Optimal decisions of price, quality, effort level and return policy in a three-level closed-loop supply chain based on different game theory approaches

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Abstract: Over the last few decades, the closed loop supply chain (CLSC) has been examined because of concerns over the environment and social liability. In this paper, we propose a joint optimisation model of pricing strategies, quality levels, effort decisions, and return policies by considering the reference price effect in a three-level supply chain under different channel power structures. To investigate the impact of different scenarios on optimal decisions and performance of a CLSC, we address five different channel power structures: centralised, vertical Nash, manufacturer Stackelberg, retailer Stackelberg, and third party Stackelberg. We present a numerical example to demonstrate the theoretical results of the developed model, and we also compare the optimal decisions to determine the best channel power structures considered. Then, to examine the impact of the key parameters on the model's behaviour, we conduct a sensitivity analysis on the main parameters, and finally, we provide a conclusion. [Received 5 October 2016; Revised 9 March 2017; Accepted 24 March 2017]

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1 Introduction

In recent years, closed-loop supply chain (CLSC) management has been increasingly considered, in part, because of growing environmental awareness and legislation and increasing resource shortages. In addition, the pressure from the Waste Electrical and Electronic Equipment (WEEE), European Union (EU), Restriction of Hazardous Substance (RoHS), and Eco-Design Requirements for Energy Using Products (EuP) exert significant influence on the establishment of CLCS, which can be used to save natural resources for future generations and thereby embraced by firms undertaking sustainable development efforts. Moreover, reuse of old or discarded items is economically attractive compared to their disposal, so we developed a model of closed-loop supply to determine the optimal price, product quality, sales and collection effort level, and buy-back price in different channel power structures.

Our research relates to four types of literature: quality of product, reference price effect, returns policy, and sales and collection efforts.

The first type of literature related to our study focuses on quality considerations. Price and quality play significant roles in achieving a competitive advantage by affecting the market competitiveness of the enterprise and the supply chain. Nowadays, in many industries, the basis of competition is changing from that of price to quality (Gans, 2002; Ren and Zhou, 2008). Consequently, in a particular market segments, competitors are trying to offer products of different quality using the same price policy. Also, quality and price affect demand and profit (Baiman et al., 2000; Banker et al., 1998). In fact, by increasing the quality of products, demand increases, too; however, increased product quality may lead to a decrease in the manufacturer's profit, at a level commensurate with operational inefficiency. Because quality improvement raises total cost, the price of an improved product may increase. So, the manufacturer faces a trade-off between price and quality.

In the relevant literature, product quality improvement has been receiving increased attention in recent years. Singer et al. (2003) determined the optimal effort to invest in the quality of disposable products in a distribution channel. Chambers et al. (2006) examined the effect of variable production costs on competitive behaviour in which firms compete on quality and price. By applying mixed-integer linear programming, Rong et al. (2011) proposed a planning and distribution model in a food supply-chain framework to control the quality of product. Hsieh and Liu (2010), through different game models under different degrees of accessible information, determined an optimal strategy for the inspection and quality investment of the manufacturer and supplier. Xie et al. (2011b) proposed a make-to-order supply chain model to examine the ways risk-averse behaviour and different supply chain strategies affect quality investment and pricing strategies. De Giovanni (2011) optimised price, advertisement, and quality improvement decisions in a dynamic setting by considering cooperative and non-cooperative cases. Xie et al. (2011a) addressed the problem of supply-chain structure selection and quality investment in two competing supply chains that feature different qualities of a particular product at the identical price. Liu et al. (2015) proposed a differential game model with a focus on the product's design quality which depends on advertising effort. Giri et al. (2015) proposed a joint pricing and quality-level decision model in a two-echelon supply chain by considering a single retailer and multiple manufacturers. They also assumed that the manufacturers compete on the quality of the products and offer different prices to the retailer. Yu and Ma (2013) proposed a stochastic integrated supply chain model by examining the impact of different decision sequences on quality investment, product price, and profit. Wee and Wang (2013) extended a newsboy problem for short-life cycle products in a decentralised supply chain containing a manufacturer and retailer to obtain optimal price and order quantity. By considering a high variation in the quality of parts disassembled for remanufacturing, Dingeç and Korugan (2013) focused on quantifying the potential benefits of this decision in a remanufacturing framework by applying a queuing network model. Recently, Moshtagh and Taleizadeh (2017) extended a hybrid manufacturing/remanufacturing model by considering shortage, rework, and quality-dependent return rate in which quality of used items is considered as a random variable. Maiti and Giri (2014) optimised the price and quality of products in a three-echelon CLSC under five scenarios in which demand of each product depends on its quality. Apart from the quality of products, the quality of services has widely taken into consideration in the literature. For instance, Greenfield (2014) analysed correlation between the quality of service and competition in the airline industry by considering two different strategies.

The second type of related literature examines the reference price effect. Reference price is the customers' perceptions of the appropriate price which is made based on historical prices of the product. It plays a significant role in the purchase decisions of consumers (Dickson and Sawyer, 1990; Kalwani et al., 1990). When the reference price is lower than the current price, the customer feels a sense of loss, which affects the demand for the product. Conversely, when the reference price is greater than the current price, the customer experiences a sense of gain, which increases the demand for the product (Kalwani et al., 1990). However, existing research on CLSCs rarely offers considerations on the importance of reference price on market demand.

Among this class of related articles, eight works are particularly relevant to our study. Kopalle et al. (1996) analysed the reference price effect under different cases consisting of monopolies, duopolies, and monopolist retailers who manage two brands. Their results indicate that the optimal retail pricing decision is based on either everyday low pricing or a high low pricing strategy under different scenarios. Fibich et al. (2003) examined the impact of reference price on the pricing decisions under both finite and infinite planning horizons. They showed that the initial reference price plays an important role in retail pricing policy. Greenleaf (1995) proposed a pricing model in a dynamic setting by examining the impact of the reference price effect on single-period promotions. Zhang et al. (2013) analysed the effect of the reference price on the optimal decisions of all the channel members in a dynamic advertising model with a two-level supply chain. They also examined two different game models to obtain the optimal strategies of the retailer and the manufacturer. By considering time-and-price sensitive demand and reference price effects, Dye and Yang (2015) recently investigated a pricing model for deteriorating items in a dynamic setting. Zhang et al. (2014) proposed a dynamic pricing model for a two-echelon supply chain consisting of a manufacturer and a retailer under two different scenarios. They showed that consumers who are sensitive to the reference price effect conferred benefits in both the centralised and decentralised systems. Viglia et al. (2016) developed a dynamic pricing model to analyse the influence of hotel price sequences on hotel reference prices. Their results indicate that when competing hotels maximise their profit and set their price at the same time, customers decrease their reference price. By considering the manufacturer as a Stackelberg leader under three different reverse channels, Xu and Liu (2014) analysed how the reference price influences the supply chain members' strategies. Their results showed that as the reference price effect increases, manufacturer and retailer profits decrease and third party profit increases.

Another type of related literature concentrates on return policy. The return policy of the third party as well as the product's quality and collection efforts affect the sales and return rate of the used products. In the past, researchers have analysed the effects of different return policies on customer behaviour (Ai et al., 2012; Davis et al., 1998; Hess and Mayhew, 1997; Shulman et al., 2009). Yu and Wang (2008) optimised the return strategy and marketing decisions in a direct-selling framework. Mukhopadhyay and Setoputro (2004) developed a pricing model to investigate the impact of pricing decisions and return strategies on market demand and return rate in a direct selling situation. Bonifield et al. (2010) investigated how the quality relates to return policies. Recently, Li et al. (2013) proposed an online direct-selling model to investigate the effect of an online distributor's return policy, pricing strategy, and quality policy on customer purchase and return decisions.

The last type of related literature investigates sales and collection efforts. In a realistic situation, the retailer usually exerts sales efforts to gain market share. For example, the retailer can encourage customers to purchase by advertising product features. In another case, a third party exerts effort to collect used products through strategies such as reverse logistics services, advertising about recycling policies, and employee training programs (Gao et al., 2015). However, previous works on CLSCs rarely include considerations on the influence of advertising on demand and return quantity.

Table 1 A brief literature review

Author(s) / *Assumptions*	*Considering reverse supply chain*	*Quality of product*	*Reference price effect*	*Sales effort*	*Collection effort*	*Return policy*	*System* *Centralised*	*Manufacturer or supplier Stackelberg*	*Retailer Stackelberg*	*Third party Stackelberg*	*Vertical Nash*	*Number of supply chain members*
Hsieh and Liu (2010)		✓										2
Xie et al. (2011b)		✓					✓	✓				2
De Giovanni (2011)		✓		✓				✓				2
Xie et al. (2011a)		✓					✓	✓			✓	2
Liu and Xie (2013)		✓					✓	✓			✓	2
Yu and Ma (2013)		✓					✓	✓			✓	2
Liu et al. (2015)		✓		✓			✓	✓				2
Maiti and Giri (2014)	✓	✓					✓	✓	✓	✓	✓	3
Giri et al. (2015)		✓					✓		✓			2
Zhang et al. (2014)			✓				✓	✓				2
Zhang et al. (2013)			✓	✓			✓					2
Xu and Liu (2014)	✓		✓					✓				3
Dye and Yang (2015)			✓									1
Jena and Sarmah (2014)	✓						✓	✓				2
Li et al. (2013)	✓	✓				✓						1
Ma et al. (2013b)		✓		✓			✓	✓	✓		✓	2
Ma et al. (2013a)		✓		✓			✓	✓				2
Hong et al. (2015)	✓			✓			✓	✓				3
Chen (2015)				✓				✓				2
Gao et al. (2015)	✓			✓	✓		✓	✓	✓		✓	2
Huang et al. (2013)	✓				✓		✓	✓				3
Choi et al. (2013)	✓				✓		✓	✓	✓	✓		3
This paper	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	3

Nine works are particularly relevant to our investigation. Li et al. (2002) optimised marketing and investment effort levels in a two-level supply chain. They also analysed the impact of local advertising, brand name investment, and sharing policy on advertising decisions in these models. Taylor (2002) discussed the model of supply chain coordination, which can be achieved with rebate and returns contracts. Taylor also assumed that demand expansion depends on sales effort. Ma et al. (2013b) optimised effort decisions and profits under different supply-chain power structures in which demand expansion depends on advertising. They found optimal strategies when the retailer invests in marketing efforts under a retailer Stackelberg model and the manufacturer invests in quality efforts under a manufacturer Stackelberg model. Yue et al. (2013) developed a joint pricing and advertising model by applying a game theory approach in which both the manufacturer and retailer offer price discounts. Chen (2015) extended a comparative dual-channel supply chain model to analyse the impact of pricing strategies and cooperative advertising on the system. Chen and Lin (2013) investigated the impact of advertising on the hotel industry, especially room revenue. Their results indicate that advertising affects room price significantly rather than quantity demanded. Guo et al. (2013) incorporated application of online reservation into hotel industry on the basis of market segmentation for service products. Hong et al. (2015) proposed an optimisation model of pricing, advertising, and collection decisions under three reverse collection formats in centralised and decentralised supply chains. Gao et al. (2015) proposed a pricing and effort decision model in a CLSC to analyse how different channel power structures affect optimal decisions and profits. To establish this model, they assumed that market demand depends on retail price, sales effort, and collection effort.

A brief literature review is presented in Table 1.

Five different game theory models of centralised and decentralised CLSCs, respectively, are used to examine the effect of price, reference price, product quality, and return policy as well as sales and collection efforts to increase demand and return quantity. We use these models to answer the following questions:

1 What are the optimal price, product quality, and effort-level decisions and profits under different game models?

2 Which channel power structure is the best under different conditions from the perspective of the whole CLSC and each member?

3 How does the manufacturer react to the quality trade-off investment in five different game models? How do different channel power structures influence the quality of products?

4 What is the effect of the sales and collection efforts on optimal decisions and profits? What is the influence of the channel power structure on sales and collection efforts?

5 How is the supply chain's optimal performance affected by consumer behaviour that is sensitive to the reference price effect?

Generally, this paper extends the work of Maiti and Giri (2014) by considering the effect of advertising, collection effort, reference price, and return policy. Specifically, this paper contributes to the literature on CLSCs in the following ways:

1 According to the literature review, no researchers have studied CLSC models by simultaneously considering selling price, quality of product, sales and collection effort, buy-back price, and reference price effect. To fill this gap, we investigate a CLSC model by considering selling price, quality of product, sales and collection effort, and buy-back price as decision variables.

2 To the best of our knowledge, very few researchers have studied sales and collection effort decisions in a CLSC under different power channel structures. We found one work in the literature that reports on the simultaneous sales effort in a forward-channel supply chain and the collection effort in a reverse supply chain.

Therefore, we consider, respectively, sales and collection efforts that affect demand expansion and quantity of returns.

3 Although buy-back price plays an important role in the quantity of returns, very few studies show the impact of buy-back price on the quantity of returns. So, like Li et al. (2013), we assume that return quantity depends on buy-back price; however, unlike Li et al., who did not consider the sales effort, reference price, and some other parameters that influence demand and return quantity expansions, we consider these variables in our model.

4 As explained in the literature review on CLSCs (Table 1), a few authors considered reference price effect; however, these studies did not investigate different channel power structures or other parameters that may affect demand and return-quantity expansions. So, in this paper, we study the reference price effect under five different game models.

The rest of this paper is organised as follows. Assumptions and notations are presented in Section 2. In Section 3, description and formulation of the general model under five different scenarios are presented. Computational experiments and sensitivity analyses are provided in Sections 4 and 5. Section 6 concludes this paper.

2 Notations and assumptions

The following notations are used for developing the mathematical models:

ω^k unit wholesale price

p^k unit retail price

r^k average recycling price of used products collected by the third party

D the demand faced by the retailer

R the returned quantity of used products

c_m unit manufacturing cost of end product from original materials

c_r unit manufacturing cost of end product from used products

Δ unit cost saved by recycling ($\Delta = c_m - c_r$)

c_q quality improvement cost of the manufacturer

g^k sales effort of the retailer

y^k collection effort of the third party

q^k product quality of newly produced items

q_r product quality of returned products

r_e reference price

b average recycling price of returned items paid by the manufacturer to the third party

Π_i profit function for supply chain member i.

Superscript $k \in \{C, MS, RS, TS, VN\}$ refers to centralised system, manufacturer Stackelberg system, retailer Stackelberg system, third-party Stackelberg system, and the vertical Nash game, respectively. Subscript $i \in \{M, R, T, C\}$ refers to the manufacturer, the retailer, the third party, and the whole supply chain, respectively.

This model is based on these following assumptions.

1 Linear dependency between demand and price is considered in this paper. A similar linear approximation approach to a general demand curve was used in many other works in the literature, such as (Choi et al., 2013; Gao et al., 2015; Giri et al., 2015; Li et al., 2013; Xu and Liu, 2014) to simplify the entire pricing problem. Going into the depth, the profit-maximising price-cost margin is inversely related to the firm's price elasticity of demand (Pindyck and Rubinfeld, 2005). However, owing to lack of history of demand, calculation of firm's price elasticity is very difficult in many newly launched products or newly offered services such as emergent market for internet music downloads. Therefore, rather than predicting the elasticity of demand directly, firms try to utilise an alternative approach that approximates the demand curve as a linear function.

   The market demand D is linearly dependent on the own sale price, quality of the product, buy-back price, sales effort, and reference price, where $\alpha, \gamma, \upsilon, \delta > 0$. It is adopted in the literature as representative of quality considerations (Maiti and Giri, 2014), return policies (Li et al., 2013), sales effort (Gao et al., 2015) and reference price effect (Xu and Liu, 2014). Therefore, the demand can be written as following:

$$D = d - \alpha p + \beta q + \upsilon r + kg - \delta\left(p - r_e\right) \tag{1}$$

   where D represents the demand faced by the retailer. d is the basic market demand for the product and is significantly greater than the other parameters of the model. α represents the self-price elastic coefficient, representing the sensitivity of the demand of the channels to its own price. β represents the degree of demand sensitivity to quality as measured by the impact of product quality on demand. υ describes demand sensitivity to the buy-back price. According to Li et al. (2013), most consumers consider return policies before making buying decisions. Actually, concerns about return policy may prevent the customer from purchasing the product. k represents customers sensitivity to sales effort. δ describes the reference price effect, which represents demand sensitivity to reference price effect.

2 The returned quantity R is linearly dependent on the buy-back price and collection effort, where $\varphi, l > 0$ represents sensitivity to the return policy (Li et al., 2013) and collection effort (Gao et al., 2015), respectively, as adopted in the literature. Therefore, the returned quantity function can be expressed as follows:

$$R = \phi + \varphi r + ly \tag{2}$$

   where R denotes quantities of returns from the third party. ϕ represents the basic return quantity, which is less than the basic market demand of the product and is greater than the other parameters of the return function.

3 There is no difference between new products and remanufactured items, so they can be sold at the same price. A similar assumption is made in Choi et al. (2013), Gao et al. (2015), Hong et al. (2015), Maiti and Giri (2014), Savaskan et al. (2004) and

Savaskan and van Wassenhove (2006). For example, the Kodak single-use camera is an instance for this assumption, and the manufacture sells both new and remanufactured product to the retailer at the wholesale price (Atasu et al., 2013). Another example for this case is the Xerox high-value copiers where Xerox collects used copiers directly and remanufactures them as new copiers.

4 The information is symmetric.

5 The cost of remanufacturing a returned product is less than manufacturing a new one; i.e., $c_m > c_r$.

6 The reference price is greater than the cost of producing a new item from raw materials; i.e., $r_e > c_m$.

7 The quality of a returned item is q_r, and the quality of a manufactured item is q such that $q_r < q$. Returned items that satisfy the minimum allowed quality level (i.e., q_r) are suitable for remanufacturing. Moreover, the quality of a newly reproduced item is the same as the quality of a newly produced item and is equal to q.

8 The fixed transfer price per unit is less than the saving generated per unit by remanufacturing. While the transfer price per unit is higher than the unit buy-back cost (i.e., $\Delta > b > r$). In other words, if $\Delta > b > r$ is not satisfied, it is obvious that remanufacturing process would not be economically viable.

9 In reality, the buy-back price may be variable, and depends on the quality of returns. However, in order to simplify the model and generate more managerial insights, we follow the extensive literature in related topics of remanufacturing, and build our model under the deterministic setting (see, e.g., Choi et al., 2013; Gao et al., 2015; Huang et al., 2013; Maiti and Giri, 2014).

10 Each member of the CLSC has an interest in cooperating in the integral system. For feasibility of the model, we assume that $p > \omega > 0$.

3 Model formulation and solution

We consider a CLSC consisting of a manufacturer, retailer, and third party. In forward flow, the manufacturer sells a product to the retailer at the wholesale price ω. Then, the retailer sells the product to the end customer at the selling price p. To attract customers and increasing the market demand, the retailer invests in sales effort activities. The retailer's investment in sales effort g is assumed to be an increasing and convex function of g, and defined as a quadratic cost function written as $0.5c_1g^2$.

In reverse flow, the third party collects used products at an average price r and delivers them to the manufacturer at an average price b. Just as the sales efforts in the forward supply chain, we assume that the third party invests in collection efforts to increase the quantity of returns.

The third party's investment in collection effort is assumed to be an increasing and convex function of y, and defined as a quadratic cost function written as $0.5c_2y^2$. Remanufacturing commences after the manufacturer collects products from the third party. The average remanufacturing cost c_r is less than the cost of producing a new product, c_m. The unit cost $c_m - c_r$ can be saved through remanufacturing. The conceptual figure of the proposed problem is depicted in Figure 1.

Since the produced items are not perfectly pure, we assume that the product quality is q ($0 < q < 1$). Although the quality of used product is assumed to be q_r, which is smaller than the quality of newly produced items, the quality is upgraded to q during remanufacturing. We assume the investment for quality as a quadratic cost function expressed as $c_q q_r^2$ for manufacturing process and as $c_q(q^2 - q_r^2)$ for remanufacturing process which indicates that the improvement of the quality of a subsequent unit is more difficult and costly than improvement of the prior one.

Figure 1 The conceptual figure of the closed loop supply chain (see online version for colours)

In the centralised system, both sale and collection decisions are made by a central decision maker. However, in the decentralised models including the manufacturer Stackelberg system, retailer Stackelberg system, third-party Stackelberg system, and the vertical Nash game, selling price and sales effort are determined by the retailer, buy-back price and collection effort are decided by the third party, and wholesale price and optimal quality level of product are obtained by the manufacturer. However, it should be noted that decision sequences are different in each CLSC structure, yielding different values of decision variables.

The profit function of the manufacturer, retailer and third party are given by:

$$\Pi_M = D\omega - (D-R)c_m - Rc_r - Rb - (D-R)c_q q^2 - Rc_q\left(q^2 - q_r^2\right) \tag{3}$$

$$\Pi_R = D(p-\omega) - \frac{1}{2}c_1 g^2 \tag{4}$$

$$\Pi_T = R(b-r) - \frac{1}{2}c_2 y^2 \tag{5}$$

3.1 Centralised CLSC model (model C)

In the centralised CLSC model, all members of the supply chain are integrated as a whole system and cooperatively make decisions on the sale price, product quality, and sales effort in the forward channel as well as the and collection effort and buy-back price in the recycling channel. They consider both channels simultaneously to maximise the profit of the whole system.

The total profit of the centralised model is given by

$$\begin{aligned}\Pi_C = {} & Dp + (D-R)c_m - Rc_r - Rr - (D-R)c_q q^2 - Rc_q\left(q^2 - q_r^2\right) \\ & -\frac{1}{2}c_1 g^2 - \frac{1}{2}c_2 y^2\end{aligned} \tag{6}$$

Proposition 1: When conditions (7) and (8) are satisfied, we find a unique optimal solution for the optimisation problem of the centralised CLSC.

$$\begin{aligned} c_q > \max & \left\{ \frac{\left[-2p^2(\delta+\alpha) + 2p(kg + \upsilon r + \beta q) - 2lry - 2\varphi r^2 - c_1 g^2 - c_2 y^2\right]}{q^2\left[2d + 6\beta q + 6\upsilon r - 6p(\delta+\alpha) + 6gk + 2\delta r_e\right]}, \right. \\ & \left. \frac{\beta}{2(\alpha+\delta)} \right\} \end{aligned} \tag{7}$$

Also, to ensure $r^C < b$, we consider the following condition:

$$c_r > \frac{\begin{aligned} & c_2 c_q\left[\left(8c_1\phi - 8c_1 c_q \varphi q_r^2\right)(\alpha+\delta)^2 \right. \\ & \left. + \left(4c_q\varphi k^2 q_r^2 - 4c_1\delta\upsilon r_e - 4\phi k^2 - 4c_1 d\upsilon\right)(\alpha+\delta)\right] \\ & + c_q b\left[\left(-8c_1 l^2 + 16c_1 c_2\varphi\right)(\alpha+\delta)^2 + \left(4l^2k^2 - 8c_2\varphi k^2 - 4c_1 c_2 \upsilon^2\right)(\alpha+\delta)\right] \\ & + c_m c_q\left[\left(-8c_1 l^2 - 8c_1 c_2 \varphi + 4c_1 c_2 \upsilon\right)(\alpha+\delta)^2 + \left(4c_2\varphi k^2 - 4l^2k^2\right)(\alpha+\delta)\right] \\ & + \left(8c_1 c_q^2 l^2 q_r^2\right)(\alpha+\delta)^2 + \left(4c_q^2 l^2 k^2 q_r^2\right)(\alpha+\delta) - c_1 c_2 \upsilon \beta^2 \end{aligned}}{\left(8c_1 c_q l^2 - 8c_1 c_2 c_q \varphi\right)(\alpha+\delta)^2 + \left(4c_2 c_q \varphi k^2 - 4c_q l^2 k^2\right)(\alpha+\delta)} \tag{8}$$

Proof: See Appendix A.

Solving the first-order condition, we obtain the equilibrium results as follows:

$$p^{C^*} = \frac{\begin{array}{l} c_1c_2\left[\begin{array}{l} c_mc_q(\alpha+\delta)^2(\upsilon(\upsilon-\varphi)-2\varphi(\alpha+\delta))+c_q(\alpha+\delta)^2\begin{pmatrix}-2d\varphi+\phi\upsilon+c_r\varphi\upsilon \\ 2\delta\varphi r_e-c_q\varphi\upsilon q_r^2\end{pmatrix} \\ +\frac{1}{4}\upsilon^2\beta^2-\frac{3}{2}\varphi\beta^2(\alpha-\delta) \end{array}\right] \\ +c_mc_q(\alpha+\delta)^2\left[c_1l^2(\alpha+\delta)-l^2k^2+c_1\upsilon l^2+2c_2\varphi k^2\right]+\frac{3}{4}\beta^2l^2c_1(\alpha+\delta) \\ +c_1c_q\left[l^2(\alpha+\delta)^2\left(d-c_r\upsilon+c_q\upsilon q_r^2\right)\right]-\frac{1}{4}\beta^2l^2k^2+\frac{1}{2}c_2\varphi\beta^2k^2 \end{array}}{c_q(\alpha+\delta)^2\left[c_1c_2\left(\upsilon^2-4\alpha\varphi-4\delta\varphi\right)+2c_1l^2(\alpha+\delta)+2c_2\varphi k^2\right]} \tag{9}$$

$$r^{C^*} = \frac{\begin{array}{l} c_1c_2c_q(\alpha+\delta)\left[\left(2\phi-c_m\upsilon-2c_q\varphi q_r^2-2\varphi\left(c_m-c_r\right)\right)(\alpha+\delta)+d\upsilon\right] \\ +c_1c_ql^2\left(-2c_r+2c_qq_r^2\right)(\alpha+\delta)^2 \\ +c_2c_q\left[\left(-\phi k^2+c_q\varphi k^2q_r^2\right)(\alpha+\delta)-c_r\delta\varphi k^2\right] \\ +c_mc_q\left[2c_1l^2(\alpha+\delta)^2-\left(l^2k^2-c_2\varphi k^2\right)(\alpha+\delta)\right]-(\alpha+\delta)\left(c_qq_r^2-c_r\right)l^2k^2c_q \end{array}}{2(\alpha+\delta)c_q\left[l^2k^2-2c_2\varphi k^2-c_1c_2\upsilon^2+(\alpha+\delta)\left(2c_1c_2\varphi-c_1l^2\right)\right]} \tag{10}$$

$$y^{C^*} = \frac{\begin{array}{l} c_1c_2l\left[\begin{array}{l}(\alpha+\delta)^2\left(2\phi+2c_q\varphi q_r^2+c_m\upsilon+2\varphi\left(c_m-c_r\right)\right) \\ +(\alpha+\delta)\left(-d\upsilon-c_q\upsilon q_r^2-\upsilon^2\left(c_m-c_r\right)\right)\end{array}\right] \\ +(\alpha+\delta)l\left(-c_q\phi k^2-c_q^2q_r^2\varphi k^2-c_q\varphi k^2\left(c_m-c_r\right)-c_1c_q\delta\upsilon r_e\right)-\frac{1}{4}c_1\upsilon\beta^2l \end{array}}{(\alpha+\delta)c_q\left(l^2k^2-2c_2\varphi k^2-c_1c_2\upsilon^2+2(\alpha+\delta)\left(2c_1c_2\varphi-c_1l^2\right)\right)} \tag{11}$$

$$g^{C^*} = \frac{\begin{array}{l} k\left[\left(8c_2c_mc_q\varphi-4c_mc_ql^2\right)(\alpha+\delta)^2-\begin{pmatrix}-4c_qdl^2+8c_2c_qd\varphi \\ -8c_2c_q\delta\varphi r_e+4c_q\delta l^2r_e\end{pmatrix}(\alpha+\delta)\right] \\ +\upsilon k\begin{pmatrix}-4c_ql^2q_r^2-4c_mc_ql^2+4c_qc_rl^2-4c_2c_q\phi \\ +4c_2c_q^2\varphi q_r^2+4c_2c_q\varphi\left(c_m-c_r\right)\end{pmatrix} \end{array}}{8\left(c_1c_ql^2-2c_1c_2c_q\varphi\right)(\alpha+\delta)^2-\left(4c_ql^2k^2-8c_2c_q\varphi k^2-4c_1c_2c_q\upsilon^2\right)(\alpha+\delta)} \tag{12}$$

$$q^{C^*} = \frac{\beta}{2c_q(\alpha+\delta)} \tag{13}$$

Also, by substituting the optimal solution into equation (6), the optimal value of the total profit can be obtained.

3.2 Decentralised CLSC models

We consider a decentralised supply chain with members consisting of a manufacturer, retailer, and third party, who make their decisions, independently. That is, each makes a decision to maximise one's own profit.

3.2.1 Manufacturer Stackelberg game model (model MS)

In a manufacturer Stackelberg game, the manufacturer acts as the leader (L) with control over the CLSC, and the other members (i.e., retailer and third party) will act as the Stackelberg followers in their own chains. So, the retailer (F1) and the third party (F2) react as best they can to the manufacturer's optimal strategies. In practice, Xerox controls take-back activities of electronics industry as well as the distribution of the new products. Xerox has been a leader in reusing of lease copiers which satisfy the same strict quality standards. Similarly, IBM and Compaq motivate customers to utilise their asset recovery services, which ease disposal and replacement of end of life PCs.

3.2.1.1 Retailer's reaction

We first obtain the best reactions of the retailer as a function of decisions ω^* and q^* made by the manufacturer.

Proposition 2: If condition (14) is satisfied, then Π_R is concave in p^{MS} and g^{MS}, and a unique optimal solution is found for the optimisation problem of the retailer.

$$c_1 > \frac{k^2}{2(\alpha+\delta)} \tag{14}$$

Proof: See Appendix B.

By solving the first-order conditions $\frac{\partial \Pi_R}{\partial p^{MS}} = 0$ and $\frac{\partial \Pi_R}{\partial g^{MS}} = 0,$ the optimal reaction of the retailer can be obtained as follows:

$$p^{MS^*} = \frac{\omega^{MS} k^2 - c_1 \left(d + \upsilon r^{MS} + \beta q^{MS} + \delta r_e + \omega^{MS}(\alpha+\delta)\right)}{k^2 - 2c_1(\alpha+\delta)} \tag{15}$$

$$g^{MS^*} = \frac{k\left(-d - \upsilon r^{MS} - \beta q^{MS} - \delta r_e + \omega^{MS}(\alpha+\delta)\right)}{k^2 - 2c_1(\alpha+\delta)} \tag{16}$$

3.2.1.2 Third party's reaction

After determining the optimal decision of the retailer, we can derive the third party's best reactions:

Proposition 3: When conditions (17) and (18) are satisfied, the profit function Π_T is concave in r^{MS} and y^{MS}, and a unique optimal solution can be found for the optimisation problem of the third party.

$$c_1 > \frac{l^2}{2\varphi} \tag{17}$$

$$c_2 > \frac{l^2}{2\varphi} \tag{18}$$

Proof: See Appendix C.

By setting $\frac{\partial \Pi_T}{\partial r^{MS}}$ and $\frac{\partial \Pi_T}{\partial y^{MS}}$ to zero and solving them with respect to r^{MS} and y^{MS} simultaneously, we can derive the third party's best response as the following functions:

$$r^{MS^*} = \frac{-bl^2 - c_2\phi + c_2 b\varphi}{2c_2\varphi - l^2} \tag{19}$$

$$y^{MS^*} = \frac{\phi l + b\varphi l}{2c_2\varphi - l^2} \tag{20}$$

3.2.1.3 Manufacturer's optimal decisions

By considering the optimal reaction of the retailer and third party, the manufacturer takes the retailer and third party's reaction decisions into consideration and determines the wholesale price and product quality of items. Substituting equations (15), (16), (19) and (20) into equation (3), representing the profit of the manufacturer, and solving the first-order condition for the resulting function, we can obtain the optimal values of q^{MS^*} and ω^{MS^*} as follows:

Proposition 4: Π_M is concave in q^{MS} and ω^{MS} and when condition (21) is satisfied, a unique optimal decision of the manufacturer can be written as follows:

$$c_q > \max\left\{ \frac{\beta\omega}{q\left(gk - (\alpha+\delta)p + \delta r_e + \upsilon r + 3\beta q\right)}, \frac{\beta}{2(\alpha+\delta)} \right\} \tag{21}$$

Proof: See Appendix D.

By solving $\frac{\partial \Pi_M}{\partial q^{MS}}$ and $\frac{\partial \Pi_M}{\partial \omega^{MS}}$ simultaneously, we can derive the manufacturer's best response as the following functions:

$$q^{MS^*} = \frac{\beta}{2c_q(\alpha+\delta)} \tag{22}$$

$$\omega^{MS^*} = \frac{(\alpha+\delta)\begin{bmatrix} 4c_2c_q\left(2d\varphi - \phi\upsilon + 2\delta\varphi r_e + b\varphi\upsilon + 2c_m\varphi(\alpha+\delta)\right) \\ +4c_q l^2\left(-d - \delta r_e - b\upsilon - c_m(\alpha+\delta)\right) \end{bmatrix} + 6c_2\varphi\beta^2 - 3\beta^2 l^2}{8c_q\left(\alpha+\delta\right)^2\left(2c_2\varphi - l^2\right)} \tag{23}$$

By substituting equations (22) and (23) into equations (15) and (16), p^{MS^*} and g^{MS^*} can be obtained as follows:

$$p^{MS*} = \frac{\begin{array}{l}\left(2c_2\varphi - l^2\right)\begin{pmatrix}4c_1c_mc_q(\alpha+\delta)^3 + \left(12c_1c_q\left(d+\delta r_e\right) - 4c_mc_qk^2\right)(\alpha+\delta)^2 \\ \left(7c_1\beta^2 - 4c_qk^2\left(d+\delta r_e\right)\right)(\alpha+\delta) - 3\beta^2k^2\end{pmatrix} \\ \left(c_2\varphi - l^2\right)\left(12c_1c_qbv(\alpha+\delta)^2 - 4c_qbvk^2(\alpha+\delta)\right) - 12c_1c_2c_q\phi v(\alpha+\delta)^2 \\ +4c_2c_q\phi vk^2(\alpha+\delta)\end{array}}{-8c_q(\alpha+\delta)^2\left(k^2 - 2c_1(\alpha+\delta)\right)\left(2c_2\varphi - l^2\right)} \quad (24)$$

$$g^{MS*} = \frac{\begin{array}{l}k\left(2c_2\varphi - l^2\right)\left(4c_mc_q(\alpha+\delta)^2 - 4c_q\left(d+\delta r_e\right)(\alpha+\delta) - \beta^2\right) \\ -4c_qbvk\left(c_2\varphi - l^2\right)(\alpha+\delta) + 4c_2c_q\phi v\end{array}}{8c_q(\alpha+\delta)\left(k^2 - 2c_1(\alpha+\delta)\right)\left(2c_2\varphi - l^2\right)} \quad (25)$$

Also, the profit of manufacturer, retailer, and third party can be obtained by substituting the optimal solutions into equations (3), (4) and (5), respectively.

3.2.2 Retailer Stackelberg game model (model RS)

In this case, the retailer plays the dominant role in the CLSC and acts as the Stackelberg leader (L), and other agents, (i.e., manufacturer and third party) play the Stackelberg followers in their own chains. First, the manufacturer and the third party make their best decisions. Then, the retailer determines the optimal decisions for the chain.

3.2.2.1 Manufacturer's reaction

We first derive the manufacturer's best choices. In this case, the manufacturer tends to determine a wholesale price that optimises the manufacturing profit function. Because the profit function of the manufacturer linearly increases with the wholesale price, the best situation reflects $\omega^{RS} = p^{RS}$, whereas p^{RS} is the upper bound. As a consequence, the retailer's profit will be zero which is unacceptable. Because the wholesale price should be in the range of p^{RS} and c_m, we take the wholesale price of the manufacturer to be $(p^{RS} + c_m)/2$. A similar approach was used in Maiti and Giri (2014).

Proposition 5: Because Π_M is concave in q^{RS}, a unique optimal solution can be found for the optimisation problem of the manufacturer.

Proof: See Appendix E.

By solving $\omega^{RS} = \dfrac{p^{RS} + c_m}{2}$ and $\dfrac{\partial \Pi_M}{\partial q^{RS}} = 0,$ the optimal decisions of the manufacturer can be obtained as follows:

$$q^{RS*} = \frac{\begin{array}{l}2\left(p^{RS}(\alpha+\delta) - \left(d+\delta r_e\right) - \left(g^{RS}k + vr^{RS}\right)\right) \\ +3\sqrt{\dfrac{4c_q\begin{pmatrix}p^{RS}(\alpha+\delta) - \left(d+\delta r_e\right) \\ -\left(g^{RS}k + vr^{RS}\right)\end{pmatrix}^2 + 6\beta^2\left(p^{RS} - c_m\right)}{9c_q}}\end{array}}{6\beta} \quad (26)$$

$$\omega^{RS^*} = \frac{p^{RS} + c_m}{2} \tag{27}$$

3.2.2.2 Third party's reaction

After determining the optimal decision of the manufacturer, we derive the third party's best reactions:

Proposition 6: When conditions (28) and (29) are satisfied, the profit function Π_T is concave in r^{RS} and y^{RS} and a unique optimal solution for the optimisation problem can be found for the third party.

$$c_1 > \frac{l^2}{2\varphi} \tag{28}$$

$$c_2 > \frac{l^2}{2\varphi} \tag{29}$$

Proof: See Appendix C.

By setting $\frac{\partial \Pi_T}{\partial r^{RS}}$ and $\frac{\partial \Pi_T}{\partial y^{RS}}$ to zero and solving for r^{RS} and y^{RS} simultaneously, we can derive the third party's best response as follows:

$$r^{RS^*} = \frac{-bl^2 - c_2\phi + c_2 b\varphi}{2c_2\varphi - l^2} \tag{30}$$

$$y^{RS^*} = \frac{\phi l + b\varphi l}{2c_2\varphi - l^2} \tag{31}$$

3.2.2.3 Retailer's optimal decisions

Having the information about the decisions of the manufacturer and third party, the retailer takes the other agents' reaction decisions into consideration and determines the selling price and sales effort decisions. Substituting equations (26), (27), (30) and (31) into equation (4), and solving the first-order condition for the obtained function, we can obtain the optimal selling price and sales effort.

Proposition 7: Π_R is concave in p^{RS} and g^{RS}, and when condition (32) is satisfied, a unique optimal decision of the retailer can be found:

$$c_1 > \frac{k^2}{2(\alpha + \delta)} \tag{32}$$

Proof: See Appendix B.

The optimal retail price and sales effort can be obtained by equating $\partial\Pi_R/\partial p^{RS}$ and $\partial\Pi_R/\partial g^{RS}$ to zero and thus satisfying condition (32). Because $\partial\Pi_R/\partial p^{RS}$ and $\partial\Pi_R/\partial g^{RS}$ are nonlinear functions of p^{RS} and g^{RS}, it is very difficult to obtain the closed-form parametric solutions for this model. Therefore, we use a numerical method to obtain the optimal solution in this case. Also, the profits of manufacturer, retailer, and third party can be obtained by substituting the optimal solutions into equations (3), (4) and (5), respectively.

3.2.3 Third party Stackelberg game model (model TS)

In this case, the third party plays the dominant role in the CLSC and acts as the Stackelberg leader (L), and the other players, (i.e., manufacturer and retailer) act as Stackelberg followers in their own chains. First, the manufacturer (F1) and the retailer (F2) make their best choices, and then the third party determines the optimal decisions. In practice, the collector or the third party may act as a leader in some metal and electronics industries. In these industries, collectors have powerful market power to decide the quantity of returns and the transfer price.

3.2.3.1 Manufacturer's reaction

We first derive the manufacturer's best decision. In this case, as well as the retailer-led Stackelberg game, we assume that $\omega^{TS} = (p^{TS} + c_m)/2$.

Proposition 8: Because Π_M is concave in q^{TS}, a unique optimal solution can be obtained for the optimisation problem of the manufacturer.

Proof: See Appendix E.

By solving $\omega^{TS} = \dfrac{p^{TS} + c_m}{2}$ and $\dfrac{\partial \Pi_M}{\partial q^{TS}} = 0,$ the optimal decisions of manufacturer can be obtained as follows:

$$q^{TS^*} = \frac{2\left(p^{TS}(\alpha+\delta) - (d+\delta r_e) - \left(g^{TS}k + \upsilon r^{TS}\right)\right) + 3\sqrt{\dfrac{4c_q\begin{pmatrix} p^{TS}(\alpha+\delta) - (d+\delta r_e) \\ -\left(g^{TS}k + \upsilon r^{TS}\right)\end{pmatrix}^2 + 6\beta^2\left(p^{TS} - c_m\right)}{9c_q}}}{6\beta} \tag{33}$$

$$\omega^{TS^*} = \frac{p^{TS} + c_m}{2} \tag{34}$$

3.2.3.2 Retailer's reaction

After determining the optimal decision of manufacturer, the retailer' best reactions can be written

Proposition 9: When condition (35) is satisfied, the profit function Π_R is concave in p^{TS} and g^{TS}, and a unique optimal solution for the optimisation problem of the retailer can be found.

$$c_1 > \frac{k^2}{2(\alpha+\delta)} \tag{35}$$

Proof: See Appendix B.

By setting $\frac{\partial \Pi_R}{\partial p^{TS}}$ and $\frac{\partial \Pi_R}{\partial g^{TS}}$ to zero and solving them with respect to p^{TS} and g^{TS} simultaneously, we can derive the retailer' best reactions as follows:

$$p^{TS^*} = \frac{\omega^{TS} k^2 - c_1 \left(d + \upsilon r^{TS} + \beta q^{TS} + \delta r_e + \omega^{TS} (\alpha + \delta) \right)}{k^2 - 2c_1 (\alpha + \delta)} \tag{36}$$

$$g^{TS^*} = \frac{k \left(-d - \upsilon r^{TS} - \beta q^{TS} - \delta r_e + \omega (\alpha + \delta) \right)}{k^2 - 2c_1 (\alpha + \delta)} \tag{37}$$

3.2.3.3 Third party's optimal decisions

After getting the reactions of the manufacturer and the retailer, the third party maximises its own profit and determines the optimal decisions for the return policy and collection effort.

Proposition 10: When conditions (38) and (39) are satisfied, then Π_T is concave in r^{TS} and y^{TS} and the unique optimal decision of the third party can be written as follows:

$$c_1 > \frac{l^2}{2\varphi} \tag{38}$$

$$c_2 > \frac{l^2}{2\varphi} \tag{39}$$

Proof: See Appendix C.

By solving $\frac{\partial \Pi_T}{\partial r^{TS}}$ and $\frac{\partial \Pi_T}{\partial y^{TS}}$ simultaneously, we can derive the manufacturer's best response functions as follows:

$$r^{TS^*} = \frac{-bl^2 - c_2 j + c_2 b\varphi}{2c_2\varphi - l^2} \tag{40}$$

$$y^{TS^*} = \frac{\phi l + b\varphi l}{2c_2\varphi - l^2} \tag{41}$$

We substitute $p^{TS^*}, g^{TS^*}, r^{TS^*}$ into equation (33) and $q^{TS^*}, \omega^{TS^*}, r^{TS^*}$ into equation (36). Then, p^{TS^*} and q^{TS^*} can be obtained as follows:

$$p^{TS*} = \frac{-\sqrt{A} - c_1^2\beta^2\left(2c_2\varphi - l^2\right) - c_q\begin{bmatrix} \left(2c_2\varphi - l^2\right)\begin{pmatrix} \left(8c_1^2\left(d+\delta r_e\right) - 6c_1c_mk^2\right)(\alpha+\delta) + c_mk^4 \\ +7c_1^2c_m(\alpha+\delta)^2 - 2c_1k^2\left(d+\delta r_e\right) \end{pmatrix} \\ +\left(c_2\varphi - l^2\right)\begin{pmatrix} 8c_1^2b\upsilon(\alpha+\delta) \\ -2c_1b\upsilon k^2 \end{pmatrix} - \begin{pmatrix} 8c_1^2c_2\phi\upsilon(\alpha+\delta) \\ -2c_1c_2\phi\upsilon k^2 \end{pmatrix} \end{bmatrix}}{\left(2c_2\varphi - l^2\right)\left(-15c_1^2(\alpha+\delta)^2 + 8c_1k^2(\alpha+\delta) - k^4\right)} \tag{42}$$

$$q^{TS*} = \frac{\begin{array}{l} \begin{pmatrix} -10c_1^2c_q\beta l^2(\alpha+\delta) \\ +2c_1c_q\beta k^2 \end{pmatrix}\left(2c_2\varphi - l^2\right)\begin{bmatrix} \left(2c_2\varphi - l^2\right)\begin{pmatrix} 2c_q(\alpha+\delta)\left(d+\delta r_e\right) \\ -2c_mc_q(\alpha+\delta)^2 - \beta^2 \end{pmatrix} \\ +(\alpha+\delta)\begin{pmatrix} -2c_2c_q\phi\upsilon \\ +2c_qb\upsilon\left(c_2\varphi - l^2\right) \end{pmatrix} \end{bmatrix} \\ -2c_1c_q\beta k^2\left(2c_2\varphi - l^2\right)\begin{bmatrix} \left(2c_2\varphi - l^2\right)\left(2c_q(\alpha+\delta) - 2c_mc_q(\alpha+\delta)^2 - \beta^2\right) \\ -(\alpha+\delta)\left(2c_2c_q\phi\upsilon + 2c_qb\upsilon\left(c_2\varphi - l^2\right)\right) \end{bmatrix} \\ +10c_q\beta l^2(\alpha+\delta)\left(2c_2\varphi - l^2\right)\sqrt{A} \end{array}}{-10c_q\beta l^2(\alpha+\delta)\left(-10c_1^2c_q\beta l^2(\alpha+\delta) + 2c_1c_q\beta k^2\right)\left(2c_2\varphi - l^2\right)^2} \tag{43}$$

Also ω^{TS^*} can be obtained by substituting p^{TS^*} into equation (34) as follows:

$$\omega^{TS^*} = \frac{-\sqrt{A} - c_1^2\beta^2\left(2c_2\varphi - l^2\right) - c_q\begin{bmatrix} \left(2c_2\varphi - l^2\right)\begin{pmatrix} \begin{pmatrix} 8c_1^2\left(d+\delta r_e\right) \\ -14c_1c_mk^2 \end{pmatrix}(\alpha+\delta) + 2c_mk^4 \\ +22c_1^2c_m(\alpha+\delta)^2 - 2c_1k^2\left(d+\delta r_e\right) \end{pmatrix} \\ +\left(c_2\varphi - l^2\right)\begin{pmatrix} 8c_1^2b\upsilon(\alpha+\delta) \\ -2c_1b\upsilon k^2 \end{pmatrix} - \begin{pmatrix} 8c_1^2c_2\phi\upsilon(\alpha+\delta) \\ -2c_1c_2\phi\upsilon k^2 \end{pmatrix} \end{bmatrix}}{2\left(2c_2\varphi - l^2\right)\left(-15c_1^2(\alpha+\delta)^2 + 8c_1k^2(\alpha+\delta) - k^4\right)} \tag{44}$$

Also, by substituting equations (40), (43) and (44) into equation (37), we can obtain g^{TS^*} as follows:

$$g^{TS^*} = \frac{k\left(-c_1^2\beta^2\left(2c_2\varphi - l^2\right) - \sqrt{A}\right) - c_q\begin{pmatrix} c_1^2k\begin{bmatrix} \left(2c_2\varphi - l^2\right)\begin{pmatrix} 8(\alpha+\delta)\left(d+\delta r_e\right) \\ -8c_m(\alpha+\delta)^2 \end{pmatrix} \\ +(\alpha+\delta)\left(-8c_2\phi\upsilon + 8b\upsilon\left(c_2\varphi - l^2\right)\right) \end{bmatrix} \\ +c_1k^3\begin{bmatrix} \left(2c_2\varphi - l^2\right)\left(-2\left(d+\delta r_e\right) + 2c_m(\alpha+\delta)\right) \\ +2c_2\phi\upsilon - 2b\upsilon\left(c_2\varphi - l^2\right) \end{bmatrix} \end{pmatrix}}{2c_qc_1\left(2c_2\varphi - l^2\right)\left(-15c_1^2(\alpha+\delta)^2 + 8c_1k^2(\alpha+\delta) - k^4\right)} \tag{45}$$

where A is given as follows:

$$
\begin{aligned}
A &= (\alpha+\delta)\begin{bmatrix} 4c_1^4 c_2 \begin{pmatrix} 16c_q\varphi\beta^2\left(d+\delta r_e\right)\left(c_2\varphi-l^2\right)+4c_q b\varphi\upsilon\beta^2\left(2c_2\varphi-3l^2\right) \\ -4c_q\phi\upsilon\beta^2\left(2c_2\varphi-l^2\right) \end{pmatrix} \\ +16c_1^4 c_q\beta^2 l^4\left(\left(d+\delta r_e\right)+b\upsilon\right)+4c_1^3 c_m c_q\beta^2 k^2\left(4c_2\varphi\left(c_2\varphi-l^2\right)+l^4\right) \end{bmatrix} \\
&+4c_1^4(\alpha+\delta)^2\begin{bmatrix} \left(c_2\varphi-l^2\right)\left(4c_2c_q^2\varphi\left(d+\delta r_e\right)^2-16c_2c_mc_q\varphi\beta^2-2c_2c_q^2 b\phi\upsilon^2\right) \\ +\left(d+\delta r_e\right)\begin{pmatrix} -2c_2c_q^2\phi\upsilon\left(2c_2\varphi-l^2\right)+2c_2c_q^2 b\varphi\upsilon\left(2c_2\varphi-3l^2\right) \\ +2c_q^2 b\upsilon l^4+c_q^2 l^4 \end{pmatrix} \\ +c_2c_q^2 b^2\varphi\upsilon^2\left(c_2\varphi-2l^2\right)+4c_2^2c_q^2\phi^2\upsilon^2-4c_mc_q\beta^2 l^4 \\ +c_q^2 b^2\upsilon^2 l^4 \end{bmatrix} \\
&+8c_1^4 c_m c_q^2(\alpha+\delta)^3\begin{bmatrix} \left(d+\delta r_e\right)\left(-4c_2\varphi\left(c_2\varphi-l^2\right)-l^4\right)+b\upsilon l^2\left(3c_2\varphi-l^2\right) \\ +c_2\phi\upsilon\left(2c_2\varphi-l^2\right)-2c_2^2 b\varphi^2\upsilon \end{bmatrix} \\
&+4c_1^4 c_m^2 c_q^2(\alpha+\delta)^4\left[4c_2\varphi\left(c_2\varphi-l^2\right)+l^4\right] \\
&+\begin{pmatrix} -16c_1^3 c_2 c_q\varphi\beta^2 k^2\left(c_2\varphi-l^2\right)\left(d+\delta r_e\right)+4c_1^4 c_2\varphi\beta^4\left(c_2\varphi-l^2\right)+c_1^4\beta^4 l^4 \\ +4c_1^3 c_q\beta^2 k^2\left(\left(2c_2\varphi-l^2\right)\left(c_2\phi\upsilon-c_2 b\varphi\upsilon\right)-l^4\left(d+\delta r_e\right)-b\upsilon l^4\right) \end{pmatrix}
\end{aligned} \tag{46}
$$

Also, the profit of the manufacturer, retailer, and third party can be obtained by substituting the optimal solutions into equations (3), (4) and (5), respectively.

3.2.4 *Vertical Nash game model (model VN)*

In the Nash game model, the manufacturer, retailer, and third party simultaneously maximise their own profits and make decisions independently. Because the profit function of the manufacturer is linearly increasing with respect to wholesale price, the best reaction will be $\omega^{VN} = p^{VN}$. However, p^{VN} is the upper bound, so wholesale price ω^{VN} cannot be equal to the retail price p^{VN}. Because the wholesale price should fall in the range of p^{VN} and c_m, we set the wholesale price of the manufacturer as $\omega^{VN} = (p^{VN} + c_m)/2$.

Proposition 11: Π_M is always concave in q^{VN}. When condition (35) is satisfied, the profit function Π_R is concave in p^{TS} and g^{TS}, and when conditions (38) and (39) are satisfied, Π_T is concave in r^{TS} and y^{TS} and $r^* < b$. By satisfying conditions (35), (38) and (39), we obtain a unique optimal solution for the optimisation problem for the vertical Nash model.

Proof: See Appendixes B, C and E.

By setting $\omega^{VN} - \dfrac{p^{VN}+c_m}{2}$, $\dfrac{\partial \Pi_M}{\partial q^{VN}}$, $\dfrac{\partial \Pi_R}{\partial p^{VN}}$, $\dfrac{\partial \Pi_R}{\partial g^{VN}}$, $\dfrac{\partial \Pi_T}{\partial r^{VN}}$ and $\dfrac{\partial \Pi_T}{\partial y^{VN}}$ to zero and solving for ω^{VN}, p^{VN}, q^{VN}, r^{VN}, g^{VN} and y^{VN} simultaneously, we observed that the optimal solutions of Nash game is exactly the same as the optimal solutions of the third party's Stackelberg game, which are presented in equations (40) through (46). To avoid duplication in presentation of the equations, the optimal solutions of Nash game is not presented here.

One of the anomalies of empirical business is the failure of concavity conditions in estimating profit functions. From a theoretical perspective, concavity of the profit function is a basic tenet because concavity ensures a firm's rational behaviour for profit maximisation. However, what if these concavity conditions are not satisfied? The first approach is to make some attempts for incorporating either global or local concavity conditions into the cost function. For example, firms can change some controllable parameters such as advertising, collection effort, manufacturing, or remanufacturing costs so that profit functions satisfy concavity conditions. Another possibility is that firms can obtain a feasible solution using some optimisation programs such as Lingo and Gams.

4 Computational and practical results

This section illustrates the performance of the developed model derived in the previous section. We carry out numerical example for different channel-power structures. The following parameters are used: d = 1,100, ϕ = 50, α = 10, β = 10, l = 6, k = 6, υ = 5, φ = 8, δ = 1, c_m = 80, c_r = 50, c_q = 1, r_e = 90, b = 20, c_1 = 500, c_2 = 300, q_r = 0.05. We obtained the optimal results of the C, MS, RS, TS, and VN models as summarised in Table 2.

According to Tables 2 through 6, the centralised system is the benchmark model for the profit of the entire system as well as for the quality of product and return policies. However, the lowest selling price is presented by the retailer in the manufacturer-led Stackelberg game. The optimal selling prices in five models are satisfied in the following order: $p^{RS} < p^{C} < p^{TS} = p^{VN} < p^{MS}$. The wholesale price sequence is $\omega^{RS} < \omega^{TS} = \omega^{VN} < \omega^{MS}$. The buy-back price sequence is $r^{MS} > r^{RS} = r^{TS} = r^{VN}$. The sequence of product quality satisfies the following relationships: $q^{C} = q^{MS} > q^{TS} = q^{VN} > q^{RS}$. The relationships of optimal sales and the collection effort in different cases are given as follows: $g^{C} > g^{TS} = g^{VN} > g^{MS} > g^{RS}$, $y^{C} < y^{MS} = y^{RS} = y^{TS} = y^{VN}$. Also, the profit sequence for all players and the whole system in five models satisfies the following relationships: $\Pi_M^{MS} > \Pi_M^{RS} > \Pi_M^{VN} = \Pi_M^{TS}$, $\Pi_R^{RS} > \Pi_R^{VN} = \Pi_R^{TS} > \Pi_R^{MS}$, $\Pi_T^{MS} = \Pi_T^{RS} = \Pi_T^{TS} = \Pi_T^{VN}$ and $\Pi^{C} > \Pi^{RS} > \Pi^{MS} > \Pi^{TS} = \Pi^{VN}$. Because the optimal solutions of r and y in all decentralised models are identical (the reason is that r and y only exist in third party's profit function), the third party's profit remains unaltered.

Table 2 Optimal results of numerical example under three cases

Parameter	*C*	*MS*	*RS*	*TS*	*VN*
ω		95.94	87.98	90.57	90.57
p	98.43	103.83	95.97	101.16	101.16
r	17.48	6.78	6.78	6.78	6.78
q	0.45	0.45	0.23	0.44	0.44
g	0.22	0.09	0.09	0.13	0.13
y	0.25	0.26	0.26	0.26	0.26
Π_M		2,424.27	2,415.19	2,266.07	2,266.07
Π_R		683.04	1,363.95	1,226.99	1,226.99
Π_T		1,388.54	1,388.54	1,388.54	1,388.54
Π	6,029.01				

From the above results, we can drive the beneficial choice of the channel power structure from the perspective of the whole supply chain, manufacturer, retailer, third party, and customer. Because selling price, product quality, and buy-back price affect customer preferences, customers may choose different structures from each other. For example, customers whose first priority is price may choose a retailer-led Stackelberg as the best model, but those customers whose first priority is quality of product may choose a centralised or manufacturer-led Stackelberg as the best model.

5 Sensitivity analysis

This section examines the behaviour of the decision variables and total profits for increasing values of the key parameters shown in Tables 3 through 6. Also, comparative results are depicted in Figures 1 through 6 and the following observations are made from them. It should be noted that because the results of the third party's Stackelberg game is exactly the same as those of Nash game, the vertical Nash model is omitted.

When the basic market demand (d) increases, the profits of the manufacturer, retailer, and the whole system increase, but the profit of third party does not change as expected. Also ω^* and g^* increase while q^* does not change. In fact, by increasing the basic market demand, the manufacturer increases the wholesale price, the retailer increases the selling price, and the quality of product remains unchanged. As a result, the manufacturer's and the retailer's profits increase. Also, in a centralised system, as d increases, the buy-back price increases and collection effort decreases but they remain unchanged in other models. Moreover, the effects of d on optimal solutions and profits are shown in Figure 2.

When the self-price coefficient (α) increases, p^* decreases, and as a result, profits of the whole system, the manufacturer, and the retailer decrease, but the third party's profit does not change. Also, in all models, α has a similar effect on ω^*, q^*, g^*; that is, by increasing α, y decreases. In fact, when the effect of selling price on demand rate increases, the retailer decreases the selling price to attract more customers. At the same time, the retailer forces the manufacturer to reduce the wholesale price. Consequently, the manufacturer offers products of relatively low quality. Therefore, the system is affected by a lower demand rate. Also, as α increases, the buy-back price in a centralised system increases, and in other models it remains unchanged. The effects of α on optimal solutions and profits are shown in Figure 3.

When the quality effect (β) increases, the manufacturer increases q^* to increase customer demand. Therefore, by increasing the product quality, the manufacturer increases the wholesale price which results in an increased selling price. Therefore, the manufacturer's and the retailer's profits increase while the third party's profit, sales, and collection efforts remain almost unchanged. As β increases, the buy-back price in a centralised system increases and in other models remains unchanged. The effects of β on optimal solutions and profits are shown in Figure 4.

Table 3 Sensitivity analysis for the key parameters of the C model

Parameter	*Value*	*Value*					
		p^{C^*}	q^{C^*}	r^{C^*}	g^{C^*}	y^{C^*}	Π^{C^*}
d	900	88.61	0.45	14.38	0.10	0.31	3,366.41
	1,000	93.52	0.45	15.93	0.16	0.28	4,452.06
	1,100	98.43	0.45	17.48	0.22	0.25	6,029.02
	1,200	103.35	0.45	19.02	0.28	0.22	8,097.29
	1,300	108.26	0.45	20.57	0.34	0.19	10,656.87
α	9	104.83	0.50	19.48	0.30	0.21	8,194.32
	10	98.43	0.45	17.48	0.22	0.25	6,029.02
	11	93.18	0.42	15.84	0.16	0.28	4,539.60
	12	88.80	0.38	14.46	0.10	0.31	3,559.11
	13	85.08	0.36	13.30	0.06	0.33	2,971.37
β	8	98.32	0.36	17.47	0.22	0.25	6,014.12
	9	98.37	0.41	17.47	0.22	0.25	6,021.15
	10	98.43	0.45	17.48	0.22	0.25	6,029.01
	11	98.50	0.50	17.49	0.22	0.25	6,037.72
	12	98.57	0.55	17.49	0.22	0.25	6,047.27
δ	0.5	99.14	0.48	17.69	0.23	0.25	6,110.55
	1	98.43	0.45	17.48	0.22	0.25	6,029.02
	1.5	97.79	0.43	17.28	0.21	0.25	5,956.41
	2	97.21	0.42	17.10	0.20	0.26	5,891.51
	2.5	96.68	0.40	16.94	0.20	0.26	5,833.32
φ	6	98.64	0.45	18.40	0.22	0.23	5,595.78
	7	98.52	0.45	17.87	0.22	0.24	5,811.08
	8	98.43	0.45	17.48	0.22	0.25	6,029.02
	9	98.37	0.45	17.18	0.22	0.26	6,248.65
	10	98.31	0.45	16.95	0.22	0.26	6,469.44
r_e	50	96.47	0.45	16.86	0.20	0.26	5,339.28
	70	97.45	0.45	17.17	0.21	0.26	5,674.32
	90	98.43	0.45	17.48	0.22	0.25	6,029.02
	110	99.42	0.45	17.79	0.23	0.24	6,403.37
	130	100.40	0.45	18.10	0.24	0.24	6,797.37

Table 4 Sensitivity analysis for the key parameters of the MS model

Parameter	*Value*	*Value*									
		ω^{MS^*}	p^{MS^*}	q^{MS^*}	r^{MS^*}	g^{MS^*}	y^{MS^*}	$\Pi_M^{MS^*}$	$\Pi_R^{C^*}$	$\Pi_T^{MS^*}$	Π^{MS^*}
d	900	86.85	90.18	0.45	6.78	0.04	0.26	1,301.73	121.76	1,388.54	2,812.03
	1,000	91.40	97.01	0.45	6.78	0.07	0.26	1,748.99	345.40	1,388.54	3,482.92
	1,100	95.94	103.83	0.45	6.78	0.09	0.26	2,424.27	683.04	1,388.54	4,495.85
	1,200	100.49	110.66	0.45	6.78	0.12	0.26	3,327.57	1,134.69	1,388.54	5,850.80
	1,300	105.03	117.48	0.45	6.78	0.15	0.26	4,458.89	1,700.35	1,388.54	7,547.78
α	9	101.57	112.27	0.50	6.78	0.13	0.26	3,338.90	1,140.35	1,388.54	5,867.79
	10	95.94	103.83	0.45	6.78	0.09	0.26	2,424.27	683.04	1,388.54	4,495.85
	11	91.26	96.81	0.42	6.78	0.07	0.26	1,797.25	369.53	1,388.54	3,555.31
	12	87.29	90.88	0.38	6.78	0.04	0.26	1,391.07	166.43	1,388.54	2,946.04
	13	83.90	85.79	0.36	6.78	0.02	0.26	1,158.14	49.97	1,388.54	2,596.65
β	8	95.83	103.70	0.36	6.78	0.09	0.26	2,417.82	679.81	1,388.54	4,486.17
	9	95.88	103.76	0.41	6.78	0.09	0.26	2,420.87	681.33	1,388.54	4,490.74
	10	95.94	103.83	0.45	6.78	0.09	0.26	2,424.27	683.04	1,388.54	4,495.85
	11	96.01	103.91	0.50	6.78	0.09	0.26	2,428.04	684.92	1,388.54	4,501.50
	12	96.08	103.99	0.55	6.78	0.09	0.26	2,432.18	686.99	1,388.54	4,507.71
δ	0.5	96.48	104.63	0.48	6.78	0.10	0.26	2,439.55	1,388.93	1,388.54	4,533.51
	1	95.94	103.83	0.45	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	4,495.85
	1.5	95.452	103.11	0.43	6.78	0.09	0.26	2,393.53	1,341.75	1,388.54	4,462.37
	2	95.01	102.44	0.42	6.78	0.09	0.26	2,374.23	1,321.96	1,388.54	4,432.53
	2.5	94.60	101.83	0.40	6.78	0.09	0.26	2,356.99	1,304.30	1,388.54	4,405.89
φ	6	95.69	103.46	0.45	5.69	0.09	0.29	2,182.37	661.78	1,216.33	4,060.48
	7	95.84	103.68	0.45	6.31	0.09	0.27	2,306.25	673.90	1,300.43	4,280.59
	8	95.94	103.83	0.45	6.78	0.09	0.26	2,424.27	683.04	1,388.54	4,495.85
	9	96.02	103.96	0.45	7.14	0.10	0.26	2,538.35	690.17	1,479.31	4,707.83
	10	96.09	104.06	0.45	7.42	0.10	0.25	2,649.66	695.90	1,571.93	4,917.48
r_e	50	94.12	101.10	0.45	6.78	0.08	0.26	2,126.80	534.30	1,388.54	4,049.64
	70	95.03	102.47	0.45	6.78	0.09	0.26	2,270.98	606.39	1,388.54	4,265.90
	90	95.94	103.83	0.45	6.78	0.09	0.26	2,424.27	683.04	1,388.54	4,495.85
	110	96.85	105.20	0.45	6.78	0.10	0.26	2,586.69	764.25	1,388.54	4,739.48
	130	97.76	106.56	0.45	6.78	0.11	0.26	2,758.23	850.02	1,388.54	4,996.79

Table 5 Sensitivity analysis for the key parameters of the RS model

Parameter	*Value*	Value									
		ω^{RS^*}	p^{RS^*}	q^{RS^*}	r^{RS^*}	g^{RS^*}	y^{RS^*}	$\Pi_M^{RS^*}$	$\Pi_R^{RS^*}$	$\Pi_T^{RS^*}$	Π^{RS^*}
d	900	83.44	86.87	0.23	6.78	0.04	0.26	1,297.82	243.25	1,388.54	2,929.61
	1,000	85.71	91.42	0.23	6.78	0.07	0.26	1,742.50	689.78	1,388.54	3,820.81
	1,100	87.98	95.97	0.23	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	5,167.68
	1,200	90.26	100.52	0.23	6.78	0.12	0.26	3,315.91	2,265.77	1,388.54	6,970.22
	1,300	92.54	105.07	0.23	6.78	0.15	0.26	4,444.64	3,395.24	1,388.54	9,228.42
α	9	90.81	101.61	0.25	6.78	0.13	0.26	3,325.38	2,276.74	1,388.54	6,990.66
	10	87.98	95.97	0.23	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	5,167.68
	11	85.64	91.28	0.21	6.78	0.07	0.26	1,791.37	738.03	1,388.54	3,917.94
	12	83.65	87.31	0.20	6.78	0.04	0.26	1,387.55	332.48	1,388.54	3,108.57
	13	81.96	83.91	0.19	6.78	0.02	0.26	1,156.40	99.87	1,388.54	2,644.81
β	8	87.93	95.86	0.18	6.78	0.09	0.26	2,412.05	1,357.44	1,388.54	5,158.03
	9	87.96	95.91	0.21	6.78	0.09	0.26	2,413.54	1,360.51	1,388.54	5,162.59
	10	87.98	95.97	0.23	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	5,167.68
	11	88.02	96.04	0.26	6.78	0.09	0.26	2,417.01	1,367.77	1,388.54	5,173.32
	12	88.06	96.11	0.28	6.78	0.09	0.26	2,418.99	1,371.97	1,388.54	5,179.50
δ	0.5	88.25	96.51	0.24	6.78	0.10	0.26	2,439.55	1,388.93	1,388.54	5,217.01
	1	87.99	95.97	0.23	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	5,167.68
	1.5	87.74	95.48	0.22	6.78	0.09	0.26	2,393.53	1,341.75	1,388.54	5,123.82
	2	87.52	95.03	0.21	6.78	0.09	0.26	2,374.23	1,321.96	1,388.54	5,084.73
	2.5	87.31	94.62	0.20	6.78	0.09	0.26	2,356.99	1,304.30	1,388.54	5,049.82
φ	6	87.86	95.72	0.23	5.69	0.09	0.29	2,173.43	1,321.52	1,216.33	4,711.27
	7	87.93	95.86	0.23	6.31	0.09	0.27	2,297.24	1,345.71	1,300.43	4,943.38
	8	87.99	95.97	0.23	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	5,167.68
	9	88.03	96.05	0.23	7.14	0.10	0.26	2,529.23	1,378.20	1,479.31	5,386.73
	10	88.06	96.12	0.23	7.42	0.10	0.25	2,640.49	1,389.63	1,571.93	5,602.05
r_e	50	87.08	94.15	0.23	6.78	0.08	0.26	2,118.75	1,066.96	1,388.54	4,574.26
	70	87.53	95.06	0.23	6.78	0.09	0.26	2,262.41	1,210.91	1,388.54	4,861.86
	90	87.99	95.97	0.23	6.78	0.09	0.26	2,415.19	1,363.95	1,388.54	5,167.68
	110	88.44	96.88	0.23	6.78	0.10	0.26	2,577.10	1,526.10	1,388.54	5,491.74
	130	88.90	97.791	0.23	6.78	0.11	0.26	2,748.12	1,697.36	1,388.54	5,834.02

Table 6 Sensitivity analysis for the key parameters of the TS model

Parameter	*Value*	*Value*									
		ω^{TS^*}	p^{TS^*}	q^{TS^*}	r^{TS^*}	g^{TS^*}	y^{TS^*}	$\Pi_M^{TS^*}$	$\Pi_R^{TS^*}$	$\Pi_T^{TS^*}$	Π^{TS^*}
d	900	84.50	89.00	0.44	6.78	0.05	0.26	1,271.73	222.19	1,388.54	1,388.54
	1,000	87.54	95.08	0.44	6.78	0.09	0.26	1,667.46	623.47	1,388.54	1,388.54
	1,100	90.57	101.16	0.44	6.78	0.13	0.26	2,266.07	1,226.99	1,388.54	1,388.54
	1,200	93.62	107.23	0.44	6.78	0.16	0.26	3,067.60	2,032.77	1,388.54	1,388.54
	1,300	96.65	113.31	0.44	6.78	0.20	0.26	4,072.02	3,040.78	1,388.54	1,388.54
α	9	94.33	108.66	0.49	6.78	0.17	0.26	3,076.36	2,045.40	1,388.54	6,510.30
	10	90.578	101.16	0.44	6.78	0.13	0.26	2,266.07	1,226.99	1,388.54	4,881.60
	11	87.46	94.92	0.41	6.78	0.09	0.26	1,710.82	665.47	1,388.54	3,764.82
	12	84.82	89.64	0.37	6.78	0.06	0.26	1,351.42	301.12	1,388.54	3,041.08
	13	82.56	85.12	0.34	6.78	0.03	0.26	1,145.70	91.43	1,388.54	2,625.68
β	8	90.53	101.06	0.36	6.78	0.13	0.26	2,263.09	1,215.84	1,388.54	4,867.47
	9	90.55	101.11	0.40	6.78	0.13	0.26	2,264.49	1,221.12	1,388.54	4,874.15
	10	90.57	101.16	0.44	6.78	0.13	0.26	2,266.07	1,226.99	1,388.54	4,881.60
	11	90.61	101.21	0.49	6.78	0.13	0.26	2,267.84	1,233.46	1,388.54	4,889.83
	12	90.64	101.27	0.53	6.78	0.13	0.26	2,269.78	1,240.50	1,388.54	4,898.82
δ	0.5	90.93	101.86	0.47	6.78	0.13	0.26	2,287.86	1,250.36	1,388.54	4,926.75
	1	90.58	101.16	0.44	6.78	0.13	0.26	2,266.08	1,226.99	1,388.54	4,881.61
	1.5	90.26	100.52	0.43	6.78	0.12	0.26	2,246.70	1,206.23	1,388.54	4,841.47
	2	89.96	99.93	0.41	6.78	0.12	0.26	2,229.44	1,187.73	1,388.54	4,805.70
	2.5	89.69	99.39	0.39	6.78	0.12	0.26	2,214.02	1,171.21	1,388.54	4,773.77
φ	6	90.41	100.83	0.44	5.69	0.12	0.29	2,028.98	1,189.04	1,216.33	4,434.35
	7	90.51	101.02	0.44	6.31	0.13	0.27	2,150.12	1,210.68	1,300.43	4,661.24
	8	90.58	101.16	0.44	6.78	0.13	0.26	2,266.08	1,226.99	1,388.54	4,881.61
	9	90.63	101.27	0.44	7.14	0.13	0.26	2,378.54	1,239.73	1,479.31	5,097.58
	10	90.68	101.35	0.44	7.42	0.13	0.25	2,488.55	1,249.96	1,571.93	5,310.44
r_e	50	89.36	98.73	0.44	6.78	0.11	0.26	2,002.28	961.31	1,388.54	4,352.13
	70	89.97	99.94	0.44	6.78	0.12	0.26	2,130.12	1,090.11	1,388.54	4,608.77
	90	90.58	101.16	0.44	6.78	0.13	0.26	2,266.08	1,226.99	1,388.54	4,881.61
	110	91.19	102.37	0.44	6.78	0.13	0.26	2,410.15	1,371.97	1,388.54	5,170.66
	130	91.79	103.59	0.44	6.78	0.14	0.26	2,562.34	1,525.03	1,388.54	5,475.91

Figure 2 Effects of *d* on optimal solutions and profits (see online version for colours)

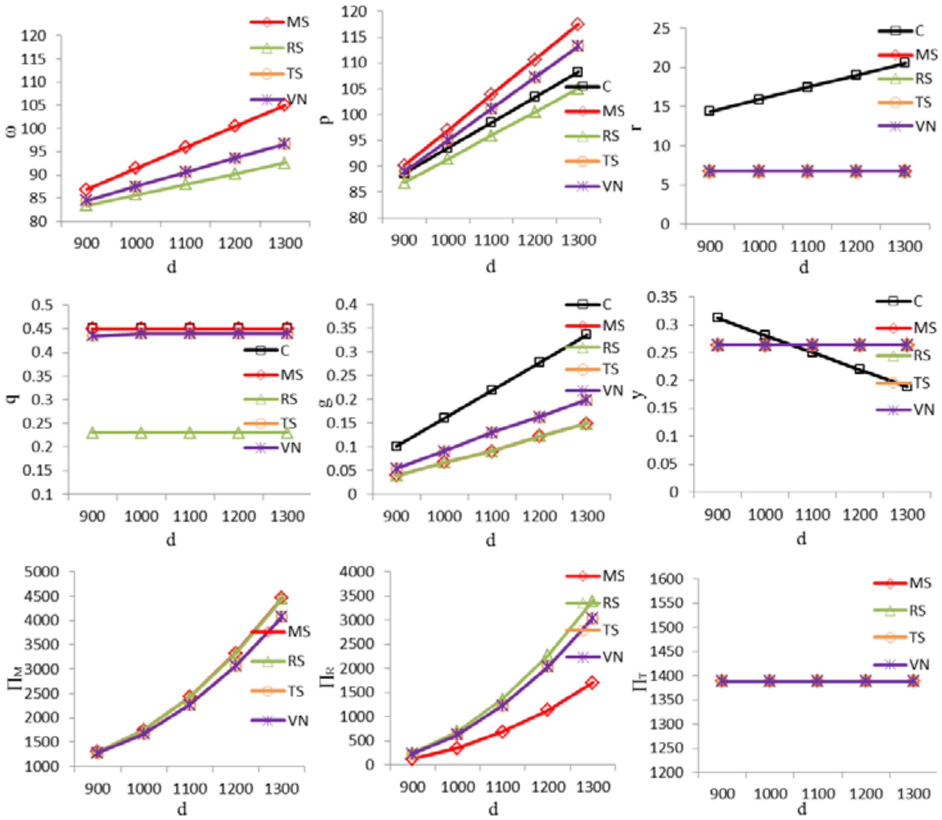


Figure 3 Effects of α on optimal solutions and profits (see online version for colours)

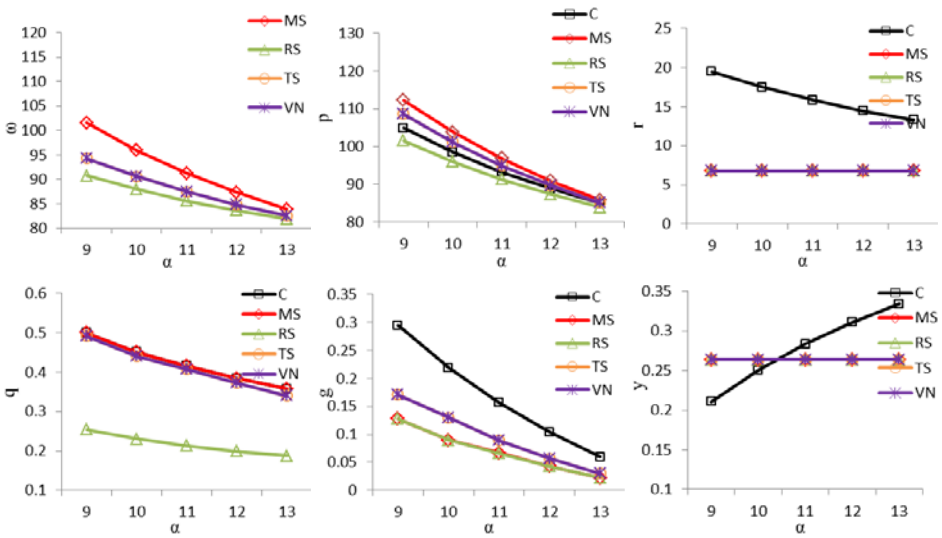


Figure 3 Effects of α on optimal solutions and profits (continued) (see online version for colours)

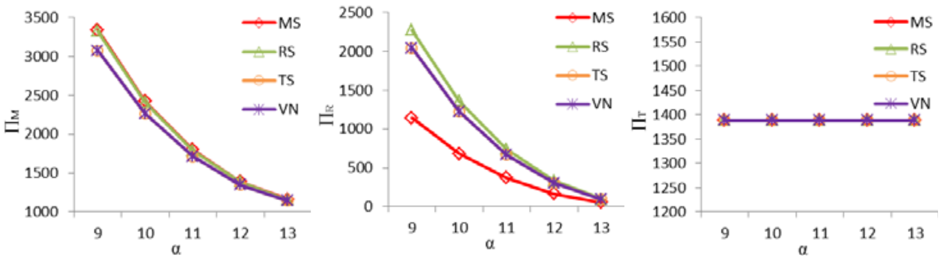
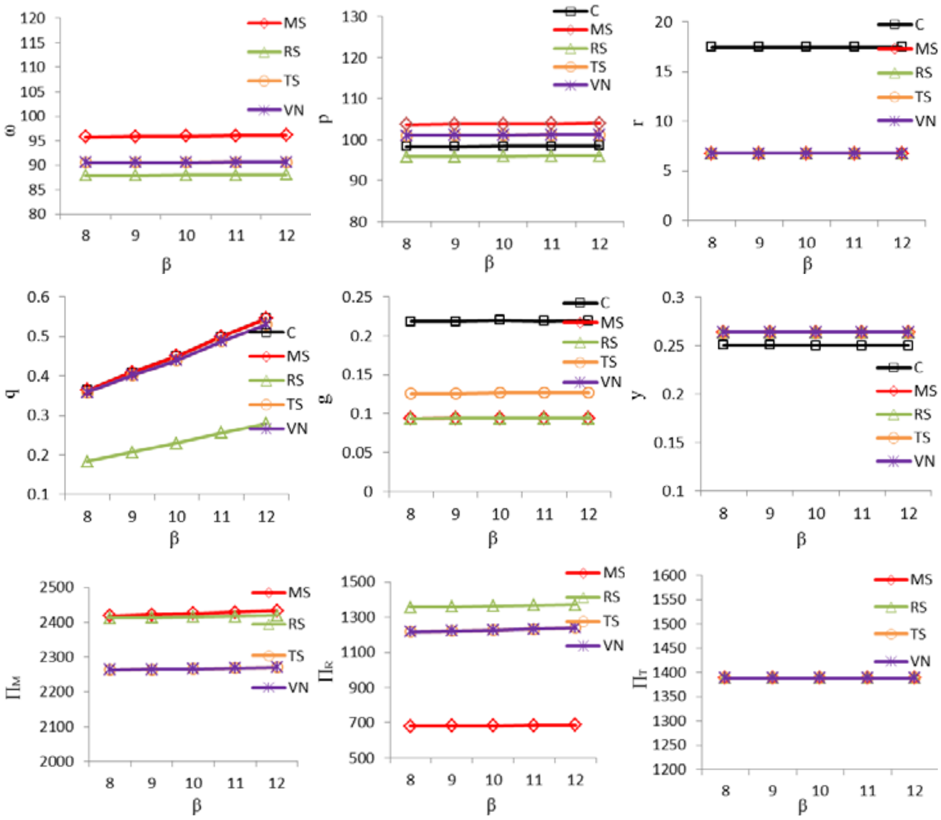
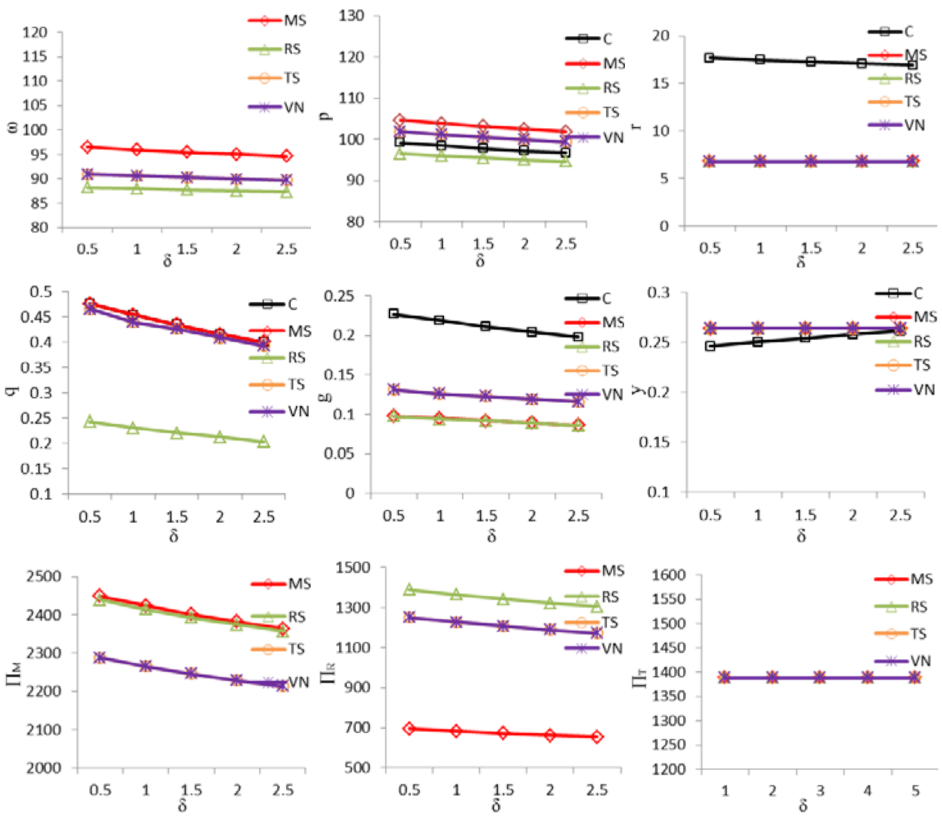


Figure 4 Effects of β on optimal solutions and profits (see online version for colours)



When the reference price effect (δ) increases in all five models, ω^*, p^*, q^*, g^*, Π_M^*, Π_R^* and Π^* decrease, and the third party's profit remains unchanged. As δ increases, the buy-back price and collection effort in a centralised system decrease and in other models remain unchanged. The results indicate that by increasing δ the sensitivity of customers to the reference price increases and the optimal strategy is to manufacture products with lower quality and lower price. This strategy results in decreased profits for the manufacturer, retailer, and the whole system. It indicates that higher sensitivity of costumers to the reference price loses both the manufacturer and the retailer. The effects of δ on optimal solutions and profits are shown in Figure 5.

Figure 5 Effects of δ on optimal solutions and profits (see online version for colours)



When the buy-back price effect (φ) increases in all decentralised models, ω^*, p^*, r^*, g^*, Π_M^*, Π_R^*, Π_T^* and Π^* increase and y^* decreases, while in a centralised model, p^*, r^* and g^* increase and y^* and Π^* decrease. The sensitivity of ω, p^* and g^* changes very little with the change of φ, which φ is a return parameter. The results indicate that in decentralised models, it is optimal that the third party buys used products at high price and makes relatively little effort to collect them. In centralised models, it is optimal that the third party buys used products at low prices and makes a relatively good effort to collect them. The effects of φ on optimal solutions and profits are shown in Figure 6.

Figure 6 Effects of φ on optimal solutions and profits (see online version for colours)

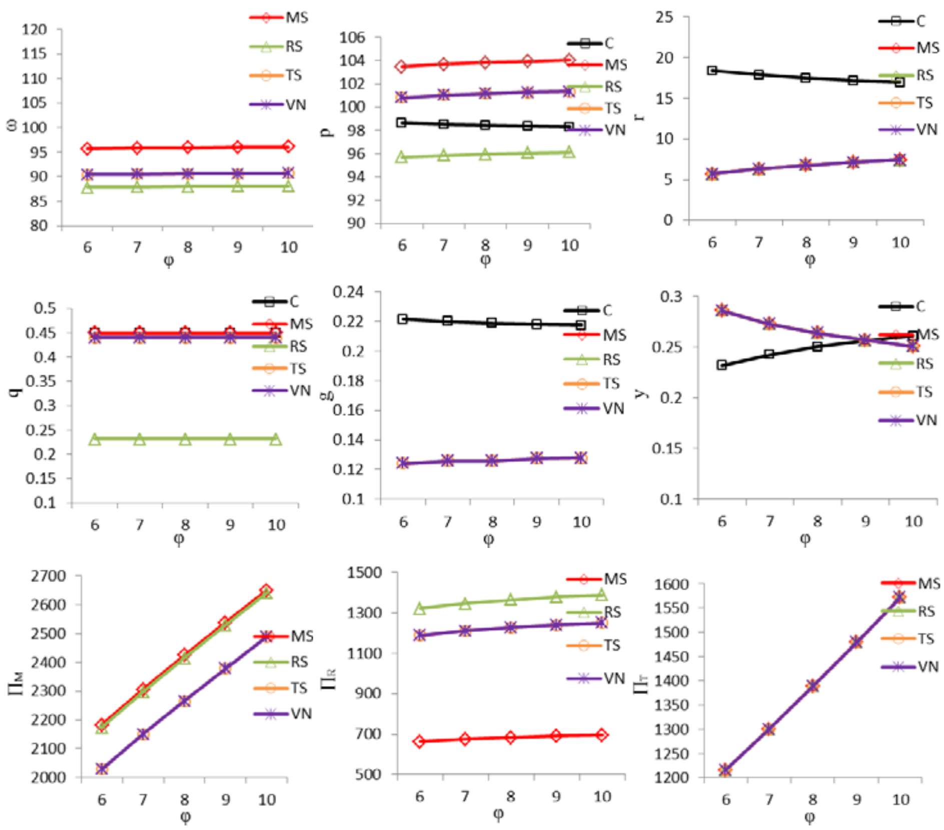


Figure 7 Effects of r_e on optimal solutions and profits (see online version for colours)

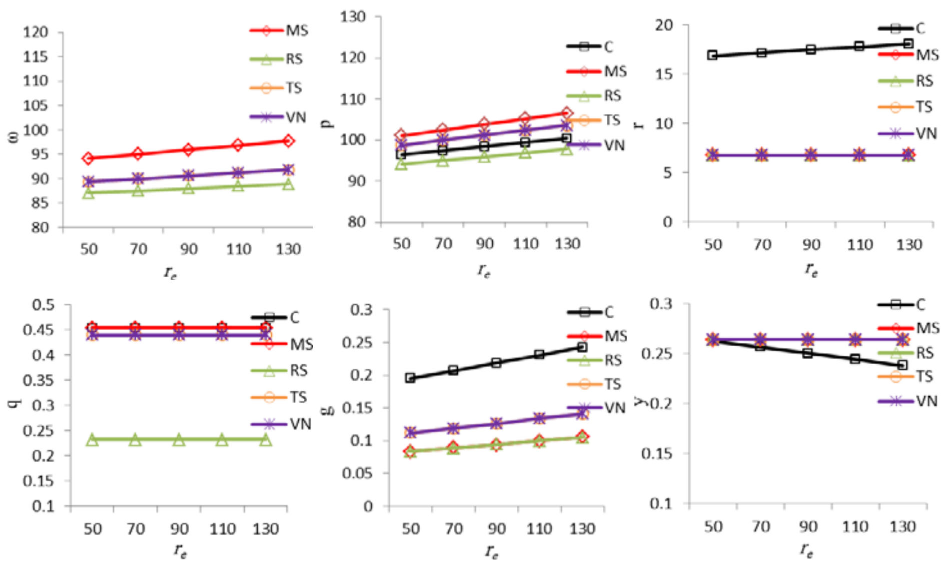
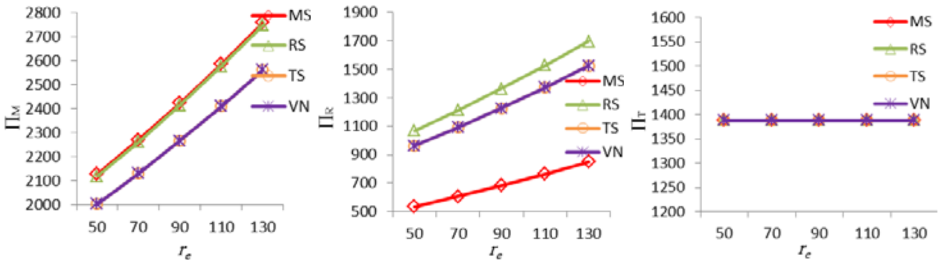


Figure 7 Effects of r_e on optimal solutions and profits (continued) (see online version for colours)



When the reference price of the customers (r_e) increases in all five models, ω^*, p^*, q^*, Π_M^*, Π_R^* and Π^* increase and the third party's profit remains unchanged. As δ increases, in a centralised system, the buy-back price increases and collection effort decreases and in other models remains unchanged. The results indicate that by increasing the reference price of the customers, the optimal strategy is to manufacture products with higher price. This strategy results in increased profits for the manufacturer, retailer, and the whole system. It indicates that higher reference price of the customers benefits both the manufacturer and retailer. The effects of r_e on optimal solutions and profits are shown in Figure 7.

6 Managerial insights and industrial applications

Consider a CLSC consisting of a manufacturer, a retailer, and a third party in which the manufacturer sells products to the retailer at wholesale price ω and the retailer sells them to the customers at retail price p. Also, in the reverse supply chain, used products are collected by a third party at buy-back price r. Finally, all the components are remanufactured and stocked as serviceable new products to satisfy a proportion of customer demand. To attract customers and increase the market demand, the retailer invests in sales effort activities. In the reverse channel, the third party invests in collection effort activities to motivate customers to return their used items. This situation can be applied to many industries which remanufacture used products for saving costs, reducing environmental issue, and satisfying government legislations, such as tire industry, carpet recycling, sand recycling, paper recycling, electronic equipment, and automotive industry. In all these industries, the companies want to know what the optimal pricing, product quality, and effort-input decisions are, which channel power structure is the best under different conditions, how to react to the quality trade-off investment in five different game models, how different channel power structures influence the quality of products, how sales and collection efforts influence the optimal decisions, and so on. All these questions can be answered by the findings in this paper. In Sections 3 and 4, we obtain the following important managerial insights:

1 In the matter of selling price, the manufacturer Stackelberg model offers higher price than other channel power structures, and the retailer Stackelberg model offers lower price than other channel power structures. Therefore, in the matter of price, the retailer Stackelberg model is more beneficial structure for customers than other structures.

2 In the matter of quality of product, the manufacturer Stackelberg model and a centralised policy are more acceptable choices than the other scenarios.

3 In the matter of buy-back price, the manufacturer Stackelberg model offers higher buy-back price to customers than other channel power structures. Therefore, in the matter of returning used items, the manufacturer Stackelberg model is more beneficial structure for customers than other structures.

4 Because the selling price, product quality, and buy-back price affect customer preferences, each customer may choose different structures. For example, customers whose first priority is price may choose a retailer-led Stackelberg as the best model while customers whose first priority is quality of product may choose a centralised or manufacturer-led Stackelberg as the best model.

5 As the reference price effect increases, the quality and price of products decrease.

6 As the return policy effect increases, the optimal prices and buy-back price increase and collectors invest less money on collecting used products; As a result, the profits of the manufacturer, retailer, and the whole system increase.

7 As the reference price of the customer increases, the manufacturer prefers to sell products with higher price which results in increased profits for the manufacturer, retailer, and the whole system.

7 Conclusions

In this paper, we investigated a CLSC with a manufacturer, retailer, and third party under five different channel power structures. We considered a joint decision model that includes the price, quality level, sales and collection efforts levels, and return policy. Demand is sensitive to the selling price, quality of product, buy-back price, sales effort, and reference price effect, and return quantity, which is particularly sensitive to the buy-back price and collection effort. The optimal pricing, quality, and effort decisions were explored by establishment of the game theory models reflecting the centralised CLCS, Nash game, and three different Stackelberg games (led by manufacturer, retailer, and third party, respectively).

Our main contribution is the exploration of a CLCS model in which we simultaneously considered the selling price, quality of product, sales and collection effort, buy-back price, and reference price effect under different scenarios. These results are unique among studies in the literature. In addition, applications of various game theory approaches are other merits of this study; managers can decide the structure to adopt in order to maximise their profit. More importantly, our results also can be used as decision tools for choosing marketing strategies based on the price, quality level, effort level, and return policy under different interactions. After formulating the problem, we used numerical examples to illustrate the theoretical results. Furthermore, we conducted a sensitivity analysis on the results of the numerical example with respect to key parameters. In this manner, we derived some managerial insights. The results in the numerical examples reveal the following important managerial insights:

1 The centralised policy is always the best from the perspective of supply chain.

2 If we consider selling price as the most important factor, then the manufacturer Stackelberg model is better than other channel power structures.

3 If we consider the quality of product as a key parameter, the manufacturer Stackelberg model and a centralised policy are more acceptable choices than other scenarios.

4 As the sensitivity of demand to reference price and to quality level decrease and increases respectively, the profits of the manufacturer, third party, and the whole system increase, too. Increasing the sensitivity of customers to selling price results in decreased profits for the manufacturer, third party, and whole system.

5 As the reference price of the customer increases, the manufacturer prefers to sell products with higher price which results in increased profits for the manufacturer, retailer and the whole system.

The model developed in this paper has a few limitations that could be ameliorated in future research. We assumed a situation of a CLCS with one retailer, and future studies could look at a circumstance with multiple retailers. We assumed that products are sold only through the retailer channel and are collected only by the third party. However, nowadays, in realistic situations, products are sold both through internet and retail channel and used items can be collected by third party, retailer, and manufacturer. Therefore, by studying different channel structures, such as a dual channel supply chain in the forward and reverse logistics, researchers could investigate interesting topics. In addition, in the context of the CLSC, government subsidy can play a prominent role in motivation for collection of used items either by collectors or manufacturers. However, this issue was not covered in this paper, and another potential future study can be consideration of role of government in the reverse process. Finally, an interesting extension to this work includes consideration of coordination mechanisms such as revenue sharing or risk sharing contracts (Choi et al., 2013).

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Appendix A

Proof of Proposition 1

The Hessian matrix for Π_C with respect to p, r, q, y and g is given by:

$$H_C = \begin{bmatrix} -2(\alpha+\delta) & 0 & k & \gamma + 2c_q q(\alpha+\delta) & \upsilon \\ 0 & -c_2 & 0 & 0 & -l \\ k & 0 & -c_1 & -2c_q kq & 0 \\ \gamma + 2c_q q(\alpha+\delta) & 0 & 2c_q kq & -2c_q\begin{pmatrix} d+gk-\alpha p+\upsilon r \\ +\gamma q-\delta(p-r_e) \end{pmatrix} -4c_q\beta q & \begin{matrix} 2c_q q(2\varphi-\upsilon) \\ -4\varphi c_q q \end{matrix} \\ \upsilon & -l & 0 & 2c_q q(2\varphi-\upsilon)-4\varphi c_q q & -4\varphi \end{bmatrix} \tag{47}$$

To prove that the joint total profit is concave in p, r, q, y and g, we show that $x \cdot H_C \cdot x^T < 0$ where $x = [p, y, g, q, r]$ By solving $x \cdot H_C \cdot x^T < 0$, we obtain the following condition:

$$c_q > \frac{\left[-2p^2(\alpha+\delta)+2p(kg+\upsilon r+\gamma q)-2lry-2\varphi r^2-c_1 g^2-c_2 y^2\right]}{q^2\left[2d+6\beta q+6\upsilon r+6p(\alpha+\delta)+6gk+2\delta r_e\right]} \tag{48}$$

Therefore, if the above condition is satisfied, H_C will be a negative definite Hessian matrix. Thus q^* and r^* can be obtained by solving the first-order condition as follows:

$$q^* = \frac{\beta}{2c_q(\alpha+\delta)} \tag{49}$$

$$r^* = \frac{\begin{matrix} c_1 c_2 c_q(\alpha+\delta)\left[\left(2\phi - c_m\upsilon - 2c_q\varphi q_r^2 - 2\varphi(c_m - c_r)\right)(\alpha+\delta)+d\upsilon\right] \\ +c_1 c_q l^2\left(-2c_r+2c_q q_r^2\right)(\alpha+\delta)^2 \\ +c_2 c_q\left[\left(-\phi k^2+c_q\varphi k^2 q_r^2\right)(\alpha+\delta)-c_r\delta\varphi k^2\right] \\ +c_m c_q\left[2c_1 l^2(\alpha+\delta)^2-\left(l^2k^2-c_2\varphi k^2\right)(\alpha+\delta)\right] \\ -(\alpha+\delta)\left(c_q q_r^2-c_r\right)l^2k^2c_q \end{matrix}}{2(\alpha+\delta)c_q\left[l^2k^2-2c_2\varphi k^2-c_1c_2\upsilon^2+(\alpha+\delta)\left(2c_1c_2\varphi-c_1 l^2\right)\right]} \tag{50}$$

To ensure $0 < q^* < 1$, we solve $0 < q^* = \dfrac{\beta}{2c_q(\alpha+\delta)} < 1$ to yield the following expression:

$$c_q > \frac{\beta}{2(\alpha+\delta)} \tag{51}$$

By integrating conditions (48) and (51), we obtain:

$$c_q > \max\left\{\frac{\left[2p^2(\alpha+\delta)+2p(kg+\upsilon r+\beta q)-2lry-2\varphi r^2-c_1g^2-c_2y^2\right]}{q^2\left[2d+6\beta q+6\upsilon r+6p(\alpha+\delta)+6g_2k+2\delta r_e\right]}, \frac{\beta}{2(\alpha+\delta)}\right\} \tag{52}$$

Moreover, to ensure $r < b$, by substituting the optimal solution of r in $r < b$, we solve to get the following expression:

$$c_r > \frac{\begin{array}{l} c_2c_q\left[\begin{array}{l}\left(8c_1\phi-8c_1c_q\varphi q_r^2\right)(\alpha+\delta)^2\\ +\left(4c_q\varphi k^2q_r^2-4c_1\delta\upsilon r_e-4\phi k^2-4c_1d\upsilon\right)(\alpha+\delta)\end{array}\right]\\ +c_qb\left[\left(-8c_1l^2+16c_1c_2\varphi\right)(\alpha+\delta)^2+\left(4l^2k^2-8c_2\varphi k^2-4c_1c_2\upsilon^2\right)(\alpha+\delta)\right]\\ +c_mc_q\left[\left(-8c_1l^2-8c_1c_2\varphi+4c_1c_2\upsilon\right)(\alpha+\delta)^2+\left(4c_2\varphi k^2-4l^2k^2\right)(\alpha+\delta)\right]\\ +\left(8c_1c_q^2l^2q_r^2\right)(\alpha+\delta)^2+\left(4c_q^2l^2k^2q_r^2\right)(\alpha+\delta)-c_1c_2\upsilon\beta^2\end{array}}{\left(8c_1c_ql^2-8c_1c_2c_q\varphi\right)(\alpha+\delta)^2+\left(4c_2c_q\varphi k^2-4c_ql^2k^2\right)(\alpha+\delta)} \tag{53}$$

Therefore, if conditions (52) and (53) are satisfied, then Π_C is concave in p, r, q, y and g, and p, r, q, y and g are optimal solutions.

Appendix B

Proof of Propositions 2, 7, 9 and 11

The Hessian matrix associated with the profit function Π_R is given by

$$H_R = \begin{bmatrix} -2(\alpha+\delta) & k \\ k & -c_1 \end{bmatrix} \tag{54}$$

To prove that the joint total profit is concave in p and g, we show that $|H_{1R}| < 0$ and $|H_{2R}| > 0$. According to Assumption 2, $|H_{1R}| = -2(\alpha + \delta) < 0$ is satisfied. Also by solving $|H_{2R}| = 2c_1(\alpha + \delta) - k^2 > 0$, we can obtain the following condition:

$$c_1 > \frac{k^2}{2(\alpha+\delta)} \tag{55}$$

Therefore, if condition (55) is satisfied, then H_R will be a negative definite Hessian matrix.

Appendix C

Proof of Propositions 3, 6, 10 and 11

The Hessian matrix associated with the profit function Π_T is given by:

$$H_T = \begin{bmatrix} -c_2 & -l \\ -l & -2\varphi \end{bmatrix} \quad (56)$$

To prove that the joint total profit is concave in r and y_1, we show that $|H_{1T}| < 0$ and $|H_{2T}| > 0$. Because $c_2 > 0$, we see that $|H_{1T}| = -2c_1 < 0$ is always satisfied. Also, by solving $|H_{2T}| = 2c_2\varphi - l^2 > 0$, we obtain the following condition:

$$c_1 > \frac{l^2}{2\varphi} \quad (57)$$

Therefore, if condition (57) is satisfied, Π_T will be a negative definite Hessian matrix. Moreover, to ensure $r^* < b$, by substitution of the optimal solution of r^* in $r^* < b$, we solve to obtain the following condition:

$$c_2 > \frac{l^2}{2\varphi} \quad (58)$$

Appendix D

Proof of Proposition 4

The Hessian matrix associated with the profit function H_M is given by

$$H_M = \begin{bmatrix} 0 & \beta \\ \beta & -2c_q \begin{pmatrix} d + gk - \alpha p + \upsilon r \\ +\gamma q - \delta(p - r_e) \end{pmatrix} - 4c_q \beta q \end{bmatrix} \quad (59)$$

To prove that the joint total profit is concave in ω and q, we show that $x \cdot H_C \cdot x^T < 0$ where $x = [\omega, q]$.

By solving $x \cdot H_C \cdot x^T < 0$, we obtain the following condition:

$$c_q > \frac{\beta\omega}{q(gk - (\alpha + \delta)p + \delta r_e + \upsilon r + 3\beta q)} \quad (60)$$

Moreover, to ensure $0 < q^* < 1$, we solve $0 < q^* = \dfrac{\beta}{2c_q(\alpha - \delta)} < 1$ to obtain the following expression:

$$c_q > \frac{\beta}{2(\alpha - \delta)} \quad (61)$$

By integrating conditions (60) and (61), we obtain:

$$c_q > \max\left\{ \frac{\beta\omega}{q(gk - (\alpha + \delta)p + \delta r_e + \upsilon r + 3\beta q)}, \frac{\beta}{2(\alpha + \delta)} \right\} \quad (62)$$

Appendix E

Proof of Propositions 5, 8, and 11

Taking the second derivatives of Π_M with respect to q, we obtain:

$$\frac{\partial^2 \Pi_M}{\partial q^2} = -2c_q \left(d - (\alpha - \delta) p + \upsilon r + 3\beta q - \delta r_e \right) < 0$$

Therefore, the manufacturer's profit is a concave function over q.