Transportmetrica A: Transport Science

ISSN: 2324-9935 (Print) 2324-9943 (Online) Journal homepage: www.tandfonline.com/journals/ttra21

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To cite this article: Hyemin Yoo, Junhyeok Lee & Ilkyeong Moon (2026) An integrated approach for an aircraft routing and fuel tankering problem, Transportmetrica A: Transport Science, 22:1, 2392164, DOI: 10.1080/23249935.2024.2392164

To link to this article: https://doi.org/10.1080/23249935.2024.2392164

Published online: 19 Aug 2024.

TRANSPORTMETRICA A: TRANSPORT SCIENCE
2026, VOL. 22, NO. 1, 2392164
https://doi.org/10.1080/23249935.2024.2392164

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An integrated approach for an aircraft routing and fuel tankering problem

Hyemin Yoo[a], Junhyeok Lee[b] and Ilkyeong Moon [c,d]

[a]Manufacturing Execution System Team, Samsung Electronics, Yongin-si, Korea; [b]Memory Innovation Team, Samsung Electronics, Hwaseong-si, Korea; [c]Department of Industrial Engineering, Seoul National University, Seoul, Korea; [d]Institute of Engineering Research, Seoul National University, Seoul, Korea

ABSTRACT

In the highly competitive airline industry, it is crucial for airlines to minimise operating costs and strengthen their competitiveness. Fuel costs constitute a substantial portion of airline operating costs and are directly related to airlines' profits. In this research, we support the airlines' fuel management by presenting a model that integrates aircraft routing and fuel tankering decisions. The effectiveness of fuel tankering can be enhanced by considering the aircraft routing decisions together, leading to a significant cost reduction for airlines. The integrated model is developed as a mixed-integer linear programming model. The symmetry-breaking methods and decomposition-based heuristic algorithm are also proposed to alleviate the computational burden. A set of computational results illustrates the significant cost-saving effects of the proposed model and the heuristic algorithm.

ARTICLE HISTORY
Received 2 January 2024
Accepted 6 August 2024

KEYWORDS
Aircraft routing; fuel tankering; fuel management; decomposition approach; heuristic algorithm

1. Introduction

According to a report by Airlines for America (2023), fuel cost is the second largest component of airlines' operating costs, following labour costs, and accounts for an average of 22 percent of airlines' operating costs. For instance, *United Airlines* spent 13.1 billion U.S. dollars on fuel costs in 2022 (United Airlines 2023). Considering the scale of the fuel costs, even a slight percentage improvement can lead to significant cost savings from an overall perspective. As fuel costs are critical factors of cost structure, airlines use several fuel management strategies such as optimising cruise speed and flight altitude or fuel hedging (Morrell and Swan 2006).

Fuel tankering is one of the fuel management strategies. Fuel tankering is a refueling strategy that leverages the fuel price differences among airports. When employing fuel tankering, the aircraft refuels more than necessary when departing from an airport with a lower fuel price. Consequently, it refuels less at the subsequent airport, which has higher fuel prices.

Figures 1 and 2 illustrate the concept of fuel tankering. Assume that an airline operates round-trip flights between Airports A and B, as seen in Figure 1. Airport A has a low fuel

CONTACT Ilkyeong Moon ikmoon@snu.ac.kr

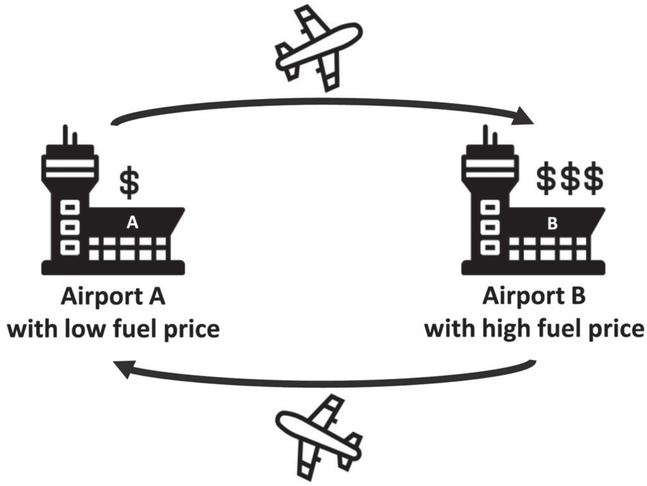
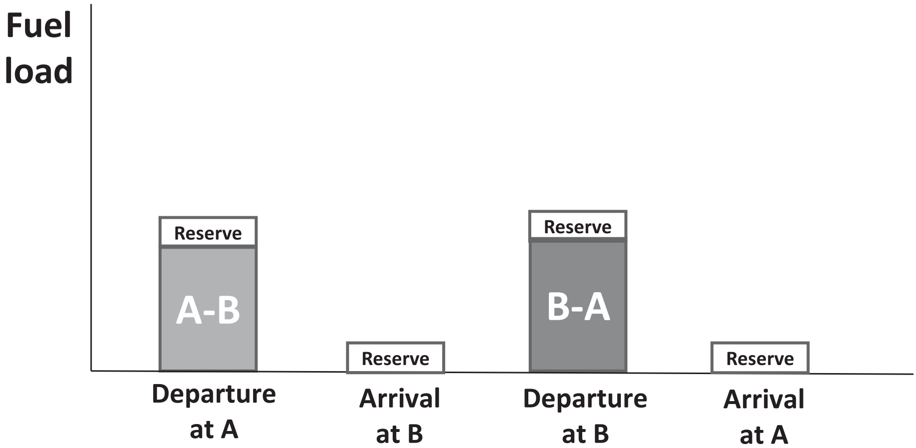


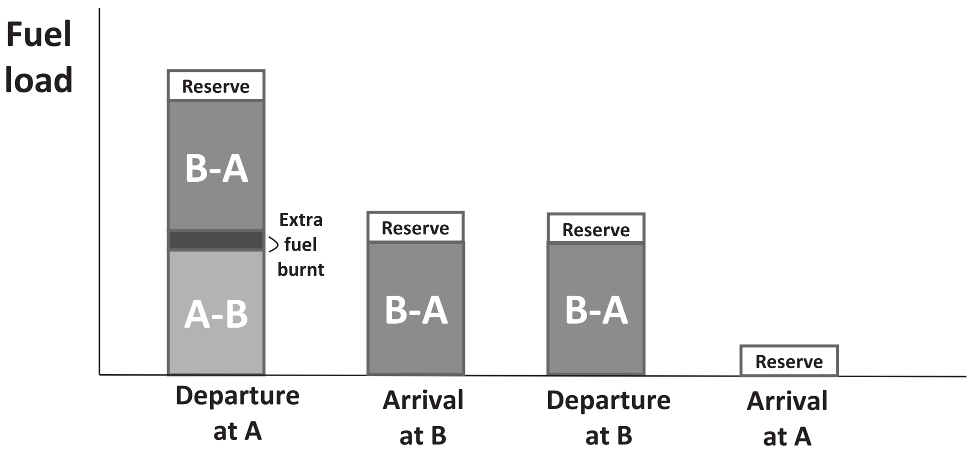
Figure 1. Round-trip flights under jet fuel price differences.

price, and Airport B has a high fuel price. Under this circumstance, the basic refueling plan is when an aircraft refuels only the required amount for the flight leg. Figure 2(a) illustrates the fuel load for round-trip flights under the basic refueling plan. An aircraft refuels with as much fuel as needed for each flight leg, with a reserve of safety fuel for emergencies. In contrast, Figure 2(b) shows the fuel load under the fuel tankering strategy. When an aircraft departs from Airport A, where the fuel price is low, it also fills up on fuel for the second flight leg departing from Airport B, where the fuel price is high. A larger difference in fuel prices between two airports yields significant cost savings. This practice can be applied not only to simple round-trip flights but also to general flights. Additionally, partial tankering, which refuels only part of the fuel required for the next flight, is also possible. However, there exist trade-offs between fuel weight and fuel efficiency. As the fuel weight increases, fuel efficiency decreases. In other words, when more fuel is taken on than what is needed for the immediate flight leg, more fuel is consumed in proportion to the amount of extra fuel (ICAO 2014). Thus, fuel tankering should be done only when the savings outweigh the wastage of fuel due to reduced fuel efficiency. When these trade-offs are not considered, fuel tankering could potentially result in losses from a cost perspective.

In summary, fuel tankering is the strategy of purchasing fuel at a low price that would otherwise be purchased at a high price. While accepting a slight compromise in fuel efficiency, airlines refuel more than necessary at airports with lower fuel prices. Furthermore, the effect of the fuel tankering strategy depends on the airports' fuel price difference. Generally, the larger the fuel price difference, the more significant the effect of the fuel tankering on an airline's net fuel cost. Fuel prices can differ across airports for various reasons. Differences can fluctuate between countries and even within the same country. Price differences can occur based on purchasing power, taxes, supply system, geographical location, and other factors. (Rietveld and van Woudenberg 2005; Unit 2019). Figure 3 shows the differences in jet fuel prices across states in the United States in March 2022 (U.S. Energy Information Administration 2022). Utah has the highest price among the states, about 21 percent higher than Michigan, which has the lowest price. In this regard, fuel tankering is



(a) Fuel load without fuel tankering strategy



(b) Fuel load with fuel tankering strategy

Figure 2. Fuel load depending on fuel tankering strategy. (a) Fuel load without fuel tankering strategy (b) Fuel load with fuel tankering strategy.

valid from a practical standpoint and is widely used in commercial airlines. Furthermore, fuel tankering serves not only as a strategy for cost reduction; it also plays a role in counteracting supply risks such as strikes, natural disasters, political issues, and problems in refueling facilities. In cases of sudden surges in fuel prices or limitations in fuel supply, fuel tankering can mitigate those risks.

Fuel tankering can achieve greater effectiveness when considered in tandem with aircraft *routes*, the orders of flight legs that each aircraft performs. As mentioned earlier, the fuel price differences between airports along a given route have a considerable impact on the effectiveness of fuel tankering. If the fuel price difference between airports is slight,

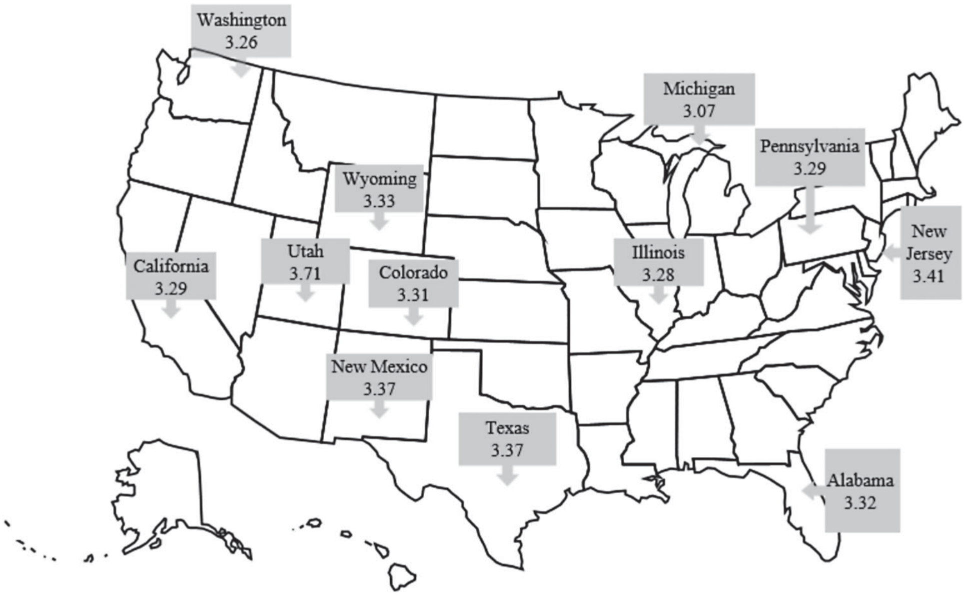


Figure 3. Jet fuel prices across the U.S. ($/gallon).

the benefits from fuel tankering can be marginal. In contrast, when fuel prices at each airport along the route are appropriately arranged, a significant amount of fuel costs could be saved. Motivated by these observations, we propose a novel framework integrating two principle decisions for airline operations: *aircraft routing* and *fuel tankering* decisions. The proposed framework aims to minimise fuel costs by simultaneously optimising aircraft routing and fuel tankering decisions.

However, there are several research gaps that need to be dealt with. The first research gap is that previous studies could not develop an effective mathematical model that considers both fuel tankering and aircraft routing. Because the aircraft routing problem belongs to the NP-hard problem, dealing with the aircraft routing problem alone requires a significant computational burden. Furthermore, integrating fuel tankering decisions with the aircraft routing problem makes the problem more challenging. To the best of our knowledge, Kheraie and Mahmassani (2012), in the most relevant paper to our study, also developed a mixed-integer linear programming (MIP) model to consider aircraft routing and fuel tankering simultaneously. However, their model is vulnerable to the realistic problem scale. The detailed limitations will be introduced in Section 2.3. The second research gap is that it lacks an algorithm that could derive promising solutions within a reasonable time for large-sized problems, allowing the proposed approach to be applied in real practice. Lastly, there is a lack of studies that provide guidelines or operational insights instructive to airline practitioners regarding the advantages of optimising fuel tankering decisions.

Our research is designed to fill these research gaps by presenting the following main contributions. First, we propose an integrated model for an aircraft routing and fuel tankering problem. Its significance comes from the scale of airlines' fuel expenses. Furthermore, we seize the symmetry related to the aircraft within the mathematical model and propose symmetry-breaking methods to reduce the computational burden. In detail, a method for assigning the initial solution is presented, and a symmetry-breaking constraint is presented. Second, we develop a heuristic algorithm, which decomposes the problems and solves them sequentially while considering the interdependence between the two problems. It is designed to provide an overwhelming advantage in terms of computational efficiency while maintaining reasonable solution quality. Third, we observe that utilising the proposed approach leads to significant fuel cost savings compared to benchmark approaches. Moreover, by conducting comprehensive experiments in various settings, we present several operational insights into how the airlines benefit from utilising our approach.

The remainder of this work is organised as follows. Section 2 provides a literature review of the aircraft routing problem and the fuel tankering problem. In Section 3, a mathematical formulation and symmetry-breaking methods are presented. The decomposition-based heuristic algorithm for reducing computation time and bounds for the model is proposed in Section 4. In Section 5, computational experiments are conducted to show the effectiveness of the proposed model and algorithm. Section 6 concludes this research.

2. Literature review

2.1. Fuel tankering problem

As emphasised in Section 1, fuel cost directly affects airlines' overall operating costs. Consequently, there are various research areas dedicated to fuel management. These include strategies such as fuel hedging to mitigate the risk of fuel price fluctuations, route optimisation to minimise fuel consumption, and contract-based discount policies with fuel suppliers (Bazargan 2016). Additionally, diverse research has been conducted on fuel tankering strategies to further enhance cost savings without requiring substantial investment or complex changes.

For an early reference of fuel tankering, Diaz (1990) outlined the fuel tankering problem and formulated the model with a vendor constraint. This constraint limits the amount of fuel purchased under a single vendor system. The model was extended to multiple vendor systems in the study of Stroup and Wollmer (1992). The proposed model also considered station constraints, which restrict the amount of fuel purchased at each station. They addressed fuel consumption as a linear function of fuel amount at take-off and its parameters to be given. Zouein, Abillama, and Tohme (2002) identified the fuel tankering model as a conventional multiple-period capacitated inventory model. The total amount of fuel on the aircraft must satisfy the fuel tank capacity and conservation equation between two consecutive flight legs. The authors conducted a regression analysis to establish a relationship between fuel consumption and the take-off weight of aircraft. On the other hand, Guerreiro Fregnani, Müller, and Correia (2013) assumed that flight distance and altitude are more crucial to fuel efficiency than the take-off weight. They derived the bi-dimensional polynomial function of distance and altitude.

Abdelghany, Abdelghany, and Raina (2005) differentiated the fuel tankering strategy into single-stage or multi-stage. In single-stage fuel tankering, the excess fuel is entirely consumed by the immediate subsequent flight leg. On the other hand, in multi-stage fuel tankering, excess fuel can be used across multiple subsequent flight legs. Obviously, because the single-stage strategy is a special case of the multi-stage strategy, the multi-stage strategy always dominates the single-stage strategy from a cost reduction perspective. The authors also performed experiments considering various operational conditions and fuel price scenarios along the route (e.g. saw edge, uphill, and cone-shaped patterns). Deo, Silvestre, and Morales (2020) analysed the effects of adopting a fuel tankering strategy considering cost index flying and utilising intermediate optional refueling stops. In order to derive tankering solutions, they developed a MIP model based on the network modelling strategies. Hassan et al. (2021) incorporated a stepped discount policy where the unit cost is discounted in a stepped manner based on the quantity of fuel bought. The authors also considered the stochasticity of flight time by applying the beta distribution. Zhang (2022) suggests a dynamic programming model for a single aircraft with a fixed route. Each flight leg along the route serves as a stage in the multi-stage planning problem. They showed that a developed model could lead to cost savings using real-world airline data.

2.2. Aircraft routing problem

The aircraft routing problem is one of airlines' major operation planning processes (i.e. *flight scheduling, fleet assignment*, and *crew scheduling*). As a result of flight scheduling and fleet assignment, a set of flight legs to be performed by each aircraft type is determined. Based on this, the aircraft routing problem is solved separately for each aircraft type, deriving the order of flight legs that each aircraft performs, called routes. The routes are usually operated in a cyclic manner for a certain period. These routes must enable airlines to execute all flight legs determined in the fleet assignment decision. Thus, the aircraft routing problem has been addressed as a feasibility problem that aims to find routes to perform given flight legs with available aircraft resources (Gopalan and Talluri 1998; Grönkvist 2005; Talluri 1998). The aircraft routing problem has been extensively studied from various perspectives. In our research, we investigate existing studies related to aircraft routing and categorise them into two main streams: (1) stand-alone studies focussing solely on aircraft routing with maintenance schedules, and (2) integrated studies considering other aspects of operation planning with aircraft routing.

First, when aircraft routing is solely considered, the corresponding research has mainly concentrated on examining how realistically maintenance schedules are incorporated into the routes. In early studies, Gopalan and Talluri (1998) modelled a periodic (e.g. three to four days) maintenance routing problem and proposed a polynomial-time algorithm. The polynomial time algorithm was shown to be effective not only in the static infinite-horizon model but also in the dynamic finite-horizon model, where the routes for a single day can vary across the planning horizon. Based on the results of this study, Talluri (1998) proved that the four-day maintenance routing problem is NP-hard and designed an Euler tour heuristic algorithm. Başdere and Bilge (2014) addressed a weekly maintenance routing model that tracks the time remaining until the maintenance check criteria. They also considered the capacity of the maintenance facility. Safaei and Jardine (2018) introduced generalised maintenance constraints to deal with any maintenance check and guarantee maintenance opportunities. Further comprehensive reviews of aircraft maintenance routing are covered by Ma et al. (2022) and Temucin, Tuzkaya, and Vayvay (2021).

Second, the aircraft routing problem has been studied in integration with other problems without losing feasibility. Integrating the aircraft routing problem with other airline planning processes can create more opportunities to reduce airline operating costs or achieve various objectives. Barnhart et al. (1998) made a significant contribution by integrating fleet assignment and aircraft routing while introducing the branch-and-price framework. They proposed a flight string model in which a *string* is a sequence of flight legs that start and end at the maintenance feasible airports. Haouari, Aissaoui, and Mansour (2009) formulated a fleet assignment and aircraft routing model, considering a deadhead flight, a flight that repositions aircraft for maintenance. Haouari et al. (2011) and Zeghal et al. (2011) also addressed the same problem as the study of Haouari, Aissaoui, and Mansour (2009). Liang and Chaovalitwongse (2013) presented a network-based model for integrating a weekly aircraft maintenance routing problem with the fleet assignment problem and offered tight linear programming (LP) relaxation. Sherali, Bae, and Haouari (2013) presented these integrated models as mixed-integer linear programming (MILP), considering several operating characteristics such as optional flight legs and multiple fare classes. In addition to flight scheduling or fleet assignment, crew scheduling has been studied widely (Radman and Eshghi 2023; Saemi et al. 2022). Cordeau et al. (2001) incorporated the crew scheduling problem with the aircraft routing problem. They compared crew costs between integrated planning and sequential planning. Dunbar, Froyland, and Wu (2012) designed a method to elaborately calculate the cost of propagated delay for an integrated aircraft routing and crew scheduling problem. This study was extended to re-timing decisions by considering the stochastic delay in the study of Dunbar, Froyland, and Wu (2014).

2.3. Aircraft routing with fuel tankering and contributions of this study

Research on solving both the aircraft routing problem and the refueling problem has been widely undertaken in aerial refueling systems. Aerial refueling is a system in which aircraft are refueled in mid-air through feeders (tanker aircraft) during the flight. Aerial refueling is more expensive than conventional ground refueling due to the operation of feeders. However, it is particularly considered in long-haul military flights, as it does not require landing on the ground. Although aerial refueling points are fixed, these points are selectively determined based on the required amount of fuel and travel distance along the route, which inherently means that the aerial refueling problem tackles a routing problem.

Kannon et al. (2015) determined the route and aerial refueling operation when the departure and destination are given. The proposed model minimised the distance travelled and the number of aerial refueling operations. Ferdowsi, Maleki, and Rivaz (2020) extended this model to reflect the uncertainty of refueling costs by considering costs as interval numbers. The refueling costs are probabilistically determined based solely on which refueling point is selected in the route; therefore, the formulation includes only binary variables related to the routing problem. Hansknecht et al. (2023) focussed on the feeder aiming to minimise the fuel consumption and the number of feeders. The model was decomposed into two stages: deriving routes serving aircraft and assigning the routes to each feeder. In addition, unmanned aerial vehicles (UAVs) are widely considered in the research area aimed at deriving routes while considering various forms of refueling or charging (Maini et al. 2019; Ribeiro et al. 2022). Although previous studies share the commonality of addressing both the aircraft routing problem and the refueling problem, there are significant structural differences in the problem. As previously mentioned, the aircraft can selectively include the aerial refueling points. Furthermore, previous studies aimed to determine the order of visits between given destinations. In contrast, the flight legs addressed in our study incorporate the concept of time. This means that our model derives the route by connecting flight leg segments, which have predefined departure and arrival times as well as departure and arrival locations.

To the best of our knowledge, only the study by Kheraie and Mahmassani (2012) approaches our research most relevantly, by addressing the aircraft routing problem and the fuel tankering problem to minimise fuel costs for commercial airlines. Our research aims to improve their shortcomings in two aspects. First, their model is fundamentally vulnerable when dealing with larger problem sizes. The authors presented the string-based aircraft routing model based on the research of Barnhart et al. (1998). However, the string-based model has a barrier with its complexity. According to Barnhart et al. (1998), even with just 100 flight legs, the aircraft routing problem alone generates over a million strings. Methodological support is essential to complement this, but this is missing in their work. As a result, their experiment was conducted with only 18 flight legs. Second, their experiments failed to demonstrate the effectiveness of the model. The presented experiments lacked quantitative analysis regarding how much cost benefit the model offers or what specific effects it brings about.

In this paper, we aim to fill the above research gaps. First, we introduce the methodologies to reduce complexity and derive solutions within an appropriate time. We suggest methods to break the symmetry in the mathematical model and develop a heuristic algorithm to handle the larger size. In our experiment, we obtained an exact solution from a flight leg size that was five times larger than in the previous study. Moreover, it was shown that using the proposed heuristic algorithm, a solution can be derived from a flight leg size that was 31 times larger. Second, we conduct thorough experiments to demonstrate our model quantitatively. In order to evaluate how our model can reduce the fuel cost, we define the lower bound on the fuel cost and present the objective value of the myopic case. Additionally, we demonstrate the strengths of our model, not only in cost reduction but also in its performance under risk scenarios (i.e. in the event of an increase in fuel prices or a restricted supply).

3. Problem description and mathematical model

Our model derives aircraft routes and refueling plans along these routes, aiming to minimise total fuel cost. This section includes a problem description and a mathematical model for an integrated model. Section 3.1 presents the network structure. Section 3.2 illustrates a detailed description of the aircraft routing problem and fuel tankering problem, and Section 3.3 formulates a mathematical model. In Section 3.4, we seize the symmetry in the model and represent two symmetry-breaking methods.

3.1. Connection network

Various network structures can be applied to airline planning problems, and our model is formulated on *connection network*. Barnhart et al. (1998) employed the connection network in the aircraft routing problem for early works.

For our network $G = (N, A)$, N and A represent the set of nodes and arcs, respectively. As illustrated in Figure 4, there are three types of nodes ($N = \mathcal{L} \cup \mathcal{S}^+ \cup \mathcal{S}^-$). A node in $\mathcal{L}$ represents each flight leg. Additionally, each airport has two nodes. One is its source node,

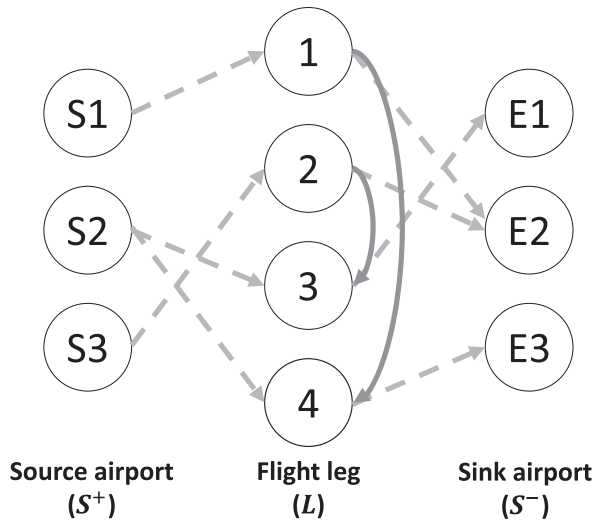


Figure 4. Simple example of connection network.

and the other is its sink node, each belonging to $\mathcal{S}^+$ and $\mathcal{S}^-$, respectively. $\mathcal{S}^+$ signifies the set of airports where each aircraft is positioned at the start of the planning horizon, while $\mathcal{S}^-$ denotes the set of airports where the aircraft will be located at the end of the planning horizon.

An arc represents a *feasible connection* between two nodes. There are two types of feasible connections: one represents connections between flight leg nodes (solid line), and the other represents connections between flight leg nodes and airport nodes (dashed line). The arc between flight leg nodes is defined when two flight legs can operate consecutively. Two flight legs can be operated consecutively when satisfying two conditions regarding airports and time. Suppose there are two flight legs, j and k. First, the arrival airport of flight leg j should be identical to the departure airport of flight leg k. Second, the departure time of flight leg k should be later than the arrival time of flight leg j. *Turnaround time* should also be guaranteed. The turnaround time refers to the minimum required time to prepare for the next flight. For example, the arc $(2, 3)$ in Figure 4 means that an aircraft that has operated Flight leg 2 can perform Flight leg 3 in succession. For the second arc type, the feasible connection between flight leg nodes and airport nodes is defined based on the departure and arrival airports of the flight legs. Each flight leg node has feasible connections from its departure airport node and toward its arrival airport. Flight leg 2 in Figure 4 operates from Airport 3 to Airport 2. And corresponding arcs $\{(S3, 2), (2, E2)\}$ are generated. When a positive flow exists from $\mathcal{S}^+$ to $\mathcal{L}$, an aircraft operates that flight leg as its first one within the planning horizon. Similarly, when there is a positive flow from $\mathcal{L}$ to $\mathcal{S}^-$, an aircraft operates that flight leg as its last one within the planning horizon.

The number of arcs is directly related to the number of decision variables and constraints. Hence, minimising unnecessary arcs is crucial. We eliminate unnecessary arcs between airport nodes and flight leg nodes, as follows. First, based on the information of the given flight leg, track the number of aircraft in operation throughout the planning horizon, as shown in the graph of Figure 5. Next, determine the time points, t_a and t_b, when all aircraft

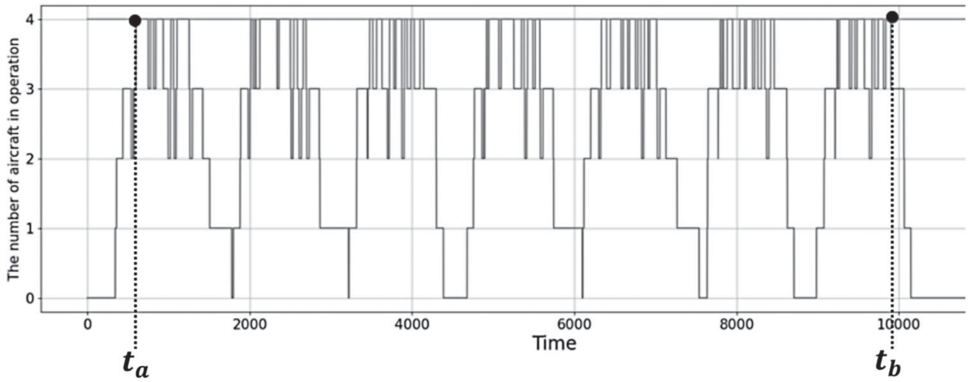


Figure 5. The number of aircraft in operation throughout the planning horizon.

are in operation for the first and last times, respectively. In the example of Figure 5, when an airline owns four aircraft, t_a and t_b correspond to the first and last time periods where the value is four. In order to operate all aircraft at time t_a, each aircraft must have initiated its route before t_a. Consequently, after t_a, no flow should exist from source airport nodes ($\mathcal{S}^+$) to flight leg nodes ($\mathcal{L}$); thus, no arcs should be generated. Similarly, for all aircraft to be operational at time t_b, no aircraft should finish its route before t_b. Therefore, no arcs should be created from flight leg nodes ($\mathcal{L}$) to the sink airport nodes ($\mathcal{S}^-$) before t_b. In conclusion, this approach prevents the generation of arcs between airport nodes and flight leg nodes.

3.2. *Aircraft routing problem and fuel tankering problem*

Aircraft routing is a decision that comes after fleet assignment. In other words, the aircraft routing problem takes the results of fleet assignment as its input. Fleet assignment derives all flight legs that specific aircraft types need to operate. Therefore, aircraft routing is solved per aircraft type, and we assume that within each problem, multiple aircraft share the same specifications. The time horizon is designed from 0 to 11,520 min (eight days) to cover cases where the aircraft lands after midnight on the last day. The route is obtained through a sequence of feasible connections on the connection network and must satisfy the following conditions. All flight legs are covered precisely once by one aircraft, and the number of aircraft located at the start and end of the planning horizon must be equal for all airports to repeat the routes cyclically on a weekly basis.

When determining the refueling amount with fuel tankering, it is assumed that the fuel prices at each airport are deterministic and given. While routes should be determined several weeks to months in advance, the refueling amount can be adjusted prior to flight operations. Therefore, even if fuel prices fluctuate after decision-making, airlines can fine-tune the refueling amount by solving the fuel tankering problem again with changed fuel prices.

Aircraft are regulated to be equipped with safety fuel for unexpected delays or emergencies. Although there are various subcomponents of safety fuel, this study considers the amount of fuel that can sustain an aircraft for 30 min. Furthermore, it is crucial to consider the trade-off for additional fuel consumption when refueling more than necessary to operate the next flight leg. A report by ICAO (2014) states that additional fuel consumption due to the additional fuel carried is usually 2.5 to 4.5 percent of the extra fuel weight per hour of flight. Thus, the additional fuel consumption is linearly proportional to the amount of extra fuel loaded and the flight time. Extra fuel weight is calculated by the net fuel weight minus trip fuel and safety fuel, where the 'trip fuel' means the right amount of fuel required for the flight leg. Also, the model tackles practical constraints such as maximum take-off weight (MTOW), maximum landing weight (MLW), and tank capacity. Such specifications are provided by the aircraft manufacturer. The tank capacity refers to the limitation concerning the fuel itself, whereas MTOW and MLW are limitations related to the total weight of the aircraft. When calculating the total weight of the aircraft, its operating empty weight (OEW) and payload are added together with the fuel weight. The payload is the weight of passengers and their luggage. It is assumed to be identical for all flight legs by considering the average load factor, which represents the ratio of the average number of passengers boarding to the total seat capacity. The payload can be calculated by multiplying the total number of seats by the average load factor and average weight per passenger.

3.3. Mathematical formulation

The mathematical formulation for the integrated model includes the aircraft routing problem and the fuel tankering problem together. A summary of the indices, sets, parameters, and decision variables used in the proposed MILP model is as follows:

Indices and sets

$\mathcal{S}$ set of airports, $s \in \mathcal{S} = \{1, 2, \ldots, S\}$
$\mathcal{S}^+$ set of source airports, $s^+ \in \mathcal{S}^+ = \{1, 2, \ldots, S^+\}$
$\mathcal{S}^-$ set of sink airports, $s^- \in \mathcal{S}^- = \{1, 2, \ldots, S^-\}$
$\mathcal{F}$ set of aircraft, $i \in \mathcal{F} = \{1, 2, \ldots, F\}$
$\mathcal{L}$ set of flight legs, $l \in \mathcal{L} = \{1, 2, \ldots, L\}$
$\mathcal{A}$ set of feasible connections (j, k), $j \in \mathcal{S}^+ \cup \mathcal{L}$, $k \in \mathcal{S}^- \cup \mathcal{L}$

Parameters

f_j fuel price at departure airport of flight leg j
C tank capacity
P payload weight of the aircraft
K safety fuel that the aircraft should keep in reserve
U_O operating empty weight (OEW) of the aircraft
U_T maximum take-off weight (MTOW)
U_L maximum landing weight (MLW)
w_j trip fuel of flight leg j
d_j flight time of flight leg j
α fuel consumption rate of extra fuel
M large number

Decision variables

y^i_{jk} 1 if a feasible connection (j, k) is included in the route of aircraft i 0 otherwise
x^i_{jk} amount of fuel refueled on aircraft i at departure airport of flight leg k along the feasible connection (j, k)

r^i_{jk} amount of fuel remaining in the aircraft i when it landed at arrival airport of flight leg j along the feasible connection (j, k)

By considering the above problem descriptions and notations, an MILP model is formulated as follows:

$$\min \quad \sum_{i\in\mathcal{F}} \sum_{(j,k)\in\mathcal{A}} f_k x^i_{jk} \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{F}} \sum_{k:(j,k)\in\mathcal{A}} y^i_{jk} = 1, \quad \forall\, j \in \mathcal{L}, \tag{2}$$

$$\sum_{i\in\mathcal{F}} \sum_{k:(j,k)\in\mathcal{A}} y^i_{jk} = \sum_{i\in\mathcal{F}} \sum_{k:(k,j)\in\mathcal{A}} y^i_{kj}, \quad \forall\, j \in \mathcal{L}, \tag{3}$$

$$\sum_{j:(s^+,j)\in\mathcal{A}} y^i_{s^+j} = \sum_{j:(j,s^-)\in\mathcal{A}} y^i_{js^-}, \quad \forall\, i \in \mathcal{F}, \forall\, s \in \mathcal{S}, \tag{4}$$

$$\sum_{s^+\in\mathcal{S}^+} \sum_{j:(s^+,j)\in\mathcal{A}} y^i_{s^+j} \le 1, \quad \forall\, i \in \mathcal{F}, \tag{5}$$

$$x^i_{jk} + r^i_{jk} \le C\, y^i_{jk}, \quad \forall\, i \in \mathcal{F}, \forall\, (j,k) \in \mathcal{A}, \tag{6}$$

$$x^i_{jk} + r^i_{jk} \le (U_T - U_O - P) y^i_{jk}, \quad \forall\, i \in \mathcal{F}, \forall\, (j,k) \in \mathcal{A}, \tag{7}$$

$$r^i_{jk} \le (U_L - U_O - P) y^i_{jk}, \quad \forall\, i \in \mathcal{F}, \forall\, j \in \mathcal{L}, \forall\, (j,k) \in \mathcal{A}, \tag{8}$$

$$r^i_{jk} \ge K\, y^i_{jk}, \quad \forall\, i \in \mathcal{F}, \forall\, j \in \mathcal{L}, \forall\, (j,k) \in \mathcal{A}, \tag{9}$$

$$(1 - \alpha\, d_k)(x^i_{jk} + r^i_{jk} - w_k) + \alpha\, K\, d_k\, y^i_{jk} \le \sum_{l:(k,l)\in\mathcal{A}} r^i_{kl}, \quad \forall\, i \in \mathcal{F}, \forall\, (j,k) \in \mathcal{A}, \tag{10}$$

$$(1 - \alpha\, d_k)(x^i_{jk} + r^i_{jk} - w_k) + \alpha\, K\, d_k + M(1 - y^i_{jk}) \ge \sum_{l:(k,l)\in\mathcal{A}} r^i_{kl}, \quad \forall\, i \in \mathcal{F}, \forall\, (j,k) \in \mathcal{A}, \tag{11}$$

$$r^i_{s^+j} = 0, \quad \forall\, i \in \mathcal{F}, \forall\, s^+ \in \mathcal{S}^+, \forall\, (s^+, j) \in \mathcal{A}, \tag{12}$$

$$y^i_{jk} \in \{0, 1\}, \quad \forall\, i \in \mathcal{F}, \forall\, (j,k) \in \mathcal{A}, \tag{13}$$

$$x^i_{jk}, r^i_{jk} \ge 0 \quad \forall\, i \in \mathcal{F}, \forall\, (j,k) \in \mathcal{A} \tag{14}$$

The objective function (1) minimises the total fuel cost. Constraints (2)–(5) cover the aircraft routing decision, and Constraints (6)–(12) cover the fuel tankering decision under the routing decision variable y^i_{jk}. Constraint (2) ensures that all flight legs are covered precisely once, and Constraint (3) states the flow continuity of the network. Constraint (4) enforces that for each airport, s, the number of aircraft located at the beginning and end of the planning horizon is the same. Constraint (5) represents that the aircraft can depart through, at most, one airport. Constraints (6), (7), and (8) state the restriction of tank capacity, MTOW, and MLW, respectively. As the OEW and payload are deterministic in this model, Constraints (7) and (8) are substituted as constraints on the fuel weight at takeoff and landing. Constraint (9) represents that the aircraft reserves enough fuel for the following flight leg, including safety fuel. Constraints (10) and (11) express conservation constraints of the fuel. In line with conventional conservation equations of an inventory management problem, when flight legs j and k are operated consecutively, the sum of fuel remaining after flight leg j and the fuel taken on prior to flight leg k should match the sum of fuel consumed during flight leg k and the fuel remaining after flight leg k. An equality constraint was divided into two inequality constraints. Constraint (12) ensures that the tank is empty at the beginning of the planning horizon. Constraints (13) and (14) represent the integrality and non-negativity constraint, respectively.

3.4. Symmetry-breaking methods

The aircraft routing problem determines routes for aircraft of the same type. Because the aircraft routing problem deals with the same type of aircraft, the objective function value will be indifferent no matter which aircraft performs a specific route. Thus, the aircraft instance causes symmetry in the mathematical formulation. When there are n routes and F aircraft, a total of ${}_FP_n = \frac{F!}{(F-n)!}$ solutions have the equivalent objective function value, and this symmetry increases the computational burden. To alleviate the symmetry, this section presents two approaches. First, we propose a method for assigning initial solutions based on the feasible connections between flight legs. Second, we propose symmetry-breaking constraints appropriate to our mathematical model.

The goal of the method for assigning the initial solution is to identify the flight legs that must be the first flight leg for any route. Based on the information in the set of feasible connections, $\mathcal{A}$, we can find the flight legs that are not subsequent to any other flight leg (e.g. $\{k \in \mathcal{L} \mid (j,k) \notin \mathcal{A}, \forall j \in \mathcal{L}\}$). For a certain flight leg k, if flight leg j with $(j,k) \in \mathcal{A}$ does not exist, this flight leg k cannot be performed after another leg. Therefore, flight leg k becomes the first flight leg of any route. Essentially, all these flight legs must be operated by distinct aircraft; hence, each of them is explicitly assigned a unique aircraft index from 1. On the other hand, a flight leg that can be connected from another flight leg is not assigned a new aircraft index. This is because it is uncertain whether this flight leg is the first flight leg performed by a new aircraft or a follow-up flight leg. Once identified flight legs are assigned to specific aircraft indices, the routes are operated using the aircraft assigned to the first flight, thus eliminating symmetry among them.

The procedure to identify the flight legs that must be the first flight leg and to assign distinct aircraft is as follows. First, initialise the set *Init*, which is the same set as $\mathcal{L}$, and eliminate flight legs that can be subsequent flight legs. Finally, we sequentially assign them starting from aircraft indexed at 0 for the flight legs in *Init*. The detailed explanation is presented in Algorithm 1.

Figure 6 illustrates the feasible connections between seven flight legs. Two flight legs, 1 and 2, are not connected by any other flight legs. Then, we assign an aircraft with an index of 1 to flight leg 1. Similarly, an aircraft with an index of 2 is assigned to flight leg 2.

If a aircraft are assigned, the number of solutions with the same objective function will decrease from ${}_FP_n$ to ${}_{F-a}P_{n-a}$. If all aircraft operate, the number of solutions decreases from

Algorithm 1 Symmetry-breaking method (initial flight leg assignment)

Initialization: $Init \leftarrow \mathcal{L},\ i \leftarrow 0$
for $l = 1, \cdots, L$ **do**
  if $l \in Init$ **then**
  end
  $Subsequent \leftarrow \emptyset$
  for $(j, k) \in \mathcal{A}$ **do**
    if $j = l$ **then**
      $Subsequent \leftarrow Subsequent \cup \{k\}$
    end
  end
  for $k \in Subsequent$ **do**
    if $k \in Init$ **then**
      $Init \leftarrow Init \setminus \{k\}$
    end
  end
end
for $j \in Init$ **do**
  Let s be the source airport of flight leg j
  $y^i_{sj} \leftarrow 1$
  $i \leftarrow i + 1$
end

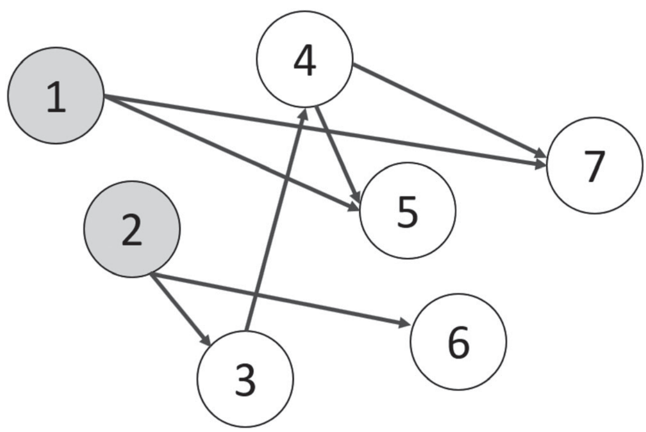


Figure 6. Example for assignment of initial solutions.

$F!$ to $(F - a)!$. Allocating just a few aircraft significantly eases the symmetry. The effectiveness of the method can be influenced by factors like aircraft utilisation and the composition of flight legs. However, its application process remains intuitive and effective.

For the second method, a symmetry-breaking constraint is presented. Symmetry-breaking has been extensively researched in the broader scope of the vehicle routing problem for a long time (Darvish, Coelho, and Jans 2020). One way to introduce it is to add the straightforward constraints as follows:

$$\sum_{(j,k)\in\mathcal{A}} y_{jk}^{i} \geq \sum_{(j,k)\in\mathcal{A}} y_{jk}^{i+1} \quad \forall\, i \in \mathcal{F} \tag{15}$$

The constraints address that an aircraft with a higher index cannot be utilised when a lower index aircraft is not utilised. Furthermore, the constraint limits the aircraft with a lower index to be assigned more flight legs. Note that the aircraft assigned for the initial solution should be excluded from the constraint. The aircraft indices for the initial solutions are compulsorily assigned starting from 1. However, the flight legs assigned to lower-indexed aircraft might be included in routes with fewer flight legs in the optimal solution.

4. Decomposition-based heuristic algorithm

The integrated model can quickly find optimal solutions to small-sized problems. However, for major commercial airlines with numerous aircraft and flight legs, obtaining optimal solutions becomes impossible through the integrated model. Therefore, there is a need to develop a solution approach that can yield good results even for larger problem sizes. This section presents a decomposition-based heuristic algorithm to derive solutions within a reasonable time. This algorithm decomposes the aircraft routing problem and fuel tankering problem and solves them sequentially. Once the routes are derived, the fuel tankering problem is an LP model, which can be easily solved with commercial solvers. However, the aircraft routing problem remains challenging because it is a pure integer programming model. Furthermore, when the two problems are decomposed, the aircraft routing problem does not include variables related to the fuel cost. Therefore, to derive the routes that maximise the effect of fuel tankering, we propose a *reward-maximising objective function* reflecting the characteristics of fuel tankering.

Before structuring the rewards of the decomposed aircraft routing problem, we can address the condition where fuel tankering is beneficial. When two flight legs are operated consecutively, fuel tankering is effective only when the fuel price at the departure airport of the first flight leg is lower than the fuel price at the departure airport of the second flight leg (which is the same as the arrival airport of the first flight leg). In other words, when fuel prices at the departure and arrival airports are given as f_{dep} and f_{arr} for a particular flight leg l, f_{dep} should be lower than f_{arr}. The trade-off resulting from extra fuel should also be considered, not just the fuel prices. To consider the trade-off, let x be the extra fuel loaded for fuel tankering. If an aircraft refuels extra fuel, x, before departing on flight leg, l, then only $(1-\alpha\, d_l)x$ of this extra fuel would remain upon arrival at the arrival airport. Hence, it is advantageous only when refueling x with f_{dep} is cheaper than refueling $(1-\alpha\, d_l)x$ with f_{arr}. The condition represented by the relationship between the three parameters, f_{dep}, f_{arr}, and d_l, is as follows:

$$f_{dep} < f_{arr}(1-\alpha\, d_l).$$

The flight legs that satisfy the above conditions could be the candidates for fuel tankering. Among these candidates, the combination of connected flight legs on the route determines whether fuel tankering should be done and to what extent.

The $reward_{jk}$ represents the reward when binary decision variable y_{jk}^{i} is 1. The variable y_{jk}^{i} indicates whether flight legs, j and k, are consecutive. As illustrated in Figure 7, for each

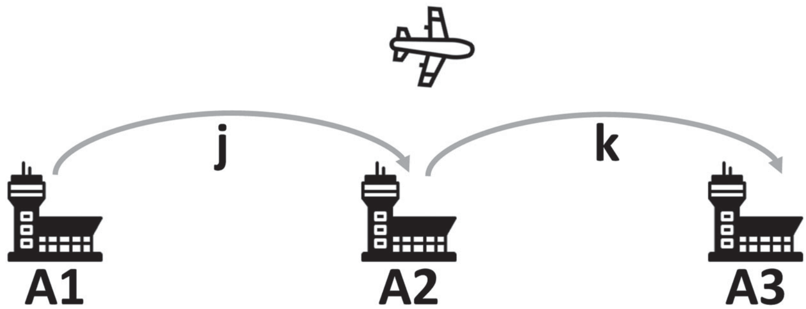


Figure 7. Operation of consecutive flight legs j and k.

decision variable, y^i_{jk}, an aircraft visits three airports ($A1$, $A2$, and $A3$) through two flight legs, j and k. From the previously derived condition, it was observed that the fuel prices at each airport and the flight times affect the validity of fuel tankering. Thus, we classify these factors and divide the case of the decision variable, y^i_{jk}. Fuel prices are classified into two categories, high and low, based on their average, while flight times are classified as long and short. Note that a flight leg with a long flight time often requires refueling most of the fuel tank, whereas a flight leg with a short flight time typically does not require even half of the fuel tank. Consequently, with fuel prices at three airports (f_{A1}, f_{A2}, and f_{A3}) and two flight times (d_j and d_k), the variable y^i_{jk} belongs to one of 32 possible cases.

The reward reflects the prioritisation of which flight leg k is preferable as the subsequent flight connected to a fixed flight leg j. When flight leg j is fixed, the categories of f_{A1}, d_j, and f_{A2} are also fixed. Therefore, for each flight leg j, priorities are assigned to four dependent cases based on d_k and f_{A3}. The detailed rewards are described in Table 1. The reward is generated only when f_{A2} is included in the higher-priced category. This is because if f_{A2} is included in the lower-priced category, the airline can refuel as much as needed for flight leg k, regardless of its connection to flight leg j. Therefore, all cases where f_{A2} is in the lower-priced category are set to a reward of 0.

For the remaining cases, two conditions are established to determine the reward. The ultimate goal of the reward system is to construct routes that maximise the economic benefits of fuel tankering. Therefore, the first condition is to use as much tankered fuel as possible if fuel tankering is feasible. The second condition is to return to a lower-priced airport as soon as possible after the tankered fuel is exhausted, or if fuel tankering is not possible.

We introduce a simple example considering rewards from No. 1 to No. 4 in Table 1, where f_{A1} is low, f_{A2} is high, and d_j is short. When the flight leg j conducts fuel tankering, a long d_k is preferred, according to the first condition. As d_j is short, the long d_k allows for the maximum utilisation of tankered fuel (rewards of No. 2 and 4 $>$ rewards of No. 1 and 3). When d_k is short, a high f_{A3} is preferred to exhaust all tankered fuel on the subsequent flight leg of flight leg k, rather than checking for new fuel tankering opportunities at low f_{A3} (reward of No. 3 $>$ reward of No. 1). Lastly, according to the second condition, it is preferable to arrive at a low f_{A3} after using all of the tankered fuel on long d_k (reward of No. 2 $>$ reward of No. 4). Therefore, rewards are given in the order of No.2, No.4, No.3, and No.1.

Based on these rewards, reward-maximising aircraft routing problems are formulated as follows:

Table 1. *Reward$_{jk}$* in the aircraft routing problem of a heuristic algorithm.

No.	f_{A1}	d_j	f_{A2}	d_k	f_{A3}	Reward
1	↓	←	↑	←	↓	1
2				→	↓	4
3				←	↑	2
4				→	↑	3
5	↓	→	↑	←	↓	4
6				→	↓	3
7				←	↑	2
8				→	↑	1
9	↑	←	↑	←	↓	4
10				→	↓	3
11				←	↑	2
12				→	↑	1
13	↑	→	↑	←	↓	4
14				→	↓	3
15				←	↑	2
16				→	↑	1
17	↓	←	↓	←	↓	0
18				→	↓	0
19				←	↑	0
20				→	↑	0
21	↓	→	↓	←	↓	0
22				→	↓	0
23				←	↑	0
24				→	↑	0
25	↑	←	↓	←	↓	0
26				→	↓	0
27				←	↑	0
28				→	↑	0
29	↑	→	↓	←	↓	0
30				→	↓	0
31				←	↑	0
32				→	↑	0

Note: ↑: *high fuel price*, ↓: *low fuel price*,
←: *short flight time*, →: *long flight time*

Reward-maximizing aircraft routing problem

$$\max \quad \sum_{i \in \mathcal{F}} \sum_{k:(j,k) \in \mathcal{A}} y^i_{jk} \cdot reward_{jk} \tag{16}$$

$$\text{s.t.} \quad \text{Constraints (2)} - \text{(5), (13)} \tag{17}$$

Let $\bar{y}^i_{jk}$ be the solution to the above problem. For each aircraft, i, a set of connections, λ_i, corresponding to the route is created (i.e. $\lambda_i = \{(j,k) \mid \bar{y}^i_{jk} = 1\}$). The decision variables x^i_{jk} and r^i_{jk} of the fuel tankering problem are created only for connections (j,k) belonging to λ_i for each aircraft, i. Therefore, the number of decision variables is significantly reduced compared to the existing model that generates decision variables for all feasible connections in $\mathcal{A}$. The formulation follows the objective function and constraints in the fuel tankering problem of the integrated model in Section 3.3. The fuel tankering problem can be solved under predetermined routes, λ_i as follows:

Fuel tankering problem

$$\min \quad \sum_{i \in \mathcal{F}} \sum_{(j,k) \in \lambda_i} f_k x^i_{jk} \tag{18}$$

$$\text{s.t.} \quad x^i_{jk} + r^i_{jk} \le C, \quad \forall\, i \in \mathcal{F},\ \forall (j,k) \in \lambda_i, \tag{19}$$

$$x^i_{jk} + r^i_{jk} \le (U_T - U_O - P), \quad \forall\, i \in \mathcal{F},\ \forall (j,k) \in \lambda_i, \tag{20}$$

$$r_{jk} \le (U_L - U_O - P), \quad \forall\, i \in \mathcal{F},\ \forall (j,k) \in \lambda_i, \tag{21}$$

$$r^i_{jk} \ge K, \quad \forall\, i \in \mathcal{F}, \forall j \in \mathcal{L},\ \forall (j,k) \in \lambda_i, \tag{22}$$

$$(1 - \alpha\, d_k)(x^i_{jk} + r^i_{jk} - w_k) + \alpha K\, d_k = \sum_{l:(k,l)\in\mathcal{A}} r^i_{kl}, \quad \forall\, i \in \mathcal{F},\ \forall (j,k) \in \lambda_i, \tag{23}$$

$$r^i_{s^+j} = 0, \quad \forall\, i \in \mathcal{F},\ \forall s \in \mathcal{S}^+,\ \forall (s^+, j) \in \lambda_i, \tag{24}$$

$$x^i_{jk}, r^i_{jk} \ge 0, \quad \forall\, i \in \mathcal{F},\ \forall (j,k) \in \lambda_i \tag{25}$$

The overall process of solving the decomposed model is presented in Algorithm 2.

To evaluate the performance of the heuristic algorithm, we define the lower bound on the fuel cost and present the objective value of the myopic case. The myopic cases can be implemented by only refueling aircraft precisely as needed without fuel tankering. In this case, net fuel cost is fixed regardless of the routes because all flight legs are given as an input. The objective value of the myopic case can be calculated by the sum of trip fuel for all given flight legs with consideration of safety fuel. Furthermore, we can measure the 'Cost-saving Rate,' indicating the rate of cost reduction compared to the objective value of the myopic case.

The lower bound of the model is also developed. It is when the net required fuel over the planning horizon is allocated in order of lower-priced fuel, considering the visit frequency to each airport and the tank capacity. First, compute the net required fuel for operating all flight legs, which is $\sum_{l\in\mathcal{L}} w_l$. Then, calculate the refueling amount at each airport by multiplying the aircraft's tank capacity by the number of flight legs departing from that airport. Fill the net required fuel starting from the airport with the lowest fuel price. The costs are calculated accordingly. Denote the number of flight legs departing from the airport s as D_s. Additionally, assume that $\mathcal{S}$ is the set of airports (without distinguishing between source or sink) and is indexed in ascending order of its fuel prices. The lower bound is defined as follows:

$$LB = \sum_{s=1}^{n-1} C\, D_s\, f_s + \min\left\{ \sum_{l\in\mathcal{L}} w_l - \sum_{s=1}^{n-1} C\, D_s, C\, D_n \right\} f_n,$$

$$\text{where } n \in \mathcal{S} : \sum_{l\in\mathcal{L}} w_l = \sum_{s=1}^{n-1} C\, D_s + \min\left\{ \sum_{l\in\mathcal{L}} w_l - \sum_{s=1}^{n-1} C\, D_s, C\, D_n \right\}$$

This lower bound is impossible to reach in the model we proposed. First, we assume α as zero because the lower bound is derived without the knowledge of the route. If the route is unknown, the fuel consumption considering α cannot be calculated as the amount of extra fuel at each flight remains unknown. Second, the lower bound does not consider the feasibility of the route. Refueling only at airports with lower fuel prices is practically impossible. Nevertheless, this method can be intuitively calculated and used to evaluate solution quality.

Algorithm 2 Decomposition-based heuristic algorithm

Solve Reward-maximizing aircraft routing problem

Step 1: Define the decision variable y^i_{jk} after eliminating unnecessary arcs between airport nodes and flight leg nodes;
Step 2: Generate the reward for each feasible connection based on the information of given flight legs and the fuel prices;
Step 3: Formulate the reward-maximizing aircraft routing problem;
Step 4: Employ the symmetry-breaking method (1): Identify the flight legs that must be the first flight leg of any route. These assignments act as constraints in the formulation;
Step 5: Employ the symmetry-breaking method (2): These statements act as constraints in the formulation;
Step 6: Solve the problem, and let $\bar{y}^i_{jk}$ be the solution;
Step 7: Derive the routes for each aircraft $\lambda_i \leftarrow \{(j,k) \mid \bar{y}^i_{jk} = 1\}$;

Solve Fuel tankering problem

Step 1: Define the decision variables x^i_{jk} and r^i_{jk} along the derived routes, λ_i;
Step 2: Formulate the fuel tankering problem;
Step 3: Solve the problem;

Return: Routes for each aircraft (λi), refueling amounts for each flight leg (x^i_{jk}), and total fuel cost;

5. Computational experiments

5.1. Description of the test cases

The test cases for experiments were collected from domestic flight legs of U.S. commercial airlines. We implicitly consider maintenance checks (e.g. A-check), assuming the maintenance check is undertaken overnight. Fuel prices for each airport are collected from the fixed base operators (FBO) in the U.S. Parameters such as OEW, MTOW, MLW, and the number of seats are based on information provided by the aircraft manufacturer. In addition, the average load factor and average weight of passengers and cabin luggage were collected to derive the payload of the aircraft (EASA 2021; EASA, EEA, and EUROCONTROL 2019).

Five test cases of different sizes were examined in the experiments. The characteristics of test cases are indicated in Table 2 along the number of flight legs, aircraft, airports, feasible connections, and average flight time in hours. Additionally, considering MLW, the aircraft may land with extra fuel to a maximum of 47 to 58% of the tank capacity for each test case. All the numerical experiments were carried out with an AMD Ryzen 3500X 6-Core CP, a 3.59 GHz processor, and 16GB of RAM. Every mathematical model was developed with the CPLEX solver. Every model was solved within a time limit (i.e. 24 h).

5.2. Performance of the proposed model and algorithm

The first experiment evaluates the performance of the proposed model and algorithms. Table 3 summarises the comparison between the myopic case, the integrated model, and the decomposed model using a heuristic algorithm for five test cases. As tackled in

Table 2. Characteristics of test cases.

No.	$\|\mathcal{L}\|$	$\|\mathcal{F}\|$	$\|\mathcal{S}\|$	$\|\mathcal{A}\|$	Avg. flight time (hr)
1	71	3	6	721	3.7
2	90	4	14	790	3.6
3	190	10	7	4,125	4.1
4	353	13	28	13,414	2.7
5	558	14	34	18,810	1.3

Table 3. Experimental results.

	Test case				
	1	2	3	4	5
Myopic case					
Cost ($)	2,727,478	3,331,757	8,416,765	8,022,604	6,318,882
Integrated model					
Bin Vars	2,163	3,160	41,250	176,384	263,340
Cont Vars	4,326	6,320	82,500	352,768	526,680
Total Vars	6,489	9,480	123,750	529,152	790,020
Cons	10,477	16,060	200,766	879,314	1,287,112
CPU (sec)	3.44	1,178.41	86,400*	–	–
Cost ($)	2,664,242	3,241,846	8,223,157	–	–
LB Gap (%)	4.68	6.15	8.69	–	–
Cost-saving Rate (%)	2.37	2.77	2.35	–	–
Decomposed model with heuristic algorithm					
Bin Vars	2,163	3,160	41,250	176,348	263,340
Cont Vars	146	188	400	1,085	1,144
Total Vars	2,309	3,348	41,650	177,433	264,484
Cons	718	1,106	4,605	10,307	17,942
CPU (sec)	0.06	0.50	16.21	58.33	906.25
Cost ($)	2,664,242	3,242,733	8,217,318	7,574,693	5,857,450
LB Gap (%)	4.68	6.18	8.61	7.66	8.57
Cost-saving Rate (%)	2.37	2.75	2.37	5.91	7.88

Note: *Best solution found within the time limit.
– No integer solution was found within the time limit.

Section 4, the myopic case stands for the refueling methods, where fuel is loaded as needed for each flight leg without fuel tankering. The number of decision variables and constraints and the lower bound gap, 'LB Gap', were used as experimental indicators. As mentioned earlier, the Cost-saving Rate represents the rate of reduced fuel cost by the presented model and algorithm. The following Cost-saving Rate represents an important metric indicating how economically efficient the presented model is compared to the myopic case.

$$\text{Cost} - \text{saving Rate} := \left(1 - \frac{\text{Best solution (OBJ by eachapproach)}}{\text{OBJ by myopic case}}\right) \times 100(\%)$$

In all test cases, both models drew a decrease of more than 2.3 percent in net fuel cost. This demonstrates that meaningful cost reductions can be achieved solely by modifying routes and refueling amounts without additional investments. In the integrated model, where the aircraft routing problem and fuel tankering problem are solved in a single problem, the optimal solution was obtained for test cases 1 and 2. However, for the remaining test cases, it could not be obtained within the time limit or memory limit. On the contrary, the decomposed model solved with a heuristic algorithm found a solution in about 900 s for the largest test case. In addition, in test case 3, the decomposed model found a better solution in just 16 s compared to the best solution found by the integrated model within the time limit. By decomposing the problem, the second stage, the fuel tankering problem, generates decision variables only under the routes derived from the first stage, the aircraft routing problem. The number of continuous decision variables in the fuel tankering problem is significantly reduced compared to the integrated model. At its most extreme, the fuel tankering decision variables for test case 5 decreased from around 520,000 to 1,100. Furthermore, it becomes a pure LP model with the CPU time converging to 0.

For test cases 1 and 2, the solutions obtained from the integrated model were either identical to or exhibited negligible differences compared to those derived from the heuristic algorithm. Moreover, for test cases 3, 4, and 5, where solutions were obtained solely from the heuristic algorithm, considering that the LB shows unobtainable values and exhibited variations depending on the data, it can be regarded as demonstrating an acceptable quality.

Notably, the Cost-saving Rate of test cases 4 and 5 was clearly high, at 5.91 percent and 7.88 percent, respectively. This is related to the average flight time. As mentioned before, the amount of additional fuel consumption due to the extra fuel is proportional to the flight time of the given flight leg. Test case 4 and especially test case 5 have shorter flight times than other test cases. Thus, airlines should actively consider implementing strategies for short-haul routes as a priority.

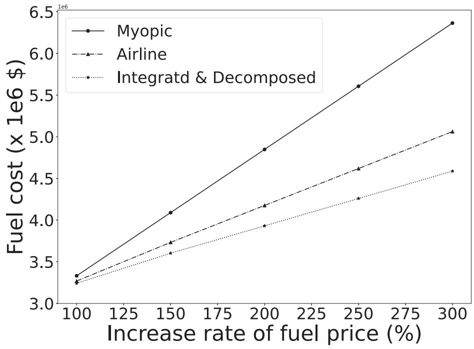
5.3. Performance of the proposed model under risk scenarios

Fuel tankering is a strategy in which fuel that should be refueled at a particular airport is refueled at a prior airport. This mechanism not only aims to save fuel costs but also serves to mitigate risks related to fuel supply. In practice, airlines utilise fuel tankering as one of their strategies to address these issues. This experiment investigates how well the proposed model can address such risk scenarios.

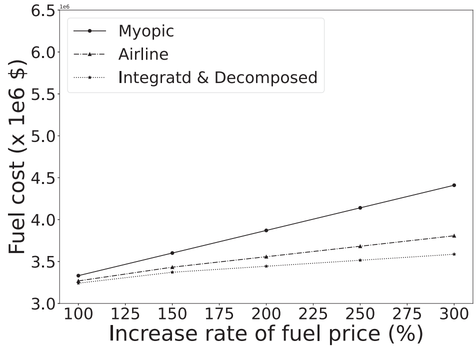
Two risk scenarios were defined in the experiments. The first risk involves an increase in fuel price at a particular airport, and the second risk involves a limitation in fuel supply to a particular airport. Additionally, for each risk scenario, two airports are chosen as the ones exposed to the risk: One is an airport from which a large number of flight legs depart (which we call a high-frequency airport), and the other is an airport from which a medium number of flight legs depart (a medium-frequency airport). An experiment was conducted on test case 2 to obtain the optimal solution in the integrated model.

In the price risk scenario, we showcased the trend of total fuel cost as the fuel price increased from 100 percent to 300 percent. In addition, to demonstrate the impact of solving fuel tankering and aircraft routing together, we introduced a new benchmark model. The graph labelled 'Airline' in Figure 8 represents the result of only performing fuel tankering on the current routes operated by the airline. By comparing these two models, we can demonstrate the effectiveness of incorporating aircraft routing with fuel tankering.

For both Figure 8(a,b), due to the marginal differences between the solutions obtained from the integrated model and the decomposed model with the heuristic algorithm, they were represented under a single label. In Figure 8(a), where the price rises in a high-frequency airport, fuel cost shows a steeper increase than in Figure 8(b). Airports from which many flight legs depart are more affected by rising fuel prices. Even so, solving the aircraft routing problem and the fuel tankering problem together helps mitigate the impact of such price surges. It is evident that when addressing these problems jointly, the fuel cost



(a) Fuel cost under price risk at high-frequency airport



(b) Fuel cost under price risk at medium-frequency airport

Figure 8. Effect of increasing price on the fuel cost. (a) Fuel cost under price risk at high-frequency airport (b) Fuel cost under price risk at medium-frequency airport.

increases more gradually compared to both the myopic case and the fuel tankering model under the airline's existing routes.

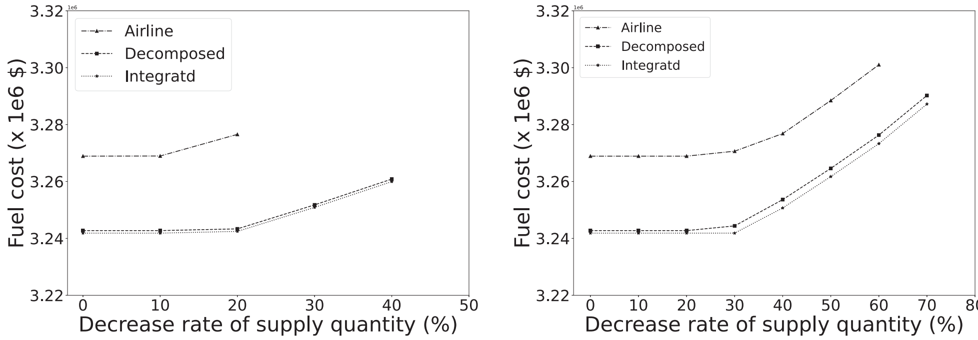
The risk of a price surge might significantly affect the fuel cost. However, it does not directly impact the feasibility of operations. On the other hand, when the supply quantity is limited, it not only affects the fuel cost but also impacts the feasibility of operations. In this experiment, we examined how decreasing supply quantity affects the variations and feasibility of the fuel cost. We set the sum of trip fuel for all flight legs departing from the particular airport over the planning horizon as the baseline supply level of the airport. When the supply quantity is limited, the myopic case is invalid, which refuels only with fuel immediately needed. Hence, it was excluded from consideration.

Both Figure 9(a,b) exhibit a steady cost initially, followed by a gradual increase over time as the risk intensifies. In the initial phase, the fuel cost remained steady. Because the optimal solution, obtained when supply quantity is not limited, was not entirely using the baseline supply level. Therefore, the decrease in supply quantity does not affect the fuel cost. However, as the supply quantity became more limited, the costs gradually increased. This indicates that, even with some compromises on the fuel cost, the proposed model eventually yielded feasible routes and refueling amounts. It is also notable that considering both aircraft routing and fuel tankering gives more flexibility with harder constraints.

5.4. Sensitivity analysis

The third experiment analyses the sensitivity of the parameters in the mathematical model. The two key parameters, α, and the aircraft's capacity (tank, MTOW, and MLW) are considered.

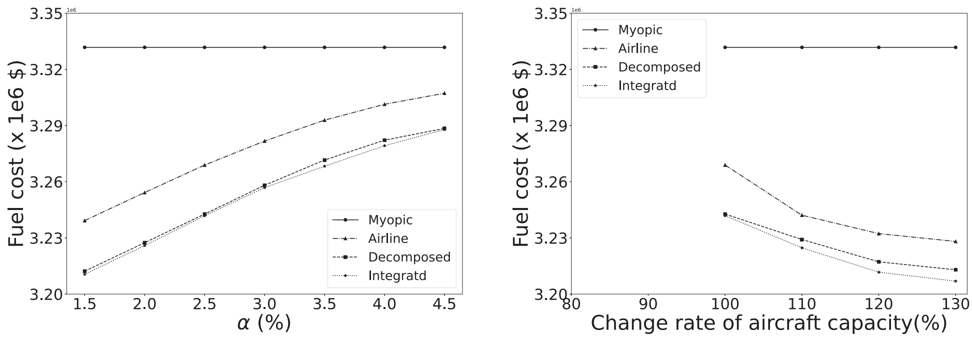
The α is one of the crucial parameters determining the fuel tankering strategy. As α increases, the fuel efficiency during fuel tankering decreases. In other words, as α increases, fuel tankering becomes effective only for flight legs with significant fuel price differences or shorter flight times. This trend is evident in Figure 10(a). The myopic case, which does not refuel beyond trip fuel, remains unaffected by α. In comparison, the other two cases show an increase in fuel cost correlating with α. Even though an increase in α results in fewer



(a) Fuel cost under supply risk at high-frequency airport

(b) Fuel cost under supply risk at medium-frequency airport

Figure 9. Effect of decreasing supply on the fuel cost. (a) Fuel cost under supply risk at high-frequency airport (b) Fuel cost under supply risk at medium-frequency airport.



(a) Effect of α on the fuel cost

(b) Effect of aircraft's capacity on the fuel cost

Figure 10. Effect of parameter variations on the fuel cost. (a) Effect of α on the fuel cost (b) Effect of aircraft's capacity on the fuel cost.

flight legs benefiting from fuel tankering, fuel tankering remains more advantageous than maintaining the myopic case. Additionally, considering aircraft routing and fuel tankering decisions together helped identify potential cost-saving routes, resulting in greater cost-saving than solely focussing on fuel tankering.

Indeed, α, a parameter related to fuel efficiency, varies based on the aircraft's type, size, and age. However, the aircraft routing problem is a problem of deriving a route for the same aircraft type and size. Therefore, excluding these factors, expanding the model by adjusting α according to the age of the aircraft could be a further extension.

The second analysis focuses on parameters related to the aircraft's capacity. As an aircraft's tank capacity increases, the upper limit of refuelable fuel also expands, enhancing the extent of fuel tankering. It is essential to consider the tank capacity with parameters related to weight proportionally, such as MTOW and MLW. Thus, we investigate the effects related to capacity by simultaneously adjusting the change rate for three parameters: tank capacity, MTOW, and MLW. As shown in Figure 10(b), even a 10 percent decrease in the given capacity did not find feasible solutions. This is because some flight legs require nearly full-capacity usage. Conversely, as the capacity increased, costs gradually decreased. It is also important to note that as the capacity increases, the slope of the decreasing fuel cost diminishes. This is because there are limits to cost savings through fuel tankering. The results imply that having an appropriate capacity for extra fuel is more reasonable than excessively expanding the capacity.

5.5. Operational insights and guidelines for utilising the proposed model

In this section, we present several operational insights that could be instructive to practitioners, as well as guidelines regarding how to utilise our model in an airline's decision process.

Operational insights:

(1) Based on the experimental results in Section 5.2, we could observe that determining aircraft route and fuel tankering decisions simultaneously results in fuel cost savings compared to solely optimising the fuel tankering decisions. The integrated approach was robust under two risk scenarios: surging fuel prices and a sudden decrease in fuel supply. Furthermore, the developed heuristic algorithm could derive not only near-optimal solutions for small- and medium-sized problems but also for scalable to large-sized problems where the commercial optimisation solver could not solve them within proper computation times. Therefore, these results highlight the usefulness of the proposed optimisation model, which could reduce airlines' operating costs. In addition, we expect that flight dispatchers, who have to consider a large number of flight legs and airports, can effectively utilise our heuristic algorithm in real operations.

(2) As stated in Belobaba, Odoni, and Barnhart (2015), airlines' major operations planning process is initiated as the following process: flight scheduling → fleet assignment → aircraft routing → crew scheduling. Afterward, other operations processes, such as refueling, are planned on the basis of the derived major operations plans. However, because fuel costs account for a large portion of airlines' operating costs, we recommend that airlines put more emphasis on determining refueling decisions. Hence, airlines suffering from high fuel costs could save significant operating costs by planning fuel tankering with other major operations (e.g. aircraft routing) during the airline's planning process.

Guidelines for utilising the proposed model:

(1) Our integrated model decides the aircraft route and fuel tankering three to six months before actual flights (when the current airlines decide the aircraft routes). The aircraft route rarely changes, except in cases of extreme airline disruption. This is because various subsequent processes, such as crew scheduling, are based on the predetermined aircraft route. However, on the day of actual operation, adjustments regarding the refueling amounts are sometimes necessary to respond to unpredictable situations (e.g. fuel shortages, severe weather conditions, or airport congestion). In other words, even though our model determines decisions for aircraft routing and fuel tankering simultaneously, re-optimising the refueling amount

could be inevitable when an irregular situation occurs. Therefore, in this case, we recommend solving only the fuel tankering problem to re-optimise the refueling amount under a predetermined aircraft route, which is derived from the previous integrated model. Because the fuel tankering problem is an LP problem, it can be solved within short computation times and is scalable to large-scale instances.

(2) Practitioners who use our model should cooperate with the following two stakeholders: (1) schedulers in charge of fleet assignment, and (2) airline fuel suppliers. First, as shown in Figure 10(a), the value of α, related to the aircraft type and age, highly affects the fuel cost. In other words, the advanced and new aircraft, which have a low value of α, could result in significant fuel cost savings for airlines. However, because operating a number of new aircraft could incur high operating costs for airlines, practitioners should cooperate with schedulers in charge of fleet assignment to effectively assign aircraft to flights. Second, as presented in Figure 9, our model is capable of deriving feasible routes and refueling amounts when the airports' fuel supplies are insufficient, compared to other approaches. Nonetheless, if the fuel supplies are extremely limited, practitioners cannot obtain feasible solutions even if our model is utilised. Therefore, before utilising our model, practitioners should cooperate with fuel suppliers to figure out the available amount of each airport's fuel supply. After that, if an appropriate fuel supply can be acquired in advance, we recommend using our model to derive promising solutions for aircraft routes and refueling amounts.

6. Conclusions

In this research, the fuel tankering decision, one of the airline's cost-saving strategies, was integrated with the aircraft routing decision. Considering both problems together, it has been shown that there is a potential for greater benefits than solely focussing on the fuel tankering decision. Compared to existing studies, we presented symmetry-breaking methods and a heuristic algorithm to alleviate the computational burden. The cost-saving effect of the proposed model was proved through experiments, and it was observed that the effect significantly depends on the flight time of the given flight legs. Therefore, airlines should focus on implementing this strategy primarily on short-haul routes. Additionally, experiments under risk scenarios highlighted the model's ability to cope with situations such as a surge in fuel prices or limitations in fuel supply. It is also worth emphasising that both aircraft routing and fuel tankering decisions do not incur additional costs or investment depending on the results. Saving operating costs without investment appeals to airlines and is important to this research problem.

Nevertheless, our research has several limitations that should be addressed in future research, as follows:

- Our model focuses on minimising the fuel cost when deriving routes and refueling amounts. However, in real practice, airlines could consider various objectives based on their priorities and strategies (e.g. enhancing operational convenience or reducing delay propagation). Therefore, an important issue to resolve for future studies is accommodating those various objectives into the mathematical model to derive valuable business insights. We recommend utilising the multi-objective optimisation model to address the proposed issue.

- We assume that fuel prices for each airport are given in the current mathematical model. However, the fuel price is uncertain because it could vary depending on the political and economic environment. Because of this uncertainty, additional fuel costs could be incurred if airlines use our model with an imprecise value for the fuel price parameter, f_j. Therefore, future research should target developing a methodology to estimate the fuel price precisely based on past data. Furthermore, extending the current model into the robust optimisation model could be another interesting topic for future research to find the optimal solution under the worst-case scenario.
- Lastly, one important issue in the airline industry is the EU's Sustainable Aviation Fuel (SAF) policy, which aims to reduce carbon emissions. Although the price of SAF is relatively high, it is eco-friendly and emits less carbon dioxide than traditional aviation fuel. Because of these advantages, the EU is trying to initiate a regulation requiring airlines to incorporate a certain portion of SAF in aviation fuels. Therefore, reflecting the SAF's regulations in the optimisation model could be promising for future research, allowing us to derive business implications that could be instructive to airline practitioners.

Acknowledgments

The authors are grateful for the valuable comments from the associate editor and four anonymous reviewers.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the National Research Foundation of Korea (NRF) grants funded by the Ministry of Science and ICT (No. RS-2023-00218913 and No. NRF-2021R1A4A1029124).

ORCID

Ilkyeong Moon http://orcid.org/0000-0002-7072-1351

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