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Coordinated logistics with trucks and drones for premium delivery

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ABSTRACT

Last-mile delivery, the final stage of the delivery process before a package arrives at a customer's address, has emerged as an important business opportunity for inland transportation. As people's perception of last-mile delivery has changed, more customers are using premium delivery services to get the delivery at a specific time rather than just pursuing free shipping. In order to satisfy the new trend of logistics, we propose a delivery system in which several drones and trucks work together to provide service to customers within given time windows. In addition, pair constraints, which considered a truck and a drone as a pair, are relaxed to determine more flexible delivery plans. A three-stage savings-based heuristic procedure was also developed based on the concept of savings to determine the operations of trucks and drones in real-world settings. We found that the drone efficiently responds to urgent delivery requests from customers and that a delivery system that utilises both trucks and drones provides substantial benefits.

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Vehicle routing problem; drone; last-mile delivery; savings algorithm

1. Introduction

The spread of COVID-19 has led to an explosive surge in demand for e-commerce. People are accustomed to the convenience of e-commerce and have begun to depend on last-mile delivery to stock up on necessities. Last-mile delivery is the last stage of the delivery process when goods are transported from a distribution hub to the final delivery destination. It is the most inefficient and time-consuming part of the logistics setup because the final leg of delivery typically involves numerous destinations with smaller payload sizes. However, it is very crucial, as last-mile delivery is the key to customers' overall satisfaction. The need for faster and more economical last-mile delivery is under pressure. This social and environmental transformation has accelerated logistics companies to expand their delivery services and pursue innovation.

Although some research (e.g. Haas and Friedrich 2021; Lamb, Wirasinghe, and Waters 2022; Liu et al. 2023; Montecinos et al. 2021; Wang et al. 2016) proposed innovative

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delivery services, last-mile delivery is still not realised to full efficiency due to a lack of economies of scale and the complexity of the concept. For innovation in last-mile delivery, the emerging technology of unmanned aerial vehicles, also known as drones, has offered a new potential for optimal operation in logistics. Pugliese, Guerriero, and Macrina (2020) analysed the drawbacks and the benefits of using drones in the delivery process. Compared to trucks, which are the most traditional means of transportation, drones have attractive advantages, such as their ability to avoid traffic congestion and their ability to fly and deliver to areas where roads do not exist, such as islands and mountainous areas (Wang and Sheu 2019). Another advantage of using drones is their autonomous swarm operation capability, which enables the system to execute multiple tasks simultaneously at a low cost (Park, Nielsen, and Moon 2020). Although drones offer significant benefits, they also have some operational constraints due to their shortcomings. For example, the delivery capacity of drones is technically restricted to just one or a few parcels, and their delivery range is significantly limited, as drones rely on relatively small batteries. Traditional vehicles can make up for the shortcomings of drone operation. The complementary nature of the two vehicles (trucks and drones) has been the driving force behind a novel delivery method called *coordinated logistics with a truck and a drone* (Carlsson and Song 2018).

In addition to the collaboration of trucks and drones, another significant trend change in delivery is worth mentioning. In the past, customers mostly wanted free delivery when they ordered goods from suppliers or retailers. Recently, more and more customers have been willing to pay a premium if they can receive goods in a faster way. Moreover, the delivery demand for city express service has increased in keeping pace with the economic growth (Chen, Chou, and Hung 2019). Improving the customer experience level has become a consensus among logistic companies (Liu et al. 2021), so they are providing various types of delivery services to deal with trend changes. For example, logistics companies such as Coupang and Market Kurly in Korea are offering amazing services that deliver until dawn the next day. Therefore, time window constraints are necessary in order to support routing decisions that take into account special requests regarding delivery time. By setting the time window of customers requesting expedited delivery to the beginning of the planning horizon, delivery to these customers is given a high priority. To pursue efficient delivery while meeting these customers' special rush needs, we developed a vehicle routing problem with time windows and drones (VRPTW-D), in which the service rendered at each customer starts within the associated time interval. In this step, we attempted to model a delivery system in which drones are attached to trucks and dispatched to deliver a single package while the truck continues to serve other customers. After the drone delivers its payload, it should return to the truck or the depot for battery replacement and then prepare packages for the next delivery.

The contributions of our work are manifold. First, we studied new variants of the VRP, that is, the VRPTW-D. To the best of our knowledge, the VRPTW-D is the most generalised mathematical model that takes into account both time windows and drones. Specifically, pair constraints, which consider a truck and a drone as a pair, are relaxed because considering a truck and a drone as a set provides limited delivery routes and may not achieve global optimality. Therefore, the VRPTW-D defined in this study allows the drone to return to a different truck than the one from which it launched. Second, finding the optimal solution through a mixed-integer linear programming (MILP) formulation in a reasonable time is only possible on a small scale. However, instances encountered in real-world settings are usually complicated and large-scale, which means that efficient heuristic algorithm development is required in practice. To address larger instances of the VRPTW-D efficiently, we developed a three-stage savings-based heuristic (TSH) algorithm that utilises the nature of the routing problem. Finally, we performed a sensitivity analysis of the relevant drone parameters in order to provide insights from the perspective of management. We showed that applying drones can reduce operating costs, highlighting the advantages of using drones in last-mile delivery.

The remainder of this paper is organised as follows. Research on drones actively underway in academia is summarised in Section 2. The problem description and the mathematical model of the VRPTW-D are proposed in Section 3. In Section 4, we propose the TSH and analyse how it effectively exploits the problem structure of the VRPTW-D. The computational experiments and their results are summarised in Section 5. Finally, Section 6 concludes this study.

2. Literature review

Drone research is emerging as a new field of study within routing problems. Related studies in this field have mostly been published recently, and significant advances have been made for different variants of coordinated logistics with a truck and a drone. Research related to drone-aided routing is thoroughly surveyed in Macrina et al. (2020), including a travelling salesman problem with drones (TSP-D), a vehicle routing problem with drones (VRP-D), drone delivery problems, and carrier-vehicle setups. In addition, academic contributions to drone routing problems are well analysed in Rojas Viloria et al. (2021). Moshref-Javadi and Winkenbach (2021) also provided a comprehensive review of the extant research on drone logistics. Readers interested in this field are encouraged to read Rojas Viloria et al. (2021), Moshref-Javadi and Winkenbach (2021), and Macrina et al. (2020). Here, only works that are deeply related to this study are summarised.

Murray and Chu (2015) offers seminal work in this field of study. They developed MILP formulations for two delivery-by-drone problems, the flying sidekick TSP (FSTSP) and the parallel drone scheduling TSP. They also developed two simple heuristic algorithms. Starting with their research, studies in the field began in earnest. Agatz, Bouman, and Schmidt (2018) studied TSP-D with the objective of minimising the logistics cost. In this problem, a key difference to the FSTSP is that the truck can wait for the drone in the same position from which the drone was launched. Moreover, they provided a theoretical bound on the maximum attainable gains that could be achieved by using the two different vehicles simultaneously. They constructed an integer programming model and developed a route-first, cluster-second heuristic algorithm. Bouman, Agatz, and Schmidt (2018) provided an exact algorithm for the TSP-D based on dynamic programming. They showed that restricting truck movement while drones are on delivery missions significantly reduced the computation time with relatively little impact on the overall solution quality. They also highlighted that their approach was able to solve large problems that the integer programming presented by Agatz, Bouman, and Schmidt (2018) could not handle. Yurek and Cenk Ozmutlu (2018) proposed an iterative algorithm by decomposing the TSP-D into two stages. They solved a MILP model in order to determine the drone route by fixing the truck route, and then the assignment decisions were made. They verified the proposed algorithm's efficiency by solving the problem with a setup of 12 customers in a reasonable time, whereas existing studies optimally solved problems with a maximum of 10 customers. Poikonen, Golden, and Wasil (2019) presented four heuristics to solve the TSP-D based on the branch-and-bound scheme. Their algorithms could solve the instances of a practical size in a reasonable amount of computing time. Murray and Raj (2020) extended the FSTSP to the version of the problem that assigned one truck and multiple heterogeneous drones to deliver parcels. They proposed a three-phase heuristic to solve the 100-customer instances. Luo et al. (2021) developed a multi-visit TSP-D and showed great potential to reduce costs when drones can visit multiple locations, deliver a higher payload, and travel a longer distance. Zhao et al. (2022) proposed robust TSP-D considering the risk of synchronisation failure associated with uncertain travel time and extended route-first cluster-second methods to solve their robust model of TSP-D. Their approach reduced the synchronisation risk a lot while increasing the expected makespan slightly.

Research on variants of the TSP-D has been actively conducted. Carlsson and Song (2018) worked on another variant of the TSP-D called the Horsefly Routing Problem. In this problem, compared to the TSP-D, a truck can collaborate with one or more drones. They used a continuous approximation (CA) technique to determine the best set of parameters that resulted in the minimum completion time of all truck-drone deliveries in the Euclidean plane. One of their key findings was that the benefit of using a drone along with a truck is proportional to the square root of the relative velocity between the truck and the drone. Ha et al. (2018) proposed a new variant of the TSP-D to minimise operational costs, including costs incurred by transportation and waiting time. They also proposed the advanced heuristic adapted from the algorithm proposed by Murray and Chu (2015) and high-performance greedy randomised adaptive search procedure (GRASP). Kim and Moon (2018) developed the travelling salesman problem with a drone station. A drone station is a facility in which drones are deployed and easily installed in any place. They proved that their model can be divided into the models for the travelling salesman problem and the parallel machine scheduling problem under special conditions. They proved that decomposition approaches effectively deal with the complexity of their problem. Ha et al. (2020) recently proposed a hybrid genetic search with dynamic population management and adaptive diversity control to solve TSP-D in which the objective is to either minimise the total operational cost or minimise the completion time for the truck and drone. Their algorithm outperformed existing solution approaches in terms of solution quality and found many new best solutions.

Wang, Poikonen, and Golden (2017) introduced the VRP-D as a generalisation of the TSP-D and derived worst-case bounds for the ratios of the total delivery times with or without drones. The worst-case results depended on the number of drones per truck and the speed of the drones relative to the speed of the truck. Poikonen, Wang, and Golden (2017) extended the results of Wang, Poikonen, and Golden (2017) by relaxing the assumptions about the limited battery life, different distance metrics, and operational expenditures of deploying drones and trucks, respectively. Wang and Sheu (2019) proposed an arc-based integer programming model for the VRP-D and reformulated it as a path-based model. One of the features of the model was that a backup drone could be utilised because they assumed a service hub. Thus, there was no need to consider synchronisation between the two vehicles. They developed a branch-and-price algorithm that included a pulse algorithm. Sacramento, Pisinger, and Ropke (2019) defined a problem similar to the FSTSP, which featured the capacitated multiple-truck case with the maximum duration and minimising operational cost as the objective function. They proposed the adaptive large neighbourhood search (ALNS) procedure and several problem-specific destroy-and-repair methods in order to solve large instances. They performed a detailed sensitivity analysis on several drone parameters of interest and investigated how beneficial the inclusion of the drone-delivery option was. Schermer, Moeini, and Wendt (2019) proposed an extension of the VRP-D in which a drone could be retrieved at some discrete points located on each arc. They also developed a hybrid algorithm based on Variable neighbourhood search (VNS) and Tabu search (TS), in order to solve large-scale instances. Schermer, Moeini, and Wendt (2019) proposed a matheuristic by exploiting the structure of the VRP-D. As part of the matheuristic, they introduced the drone assignment and scheduling problem that found an optimal assignment and schedule of drones to minimise the makespan. Kitjacharoenchai, Min, and Lee (2020) proposed a two-echelon vehicle routing problem with drones, which extended the FSTSP by allowing multiple trucks and drones to make deliveries while taking into account the capacities and two efficient heuristic algorithms to solve the problem.

Recently, new variants of problems are continuously developed. Panadero et al. (2020) considered the stochastic team orienteering problem that takes surveillance observations of target locations, in which travel times follow generic probability distributions. They also developed a novel simulation-optimisation algorithm based on simple heuristics to find near-optimal solutions. Nguyen et al. (2021) newly developed min-cost Parallel Drone Scheduling Vehicle Routing Problem and proposed Ruin and Recreate algorithm. They validated their model and algorithm with intensive experiments and found 26 new best-known solutions. Campbell et al. (2021) studied length-constrained K-drones arc routing problem, a continuous optimisation problem where homogeneous drones work together. In their problem, drones can fly directly between any two points without following the edges of the graph. They developed a mathematical formulation and two solution methods, a branch-and-cut algorithm and a matheuristic. Nadizadeh, Sabzevari Zadeh, and Bashiri (2023) proposed an extension of the line-haul feeder location-routing problem, where large vehicles are synchronised with small vehicles throughout the delivery process. They studied truck-motorcycle collaboration, but their methodology could be extended to truck-drone collaboration. Wang et al. (2022) explored the Piggyback Transportation Problem, inspired by Amazon's last-mile concept, the flying warehouse. They proved several properties of their problem and provided a simple and efficient heuristic solution procedure. Raeesi and Zografos (2020) developed the electric VRPTW that considered two synchronised levels for the electric vehicles that perform the delivery missions and for the other type of vehicles that are responsible for swapping the depleted batteries caused by the running of electric vehicles on-the-fly.

The existing study that seems most closely related to our study is Pugliese and Guerriero (2017). They introduced the vehicle drone routing problem with time windows and provided the mathematical model of the proposed problem. They used a commercial solver to reach a solution for randomly generated instances with five or ten customers and did not provide a methodology to address the high complexity of the problem. Pugliese, Macrina, and Guerriero (2020) extended the work of Pugliese and Guerriero (2017) and solved instances with up to 15 customers. In addition, a heuristic based on a two-phase strategy and a multi-start framework is developed. Another study that considered both drones and time constraints can be found in Ham (2018). Ham (2018) studied a multi-truck and multi-drone scheduling problem constrained by time windows, drop-off/pickup synchronisation, and multi-visits. This problem is uniquely modelled as an unrelated parallel machine scheduling problem with a sequence-dependent setup. A constraint programming (CP) approach is proposed, and CP formulations are further improved by using variable ordering heuristics. Recently, Coindreau, Gallay, and Zufferey (2021) also developed a minimum-cost vehicle routing problem with time windows and drones. They analysed the benefits and cost structures of the coordinated logistics concept. However, a maximum of one drone can be embedded into a truck in their model. Kuo et al. (2022) also developed a model for the vehicle routing problem with drones considering the customer time windows and a simple variable neighbourhood search algorithm.

The differences between similar studies are summarised in Table 1. Previous studies that did not develop a solution approach were marked as *No* in the last column. Our model is the most generalised as it considers all practical constraints listed in the table. A distinguishing feature of this study, with respect to recent literature, is that pair constraints are relaxed. In particular, unlike previous studies that considered time windows and drones, it is possible for a drone to land on another truck than the truck it originally launched from. Therefore, our model is not aimed at local cooperation in the way that drones are assigned to each truck but is geared instead toward global cooperation for all vehicles. In Wang and Sheu (2019), drones can land in places other than the truck they launched from, but it is hard to say that pair constraints are relaxed completely. In their model, drones should land at a service hub to travel with another truck. Thanks to service hubs, the synchronisation between the two types of transportation is not considered. On the other hand, in our model, the drone can land on another truck without special-purpose facilities, and the two types of transportation achieve perfect coordination. In this regard, our study allows for more efficient delivery routes and provides the most generalised model for the VRPTW-D. Further discussion on the originality from a modelling perspective is described in Section 3.2.

3. Problem description and mathematical model

The VRPTW-D determines the cooperative delivery route of the two types of vehicles, trucks and drones. The delivery plan should ensure the maximum duration constraints that each drone can travel and synchronisation requirements between a truck and a drone. During delivery, a truck can launch a drone when serving a customer, and the drone performs a delivery for another customer and returns to the vehicle at a different location. Sometimes, it can be efficient for a drone to complete a delivery and return to the same location. Our model allows this type of delivery route only at the depot. At a customer node, it may be impossible for a truck to remain stationary for long periods of time. Therefore, we assumed that a drone would return to a different customer node from which it departed.

Optimizing delivery routes for coordinated logistics brings new modelling mechanisms between routes of the two echelons. Thus far, the methods generally used for designing routes of two-echelon networks are the two-echelon vehicle routing problem (2E-VRP) and the truck and trailer routing problem (TTRP). The 2E-VRP optimises how goods are first transported from a central depot to satellite facilities by large vehicles and then delivered to customers by small vehicles (Perboli, Tadei, and Vigo 2011). The TTRP optimises delivery routes of customers served by trucks and trailers (Chao 2002). In the TTRP, the truck detaches the trailer and parks it at the root customer of the subtour in order to attach the

Table 1. Summary of the routing problems with drones in the literature.

Reference	Objective function	Time windows	Drone capacity	Truck capacity	Synchronisation	Pair constraints	Solution approach
Murray and Chu (2015)	makespan	No	No	No	Yes	Yes	heuristic
Carlsson and Song (2018)	makespan	No	No	No	Yes	Yes	CA
Pugliese and Guerriero (2017)	makespan	Yes	No	Yes	Yes	Yes	No
Poikonen, Wang, and Golden (2017)	makespan	No	No	Yes	Yes	Yes	No
Wang, Poikonen, and Golden (2017)	makespan	No	No	Yes	Yes	Yes	No
Bouman, Agatz, and Schmidt (2018)	makespan	No	No	No	Yes	Yes	DP
Ha et al. (2018)	costs	No	Yes	No	Yes	Yes	GRASP
Ham (2018)	makespan	Yes	No	No	No	Yes	CP
Yurek and Cenk Ozmutlu (2018)	makespan	No	No	No	Yes	Yes	iterative algorithm
Kim and Moon (2018)	makespan	No	Yes	No	No	Yes	decomposition
Poikonen, Golden, and Wasil (2019)	costs	No	No	No	Yes	Yes	exact/heuristic
Sacramento, Pisinger, and Ropke (2019)	makespan	No	Yes	Yes	No	Yes	ALNS
Schermer, Moeini, and Wendt (2019)	makespan	No	No	Yes	Yes	Yes	VNS+TS
Schermer, Moeini, and Wendt (2019)	makespan	No	No	No	Yes	Yes	matheuristic
Wang and Sheu (2019)	costs	No	Yes	Yes	Yes	No	exact
Liu et al. (2020)	costs	No	Yes	Yes	Yes	Yes	heuristic
Murray and Raj (2020)	makespan	No	Yes	No	Yes	Yes	heuristic
Coindreau, Gallay, and Zufferey (2021)	costs	Yes	Yes	No	Yes	Yes	ALNS
Luo et al. (2021)	makespan	No	Yes	No	Yes	Yes	TS
Kuo et al. (2022)	costs	Yes	Yes	Yes	Yes	Yes	VNS
This study	**costs**	**Yes**	**Yes**	**Yes**	**Yes**	**No**	**heuristic**

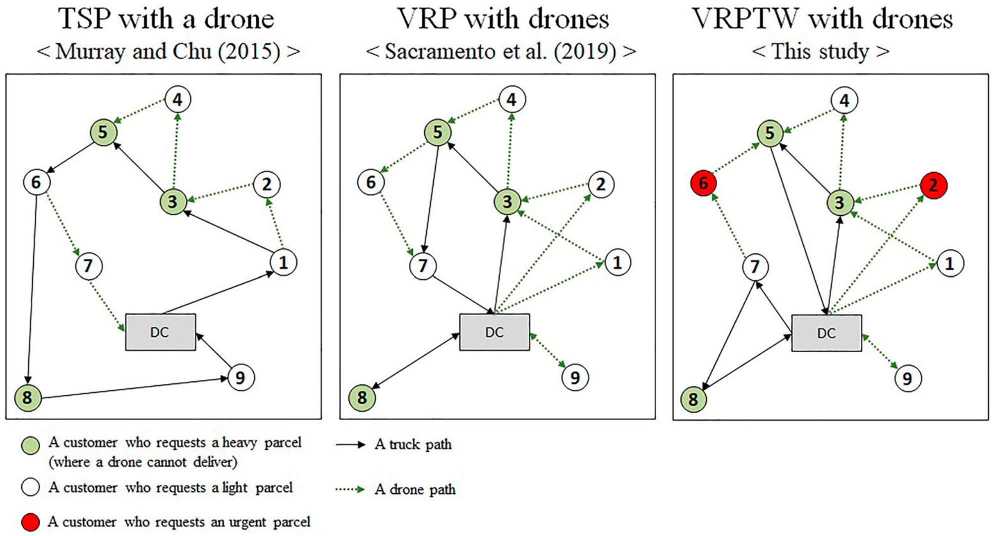

Figure 1. Overview and originality of the VRPTW-D.

trailer again after serving the customers on the subtour. So, in these variants of the problem, a subtour starts at a node that is a satellite facility or root customer and finishes at the same node. On the other hand, in routing problems with drones, a subtour can be finished at a node different from the node from which the subtour started.

Figure 1 presents an overview of the VRPTW-D and highlights the originality of the problem. Unlike the TSP-D and the VRP-D, the VRPTW-D considers the use of multiple trucks and drones. As multiple drones take over some delivery missions, the truck route has been simplified, and the overall delivery plan has become cost-efficient. In addition, the drone, which made its direct delivery to the isolated customer, Customer 9, significantly shortened the truck's route. For illustrative purposes, we will temporarily refer to demands for which the due date of the given time window is at the beginning of the planning horizon as urgent parcels. In the VRPTW-D, the routes of trucks were changed from VRP-D to meet the urgent delivery needs of Customer 6, whose location was far from the warehouse. It should be noted that in our system, the drone that completed its delivery to Customer 6 can return to a different truck than the one it started from. Delivery routes can be flexibly planned while relaxing restrictions on the combination of drones and trucks.

To formulate the VRPTW-D, the following assumptions are made regarding the behaviour of the drones and the cooperation between trucks and drones. First, a drone can visit only one customer per flight, but the truck may visit multiple customers while the drone is in flight. This assumption is necessary because of the physical limitations of the drone. Drones still lack delivery coverage and transport capacity compared to conventional vehicles. Therefore, this assumption is a practical constraint required for the actual application of drones. Indeed, most of the outstanding papers in this field also use similar assumptions (Agatz, Bouman, and Schmidt 2018; Murray and Chu 2015; Pugliese, Macrina, and Guerriero 2020; Schermer, Moeini, and Wendt 2019). Second, picking up parcels or replacing batteries in drones is only possible at customer locations or depots; that is, drones cannot leave any other deliveries when the truck is in motion. Third, the set-up time for preparing the drone for new drone delivery is negligible. This is because drone-deliverable parcels are easy for drivers to handle and because delivery preparation time can be reduced by replacing drone batteries rather than by charging them. Fourth, each truck can carry enough batteries to charge drones, and the batteries do not affect the capacity of the truck. The remaining assumptions are the same as the assumptions required to define the general VRPTW.

The VRPTW-D is an extension of the VRPTW and minimises the total costs of providing transport services by determining optimal customer assignments for drones working in tandem with trucks. The overall routing costs involved fixed vehicle utilisation costs and variable logistics costs. As mentioned in Sacramento, Pisinger, and Ropke (2019), advances in technology relatively reduce the importance of fixed costs. Therefore, the objective of the VRPTW-D is to find the shortest tour, in terms of the total cost incurred by total travel time required, to serve all customer locations by either the truck or the drone.

3.1. Mathematical formulation

The VRPTW-D is formulated over a complete graph $G = (N, A)$, where $N = \{0, \ldots, n+1\}$ is the node set and A is the arc set. Although only a single depot location exists, for the convenience of mathematical formulation, we assign it to two unique node numbers. That is, vehicles depart from the depot at node 0 and return to the depot at node $n+1$. Thus, $C = N \setminus \{0, n+1\}$ becomes the set of customer nodes. To further facilitate the network structure of the problem, let $N_+ = \{0\} \cup C = \{0, \ldots, n\}$ represent the set of nodes from which a truck or a drone may depart, and let $N_- = C \cup \{n+1\} = \{1, \ldots, n+1\}$ represent the set of nodes to which a vehicle may return. The quantity of demand that has to be delivered to customer $i \in C$ is given by $q_i \geq 0$.

The truck, which is a ground vehicle, $g \in GV = \{1, \ldots, |GV| = m\}$, is assumed to be homogeneous, meaning that $|GV|$ vehicles have the same transport speed, capacity, $Q^{GV} \geq 0$, and identical cost efficiency. A truck moving from node i to j incurs travel time, $t_{ij} \geq 0$, and travel cost, $c_{ij} \geq 0$. Each truck can carry up to ζ drones. The drone, $d \in D = \{1, \ldots, |D|\}$ is also assumed to be homogeneous and α times faster and more cost-efficient than the truck. Thus, a drone moving from node i to j incurs travel time $\tau_{ij} = t_{ij}/\alpha \geq 0$ and travel cost $\rho_{ij} = c_{ij}/\alpha \geq 0$. The reason for this is that the drone is not affected by any traffic congestion and can use shortcuts. Hence, faster delivery could potentially be made by drones. Cost efficiency comes from reduced labor costs from unmanned operations and the nature of using electricity. Along with the advantages of drone operation, the physical limitations of drones should also be considered. Each drone has a shipping range limited to a maximum travel time, $\overline{\tau}$, and a relatively low capacity, $Q^{GV} > Q^D \geq 0$. $C_{Dr} := \{n_i \in N \mid q_i < Q^D\}$ denotes the subset of customers who can be serviced by a drone.

The time dimension has been incorporated in the VRPTW-D in the form of customer-imposed time window constraints. Each customer, i, has a time window, $[e_i, l_i]$. Time window constraints are assumed to be hard constraints and restrict the start of service at a customer point to begin at e_i, or later than e_i and to begin earlier than l_i, or at l_i. The vehicle may arrive before the time windows open, but the customer cannot be serviced until the time windows open. The vehicle is not allowed to arrive after the time window has closed. The service time for trucks and drones required by the customer, i, is s_i. To simplify notation, zero demands and zero service times are defined for the depot (i.e.

$q_0 = q_{(n+1)} = s_0 = s_{(n+1)} = 0$). Furthermore, a time window is associated with them (i.e. $[e_0, l_0] = [e_{(n+1)}, l_{(n+1)}]$, where e_0 and l_0 are the earliest possible departure time from the depot and the latest possible arrival time at the depot, respectively). Given that the travel time matrix satisfies the triangle inequality, feasible solutions exist only if $e_0 \leq \min_{i \in N_-}\{l_i - \tau_{0i}\}$, and $l_0 \geq \max_{i \in N-}\{\max\{e_0 + \tau_{0i}, e_i\} + s_i + \tau_{i,n+1}\}$. The following decision variables are used for modelling the VRPTW-D. A description for each of the decision variables follows:

$$x_{ij}^g = \begin{cases} 1, & \text{if truck } g \in GV \text{ travels arc } (i,j) \in A \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$y_{ij}^d = \begin{cases} 1, & \text{if drone } d \in D \text{ travels arc } (i,j) \in A \text{ independently} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$z_{ij}^{gd} = \begin{cases} 1, & \text{if truck } g \in GV \text{ carries drone } d \in D \text{ and travels arc } (i,j) \in A \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

$$o_i^g = \begin{cases} 1, & \text{if a drone delivers goods from truck } g \in GV \text{ to customer } i \in C \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

$$S_i^g : \text{Start time of service at node } i \in N \text{ when serviced by truck } g \in GV \tag{5}$$

$$S_i^d : \text{Start time of service at node } i \in N \text{ when serviced by drone } d \in D \tag{6}$$

Based on the sets, parameters, and decision variables defined above, the mathematical model of the VRPTW-D is developed. The VRPTW-D can be formulated as the following network flow model with time window and capacity constraints. The MILP formulation of the VRPTW-D is proposed, where Equation (1) is the objective function and the constraints are given by Constraints (2)–(21).

$$\min \quad \sum_{(i,j) \in A} \sum_{g \in GV} c_{ij} x_{ij}^g + \sum_{(i,j) \in A} \sum_{d \in D} \rho_{ij} y_{ij}^d \tag{1}$$

$$\text{s.t.} \quad \sum_{g \in GV} \sum_{i \in N_+, i \neq j} x_{ij}^g + \sum_{d \in D} \sum_{i \in N_+, i \neq j} y_{ij}^d \geq 1, \quad \forall j \in C \tag{2}$$

$$\sum_{j \in N_-} x_{0j}^g = 1, \quad \forall g \in GV \tag{3}$$

$$\sum_{j \in N_-} \left(y_{0j}^d + \sum_{g \in GV} z_{0j}^{gd} \right) = 1, \quad \forall d \in D \tag{4}$$

$$\sum_{i \in N_+} x_{i,n+1}^g = 1, \quad \forall g \in GV \tag{5}$$

$$\sum_{i \in N_+} \left(y_{i,n+1}^d + \sum_{g \in GV} z_{i,n+1}^{gd} \right) = 1, \quad \forall d \in D \tag{6}$$

$$\sum_{i \in N_+, i \neq j} x_{ij}^g = \sum_{k \in N_-, j \neq k} x_{jk}^g, \quad \forall j \in C, \quad \forall g \in GV \tag{7}$$

$$\sum_{i\in N_+, i\neq j}\left(y_{ij}^d + \sum_{g\in GV} z_{ij}^{gd}\right) = \sum_{k\in N_-, k\neq j}\left(y_{jk}^d + \sum_{g\in GV} z_{jk}^{gd}\right), \quad \forall j\in C, \quad \forall d\in D \tag{8}$$

$$\sum_{d\in D} z_{ij}^{gd} \leq \zeta x_{ij}^g, \quad \forall (i,j)\in A, \quad \forall g\in GV \tag{9}$$

$$\sum_{d\in D} y_{ij}^d \leq \sum_{g\in GV}\sum_{h\in N_+}(x_{hi}^g + x_{hj}^g) + \sum_{g\in GV}\sum_{k\in N_-}(x_{ik}^g + x_{jk}^g), \quad \forall (i,j)\in A \tag{10}$$

$$\sum_{i\in N_+}\tau_{ij} y_{ij}^d + \sum_{k\in N_-}\tau_{jk} y_{jk}^d \leq \bar{\tau}, \quad \forall j\in C, \quad \forall d\in D \tag{11}$$

$$S_i^g + s_i + t_{ij} - S_j^g \leq (1 - x_{ij}^g) MGV_{ij}, \quad \forall (i,j)\in A, \quad \forall g\in GV \tag{12}$$

$$S_i^d + s_i + \tau_{ij} - S_j^d \leq (1 - y_{ij}^d) MD_{ij}, \quad \forall (i,j)\in A, \quad \forall d\in D \tag{13}$$

$$S_i^d + s_i + t_{ij} - S_j^g \leq (1 - z_{ij}^{gd}) MGV_{ij}, \quad \forall (i,j)\in A, \quad \forall g\in GV, \quad \forall d\in D \tag{14}$$

$$e_i \leq S_i^g \leq l_i, \quad \forall i\in N, \quad \forall g\in GV \tag{15}$$

$$e_i \leq S_i^d \leq l_i, \quad \forall i\in N, \quad \forall d\in D \tag{16}$$

$$\sum_{i\in N_+} q_j y_{ij}^d \leq Q^D, \quad \forall j\in C, \quad \forall d\in D \tag{17}$$

$$\sum_{h\in N_+} x_{hi}^g + \sum_{d\in D} y_{ij}^d \geq 2 o_j^g, \quad \forall i\in N_+, \quad \forall j\in C, \quad \forall g\in GV \tag{18}$$

$$\sum_{j\in N_-} q_j\left(\sum_{i\in N_+} x_{ij}^g + o_j^g\right) \leq Q^{GV}, \quad \forall g\in GV \tag{19}$$

$$x_{ij}^g, y_{ij}^d, z_{ij}^{gd} \in \mathbb{B}, \quad \forall (i,j)\in A, \quad \forall g\in GV, \quad \forall d\in D \tag{20}$$

$$o_i^g \in \mathbb{B}, S_i^g, S_i^d \in \mathbb{R}^+, \quad \forall i\in N, \quad \forall g\in GV, \quad \forall d\in D \tag{21}$$

Objective function (1) aims at minimising the total route cost based on total travel time. Constraint (2) ensures that each customer is assigned to routes. In other words, all transportation requests must be satisfied. Constraints (3)–(6) ensure that the trucks and drones that leave the depot should return to the depot. Constraints (3) and (4) force all trucks and drones to move from node 0, the depot, to perform delivery missions, and Constraints (5) and (6) ensure that all trucks and drones return to the depot. Unused vehicles go directly from the depot at Node 0 to the depot at Node $n+1$. Constraints (7) and (8) are path-flow constraints for trucks and drones, respectively. Constraints (8) controls the flow of drones considering the case where the drone moves with the truck and the case where the drone delivers independently. Constraints (9) limits the number of drones a truck can carry. Constraints (10) allows the drone to deliver to only one customer during a single flight. Constraints (11) guarantees the feasibility of drone flying duration. Next, Constraints (12)–(16) guarantee the schedule feasibility with respect to time windows. Constraints (12) and (13) calculate start times of service when trucks and drones deliver independently, and Constraints (14) synchronise departure times when moving together. The values of S_i^g and

S_i^d are meaningless whenever customer i is not visited by truck g and drone d, respectively. To get better lower bounds and accelerate problem-solving, $MGV_{ij}, (i,j) \in A$ and $MD_{ij}, (i,j) \in A$ are set to $l_i + s_i + t_{ij} - e_j$ and $l_i + s_i + \tau_{ij} - e_j$, respectively. It is also possible to simply apply large constants, $MGV_{ij} = MD_{ij} = \max_{(i,j)\in A}\{l_i + s_i + t_{ij} - e_j\}$. In addition, Constraints (12)–(14) act as sub-tour elimination constraints. Constraints (15) and (16) adjust the arrival times of a truck and a drone, respectively, to ensure that services can be started within a given time window. Constraints (17)–(19) are capacity constraints related to the weight that can be carried in each type of vehicle. Constraints (17) prevents drones from delivering to customers whose quantity of demand exceeds the transport capacity of drones. Specifically, Constraints (18) is a linking constraint on which truck the drone replenishes supplies, and Constraints (19) limits the transport capacity of the truck. Finally, the arc-flow variables are subject to binary requirements that can be expressed as in Constraints (20), and the other decision variables are subject to nonnegative integer requirements that can be expressed as in Constraints (21).

3.2. Discussion of VRPTW-D

The VRPTW-D is $\mathcal{NP}-$hard in the strong sense. This is because if drones are not available, then the VRPTW-D is equivalent to the VRPTW, which is a well-known $\mathcal{NP}-$hard combinatorial optimisation problem. Several other features of the model warrant some discussion in order to indicate the practicality for which the VRPTW-D differs from the related models found in the existing literature.

A recently developed VRP considering the drone is particularly concerned with minimising the delivery completion time. Many studies evaluate the performance by comparing the completion time of the truck-and-drone delivery system with the classical delivery system. In this study, however, the use of completion time as a performance measure may distort the original intention. This is because the newly considered time window constraints significantly affect the complete time of delivery. If there is a customer who wishes to receive a late delivery, the delivery completion time will be large regardless of other customers and previous delivery schedules. The VRPTW-D can provide a meaningless solution if the objective function is not set properly. Therefore, setting the minimisation of total travel time or distance as the objective function in the VRPTW-D is necessary in terms of the validity of the model, as well as in terms of economic perspectives.

Figure 2 illustrates another rationale for setting total travel time as the objective function. It is better to use more drones in the decision of cooperative delivery with the aim of minimising delivery completion time. Under the assumption that drones take less travel time than trucks, the drone route in an optimal solution serves to improve the objective value. This can be easily proved by applying the triangular inequality inductively. However, the situation is different for cooperative delivery in terms of minimising total travel time or distance. This is because each time a drone is assigned to a new shipment, a newly generated path of a truck adversely affects the objective function. Therefore, in order to determine the route that minimises total travel time or distance, the delivery mission must be distributed more carefully between trucks and drones.

We assign a single depot to two unique node numbers for the convenience of mathematical formulation. This very conventional way of formulation also plays an important role in planning cooperative delivery. Since the unused truck is considered to be moving to two

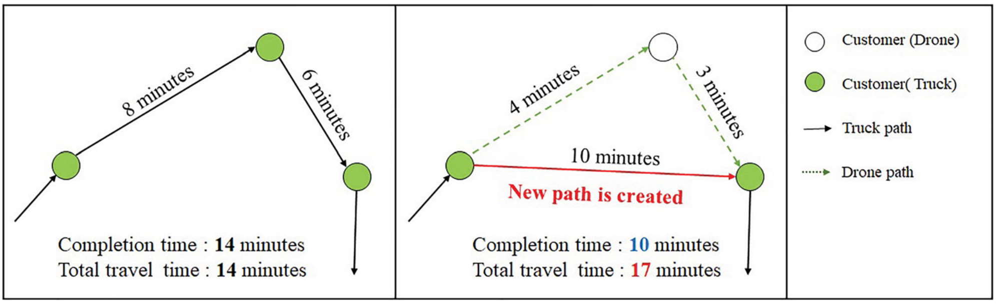

Figure 2. Difference between total travel time and completion time as an objective function.

different depot nodes, the drone can be used for additional deliveries. In other words, the drone can deliver directly to nearby customers around the depot. A drone's direct delivery eliminates inefficient truck routes and significantly improves objective functions by performing deliveries to remote isolated customers.

Another unique feature of the model is that the decision variables are defined in a different manner from previous studies. Conventionally, the physical limitations of drones are implicitly included in the structure of decision variables. In contrast to this, we utilised the general form of arc-flow decision variables. Similar approaches can also be found in Wang and Sheu (2019). These types of decision variables are easy to generalise in the model if the flight limitation and the number of drones assigned to each truck are relaxed. Additionally, the number of variables regarding the drone operations required for modelling the VRPTW-D can be reduced. Whereas existing studies required decision variables of n^3 to determine drone operations, only those of n^2 are required in this approach.

Finally, we will discuss the originality that makes this study different from other TSP-D and VRP-D studies. Research that considers cooperation between drones and trucks has not long been in the limelight. It has been mainstream up until recently to research and analyse the tractable model using strong assumptions. Our study considers additional flexibility in the delivery route of drones. Drones can return to a different truck than the one from which they originally started delivering. Such a setup raises the need for new solution approaches. Given that the VRPTW-D is a problem in which new constraints are considered from the cost minimisation perspective, not from the popular makespan perspective, it is meaningful to develop an intuitive solution approach as an initial attempt. Therefore, in Section 4, we developed a simple and efficient heuristic algorithm that can solve VRPTW-D considering not only intra-route but also inter-route cooperative delivery.

4. Solution approach for the VRPTW-D

Given the intrinsic complexity of the VRPTW-D, it is difficult to find an optimal solution of the VRPTW-D for practical-sized instances in a way that solves a mathematical model directly. Therefore, the development of heuristics would be of interest for practical applications. For solving the VRPTW-D, we use an advanced RFCS heuristic approach (e.g. Beasley 1983), where an initial solution is obtained by solving the VRPTW, and the solution is improved using the developed heuristic algorithms. More precisely, we adapt the parallel Clarke-Wright-savings heuristic (e.g. Clarke and Wright 1964) as an improvement procedure. Agatz, Bouman, and Schmidt (2018) also developed the RFCS heuristics based on local search and dynamic programming to solve the TSP-D. However, their algorithm can be applied to cases in which only one truck is utilised, so it is not suitable for solving the VRPTW-D. Moreover, existing solution approaches, including the one developed by Agatz, Bouman, and Schmidt (2018), do not take into account the delivery route in which the drone lands on a different truck than the truck from which it originally launched. This study proposes the TSH that adds new procedures to overcome prior limitations to solving the VRPTW-D. In Section 4.1, we explain the procedure that is used to generate an initial solution. Sections 4.2 and 4.3 are devoted to the presentation of the drone assignment algorithm and the route combination algorithm, respectively. Potential benefits and further remarks on the TSH are summarised in Section 4.4.

4.1. Finding an initial VRPTW tour

As the first step of the TSH, we seek a solution for the VRPTW that does not take drones into account. Naturally, the solution of the VRPTW can be that of the VRPTW-D. Over the past 40 years, the VRPTW has been an area of research that has attracted numerous researchers. Numerous exact algorithms, which can be classified into the following three families, branch-and-price, branch-and-cut, and reduced set partitioning, were designed for the VRPTW, producing a significant improvement on the size of the instances that can be solved to optimality. Despite decades of intensive study, only relatively small instances involving around 100 customers could be solved optimally. The scale of the problems encountered in the industrial field was sometimes too large to be handled by a mathematical approach. Therefore, to generate the initial solution, the ruin and recreate (R&R) algorithm, inspired by the work of Schrimpf et al. (2000), was used. The R&R algorithm is a generalisation of simulated annealing and is very similar to the large neighbourhood search heuristic. According to Bräysy and Gendreau (2005), the methods of Schrimpf et al. (2000) are the best ones with respect to solution quality.

The ruin procedure disintegrates the solution by removing customers in the route and generates a partial solution containing the remaining jobs. At this step, the randomly selected customer and the nearest neighbours are excluded from the route. Based on the partial solution, all jobs are re-integrated again, yielding a new solution in the recreation procedure. If the new solution is better than the old solution, it is accepted as the new best solution, whereupon a new ruin-and-recreate iteration starts. These steps are repeated until a certain termination criterion is met. Details of the implementation are referred to Schrimpf et al. (2000). If the number of available trucks is tightly given, the initial solution can be generated by temporarily considering a few spare trucks. After generating the initial solutions of the VRPTW using the R&R algorithm, improved solutions can be explored through the developed heuristics described in Sections 4.2 and 4.3.

4.2. Drone assignment algorithm

In this procedure, the VRPTW-D solution is constructed by distributing some of the deliveries to the drone. The basic idea of the heuristic is to find a delivery mission that would

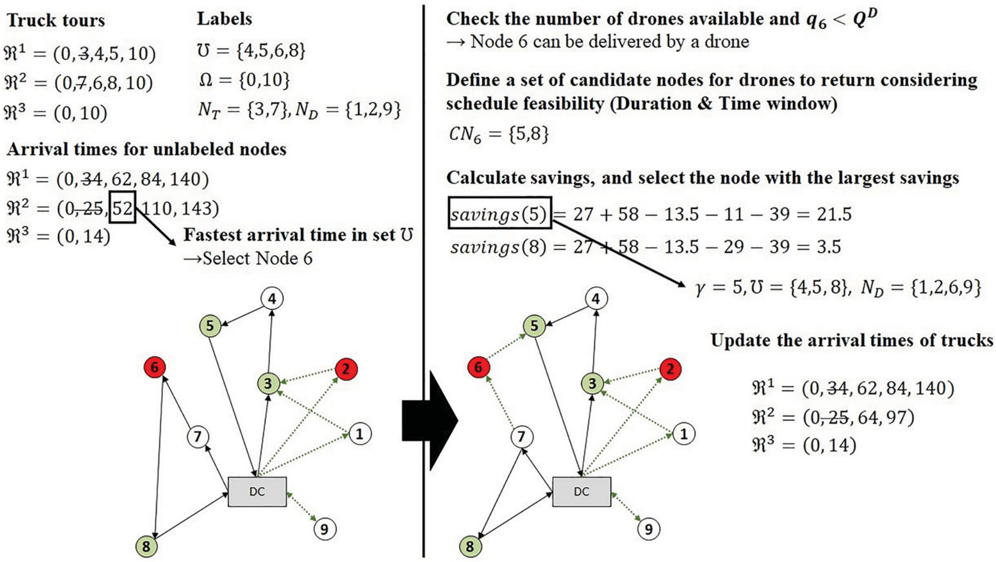

Figure 3. Example of running an algorithm that assigns a drone.

reduce total travel time when assigned to a drone. A pseudo code of the heuristic that distributes the delivery mission to the drones is summarised in Algorithm 1. To simplify notation, we renamed the nodes included in the truck tour, $\mathfrak{R}^g$, in consecutive order (i.e. $\mathfrak{R}^g = (r_0^g, \ldots, r_i^g, \ldots, r_{|V_g|-1}^g)$ where V_g is the set of nodes that truck g visits and $r_0^g = r_{|V_g|-1}^g$ refers to the depot). The overbar symbol for decision variables (e.g. $\overline{S}_{r_i^g}^g$ and $\overline{y}_{r_i^g, r_j^g}^d$) denotes the value of those decision variables given through the initial solution or previous steps in the algorithm. $N_F(r_i^g, \gamma)$ is a set of nodes to which a drone can deliver even if a drone returning to r_i^g changes its returning node to γ. On the other hand, $N_I(r_i^g, \gamma)$ is a set of nodes where the assigned drone delivery is no longer possible. For the unlabelled nodes that have the same earliest arrival time, the algorithm chooses the node in the truck route with a lower index number first.

This algorithm performs inter-route improvements. In other words, the algorithm finds the drone's return point among all unlabelled nodes, not just within the same truck route. Figure 3 shows the illustrative example of running the developed algorithm. In this iteration, Node 6 is selected. After checking whether the drone can deliver to the node, candidate nodes to return to are searched in consideration of the physical limitations of the drone. One of the candidate nodes is Node 5, which is included in the delivery route of another truck. Based on the calculated savings, a decision is made on whether to allocate drones. The drone departs from Node 7, delivers to Node 6, and returns to Node 5, not to Node 8, where the truck that the drone had departed from is located. In each iteration, nodes are labelled sequentially one by one, and only nodes that are visited later than the just-labelled node are updated. Therefore, a drone assignment algorithm can be run in $O(n^2)$ time. While Algorithm 1 is running, the order in which the nodes are visited remains unchanged, and the feasibility of the solution is always guaranteed.

Algorithm 1: Heuristic algorithm for assigning drones

Input :

$$\text{VRPTW solution}: \begin{bmatrix} (r_0^1,\ldots,r_i^1,\ldots,r_{|V_1|-1}^1), \\ \vdots \\ (r_0^g,\ldots,r_i^g,\ldots,r_{|V_g|-1}^g), \\ \vdots \\ (r_0^m,\ldots,r_i^m,\ldots,r_{|V_m|-1}^m) \end{bmatrix}$$

Sets of "*unlabelled*", "*depot*", "*truck*" and "*drone*" : $\mho, \Omega, N_T, N_D = \varnothing$

Initialization

Initialize labels for all customer nodes as "*unlabeled*" and the depot nodes as "*depot*".
$\mho = \{r_1^1, \ldots, r_i^g, \ldots, r_{|V_g|-2}^g, \ldots, r_{|V_m|-2}^m\}$,
$\Omega = \{r_0^1, r_{|V_1|-1}^1 \cdots, r_0^g, r_{|V_i|-1}^g, \ldots, r_0^m, r_{|V_m|-1}^m\}$
Set the number of drones available on each node, $\delta_{r_i^g \in N} = \zeta$
Calculate the amount of time the truck can wait for the drone on each route to maintain the feasibility of the solution, $w_{r_i^g}^g = min_{j>i}\, l_{r_j^g} - \overline{S}_{r_j^g}^g - s_{r_j^g}$

while $\mho \neq \varnothing$ **do**
  Select $r_i^g \in \mho$ with the fastest arrival time.
  if $q_{r_i^g} \leq Q^D$ & $max_{j\in\mathfrak{R}^g}\, \delta_j - min_{j\in\mathfrak{R}^g}\, \delta_j \leq \zeta$ **then**
    Find the nearest precedent *"truck" node from* r_i^g, p_i^g
    Define a set of candidate nodes for drones to return,
    $CN_{r_i^g} = \{\gamma | \tau_{p_i^g, r_i^g} + \tau_{r_i^g,\gamma} \leq \overline{\tau},\ \overline{S}_{p_i^g}^g + \tau_{p_i^g, r_i^g} + \tau_{r_i^g,\gamma} \leq \overline{S}_\gamma^g + w_{r_i^g}^g\}$
    Define the following sets for the drone path that returned to node r_i^g.
    $N_F(r_i^g, \gamma \in CN_{r_i^g}) = \{v | \sum_{d\in D} \overline{y}_{v,r_i^g}^d = 1,\ \sum_{d\in D} \overline{y}_{v_p,v}^d = 1,\ \tau_{v_p,v} + \tau_{v,\gamma} \leq \overline{\tau}\}$
    $N_I(r_i^g, \gamma \in CN_{r_i^g}) = \{v | \sum_{d\in D} \overline{y}_{v,r_i^g}^d = 1,\ \sum_{d\in D} \overline{y}_{v_p,v}^d = 1,\ \tau_{v_p,v} + \tau_{v,\gamma} > \overline{\tau}\}$
    Calculate possible savings,
    $savings(r_i^g, \gamma \in CN_{r_i^g}) = c_{ps_i^g, r_i^g} + c_{r_i^g, r_{i+1}^g} - c_{ps_i^g, r_i^g} - \rho_{ps_i^g, r_i^g} - \rho_{r_i^g,\gamma} + \sum_{v\in N_F(r_i^g,\gamma)} (\rho_{v,r_i^g} - \rho_{v,\gamma}) - \sum_{v\in N_I(r_i^g,\gamma)} savings(v, r_i^g)$
    where p_i^g is the nearest precedent *"truck" node from* r_i^g *in the same route.*
    if $max_{\gamma\in CN_{r_i^g}}\ savings(r_i^g, \gamma \in CN_{r_i^g}) \leq 0$ **then**
      Label node r_i^g as "*truck*", $\mho \leftarrow \mho \setminus \{r_i^g\}, N_T \leftarrow N_T \cup \{r_i^g\}$
    else
      Select the node γ with the largest savings. Node γ is chosen as return node for a drone and label node r_i^g as '*drone*'.
      $\mho \leftarrow \mho \setminus \{r_i^g\}, N_D \leftarrow N_D \setminus \{v \in N_I(r_i^g, \gamma)\} \cup \{r_i^g\}, N_T \leftarrow N_T \cup \{v \in N_I(r_i^g, \gamma)\}$
    end if
  else
    $\mho \leftarrow \mho \setminus \{r_i^g\}, N_T \leftarrow N_T \cup \{r_i^g\}$
  end if
  Update the number of drones available and the values of decision variables of all nodes.
end while
return VRPTW-D solution, $(\mathfrak{R}, \mathfrak{T})$

4.3. Route combination algorithm

This section describes how to adjust the number of vehicles used effectively. This procedure is unnecessary for the TSP-D, which considers only one truck. However, it is crucial in order to utilise the proper number of trucks for the VRPTW-D. The number of trucks used was initially determined by solving the classical VRPTW. If the number of trucks is kept the same as in the initial solution and then cooperative delivery routes with the drone are planned, trucks may be overused. An optimal or near-optimal delivery route can only be planned when a reasonable number of trucks is used. Therefore, a new procedure, a route combination algorithm, is proposed to determine the appropriate number of trucks used.

A simple heuristic was applied because time window constraints and the synchronisation between two vehicles significantly limited the possibility that the routes could be combined. Using complex algorithms did not result in significantly more route combinations. A route combination algorithm is based on an extension of the Clarke and Wright savings heuristic (Clarke and Wright 1964), one of the most well-known heuristics for solving the VRP. The algorithm checks the feasibility of the new route generated by combining the two routes by connecting the last customer on one route to the first customer on another route. The algorithm calculates the savings of two associated routes when the new solution is feasible and then greedily combines two routes when the saving is positive. A route combination terminates when two routes can no longer be combined and can be run in $O(m^2)$ time.

4.4. Remarks for the TSH

Solution quality and computation time are usually the two most obvious criteria used to assess the quality of an algorithm. These indicators are discussed in Section 5. Here we discuss other criteria that are also recognised as important when evaluating newly developed heuristics.

A heuristic algorithm needs to be simple in that it is easy to figure out and implement and should not be too sensitive to the parameters. Usually, an algorithm that is controlled by a few parameters is preferred. The TSH is an intuitive algorithm that applies the savings approach in Clarke and Wright (1964) to drone delivery. No parameter tuning is required, and no random choice is made while the algorithms in Sections 4.2 and 4.3 are running. While simplicity can lead to adverse effects on solution quality, the TSH algorithm strikes a reasonable balance between the performance criteria, as shown in the computational results in Section 5.

Since research in this field is still evolving, and no standardised problems have yet been defined, it becomes necessary, in order to handle various objectives and additional constraints, to develop heuristics that are sufficiently flexible. The TSH is easily able to incorporate additional constraints that may arise in practical applications. By modifying the process of calculating the savings and redefining the candidate sets, we can successfully reassign the drone's delivery mission to fit the goal of the new problem. In addition, full feasibility is guaranteed at all steps while the heuristic is running. Artificial constructions like penalty terms in the objective function are not considered at all. This property has significant implications for commercial applications.

The output of the TSH relies on the initial VRPTW solution. Using the optimal solution of the VRPTW as the initial solution generally finds a good final solution. However, a good initial solution may not lead to a near-optimal or optimal solution of the VRPTW-D. Truck routes that are too well designed hinder further allocation of drones and miss opportunities to find the optimal coordinated delivery routes for trucks and drones. Therefore, the use of the optimal solution of the VRPTW is very costly in terms of computation time but has little corresponding advantage.

5. Computational experiments

We carried out computational experiments on both small- and large-sized instances, and the computational results are provided in this section. Section 5.1 describes how the experiment was conducted and what data set was used. In Section 5.2, we discuss the performance of two solution approaches, the MILP introduced in Section 3.1 and the heuristic proposed in Section 4. The economic superiority of the coordinated delivery system compared to the truck-only delivery system is presented in Section 5.3. Afterward, we provide some sensitivity analysis on relevant drone parameters.

5.1. Description of experiments

We utilised Solomon benchmark instances to verify the performance of the developed heuristic, the TSH. Solomon (1987) introduced VRPTW benchmark instances involving 100 customers that have since been accepted as standard benchmark problems by most researchers working on related issues. Six sets of problems are generated, and the instances differ in geographical data, the tightness and positioning of the time windows, and several other factors that affect routing and scheduling. Problem sets include problems in which customers are located randomly (the R-problems), or in clusters (the C-problems). Problem sets also include a mixture of two geographic features (the RC-problems). Each problem consists of 100 customers, yet smaller problems can be created by considering only the first *n* customers. Cost and travel time will be calculated with one decimal point and truncation, a technique commonly used in this field. To check the results intuitively, the cost factor of the truck is assumed to be one. That is, the travel time and the cost incurred are expressed as the same value. The parameters c_{ij} and t_{ij} are calculated as the following equation,

$$c_{ij} = t_{ij} = \frac{\lfloor 10\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \rfloor}{10}$$

where (x_i, y_i) and (x_j, y_j) denote the coordinates for customers i and j, respectively. The units of all parameters used in the following experiments, including travel times and costs, follow the same scale in the benchmark instance.

The VRPTW-D as MILP was solved with FICO Xpress version 8.5, and the TSH was implemented in JAVA SE 8. A pilot test was conducted to find the appropriate parameters and option settings to solve VRPTW-D. Computational experiments were conducted with an AMD Ryzen 7 2700X Eight-core 3.7GHz processor with 16GB RAM in the Microsoft Windows 10 operating system. All numerical results in the following sections were rounded to the second decimal place.

Table 2. Results on 10-customer Solomon instances: summary.

Class	NP	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ^*_{10}
R1	12	12	189.29	243.43	207.26	0.84	9.49%
R2	11	11	177.05	53.10	177.74	0.78	0.39%
RC1	8	8	166.98	99.38	170.91	0.97	2.35%
RC2	8	8	159.18	92.88	162.25	0.79	1.93%
C1	9	9	55.42	61.69	55.54	0.71	0.22%
C2	8	8	120.39	22.16	122.23	0.75	1.53%

5.2. Comparing the TSH to the mathematical model

In this section, we describe the computational experiments on small-sized instances and show their numerical results. To assess the solution quality of the TSH, we compared heuristic solution values to the optimal solution values obtained by solving the mathematical model for 10-customer and 15-customer instances. The default parameter settings of the computational experiments are as follows. We assumed that the drone is twice as fast and cost-efficient as the truck (i.e. $\alpha = 2$). One truck can carry up to two drones waiting for delivery. The capacity of the drone is set to 20. For reference, the capacities of the trucks provided by the Solomon benchmark are 200, 700, or 1,000 depending on the type of problem. The maximum travel time of the drone is set to 45. The sensitivity analysis of drones set to various speeds and maximum travel times can be found in Section 5.5.

Pugliese, Macrina, and Guerriero (2020) reported that using three drones per truck does not lead to a further reduction in the cost. Therefore, up to two drones per truck were considered in this experiment. When solving the VRPTW-D as a MILP, we limited the computing time of the solver to 1,800 seconds. Originally, the number of trucks available was 25, as provided by the Solomon benchmark. However, in order to find the optimal solution within the time limit, the number of trucks available was set to four, the maximum number of trucks used when instances are solved with the VRPTW. The comparative evaluation between the two approaches in large-sized instances is meaningless because the solver often provides an absurd solution within a limited running time. Table 2 reports the class name, (Class), the number of instances in each problem class, (NP), the number of instances solved optimally, (Solved), the average objective function value of the solution found by MILP, (Obj-M), the average computing time taken to find a solution with MILP in seconds, (Time-M), the average objective function value of the solution found by TSH, (Obj-H), the average computing time taken to find a solution with TSH in seconds, (Time-H), and the average gap between two solutions in percentages, (Δ^*_{10}). We compute the optimality gap as,

$$\Delta^*_{10} = \frac{obj.\,value\,(heuristic) - \mathrm{OPT}(\mathrm{MILP})}{\mathrm{OPT}(\mathrm{MILP})}$$

where $obj.\,value\,(heuristic)$ represents objective function value of heuristic solution and OPT(MILP) is optimal solution value of an instance.

The computing times for the TSH are faster than the time needed to solve the integer programming model by a commercial solver. The TSH quickly obtained a solution, with costs lying within 3 percent of those of the optimal solutions except for the R1 class. In the R1-class instances, there was a case where the number of trucks used was drastically reduced from four to one. TSH could not find an efficient solution for this case. The solution quality of the TSH can be verified through a small gap for all tested instances. Further experiments

Table 3. Results on 15-customer Solomon instances: summary.

Class	NP	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_{15}	Δ^*_{15}
R1	12	3	301.28	1,507.87	292.97	25.24	−2.76%	7.85%
R2	11	1	300.26	1,680.28	254.7	11.65	−15.17%	3.23%
RC1	8	3	202.47	1,469.60	205.14	26.01	1.32%	3.71%
RC2	8	2	204.84	1,523.70	194.09	10.66	−5.25%	2.25%
C1	9	6	136.93	697.67	139.92	10.34	2.18%	5.56%
C2	8	7	162.84	724.94	157.98	10.86	−2.98%	0.00%

were conducted to clearly prove the performance of the TSH. The number of trucks available was set to four in the following experiments. The descriptions in each column in Table 3 are similar to those in Table 2. We compute the average gap between the two solutions in percentages as the following.

$$\Delta_{15} = \frac{obj.\,value\,(heuristic)\; - \;obj.\,value\,(MILP)}{obj.\,value\,(MILP)}$$

$$\Delta^*_{15} = \frac{obj.\,value\,(heuristic)\; - \;OPT(MILP)}{OPT(MILP)}$$

Table 3 shows that heuristic solutions are usually much better than the solutions that commercial solver provides within given computing time limits. Δ^*_{15} was computed only with instances of finding the optimal solution. In particular, TSH found all optimal solutions for the C2-class instances solved by the commercial solver. The time efficiency of the TSH is validated clearly in the larger instance. Even though the optimal solutions for most of the 15-customer instances are unknown, it is safe to say that the TSH can provide fine solutions in a short time. To sum up, the TSH provides a good solution for small-sized instances, and as the instance grows in size, it overwhelms the performance of the commercial solver.

5.3. Comparing a coordinated delivery system to truck-only delivery

We conducted a comparative analysis from an economic point of view to see the efficiency of coordinated delivery of trucks and drones. We compared the solutions found by solving the VRPTW-D and the classical VRPTW. First, an experiment was conducted on 10-customer instances to compare the optimal solutions of the two problems. Table 4 reports the class name, (Class), the number of instances in each problem class, (NP), the objective value of VRPTW, (Truck-only), average number of trucks used in the solutions of the VRPTW, (NT_0), the objective value of VRPTW-D in situations in which only one drone can be accommodated per truck, (1 dr/tr), average number of trucks used in the solutions of the VRPTW-D considering one drone/truck, (NT_1), the gap in percentage between the objective value of VRPTW-D considering one drone/truck and the VRPTW solution (Δ^1), the objective value of VRPTW-D in situations in which each truck can accommodate up to two drones, (2 drs/tr), average number of trucks used in the solutions of the VRPTW-D considering two drones/truck, (NT_2), and the gap in percentage between the objective value of VRPTW-D considering two drones/truck and the VRPTW solution (Δ^2). The gaps are calculated as follows.

$$\Delta^1, \Delta^2 = \frac{\mathrm{OPT(VRPTW)} - \mathrm{OPT(VRPTW - D)}}{\mathrm{OPT(VRPTW)}}$$

Table 4. Cost-efficiency analysis of 10-customer Solomon instances.

Class	NP	Truck-only	NT_0	1 dr/tr	NT_1	Δ^1	2 drs/tr	NT_2	Δ^2
R1	12	223.23	2.75	192.89	1.17	13.59%	189.29	1	15.20%
R2	11	193.65	1.45	177.72	1	8.23%	177.05	1	8.57%
RC1	8	172.54	2	167.37	1.875	3.00%	166.98	1.875	3.22%
RC2	8	162.86	1.75	159.19	1.625	2.26%	159.18	1.625	2.26%
C1	9	57.49	1	55.42	1	3.59%	55.42	1	3.59%
C2	8	146.81	1.5	120.39	1	18.00%	120.39	1	18.00%

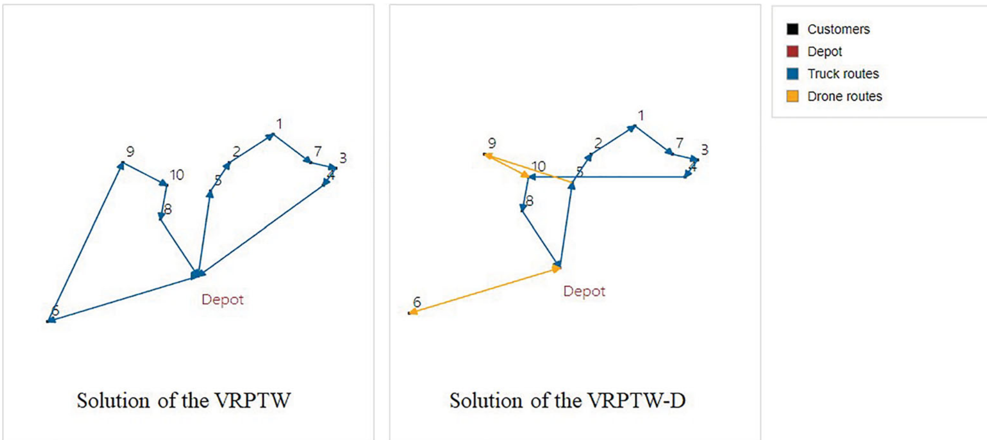

Figure 4. Comparison of the optimal solutions solved with the VRPTW and the VRPTW-D for Instance C201.

Table 4 shows that the objective values of the VRPTW-D were much lower than those of the VRPTW, which justifies the mixed use of trucks and drones. Computational results also show the difference in the performance of the coordinated delivery system according to the geographical distribution of customers. In general, the cost efficiency of drone delivery increases when customers are distant from each other. For example, one customer is far from the remaining clustered customers in 10-customer instances of Problem C2. Therefore, significant cost savings could be achieved by delivering the drone directly to the customer, as shown in Figure 4.

In addition to our objective function, we could see that the number of trucks used was reduced. This is a natural result of pursuing an optimal route from the perspective of minimising total travel time. Figure 5 shows how the number of trucks used reduces as the use of drones increases. In the optimal solution of R101 found with the VRPTW, four trucks were required, but in the optimal solution found with the VRPTW-D considering one drone/truck, two trucks were used. Furthermore, in the optimal solution found with the VRPTW-D considering two drones/truck, only one truck was used. The fixed cost of trucks and the labor cost of men to drive the trucks are very costly compared to the fixed and operating costs of drones. Therefore, reducing the number of trucks is a very significant secondary result from an economic point of view. If the objective function was to minimise the completion time of delivery, a solution exploiting all available resources would be found.

We also conducted an economic analysis for large-sized instances. The optimal solution of the VRPTW-D for large-sized instances is unknown, so the experiment was conducted

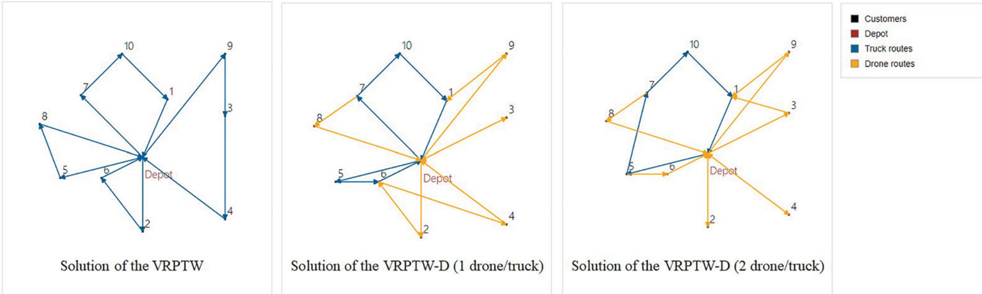

Figure 5. Comparison of the optimal solutions solved with the VRPTW and the VRPTW-D for Instance R101.

Table 5. Cost-efficiency analysis of large-sized Solomon instances.

Class	NP	NC	Truck-only	1 dr/tr	Δ^1	2 drs/tr	Δ^2
R1	12	25	463.37	453.50	2.13%	453.26	2.18%
		50	766.13	749.91	2.12%	745.20	2.73%
		100	1173.74	1148.43	2.16%	1137.42	3.09%
R2	11	25	376.20	376.27	1.55%	376.20	1.55%
		50	615.41	614.26	0.19%	610.08	0.87%
		100	872.53	856.8	1.80%	848.09	2.80%
RC1	8	25	350.24	344.24	1.71%	343.51	1.92%
		50	730.31	716.56	1.88%	714.44	2.17%
		100	1334.48	1325.08	0.71%	1318.35	1.21%
RC2	8	25	319.28	311.44	2.45%	309.72	2.99%
		50	571.63	571.56	0.01%	568.33	0.58%
		100	1000.68	989.04	1.04%	987.49	1.32%
C1	9	25	190.59	189.68	0.48%	189.59	0.52%
		50	361.69	361.28	0.11%	360.99	0.19%
		100	826.70	825.49	0.15%	825.21	0.18%
C2	8	25	214.45	213.70	0.35%	213.14	0.61%
		50	357.50	356.08	0.40%	355.99	0.42%
		100	587.37	582.83	0.77%	582.83	0.77%

using the TSH. The TSH seems to be appropriate to analyse large-sized instances, as demonstrated in the previous section. The experimental results in Table 5 present the guaranteed minimum economic effect, not the ideal cost reduction. The descriptions in each column in Table 5 are the same as in Table 4. The effect of cost reduction was less than that of the previous experiment. The main reason for this was the comparison between the optimal solution for the VRPTW and the heuristic solution for the VRPTW-D. Aside from this, one potential reason for the difference may be that in large instances, the average distance between customer locations is smaller, resulting in less benefit from allocating delivery missions to drones. Also, since the number of drones was limited, they might not have been able to adequately cover the increased number of customers. Nonetheless, the economic superiority of coordinated delivery was still evident. Therefore, successful implementation of the coordinated delivery could bring about cost efficiency.

5.4. Impact of time window constraints and pair constraints

First, we analysed the effect of the width of the time window constraints on the delivery schedule and total cost. All instances within the same type have the same customer

Table 6. Impact of time window constraints.

Class	Narrow	Obj-N	Wide	Obj-W	Δ^{tw}
R1	1,5,9,10	204.24	2,3,4,6,7,8,11,12	187.22	8.33%
R2	1,5,9	188.88	2,3,4,6,7,8,10,11	173.54	8.12%
RC1	1,6,7	171.60	2,3,4,5,8	164.83	3.95%
RC2	1,6,7	167.03	2,3,4,5,8	154.48	7.52%
C1	1,5,6,7,8,9	55.80	2,3,4	54.67	2.03%
C2	1,5,6,7,8	122.92	2,3,4	116.17	5.49%

coordinates and have different time windows of customers. Some have very narrow and tight time windows, while others have time windows that are hardly constraining. We divided instances of the Solomon benchmark into two sets and compared the optimal solutions of each set. We defined instances in which more than half of the customers in each instance had a time window longer than 40 percent of the planning horizon as instances with wide time windows. In order to remove the external influence and properly analyse the impact of the time window constraints, we experimented with instances of 10 customers for which the optimal solution was found. The values of the parameters were set the same as those of the experiments in Section 5.3, and only one drone per truck performed the delivery mission. Table 6 reports the class name, (Class), the name of instances that have narrow time windows in each problem class, (Narrow), the average objective value of instances that have narrow time windows, (Obj-N), the name of instances that have wide time windows in each problem class, (Wide), the average objective value of instances that have wide time windows, (Obj-W), and the gap in percentage between the average objective value of instances that have narrow time windows and those that have wide time windows (Δ^{tw}). We compute the average gap between the two solutions in percentages as the following.

$$\Delta^{tw} = \frac{\mathrm{OPT(Narrow)} - \mathrm{OPT(Wide)}}{\mathrm{OPT(Narrow)}}$$

As can be seen in Table 6, the time window constraint affects route planning. However, the impact of time window constraints is not as clear as the difference between the VRP and the VRPTW, as drones can be used to satisfy customers' demands for delivery flexibly. Thus, these results highlight the potential of drones in last-mile delivery, which must satisfy the various types of services requested by customers.

Regarding relaxing pair constraints, various experiments using the Solomon benchmark did not find cases in which the drone landed with another truck. The failure to observe cases where the drone moves to another truck is not only a problem with the dataset but also a feature of the solution for the routing problem. To optimise the route, customers in proximity are generally delivered by the same truck within physical constraints. Therefore, truck routes are generally far from each other, and the truck from which the drone departed is usually the closest to the drone. Considering constraints such as time windows, synchronisation, and the number of drones that can ride on each truck, it is most reasonable for a drone to return to the truck from which it launched. We observed that relaxing the constraints on the pair between the truck and the drone did not result in a significant improvement in the delivery routes and demonstrated that the assumptions and the approaches made in most previous studies were reasonable in their own way. However, we surmise that our modelling approach will be more effective in an advanced model where

Table 7. Results of sensitivity analysis for different drone speeds.

Class	NP	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$
R1	12	1181.60	1137.42	1089.72	879.85
R2	11	875.53	848.09	819.69	804.72
RC1	8	1344.74	1318.35	1285.61	804.38
RC2	8	1008.30	987.49	961.48	949.86
C1	9	826.70	825.21	813.22	802.90
C2	8	587.38	582.83	563.90	545.35

Table 8. Results of sensitivity analysis for different delivery ranges.

Class	NP	$\overline{\tau} = 10$	$\overline{\tau} = 30$	$\overline{\tau} = 50$
R1	12	1156.01	1139.36	1137.42
R2	11	848.09	848.09	848.09
RC1	8	1330.86	1318.35	1318.35
RC2	8	993.01	984.94	984.94
C1	9	825.21	825.21	825.21
C2	8	582.83	582.83	582.83

drones can deliver multiple nodes in a single flight or where synchronisation with trucks is easy.

5.5. Sensitivity analysis with the drone features

In this section, we carried out a sensitivity analysis when some fundamental parameters were varied. In addition to setting the features we wanted to analyse, we set the other parameters to the same values as in previous experiments. First, we examined the impact of the speed of the drone on the performance of a coordinated delivery system. We assumed that a drone is twice as fast and cost-efficient as a truck. In the following experiments, the relative drone speeds and cost efficiency metrics were changed from one to four. Table 7 shows that cost savings increase with the relative speed of the drone. A drone can deliver only to nodes that are close to the truck route when the speed of the drone is low. A drone travelling at a higher speed can cover more distance and deliver to nodes farther away. As the use of drones increases, it is possible to eliminate the inefficient routes of trucks that were dictated by time window constraints.

We also investigated the impact of the delivery range of drones on system performance. To this point, the maximum travel time of drones was set to 45. In the following experiments, we varied the maximum travel time of the drone between 10 and 50. Table 8 shows that the cost savings of a coordinated delivery system increased with the flying duration of the drone. The increased flying duration of the drone increased cost savings because deliveries made by drones replaced inefficient truck routes. However, flying duration did not bring about as dramatic an effect as did increasing the speed of the drone. The reason for this is that increasing the speed of the drone in our model included the effect of expanding the delivery range of the drone.

Lastly, we carried out the following experiments, changing the delivery capacity of the drone from a relatively small capacity to a capacity that could cover all customers. Table 9 shows that the cost savings of a coordinated delivery system increase with the transport

Table 9. Results of sensitivity analysis for different capacities of drones.

Class	NP	$Q^D = 10$	$Q^D = 20$	$Q^D = 30$	$Q^D = \max_{i \in N} q_i$
R1	12	1158.45	1137.42	1132.25	1130.92
R2	11	863.48	848.09	845.20	844.91
RC1	8	1332.38	1318.35	1316.08	1315.46
RC2	8	1001.08	984.94	983.38	981.13
C1	9	826.01	825.21	824.81	824.81
C2	8	582.92	582.83	582.83	582.15

capacity of the drone. As the transport capacity of drones increases, more cost-effective routes can be designed.

We conclude the discussion of computational results with some insights into the characteristics of a coordinated delivery system. As expected, we saw cost savings increase as the physical limitations of the drone were relaxed. In particular, the effect of drone speed on the efficiency of the coordinated delivery system was very strong. The limited shipping range and capacity of drones can be supplemented with the help of trucks. Even though the performances of these features were improved, there was no dramatic change in cost optimisation. On the other hand, flight speed and cost efficiency are advantages of drones that cannot be replaced by trucks. In a delivery system where only drones are available, all features of drones will be significant. However, in a coordinated delivery system with trucks and drones, it is important to strengthen the advantages of drones rather than overcome their disadvantages.

6. Conclusions

Drones have emerged as attractive options to supplement traditional delivery vehicles. Research on how to integrate drones into logistics and how to prove the effectiveness of drone delivery is constantly being conducted. In this study, we defined a new variant of the VRP, a vehicle routing problem with time windows and drones in which several trucks and drones worked together to provide service to customers within given time windows. A drone can move with a truck, take off from the truck to serve customers, and land on the same or a different truck from which it took off. The VRPTW-D was formulated based on a MILP, and we discussed the characteristics of the newly developed problem. The VRP considering drones has been optimally solved only on a very small scale. Therefore, we presented a three-stage savings-based heuristic, a simple yet time-efficient heuristic framework for solving large-sized instances of the VRPTW-D. The three levels of the TSH consisted of generating an initial solution, assigning a drone to delivery, and combining procedures to reduce the number of trucks used. Competitive results were obtained without exploiting sophisticated mathematical programming, decomposition algorithms, or metaheuristics. Heuristic solutions are often better than those obtained by the solver and not far from the optimal solution, though obtained in a shorter time.

We presented the results and analysis of our computational experiments. Results showed that the coordinated delivery system has significant economic benefits over the truck-only system. One of the weak points in our analysis is the fact that we used a simple heuristic algorithm rather than an exact algorithm, but future studies may explore the theoretical bound of the VRPTW-D and propose state-of-the-art techniques that find optimal solutions for large-sized instances. The development of more efficient heuristic approaches, including metaheuristics and machine learning algorithms, is also expected. In addition to developing solution approaches, our research leads to other opportunities for future research. For example, our model can be extended to include the use of heterogeneous drones, multi-visits of drones, and more real-world considerations. If the physical limitations on the flight of drones are overcome so that multiple nodes can be delivered in one flight, our modelling approach that relaxes pair constraints will yield more significant results. Therefore, we believe that our modelling and algorithmic contributions can be a means to overcome the limits of drone-aided routing and accelerate the commercialisation of coordinated logistics with trucks and drones.

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