M.W.P., 2002. The stochastic inventory routing problem with direct deliveries. *Transport. Sci.* 36 (1), 94–118.
- Li, J.X., Chu, F., Chen, H.X., 2011. A solution approach to the inventory routing problem in a three-level distribution system. *Eur. J. Oper. Res.* 210, 736–744.
- Li, L., Zhang, R.Q., 2015. Cooperation through capacity sharing between competing forwarders. *Transport. Res. Part E* 75, 115–131.
- Lippman, S.A., McCardle, K.F., 1997. The competitive newsboy. *Oper. Res.* 45, 54–65.
- Liu, S.C., Chen, J.R., 2011. A heuristic method for the inventory routing and pricing problem in a supply chain. *Expert Syst. Appl.* 38, 1447–1456.
- Luo, S.R., Sethi, S.P., Shi, R.X., 2016. On the optimality conditions of a price-setting newsvendor problem. *Oper. Res. Lett.* 44, 697–701.
- Murray, C.C., Gosavi, A., Talukdar, D., 2012. The multi-product price-setting newsvendor with resource capacity constraints. *Int. J. Prod. Econ.* 138, 148–158.
- Netessine, S., Rudi, N., 2003. Centralized and competitive inventory models with demand substitution. *Oper. Res.* 51 (2), 329–335.
- Niakan, F., Rahimi, M., 2015. A multi-objective healthcare inventory routing problem; a fuzzy possibilistic approach. *Transport. Res. Part E* 80, 74–94.
- Qin, Y., Wang, R.X., Vakharia, A.J., Chen, Y.W., Seref, M.M.H., 2011. The newsvendor problem: review and directions for future research. *Eur. J. Oper. Res.* 213, 361–374.
- Rubio-Herrero, J., Baykal-Gürsoy, M., Jaskiewicz, A., 2015. A price-setting newsvendor problem under mean-variance criteria. *Eur. J. Oper. Res.* 247, 575–587.
- Shao, J., Krishnan, H., McCormick, S.T., 2011. Incentives for transshipment in a supply chain with decentralized retailers. *Manuf. Service Oper. Manage.* 13 (3), 361–372.
- Song, J.H., Savelsbergh, M., 2007. Performance measurement for inventory routing. *Transport. Sci.* 41 (1), 44–54.
- Spengler, J.J., 1950. Vertical integration and antitrust policy. *J. Polit. Econ.* 58, 347–352.
- Wang, C., Chen, X., 2015. Optimal ordering policy for a price-setting newsvendor with option contracts under demand uncertainty. *Int. J. Prod. Res.* 53 (20), 6279–6293.
- Wang, C.X., Webster, S., Zhang, S.D., 2014. Robust price-setting newsvendor model with interval market size and consumer willingness-to-pay. *Int. J. Prod. Econ.* 154, 100–112.
- Xu, M.H., Chen, Y.H., Xu, X.L., 2010. The effect of demand uncertainty in a price-setting newsvendor model. *Eur. J. Oper. Res.* 207, 946–957.
- Xu, X.L., Cai, X.Q., Liu, C.L., To, C., 2007. Optimal commodity distribution for a vehicle with fixed capacity under vendor managed inventory. *LNCS* 4614, 329–339.
- Yan, S.Y., Chen, C.Y., 2011. A model and a solution algorithm for the car pooling problem with pre-matching information. *Comput. Indust. Eng.* 61, 512–524.
- Ye, T.F., Sun, H., 2016. Price-setting newsvendor with strategic consumers. *Omega* 63, 103–110.
- Yu, Y.G., Chen, H.X., Chu, F., 2008. A new model and hybrid approach for large scale inventory routing problems. *Eur. J. Oper. Res.* 189, 1022–1040.
- Zhang, W.G., Fu, J.H., Li, H.Y., Xu, W.J., 2012. Coordination of supply chain with a revenue-sharing contract under demand disruptions when retailers compete. *Int. J. Prod. Econ.* 138, 68–75.
- Zhao, Q.H., Wang, S.Y., Lai, K.K., 2007. A partition approach to the inventory/routing problem. *Eur. J. Oper. Res.* 177, 786–802.
- Zhao, X., 2008. Coordinating a supply chain system with retailers under both price and inventory competition. *Prod. Oper. Manage.* 17 (5), 532–542.
- Zhao, X., Atkins, D.R., 2008. Newsvendors under simultaneous price and inventory competition. *Manuf. Service Oper. Manage.* 10 (3), 539–546.