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# A decomposition approach for robust omnichannel retail operations considering the third-party platform channel

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ABSTRACT

In recent years, third-party platforms (3PPs), such as Amazon, have attracted considerable interest from omnichannel retailers as a sales channel option. Even though omnichannel retailers have their own offline and online channels, they have participated in the fulfillment service of 3PPs to absorb additional demand from the 3PP channel. To the best of our knowledge, no existing study has addressed the robust omnichannel retail operations utilizing the channel of 3PPs as one of a retailer's sales channels. To fill these research gaps, this paper formulates omnichannel retail operations with the 3PP channel into a multi-period stochastic optimization model. The proposed model involves the supply chain networks of the retailer and 3PP and also the production capacity constraint, which restricts replenishment quantity depending on the production capacity of each supplier. Unfortunately, the existence of the 3PP channel and the production capacity constraint increases the computational complexity; thus, the problem cannot be solved within an acceptable computational time by using the existing approach (i.e., a two-phase approach (TPA) based on robust optimization). To overcome these challenges, we propose a novel decomposition approach called DECOM. DECOM has a distinct advantage in that it can decompose the proposed problem into two small problems, one for the retailer's supply chain and the supply chain of the 3PP. We evaluate the performance of DECOM by comparing it with the TPA on a set of experiments carried out in various experimental settings. Both DECOM and TPA could provide high-quality solutions, but DECOM outperformed TPA in terms of computational efficiency. In particular, we observed that DECOM was scalable to large-scale instances. Furthermore, we explored the advantages of utilizing omnichannel retail operations and the 3PP channel by performing a sensitivity analysis. In particular, we showed the cost-saving effect resulting from the introduction of the 3PP channel in omnichannel retailing.

## 1. Introduction

With the rapid rise of digitalization and e-commerce platforms, the omnichannel strategy has become more popular with retail companies (He et al., 2022). Omnichannel refers to a strategy with multiple sales channels and can provide customers with seamless experiences no matter where they shop. Using the omnichannel service, customers can purchase or receive products in various ways, such as "buy online and pick up in-store" or "examine products in-store and buy online (showrooming)" (Jiu, 2022; Liu et al., 2023).

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According to the survey of Emma et al. (2017), among 46,000 customers, 73 percent preferred to use multiple channels during their shopping journeys. Because of customers' preferences for omnichannel service, retail companies operating brick-and-mortar stores have invested large amounts of resources in the online sales channel (e.g., apps and official online websites).

However, in recent years, several retail companies have sold their products on third-party platforms (3PP), such as *Amazon* and *Coupang*, despite having their own offline and online channels (Zhen and Xu, 2022). In real business, Coupang launched a service called the *C.AVENUE*, and many omnichannel companies, such as *Nike* and *Adidas*, have participated in this service and sold their products using 3PP. From the perspective of retailers, there are distinct advantages to adopting the 3PP channel as one of their sales channels. These advantages are as follows:

- *Self-supporting logistics service system (SLSS)*: The 3PP companies could implement logistics of fulfillment on behalf of the retailer by using their SLSS. For example, Amazon has provided a fulfillment service called *Fulfillment by Amazon* (FBA) (Lai et al., 2022).
- *Customers in the 3PP channel*: The retailer could absorb the additional demand of 3PP. A significant number of customers use 3PP to buy products online. Specifically, as of 2022, more than 197 million monthly active users use the Amazon app, and more than 27 million monthly active users use the Coupang app (Daniel, 2023). Therefore, in addition to customers who want to buy a specific product from a retailer, other users of 3PP could also buy that product while looking around the platform.

Motivated by observing the advantages retailers obtain by using 3PP, we cope with omnichannel retail operations that have adopted the 3PP channel as one of the sales channels. Moreover, we address decision and optimization problems considering demand uncertainty, which jointly determine the four types of decision over a multi-period planning horizon (i.e., replenishment, allocation, transshipment, and fulfillment). We aim to minimize the expected total cost over the planning horizon from the retailer's perspective. Of special note, we consider the following two features, which are generally considered in real business: (1) the binary decision for replenishment to accommodate fixed order costs and (2) the constraint restricting replenishment quantity depending on the production capacity of each supplier (i.e., the *production capacity constraint*).

However, there are four issues that make the proposed problem challenging. First, the retailer has to make binary replenishment decisions adaptively after demand unfolds over periods (i.e., the *adjustable binary decision*), which increases the solution space and complexity of the problem (Hanasusanto et al., 2015). Second, according to the common assumption in retail environments, the replenishment, allocation, and transshipment of products are decided before the demand is realized (*anticipative manner*), and the fulfillment is decided after demand is realized (*reactive manner*) (Jiu, 2022). Thus, the solution approach providing a good quality solution with integrating anticipative and reactive decisions is necessary. Third, the existence of the 3PP channel makes the problem larger than it would be without this channel. In addition to the retailer's supply chain for online and offline channels, the supply chain for the 3PP channel (i.e., the *3PP supply chain*) should also be considered if the 3PP channel is adopted. Fourth, the production capacity of suppliers makes the problem quickly become intractable. To the best of our knowledge, no existing study addresses the above four issues simultaneously, even though Lim et al. (2021) and Jiu (2022) dealt with the first and second issues.

In order to fill these research gaps, our study deals with a multi-period stochastic optimization model that takes into account the logistics operations of an omnichannel retailer's supply chain and the supply chain of the 3PP simultaneously. Additionally, we propose a novel decomposition method, which is called *DECOM*, to enhance computational efficiency. In this research, we attempt to answer the following research questions to derive management implications in omnichannel and 3PP research areas:

1. Does the DECOM outperform the existing algorithm, and is it scalable in realistic problem scales?
2. If it can, how does the production capacity of suppliers affect the performance of DECOM, and is DECOM effective in guaranteeing the robustness for demand uncertainty?
3. Which cost parameters play important roles for the cost-saving effect derived from adopting omnichannel retailing and the 3PP channel?

We present the main contributions of our study from the following two perspectives:

**Practical contributions:** As far as we know, this is the first study to address omnichannel retail operations under demand uncertainty and consider both the retailer's supply chain and the supply chain of the 3PP. Furthermore, we deal with the production capacity of suppliers and transshipment between logistics centers, which are two elements that have not been addressed in related existing studies. We explore the effects of adopting the omnichannel system and the 3PP channel by conducting sensitivity analysis on various cost parameters. In particular, we examine the substantial cost savings caused by employing the 3PP channel in omnichannel retail operations. Based on experimental results, we suggest managerial insights that could be instructive to practitioners who aim to set up an effective supply chain considering the omnichannel retail operations and the 3PP channel.

**Theoretical contributions:** We develop the multistage stochastic optimization model to address the proposed problem. Our model can jointly determine every decision, and the anticipative and reactive manners are implemented seamlessly as the demand unfolds over periods. The two-phase approach (*TPA*) on the robust optimization approach is the state-of-the-art method to deal with adjustable binary decisions (Lim et al., 2021). However, the TPA requires a significant computational burden to solve large-scale instances in our model. To alleviate these issues, we develop a novel approach, DECOM, which can be regarded as an extended version of the TPA. DECOM can decompose the total supply chain into two streams, one for the retailer's supply chain and the other for the 3PP supply chain, by introducing artificial variables. On a set of computational experiments, DECOM could provide high-quality solutions similar to solutions derived from the TPA. Furthermore, in terms of computational efficiency, DECOM outperforms the TPA by solving large-scale problems within a reasonable time.

This paper is organized as follows: We present the literature review related to omnichannel retail operations, the 3PP channel, and robust optimization (RO) in Section 2. We present the problem description for the omnichannel supply chain with the 3PP channel and the corresponding deterministic and stochastic optimization models in Section 3. In Section 4, we briefly explain how we customize the TPA approach for the proposed problem. In Section 5, we present the principles of the DECOM approach. In Section 6, we conduct computational experiments on various demand distributions and problem sizes to evaluate the performance of the developed DECOM approach. Furthermore, we analyze the effects of the omnichannel and the 3PP channel. Finally, in Section 7, we summarize the contributions of this study, along with further research ideas.

## 2. Literature review

The literature review will focus on three streams of research in operations management: omnichannel retail operations under uncertainty, the 3PP channel, and RO.

### 2.1. Omnichannel retail operations under uncertainty

The last few years have seen a huge growth in the number of papers published on the topic of omnichannel leverage in retail operations (Cai and Lo, 2020). Instead of reviewing all existing studies related to the omnichannel topic, we present a detailed review of recent literature regarding the optimization under uncertainty in omnichannel retailing. Many researchers used optimization methods requiring assumptions on the demand distributions. Arslan et al. (2021) studied the distribution network deployment problem, which aimed to integrate the online channel and offline retailers. To consider uncertainties that occurred in online orders, store sales, and capacities, they developed a two-stage stochastic programming model. Siawsolit and Gaukler (2021) developed a Markov decision process model to derive the optimal replenishment policy within omnichannel grocery retailing. They considered the uncertain nature of demand and shelf life of groceries in the proposed model. Abouelrous et al. (2022) addressed the multi-location inventory problem, aiming to determine the initial inventory at each location within a given planning horizon. They also simultaneously considered the stochastic online and in-store demands, which were general assumptions in omnichannel retail operations. Guo and Keskin (2023) studied the integration of strategic, tactical, and operational levels of decisions in the omnichannel supply chain. Their two-stage stochastic programming model can optimize the three types of decisions, depending on the demand realizations. Silbermayr and Waitz (2024) addressed transshipments from an offline channel to online customers to enhance the balance between supply and demand for perishable and substitutable products. They proposed a two-location newsvendor model for inventory management with substitutions for product and sales channels and one-way transshipment.

However, it is difficult to estimate the exact demand distributions within omnichannel retailing (Qiu et al., 2023). Furthermore, if a significant discrepancy between the estimated distribution and the actual demand exists, the quality of solutions derived from the estimated distribution could be poor. Under these circumstances, RO is an effective methodology because it only requires partial information about the demand (e.g., mean). We present three relevant studies that utilize the RO for omnichannel retailing. Qiu et al. (2023) addressed the problem for pricing and ordering optimization considering full-refund and no-refund policies. They also defined the demand as a linear function of the price and refund to accommodate the general case that demands depend on the prices and available return policies. Using historical data, they presented a nonlinear optimization model to cope with demand uncertainty, and the proposed model was transformed into the tractable MILP model by using the duality theory. However, the presented approach is challenging to apply in the multi-period problem, and the computational efficiency was not analyzed. Guan et al. (2024) studied a stochastic optimization model as an integrated method to address assortment planning, replenishment, and e-fulfillment problems. They developed a distributionally RO model, specifically the worst-case mean-conditional value-at-risk model. The proposed model can adjust the trade-off between profitability and risk according to the decision-maker's preference.

On the other hand, Jiu (2022), which is the most relevant study to our research, addressed the multi-period problem for robust omnichannel retailing. The study used the TPA, developed by Lim et al. (2021), to solve the problem. The TPA could provide high-quality solutions compared to existing approaches. In addition, through computational experiments on large-scale problems, the study indicated that the TPA was scalable to the problem. Our study has several differences compared with the study by Jiu (2022). One of these differences is that both transshipment decisions and production capacity are considered in our model. When inventories are insufficient, stockouts can be avoided by implementing the transshipment between logistics centers. Furthermore, suppliers usually have their production capacity, so the retailer should replenish products by considering this constraint. Therefore, our model is suitable for real-world problems and can offer more instructive management implications compared to previous literature. In addition to these two differences, the most apparent contribution of our study is that we adopt the 3PP channel in our model. In other words, when optimizing the proposed problem, the retailer's supply chain and the supply chain of the 3PP should be considered simultaneously.

### 2.2. Third-party platform (3PP) channel

We present several studies that analyze the effects of adopting the 3PP channel for retailing. By investigating the existing studies considering 3PP in retail, we observe that 3PP companies can be classified into two types depending on the existence of SLSS in those companies. For 3PP companies without the SLSS, the retailer or manufacturer who participates in 3PP can only sell their products using the platform, but the logistics of products must be implemented by themselves. On the other hand, for 3PP companies with the SLSS, the retailer can sell their products on 3PPs. Adding to that, the 3PP company implements every logistics and fulfillment procedure on behalf of the retailer. Our study considers the latter type for a 3PP company by reflecting real cases of Coupang and Amazon.

We will introduce literature that considers the 3PP company operating with SLSS. Previous studies can be classified into two types: whether research outcomes are helpful for the 3PP company, or whether they are helpful for the retailer who uses the service provided by the 3PP company. First, we present recent studies that could be helpful for the 3PP company. Lai et al. (2022) investigated the effects of FBA, which is a fulfillment service offered by Amazon, on both Amazon itself and on retailers that use this service. They developed a strategic competition model and found that FBA could alleviate price competition between Amazon and the retailer. Li and Li (2023) examined whether a 3PP company should establish the SLSS by developing game-theoretic models. Furthermore, they discussed the interaction between the SLSS of the 3PP company and the manufacturer's platform entry decision. Deshpande and Pendem (2023) studied how logistics service quality affects service ratings and retailers' sales on a platform provided by the 3PP company. By utilizing the real-world data set and developing the customer choice model, they found that logistics service quality plays an important role in the customer purchase decision.

Second, we present recent research that could be valuable for retailers who are contemplating using the services of the 3PP company. Qin et al. (2020) addressed the SLSS of 3PP, which is provided to retailers that participate in the 3PP channel. They analyzed the strategic and economic impacts of logistic service sharing and examined the equilibrium mode between 3PP and the retailer considering the logistics service level and the market potential. Zhen and Xu (2022) dealt with a research question of whether the retailer who has online and offline channels should adopt 3PP for the sales channel. In order to answer the research question, they developed a Stackelberg game model and examined the retailer's best choice on different channel competitions and the agency fee. Zhen et al. (2022) addressed a similar research question of Zhen and Xu (2022) under various supply chain structures and two directions of the spillover effect. They explored the impact of the direction of the spillover effect between sales channels by varying the degree of channel competition and assuming the agency fee for using 3PP.

The abovementioned literature only investigated whether the retailer who owns its offline and online channels should expand sales channels by utilizing the 3PP channel. However, in a setting where the retailer has determined to utilize the 3PP channel in advance, there is a lack of research investigating the optimal way to operate both the retailer's supply chain and the supply chain of the 3PP. To fill these gaps, our study addresses the problem that the retailer has determined to utilize the 3PP channel in advance. Furthermore, we aim to provide efficient logistics operations by minimizing the total expected cost from the perspective of the retailer. We adopt RO as our solution approach, and several key papers in the RO research area will be presented in the following section.

### *2.3. Robust optimization (RO)*

RO is one of the approaches that deals with uncertainty in optimization problems (Xu et al., 2023). In contrast to other approaches (e.g., stochastic programming and dynamic programming), RO does not need any assumption about the probability distribution of uncertain parameters. But instead, it assumes that the uncertainty value belongs within a predetermined set, called the *uncertainty set*. The goal of RO is to find the optimal solution under the worst-case, and the obtained solution should be guaranteed to be feasible for any realizations of uncertain parameters in the uncertainty set (Ben-Tal et al., 2009). In order to make the RO model tractable, the uncertainty set is generally defined as a convex set (e.g., box shape (Soyster, 1973) and ellipsoid (Ben-Tal and Nemirovski, 1999)).

Two types of decisions can be utilized for the multi-period decisions problem: (1) *here-and-now* and (2) *wait-and-see*. For the here-and-now scheme, every decision is determined before the planning horizon starts (i.e., before every uncertain parameter is revealed). In contrast, for the wait-and-see scheme, we can postpone making decisions until some of the uncertain parameters are revealed. Therefore, the wait-and-see decision is less conservative than the here-and-now decision because it can be adjusted flexibly according to the realized portion of uncertain parameters at each stage (Yanıkoğlu et al., 2019). However, it is complex to deal with the wait-and-see decision because of the large feasible space of adjustable variables.

The adjustable robust optimization (ARO) is developed to deal with multistage problems, which commonly assume the multi-period setting and consider adjustable variables to implement the wait-and-see decision. Because of tractability reasons, it is typical to restrict feasible space by optimizing a certain type of parameterized function. This function is usually called the *decision rule*. Several researches have used nonlinear functions for the decision rule (Bertsimas et al., 2011; Georghiou et al., 2015). However, a broad body of literature has adopted the linear function for the decision rule, which is called the *linear decision rule* (LDR). Ben-Tal et al. (2004) first presented the LDR for a production inventory problem. Because the LDR could lead problems to be reformulated to be tractable, it has attracted considerable interest in many domains , and in particular, it has been widely utilized in inventory management (Bertsimas and Thiele, 2006; See and Sim, 2010; Shin et al., 2020). The simplest version of the decision rule is the *static rule*, in which decisions are fixed regardless of the revealed uncertainties. For some cases, the static rule has proved to be optimal (See and Sim, 2010; Bertsimas et al., 2015; Marandi and Den Hertog, 2018).

The solution approaches of the abovementioned studies have focused on adjustable continuous variables. However, only a few studies developed solution approaches to deal with adjustable binary variables: the K-adaptability approach (Hanasusanto et al., 2015), the finite adaptability approach (FA) (Bertsimas and Dunning, 2016; Postek and Den Hertog, 2016), and the binary decision rule (BDR) (Bertsimas and Georghiou, 2018). In particular, Lim and Wang (2017) developed the target-oriented robust optimization (TRO) method to address the adjustable binary and continuous variables at the same time. TRO could provide a static rule that was optimal for a multi-period inventory problem. By utilizing the strength of TRO, Lim et al. (2021) developed the TPA. In the TPA, they decoupled adjustable binary variables and adjustable continuous variables for making decisions. TPA decided the adjustable binary

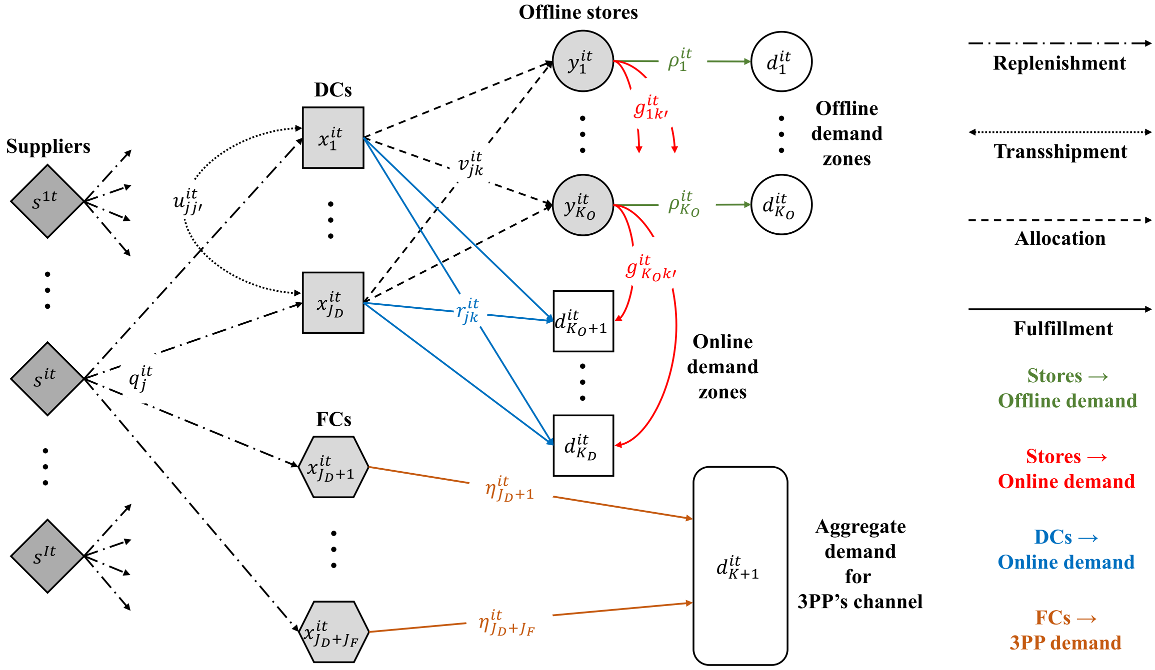


**Fig. 1.** Supply chain network of the proposed problem. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

variables by a static rule of TRO and resorted to the LDR for determining the adjustable continuous variables. The experimental results showed that the TPA outperformed existing approaches, BDR and FA, for both solution quality and computational efficiency.

Even though the TPA has shown outstanding performance compared to existing approaches, it could not be scalable to our problem. The TPA has required a significant computational burden for large-scale instances because our problem considers the retailer's supply chain (online and offline channels) and the supply chain of the 3PP (3PP channel) simultaneously. Therefore, our study develops the DECOM approach, which could be scalable to large-scale problems.

## 3. Problem description and mathematical model

### *3.1. Problem description*

We consider a model in which a retailer sells products, $i \in \mathcal{I}$, to customers through several sales channels as shown in Fig. 1. By following the assumption of Jiu (2022), we also assume that a retailer replenishes each individual product from a single supplier (i.e., each product $i$ can only be provided from the corresponding supplier $i$). Also, each supplier $i$ has a limited production capacity, $s^{it}$. Furthermore, we assume that each product $i$ can only be provided from the corresponding supplier $i$. There are three types of sales channels (1) a retailer's offline channel, (2) a retailer's online channel, and (3) the 3PP channel. The supply chain network consists of multiple capacitated logistics centers, $j \in \mathcal{J}$, and offline stores, $k \in \mathcal{K}_O$, several logistics centers, $j \in \mathcal{J}_D$, operated by the retailer, which is called *DC*, and the others, $j \in \mathcal{J}_F$, operated by the 3PP, which is called *FC*. In the case of the retailer's offline channel, we assume that the offline store $k$ is located at each offline demand zone $k$. Therefore, each demand zone is fulfilled by the corresponding offline store. For the retailer's online channel, there are multiple online demand zones. On the other hand, for the 3PP channel, we consider the aggregate demand for FCs because the 3PP company can deliver products from FCs to customers using its SLSS. It should be noted that our model can be easily extended to the general case, the multiple online demand zones for FCs, by defining the set of online demand zones for FCs. We do not anticipate any customer switching between channels if there is a stockout. We consider some features of the omnichannel supply chain network by referring to Jiu (2022). In particular, we consider one of the significant advantages of the omnichannel setup: The retailer's online demand can be fulfilled by the inventory held in the offline stores (i.e., the ship-from-store for online demands). In contrast to existing studies, we also consider transshipment between DCs and suppliers' production capacity in our model.

We accommodate the following features of the 3PP channel in our model by deeply investigating the business model of Coupang. First, products to be sold in the 3PP channel are stored in the fulfillment center of the 3PP (i.e., FC). Therefore, the demand for the 3PP channel is satisfied by inventories held in FCs. Second, even though the 3PP company implements logistics of fulfillment on behalf of the retailer, the retailer should transport the replenished products from suppliers to the 3PP company's FCs. After that, these replenished products are handled by the SLSS service of the 3PP company. Therefore, the unit replenishment cost from suppliers to FCs, $oc^t_{ij}$, $\forall j \in \mathcal{J}_F$, is determined based on the distance between supplier $i$ and FC $j$. Third, the 3PP company delivers products to satisfy demand in the 3PP channel on behalf of a retailer. In real business, the delivery fee is charged per product, $i$, $af^{it}_j, \forall j \in \mathcal{J}_F$, regardless of distances between FCs and customer demand zones. Therefore, the distance between an FC and the demand zone is not considered for the fulfillment cost in the 3PP channel. Fourth, various types of additional costs are incurred when using the 3PP channel as follows:

- *Fixed participation cost ($\lambda^s$):* A fixed participation fee could be incurred when the retailer uses the 3PP channel (Ryan et al., 2012). We implicitly accommodate this type of fee in $S^{it}_j$, $\forall j \in \mathcal{J}_F$.
- *Warehousing cost ($\lambda^o$):* Because the process for packaging and moving products is required when the 3PP company receives products in FCs, the warehousing fee is charged per product to users. Therefore, we consider this type of cost in $oc^t_{ij}$, $\forall j \in \mathcal{J}_F$.
- *Inventory holding cost ($\lambda^h$):* The 3PP company charges a storage fee to users when they store their products in FCs. Therefore, this results in higher inventory holding costs when the retailer stores products in FCs instead of in DCs. We accommodate this property in $lh^{it}_j, \forall j \in \mathcal{J}_F$.

In summary, the costs parameters $S^{it}_j, lh^{it}_j$, and $oc^t_{ij}$ for the 3PP channel ($\forall j \in \mathcal{J}_F$) are more expensive than they are for the retailer ($\forall j \in \mathcal{J}_D$). We reflect these features with parameters $\lambda^s$, $\lambda^h$, and $\lambda^o$.

We consider a multi-period problem with a finite horizon divided into period $t \in \mathcal{T}$. For each period $t$, the retailer makes decisions for the replenishment, transshipment, allocation, and fulfillment, and the following sequence of an event is repeated:

1. At the start of period $t$, the quantity of product $i$ replenished at $t - L^i_j$ period arrives at the logistics center $j$. The retailer decides how many products to order for each logistics center $j$ from each supplier $i$ (i.e., replenishment decision, $\delta^{it}_j$ and $q^{it}_j$).
2. The retailer then decides the transshipment quantity between DCs and how many products to allocate from DCs to offline stores (i.e., transshipment and allocation decisions, $u^{it}_{jj'}$ and $v^{it}_{jk}$).
3. Each type of demand is realized at the end of period $t$. The retailer decides how many products to fulfill for each type of demand, and from which DCs, FCs, and offline stores to fulfill it (i.e., fulfillment decision, $\rho^{it}_k, \eta^{it}_j, g^{it}_{kk'}$ and $r^{it}_{jk}$). If customers face a stockout, the demand gets lost, which is a general assumption in retail environments (Goedhart et al., 2023).

Fig. 1 describes the retailer's supply chain and four types of decisions. We represent four types of fulfillment decisions with the solid arrow in different colors. We utilize the notations in Table A.6 to formulate mathematical models.

The total cost incurred in the supply chain consists of eleven cost components: (1) the fixed cost to place an order, $S^{it}_j\delta^{it}_j$, (2) the per-unit ordering cost, $oc^t_{ij}q^{it}_j$, (3) the inventory holding cost for DCs and FCs, $lh^{it}_j x^{i,t+1}_j$, (4) the inventory holding cost for offline stores, $oh^{it}_k y^{i,t+1}_k$, (5) the stockout cost, $p^{it}_k z^{it}_k$, (6) the fulfillment cost from offline stores to online demand zones, $of^{it}_{kk'}g^{it}_{kk'}$, (7) the transshipment cost between DCs, $tc^{it}_{jj'}u^{it}_{jj'}$, (8) the allocation cost from DCs to offline stores, $ac^{it}_{jk}v^{it}_{jk}$, (9) the fulfillment cost from DCs to online demand zones, $ef^{it}_{jk}r^{it}_{jk}$, (10) the fulfillment cost for offline demand zones, $bf^{it}_k\rho^{it}_k$, and (11) the fulfillment cost for the aggregate demand for the 3PP channel, $af^{it}_j\eta^{it}_j$. We present the deterministic model ($\text{P}_{\text{DET}}$) where all demand is known in Appendix A.

### 3.2. Stochastic optimization model with the demand uncertainty

In this section, we present the stochastic optimization model to accommodate the demand uncertainty. We use $\tilde{d}^{it}_k$ to denote random demand $k$ for product $i$ at period $t$ for all $i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}$. The mean values of demand $\tilde{d}^{it}_k$ are denoted as $\bar{d}^{it}_k$. Also, we use $d^{it}_k$ to denote the realization of the demand. Let $\tilde{\mathbf{d}}^t = \left(\tilde{d}^{i\tau}_k, \forall i \in \mathcal{I}, k \in \mathcal{K}, \tau \in \{1, \dots, t\}\right)$ denote a collection of all demands from period 1 to period $t$, and $\tilde{\mathbf{d}}$ denotes $\tilde{\mathbf{d}}^T$. The realization of the demand $\tilde{\mathbf{d}}^t$ and $\tilde{\mathbf{d}}$ are denoted as $\mathbf{d}^t$ and $\mathbf{d}$, respectively.

In the proposed stochastic optimization model, we consider the *adjustable decision variables* to accommodate two different types of decisions (i.e., anticipative and reactive manners). The adjustable decision variables can postpone the decision until some portion of the demand is realized (i.e., wait-and-see decisions), which is different from the process that every decision should be made at the start of period 1 (i.e., here-and-now decisions). We define the adjustable decision variables as presented in Table A.6. It should be noted that only the $\delta^{it}_j(\tilde{\mathbf{d}}^{t-1})$ are the *adjustable binary variables*, and the others are the *adjustable continuous variables*. In addition, because $\delta^{it}_j(\tilde{\mathbf{d}}^{t-1}), q^{it}_j(\tilde{\mathbf{d}}^{t-1}), x^{it}_j(\tilde{\mathbf{d}}^{t-1}), u^{it}_{jj'}(\tilde{\mathbf{d}}^{t-1})$, and $v^{it}_{jk}(\tilde{\mathbf{d}}^{t-1})$ are decided at the start of period $t$, these decisions are determined based on the anticipative manner. On the other hand, because $\rho^{it}_k(\tilde{\mathbf{d}}^t), \eta^{it}_j(\tilde{\mathbf{d}}^t), g^{it}_{kk'}(\tilde{\mathbf{d}}^t), r^{it}_{jk}(\tilde{\mathbf{d}}^t)$, and $z^{it}_k(\tilde{\mathbf{d}}^t)$ are decided at the end of period $t$, these decisions are determined based on the reactive manner. For ease of the exposition, let $\delta(\tilde{\mathbf{d}}) = \left(\delta^{it}_j(\tilde{\mathbf{d}}^{t-1}), \forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}\right)$ denote a collection of the adjustable binary variables. We use notations $\boldsymbol{\pi}(\tilde{\mathbf{d}})$ and $\boldsymbol{\mu}(\tilde{\mathbf{d}})$ to denote a collection of the adjustable continuous variables determined based on the anticipative and reactive manners, respectively:

$$\boldsymbol{\pi}(\tilde{\mathbf{d}}) = \left(q^{it}_j(\tilde{\mathbf{d}}^{t-1}), x^{it}_j(\tilde{\mathbf{d}}^{t-1}), y^{it}_k(\tilde{\mathbf{d}}^{t-1}), u^{it}_{jj'}(\tilde{\mathbf{d}}^{t-1}), v^{it}_{jk}(\tilde{\mathbf{d}}^{t-1}),\ \forall i \in \mathcal{I}, j \in \mathcal{J}, j' \in \mathcal{J}, k \in \mathcal{K}_O, t \in \mathcal{T}\right),$$

$$\boldsymbol{\mu}(\tilde{\mathbf{d}}) = \left(\rho^{it}_k(\tilde{\mathbf{d}}^t), \eta^{it}_j(\tilde{\mathbf{d}}^t), g^{it}_{kk'}(\tilde{\mathbf{d}}^t), r^{it}_{jk'}(\tilde{\mathbf{d}}^t), z^{it}_{k''}(\tilde{\mathbf{d}}^t),\ \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}_O, k' \in \mathcal{K}_D, k'' \in \mathcal{K}, t \in \mathcal{T}\right).$$

If the demand is given as $\mathbf{d}$, the total cost incurred in the supply chain is defined as follows:

$$\boldsymbol{\Gamma}\left(\delta(\mathbf{d}), \boldsymbol{\pi}(\mathbf{d}), \boldsymbol{\mu}(\mathbf{d})\right) =$$

$$\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\sum_{j\in\mathcal{J}} S_j^{it}\delta_j^{it}(\mathbf{d}^{t-1}) + \sum_{j\in\mathcal{J}} oc_{ij}^t q_j^{it}(\mathbf{d}^{t-1}) + \sum_{j\in\mathcal{J}} lh_j^{it} x_j^{i,t+1}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}_O} oh_k^{it} y_k^{i,t+1}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}} p_k^{it} z_k^{it}(\mathbf{d}^t) + \sum_{j\in\mathcal{J}_D}\sum_{j'\in\mathcal{J}_D} tc_{jj'}^{it} u_{jj'}^{it}(\mathbf{d}^{t-1})\right.$$
$$\left. + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_O} ac_{jk}^{it} v_{jk}^{it}(\mathbf{d}^{t-1}) + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_D} ef_{jk}^{it} r_{jk}^{it}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}_O}\sum_{k'\in\mathcal{K}_D} of_{kk'}^{it} g_{kk'}^{it}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}_O} bf_k^{it}\rho_k^{it}(\mathbf{d}^t) + \sum_{j\in\mathcal{J}_F} af_j^{it}\eta_j^{it}(\mathbf{d}^t)\right).$$

We propose the following stochastic optimization model ($\mathrm{P_{STOC}}$) by accommodating the demand uncertainty:

($\mathrm{P_{STOC}}$)

$$\min \quad \mathbb{E}_{\tilde{\mathbf{d}}}\left[\boldsymbol{\Gamma}\left(\boldsymbol{\delta}(\tilde{\mathbf{d}}), \boldsymbol{\pi}(\tilde{\mathbf{d}}), \boldsymbol{\mu}(\tilde{\mathbf{d}})\right)\right] \tag{1}$$

$$\text{s.t.} \quad q_j^{it}(\tilde{\mathbf{d}}^{t-1}) \le \bar{q}_j^i \delta_j^{it}(\tilde{\mathbf{d}}^{t-1}), \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T} \tag{2}$$

$$\sum_{j\in\mathcal{J}} q_j^{it}(\tilde{\mathbf{d}}^{t-1}) \le s^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{3}$$

$$\sum_{i\in\mathcal{I}}\left(x_j^{it}(\tilde{\mathbf{d}}^{t-1}) + q_j^{i,t-L_j^i}(\tilde{\mathbf{d}}^{t-L_j^i-1})\right) \le \bar{x}_j, \qquad \forall j\in\mathcal{J}_F, t\in\mathcal{T} \tag{4}$$

$$\sum_{i\in\mathcal{I}}\left(x_j^{it}(\tilde{\mathbf{d}}^{t-1}) + q_j^{i,t-L_j^i}(\tilde{\mathbf{d}}^{t-L_j^i-1}) + \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{j'j}^{it}(\tilde{\mathbf{d}}^{t-1}) - \sum_{k\in\mathcal{K}_O} v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1}) - \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{jj'}^{it}(\tilde{\mathbf{d}}^{t-1})\right) \le \bar{x}_j, \qquad \forall j\in\mathcal{J}_D, t\in\mathcal{T} \tag{5}$$

$$x_j^{it}(\tilde{\mathbf{d}}^{t-1}) + q_j^{i,t-L_j^i}(\tilde{\mathbf{d}}^{t-L_j^i-1}) \ge \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{jj'}^{it}(\tilde{\mathbf{d}}^{t-1}), \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T} \tag{6}$$

$$\sum_{i\in\mathcal{I}}\left(y_k^{it}(\tilde{\mathbf{d}}^{t-1}) + \sum_{j\in\mathcal{J}_D} v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1})\right) \le \bar{y}_k, \qquad \forall k\in\mathcal{K}_O, t\in\mathcal{T} \tag{7}$$

$$\sum_{j\in\mathcal{J}_D} r_{jk}^{it}(\tilde{\mathbf{d}}^t) + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it}(\tilde{\mathbf{d}}^t) + z_k^{it}(\tilde{\mathbf{d}}^t) = \tilde{d}_k^{it}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_D, t\in\mathcal{T} \tag{8}$$

$$\rho_k^{it}(\tilde{\mathbf{d}}^t) + z_k^{it}(\tilde{\mathbf{d}}^t) = \tilde{d}_k^{it}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T} \tag{9}$$

$$\sum_{j\in\mathcal{J}_F} \eta_j^{it}(\tilde{\mathbf{d}}^t) + z_{K+1}^{it}(\tilde{\mathbf{d}}^t) = \tilde{d}_{K+1}^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{10}$$

$$x_j^{i,t+1}(\tilde{\mathbf{d}}^t) = x_j^{it}(\tilde{\mathbf{d}}^{t-1}) + q_j^{i,t-L_j^i}(\tilde{\mathbf{d}}^{t-L_j^i-1}) + \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{j'j}^{it}(\tilde{\mathbf{d}}^{t-1}) - \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{jj'}^{it}(\tilde{\mathbf{d}}^{t-1}) - \sum_{k\in\mathcal{K}_O} v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1}) - \sum_{k\in\mathcal{K}_D} r_{jk}^{it}(\tilde{\mathbf{d}}^t), \quad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T} \tag{11}$$

$$x_j^{i,t+1}(\tilde{\mathbf{d}}^t) = x_j^{it}(\tilde{\mathbf{d}}^{t-1}) + q_j^{i,t-L_j^i}(\tilde{\mathbf{d}}^{t-L_j^i-1}) - \eta_j^{it}(\tilde{\mathbf{d}}^t), \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T} \tag{12}$$

$$y_k^{i,t+1}(\tilde{\mathbf{d}}^t) = y_k^{it}(\tilde{\mathbf{d}}^{t-1}) + \sum_{j\in\mathcal{J}_D} v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1}) - \sum_{k'\in\mathcal{K}_D} g_{kk'}^{it}(\tilde{\mathbf{d}}^t) - \rho_k^{it}(\tilde{\mathbf{d}}^t), \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T} \tag{13}$$

$$q_j^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, q_j^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \delta_j^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{B}^{t-1}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T} \tag{14}$$

$$x_j^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, x_j^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}^+ \tag{15}$$

$$y_k^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, y_k^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T}^+ \tag{16}$$

$$u_{jj'}^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, u_{jj'}^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall j\in\mathcal{J}_D, j'\in\mathcal{J}_D, i\in\mathcal{I}, t\in\mathcal{T} \tag{17}$$

$$v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall j\in\mathcal{J}_D, k\in\mathcal{K}_O, i\in\mathcal{I}, t\in\mathcal{T} \tag{18}$$

$$\rho_k^{it}(\tilde{\mathbf{d}}^t) \ge 0, \rho_k^{it}(\tilde{\mathbf{d}}^t) \in \mathcal{R}^t, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T} \tag{19}$$

$$\eta_j^{it}(\tilde{\mathbf{d}}^t) \ge 0, \eta_j^{it}(\tilde{\mathbf{d}}^t) \in \mathcal{R}^t, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T} \tag{20}$$

$$g_{kk'}^{it}(\tilde{\mathbf{d}}^t) \ge 0, g_{kk'}^{it}(\tilde{\mathbf{d}}^t) \in \mathcal{R}^t, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, k'\in\mathcal{K}_D, t\in\mathcal{T} \tag{21}$$

$$r_{jk}^{it}(\tilde{\mathbf{d}}^t) \ge 0, r_{jk}^{it}(\tilde{\mathbf{d}}^t) \in \mathcal{R}^t, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, k\in\mathcal{K}_D, t\in\mathcal{T} \tag{22}$$

$$z_k^{it}(\tilde{\mathbf{d}}^t) \ge 0, z_k^{it}(\tilde{\mathbf{d}}^t) \in \mathcal{R}^t, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}, t\in\mathcal{T} \tag{23}$$

where $\mathcal{R}^\tau$ and $\mathcal{B}^\tau$ functions are mapping from $\mathbb{R}^{I\times\tau\times(K+1)}$ to $\mathbb{R}$ and $\{0,1\}$, respectively. The objective function (1) minimizes the expected total cost incurred within the supply chain. Constraint (2) represents that if products are ordered, a fixed ordering cost is incurred. Constraint (3) enforces that the total number of products replenished from supplier $i$ cannot exceed the given production capacity $s^{it}$. Constraint (4) enforces that the inventory of the FC $j$ cannot exceed its capacity, $\bar{x}_j$, after products arrive. Constraint (5) also represents the storage capacity constraint for the DC $j$ considering the replenishment, transshipment, and allocation quantities. Constraint (6) represents that the number of products transshipped from the DC $j$ to other DCs should be less than the inventory of the DC $j$. Constraint (7) restricts that the inventory of the offline store $k$ cannot exceed its capacity, $\bar{y}_k$, after products arrive. Constraints

(8), (9), and (10) ensure that the demand is satisfied by inventories held in DCs, offline stores, and FCs, respectively. Moreover, these constraints ensure that all unsatisfied demand becomes lost. Constraints (11), (12), and (13) are the balance equations for inventories of DCs, FCs, and offline stores, respectively. Finally, Constraints (14)–(23) ensure that adjustable continuous variables are non-negative real variables, except for $\delta_j^{it}(\tilde{\mathbf{d}}^{t-1})$, which are adjustable binary variables. The two advantages of adopting the 3PP channel (i.e., (1) SLSS and (2) Customers in the 3PP channel) are reflected in the proposed mathematical model with Constraints (4), (10), and (15). In detail, we accommodate the first advantage by defining the fulfillment decisions for the 3PP channel as $\eta_j^{it}(\tilde{\mathbf{d}})$, which does not consider the index for the locations of customer demand zones, $k$, because the SLSS of the 3PP company delivers product on behalf of the retailer. To accommodate the second advantage, we define Constraint (15), which ensures that the demand for the 3PP channel can only be fulfilled by the supply chain operations of the 3PP. Therefore, the retailer can absorb the additional demand by adopting the 3PP channel.

The $\mathrm{P_{STOC}}$ aims to minimize the total expected cost, and every constraint must be satisfied for all demand realizations. The $\mathrm{P_{STOC}}$ is the multistage stochastic optimization problem that is generally intractable to solve (Shapiro and Nemirovski, 2005). Traditionally, dynamic programming or multistage stochastic programming methods are used to solve the stochastic optimization problem by characterizing demand uncertainty with a known probability distribution. However, assumptions about demand distribution could be unrealistic if a decision maker has insufficient demand data. If the gap between true demand and assumed distributions is large, solutions derived from these methods could show poor performance in practice. Furthermore, the existence of $\delta(\tilde{\mathbf{d}})$ increases the computational complexity of $\mathrm{P_{STOC}}$ significantly. Before explaining the proposed approach, we briefly introduce how we customize the TPA for our problem in the following section.

## 4. A two-phase approach (TPA) based on robust optimization

TPA solves the proposed problem by decoupling binary decision variables and the continuous decision variables. The binary decisions are determined with the static rule (i.e., $\delta(\tilde{\mathbf{d}}^{t-1}) = \delta$) by utilizing a TRO (Lim and Wang, 2017) in Phase 1. In Phase 2, we adaptively decide the continuous variables by utilizing the LDR with an objective of minimizing the worst-case expected total cost (Ben-Tal et al., 2004). In order to adopt a TPA, it is assumed that the demand $\tilde{d}_k^{it}$ is $\hat{d}_k^{it}$ mean random variables and fall in a support set $\left[\underline{d}_k^{it}, \bar{d}_k^{it}\right], \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}$. Considering this assumption, the uncertainty set is defined as $D_k^{it} := \left\{d_k^{it} \mid \underline{d}_k^{it} \le d_k^{it} \le \bar{d}_k^{it}\right\}$ for each $\tilde{d}_k^{it}, \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}$.

### 4.1. Phase 1 of TPA

The binary decisions are determined by utilizing the TRO aiming to maximize the size of the uncertainty set and make a total cost lower than a predetermined cost target. Lim and Wang (2017) proved that a static rule is optimal for TRO formulation and showed that the computational burden could be reduced significantly. In order to reformulate $\mathrm{P_{STOC}}$ into the TRO model, we define the *adjustable uncertainty set* for each $\tilde{d}_k^{it}$ as $D_k^{it}(\gamma) := \left\{d_k^{it} \mid \hat{d}_k^{it} - \gamma\underline{\zeta}_k^{it} \le d_k^{it} \le \hat{d}_k^{it} + \gamma\bar{\zeta}_k^{it}\right\}$ where $\underline{\zeta}_k^{it} = \hat{d}_k^{it} - \underline{d}_k^{it}$ and $\bar{\zeta}_k^{it} = \bar{d}_k^{it} - \hat{d}_k^{it}$. For notational convenience, let $\mathbf{D}^t(\gamma) = \left(D_k^{i\tau}(\gamma),\ \forall k \in \mathcal{K}, i \in \mathcal{I},\ \tau \in \{1,\dots,t\}\right)$ and $\mathbf{D}(\gamma) = \mathbf{D}^T(\gamma)$. In addition, we define a *cost target* $\psi$ to restrict total cost to be no more than a predetermined value $\psi$ under any demand realizations. We present the TRO model, $\mathrm{P_{TRO}}$, as follows:

$$
\begin{aligned}
&(\mathrm{P_{TRO}})\\
&\gamma^* = \max\ \gamma\\
&\text{s.t.}\quad \boldsymbol{\Gamma}\left(\boldsymbol{\delta}(\mathbf{d}), \boldsymbol{\pi}(\mathbf{d}), \boldsymbol{\mu}(\mathbf{d})\right) \le \psi, \qquad \forall \mathbf{d} \in \mathbf{D}(\gamma)\\
&\qquad \text{Constraints (2)–(23)}, \qquad \forall \mathbf{d}^t \in \mathbf{D}^t(\gamma)\\
&\qquad 0 \le \gamma \le 1
\end{aligned}
$$

The objective of the model is to absorb as much uncertainty as by maximizing the sizes of the adjustable uncertainty set. We control the sizes of adjustable uncertainty set by adopting the new decision variable $\gamma$ $(0 \le \gamma \le 1)$. Simultaneously, the total cost must be lower than a cost target $\psi$ as indicated in the first constraint. The other constraints are the same as $\mathrm{P_{STOC}}$. However, the equality constraints (8)–(10) could cause an infeasibility issue if the static rule is adopted. Fortunately, we can overcome this issue by allowing Constraints (8)–(10) to be relaxed from equality to inequality as follows:

$$(\mathrm{P_{TRO-R}})$$

$$\gamma' = \max\ \gamma \tag{24}$$

$$\text{s.t.}\quad \boldsymbol{\Gamma}\left(\boldsymbol{\delta}(\mathbf{d}), \boldsymbol{\pi}(\mathbf{d}), \boldsymbol{\mu}(\mathbf{d})\right) \le \psi, \qquad \forall \mathbf{d} \in \mathbf{D}(\gamma) \tag{25}$$

$$\sum_{j\in\mathcal{J}_D} r_{jk}^{it}(\mathbf{d}^t) + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it}(\mathbf{d}^t) + z_k^{it}(\mathbf{d}^t) \ge d_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D, \qquad \forall \mathbf{d}^t \in \mathbf{D}^t(\gamma) \tag{26}$$

$$\rho_k^{it}(\mathbf{d}^t) + z_k^{it}(\mathbf{d}^t) \ge d_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O, \qquad \forall \mathbf{d}^t \in \mathbf{D}^t(\gamma) \tag{27}$$

$$\sum_{j\in\mathcal{J}_F} \eta_j^{it}(\mathbf{d}^t) + z_{K+1}^{it}(\mathbf{d}^t) \ge d_{K+1}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, \qquad \forall \mathbf{d}^t \in \mathbf{D}^t(\gamma) \tag{28}$$

$$\text{Constraints (2)–(7), (11)–(23)} \qquad \forall \mathbf{d}^t \in \mathbf{D}^t(\gamma) \tag{29}$$

$$0 \leq \gamma \leq 1 \tag{30}$$

It should be noted that Constraints (26)–(28) lead to $\gamma^* \leq \gamma'$.

We define *uncertainty variables* $n_k^{it}$ and $m_k^{it}$ falling in $N_k^{it}(\gamma) := \left\{ n_k^{it} \mid 0 \leq n_k^{it} \leq \gamma \underline{\zeta}_k^{it} \right\}$ and $M_k^{it}(\gamma) := \left\{ m_k^{it} \mid 0 \leq m_k^{it} \leq \gamma \bar{\zeta}_k^{it} \right\}$, respectively. By adopting uncertainty variables, we can tighten constraints in $\mathrm{P_{TRO-R}}$, and each demand can be represented as $d_k^{it} = \hat{d}_k^{it} - n_k^{it} + m_k^{it}$, $\forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}$. For simplicity, we use boldface notation to denote collections of $n_k^{it}, m_k^{it}, N_k^{it}(\gamma)$, and $M_k^{it}(\gamma)$ as

$$\mathbf{n}^t = \left( n_k^{i\tau},\ \forall i \in \mathcal{I}, k \in \mathcal{K}, \tau \in \{1, \dots, t\} \right), \ \mathbf{m}^t = \left( m_k^{i\tau},\ \forall i \in \mathcal{I}, k \in \mathcal{K}, \tau \in \{1, \dots, t\} \right),$$
$$\mathbf{N}^t(\gamma) = \left( N_k^{i\tau}(\gamma),\ \forall i \in \mathcal{I}, k \in \mathcal{K}, \tau \in \{1, \dots, t\} \right), \ \mathbf{M}^t(\gamma) = \left( M_k^{i\tau}(\gamma),\ \forall i \in \mathcal{I}, k \in \mathcal{K}, \tau \in \{1, \dots, t\} \right).$$

By replacing $d_k^{it}$ with $\hat{d}_k^{it} + m_k^{it}$ in Constraints (26)–(28), $\mathrm{P_{TRO-R}}$ can be approximated as follows:

$(\mathrm{P_{TRO-A}})$

$$\begin{aligned}
\gamma'' = \ & \max \ \gamma \\
\text{s.t.} \ & \boldsymbol{\Gamma}\left(\boldsymbol{\delta}(\mathbf{d}), \boldsymbol{\pi}(\mathbf{d}), \boldsymbol{\mu}(\mathbf{d})\right) \leq \psi, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \\
& \sum_{j \in \mathcal{J}_D} r_{jk}^{it}(\mathbf{d}^t) + \sum_{k' \in \mathcal{K}_O} g_{k'k}^{it}(\mathbf{d}^t) + z_k^{it}(\mathbf{d}^t) \geq \hat{d}_k^{it} + m_k^{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \\
& \rho_k^{it}(\mathbf{d}^t) + z_k^{it}(\mathbf{d}^t) \geq \hat{d}_k^{it} + m_k^{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \\
& \sum_{j \in \mathcal{J}_F} \eta_j^{it}(\mathbf{d}^t) + z_{K+1}^{it}(\mathbf{d}^t) \geq \hat{d}_{K+1}^{it} + g_{K+1}^{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \\
& \text{Constraints (2)–(7), (11)–(23)}, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \\
& 0 \leq \gamma \leq 1
\end{aligned}$$

Because constraints in $\mathrm{P_{TRO-A}}$ are tighter than those of Problem $\mathrm{P_{TRO-R}}$, it is obvious that $\gamma'' \leq \gamma'$.

We consider a static rule; thus, decisions are fixed regardless of the revealed uncertainties. Therefore, every adjustable variable is replaced with the decision variables of the deterministic problem (e.g., $\delta_j^{it}(\mathbf{d}^{t-1}) \to \delta_j^{it}$ and $\boldsymbol{\delta}(\mathbf{d}) \to \boldsymbol{\delta}$). We define the total cost for the static rule as follows:

$$\boldsymbol{\Gamma}^{\dagger}(\boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{\mu}) =$$
$$\sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left( \sum_{j \in \mathcal{J}} S_j^{it} \delta_j^{it} + \sum_{j \in \mathcal{J}} oc_{ij}^t q_j^{it} + \sum_{j \in \mathcal{J}} lh_j^{it} x_j^{i,t+1} + \sum_{k \in \mathcal{K}_O} oh_k^{it} y_k^{i,t+1} + \sum_{k \in \mathcal{K}} p_k^{it} z_k^{it} + \sum_{k \in \mathcal{K}_O} \sum_{k' \in \mathcal{K}_D} of_{kk'}^{it} g_{kk'}^{it} + \sum_{j \in \mathcal{J}_D} \sum_{j' \in \mathcal{J}_D} tc_{jj'}^{it} u_{jj'}^{it} \right.$$
$$\left. + \sum_{j \in \mathcal{J}_D} \sum_{k \in \mathcal{K}_O} ac_{jk}^{it} v_{jk}^{it} + \sum_{j \in \mathcal{J}_D} \sum_{k \in \mathcal{K}_D} ef_{jk}^{it} r_{jk}^{it} + \sum_{k \in \mathcal{K}_O} bf_k^{it} \rho_k^{it} + \sum_{j \in \mathcal{J}_F} af_j^{it} \eta_j^{it} \right).$$

The static rule can be derived by solving the following $\mathrm{P_{TRO-S}}$:

$(\mathrm{P_{TRO-S}})$

$$\gamma^s = \max \quad \gamma \tag{31}$$
$$\text{s.t.} \quad \boldsymbol{\Gamma}^{\dagger}(\boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{\mu}) \leq \psi \tag{32}$$
$$\sum_{j \in \mathcal{J}_D} r_{jk}^{it} + \sum_{k' \in \mathcal{K}_O} g_{k'k}^{it} + z_k^{it} \geq \hat{d}_k^{it} + m_k^{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \tag{33}$$
$$\rho_k^{it} + z_k^{it} \geq \hat{d}_k^{it} + m_k^{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \tag{34}$$
$$\sum_{j \in \mathcal{J}_F} \eta_j^{it} + z_{K+1}^{it} \geq \hat{d}_{K+1}^{it} + g_{K+1}^{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad \forall \mathbf{n}^t \in \mathbf{N}^t(\gamma), \mathbf{m}^t \in \mathbf{M}^t(\gamma) \tag{35}$$
$$\text{Constraints (A.2)–(A.7), (A.11)–(A.23)} \tag{36}$$
$$0 \leq \gamma \leq 1 \tag{37}$$

We could know that $\gamma^s \leq \gamma''$ because decisions with the static rule are more restrictive than adjustable decisions. Before presenting an approach to derive an optimal static rule for $\mathrm{P_{TRO-S}}$, we use the notation $\theta$ to denote a collection of uncertainty variables $n_k^{it}$ and $m_k^{it}$ (i.e., $\theta = \left( n_k^{it}, m_k^{it},\ \forall i \in \mathcal{I},\ k \in \mathcal{K},\ t \in \mathcal{T} \right)$). Given $\gamma$, let $\boldsymbol{\Theta}(\gamma)$ denote the support set of $\theta$. For ease of the exposition, we represent $\mathrm{P_{TRO-S}}$ as the following simple form:

$$\begin{aligned}
\gamma^s = \ & \max \quad \gamma \\
& \text{s.t.} \quad \mathbf{C}(\theta)\boldsymbol{\kappa} \leq \mathbf{e}(\theta), && \forall \theta \in \boldsymbol{\Theta}(\gamma) \\
& \qquad \boldsymbol{\kappa} \in \mathcal{Y}, && \forall \theta \in \boldsymbol{\Theta}(\gamma)
\end{aligned}$$

where $\mathbf{C}(\theta)$ and $\mathbf{e}(\theta)$ represent all coefficients, and $\boldsymbol{\kappa}$ and $\mathcal{Y}$ represent decision variables for the static rule and the feasible set, respectively. We present the definition of the *worst-case scenario of uncertainty* as follows:

**Definition 1** (*Worst-case Scenario of Uncertainty (WSU)*)**.** Given the coefficients $\mathbf{C}(\theta)$ and $\mathbf{e}(\theta)$ in $\mathrm{P}_{\mathrm{TRO-S}}$, we call an element $\breve{\theta}(\gamma) \in \boldsymbol{\Theta}(\gamma)$ as the WSU if for each $\boldsymbol{\kappa} \in \mathcal{Y}$ that satisfies $\mathbf{C}(\breve{\theta}(\gamma))\boldsymbol{\kappa} \le \mathbf{e}(\breve{\theta}(\gamma))$, it also satisfies $\mathbf{C}(\theta)\boldsymbol{\kappa} \le \mathbf{e}(\theta)$, $\forall \theta \in \boldsymbol{\Theta}(\gamma)$.

We reformulate $\mathrm{P}_{\mathrm{TRO-S}}$ with the WSU $\breve{\theta}(\gamma)$ by replacing the right-hand side inequality Constraints (33)–(35) from $\hat{d}_k^{it} + m_k^{it}$ to $\hat{d}_k^{it} + \gamma\bar{\zeta}_k^{it}$. Finally, the problem with the WSU is defined as the following deterministic problem:

$$
\begin{aligned}
&(\mathrm{P}_{\mathrm{STATIC}})\\
&\qquad \gamma^{\dagger} = \max \quad \gamma\\
&\qquad \text{s.t.} \quad \boldsymbol{\Gamma}^{\dagger}(\boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{\mu}) \le \psi\\
&\qquad\qquad \sum_{j\in\mathcal{J}_D} r_{jk}^{it} + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it} + z_k^{it} \ge \hat{d}_k^{it} + \gamma\bar{\zeta}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D\\
&\qquad\qquad \rho_k^{it} + z_k^{it} \ge \hat{d}_k^{it} + \gamma\bar{\zeta}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O\\
&\qquad\qquad \sum_{j\in\mathcal{J}_F} \eta_j^{it} + z_{K+1}^{it} \ge \hat{d}_{K+1}^{it} + \gamma\bar{\zeta}_{K+1}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}\\
&\qquad\qquad \text{Constraints (A.2)–(A.7), (A.11)–(A.23)}\\
&\qquad\qquad 0 \le \gamma \le 1
\end{aligned}
$$

Let $\bar{\boldsymbol{\delta}}$ denote the optimal solution of $\boldsymbol{\delta}$ obtained by solving the $\mathrm{P}_{\mathrm{STATIC}}$. Because the constraints of $\mathrm{P}_{\mathrm{STATIC}}$ are more restrictive than those of $\mathrm{P}_{\mathrm{TRO-S}}$, we have $\gamma^{\dagger} \le \gamma^{s} \le \gamma''$. Interestingly, Lim et al. (2021) shows that $\gamma^{\dagger} \ge \gamma^{s} \ge \gamma''$ in Theorem 1. Therefore, we have $\gamma^{\dagger} = \gamma''$; thus, the optimal solution of the deterministic problem $\mathrm{P}_{\mathrm{STATIC}}$ is also optimal for $\mathrm{P}_{\mathrm{TRO-A}}$.

By controlling the cost target $\psi$, a decision maker could choose the degree of conservativeness for the obtained solution. To determine the proper value for $\psi$, we utilize the following affine function of $\phi$, which is called the *target coefficient*:

$$\psi(\phi) := (1-\phi)\nu(1) + \phi\nu$$

where $\nu(1)$ and $\nu(0)$ is the optimal objective of the following deterministic problem:

$$
\begin{aligned}
&(\mathrm{P}_{\mathrm{TPA-}\nu(\gamma)})\\
&\qquad \nu(\gamma) = \min \boldsymbol{\Gamma}^{\dagger}(\boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{\mu})\\
&\qquad \text{s.t.} \quad \sum_{j\in\mathcal{J}_D} r_{jk}^{it} + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it} + z_k^{it} \ge \hat{d}_k^{it} + \gamma\bar{\zeta}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D\\
&\qquad\qquad \rho_k^{it} + z_k^{it} \ge \hat{d}_k^{it} + \gamma\bar{\zeta}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O\\
&\qquad\qquad \sum_{j\in\mathcal{J}_F} \eta_j^{it} + z_{K+1}^{it} \ge \hat{d}_{K+1}^{it} + \gamma\bar{\zeta}_{K+1}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}\\
&\qquad\qquad \text{Constraints (A.2)–(A.7), (A.11)–(A.23)}
\end{aligned}
$$

If the $\phi$ is close to zero, the conservativeness of solutions is increased; otherwise, it is decreased. Until now, we briefly introduced the principle of the TRO approach in this section. For further information, we recommend readers refer to Lim and Wang (2017) and Lim et al. (2021).

### *4.2. Phase 2 of TPA*

In Phase 2, we determine the adjustable continuous variables with fixed binary decisions $\bar{\boldsymbol{\delta}}$ obtained in Phase 1. As mentioned in Section 4.1, without any knowledge on the true demand distribution, only the mean of $\tilde{d}_k^{it}$ (i.e., $\hat{d}_k^{it}$) and the support set $[\underline{d}_k^{it}, \bar{d}_k^{it}]$ are given. In order to deal with distributional ambiguity, we adopt the solution approach proposed by Gilboa and Schmeidler (1989). We first consider $\mathcal{F}$ as a family of distributions of $\tilde{\mathbf{d}}$, and the mean support set $\hat{\mathbf{D}} = \left(\hat{D}_k^{it}, \forall k \in \mathcal{K}, i \in \mathcal{I}, t \in \mathcal{T}\right)$. Let $\mathcal{P}$ denote any distribution of $\tilde{\mathbf{d}}$ included in $\mathcal{F}$, $\mathcal{P} \in \mathcal{F}$; thus, we have $\mathbb{E}_{\mathcal{P}}\left[\tilde{\mathbf{d}}\right] \in \hat{\mathbf{D}}$. Considering a family of distributions $\mathcal{F}$, we solve the following problem with the objective of minimizing the worst-case expected total cost:

$$
\begin{aligned}
&(\mathrm{P}_{\mathrm{ARO}})\\
&\min \max_{\mathcal{P}\in\mathcal{F}} \quad \mathbb{E}_{\mathcal{P}}\left[\boldsymbol{\Gamma}\left(\boldsymbol{\pi}(\tilde{\mathbf{d}}), \boldsymbol{\mu}(\tilde{\mathbf{d}})\right)\right]\\
&\qquad \text{s.t.} \quad q_j^{it}(\mathbf{d}^{t-1}) \le \bar{q}_j^{i}\bar{\delta}_j^{it}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}, \qquad \forall \mathbf{d}^{t-1} \in \mathbf{D}^{t-1}\\
&\qquad\qquad \text{Constraints (3)–(23)}, \qquad \forall \mathbf{d}^{t} \in \mathbf{D}^{t}
\end{aligned}
$$

with the fixed binary decisions $\bar{\delta}_j^{it}$. Because it is generally intractable to solve $\mathrm{P}_{\mathrm{ARO}}$, we rely on optimizing parameterized functions, where the feasible space is restricted to linear functions (i.e., the LDR (Yanıkoğlu et al., 2019)). For each adjustable continuous variable, we define the following LDR:

$$q_j^{it}\left(\mathbf{d}^{t-1}\right) = q_j^{it,0} + \sum_{\sigma\in\mathcal{K}} \sum_{\tau=1}^{t-1} q_j^{it,\sigma\tau} d_{\sigma}^{i\tau}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}$$

$$x_j^{it}\left(\mathbf{d}^{t-1}\right) = x_j^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t-1} x_j^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}^+$$

$$y_k^{it}\left(\mathbf{d}^{t-1}\right) = y_k^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t-1} y_k^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T}^+$$

$$u_{jj'}^{it}\left(\mathbf{d}^{t-1}\right) = u_{jj'}^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t-1} u_{jj'}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, j'\in\mathcal{J}_D, t\in\mathcal{T}$$

$$v_{jk}^{it}\left(\mathbf{d}^{t-1}\right) = v_{jk}^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t-1} v_{jk}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, k\in\mathcal{K}_O, t\in\mathcal{T}$$

$$\rho_k^{it}\left(\mathbf{d}^{t}\right) = \rho_k^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t} \rho_k^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T}$$

$$\eta_j^{it}\left(\mathbf{d}^{t}\right) = \eta_j^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t} \eta_j^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}$$

$$g_{kk'}^{it}\left(\mathbf{d}^{t}\right) = g_{kk'}^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t} g_{kk'}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, k'\in\mathcal{K}_D, t\in\mathcal{T}$$

$$r_{jk}^{it}\left(\mathbf{d}^{t}\right) = r_{jk}^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t} r_{jk}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, k\in\mathcal{K}_D, t\in\mathcal{T}$$

$$z_k^{it}\left(\mathbf{d}^{t}\right) = z_k^{it,0} + \sum_{\sigma\in\mathcal{K}}\sum_{\tau=1}^{t} z_k^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}, t\in\mathcal{T}$$

If the coefficient of the LDR is given, each type of decision is determined as demand is unveiled. We present $\mathrm{P}_{\mathrm{LDR}}$ in Online Appendix A of the supplementary material. We could obtain the coefficient of the LDR by solving $\mathrm{P}_{\mathrm{LDR}}$ considering coefficients as decision variables. We develop the $\mathrm{P}_{\mathrm{LDR}}$ based on Theorem 2 in Lim et al. (2021). $\mathrm{P}_{\mathrm{LDR}}$ can be transformed to the linear deterministic model by duality theory (Ben-Tal et al., 2009). Consequently, the coefficient can be obtained by solving the linear deterministic model with a commercial solver. We present the linear deterministic model transformed from the $\mathrm{P}_{\mathrm{LDR}}$ in Online Appendix B of the supplementary material.

## 5. A decomposition approach (DECOM)

Given cost target $\psi$, three MILP models ($\mathrm{P}_{\mathrm{TPA}-\nu(0)}, \mathrm{P}_{\mathrm{TPA}-\nu(1)}$, and $\mathrm{P}_{\mathrm{STATIC}}$) and one linear programming (LP) model ($\mathrm{P}_{\mathrm{LDR}}$) must be solved for applying the TPA. However, the existence of the supply chain of 3PP and the production capacity constraint increases the complexity of the problem because two supply chains, one for the retailer and the other for the 3PP, should be considered simultaneously. Therefore, it requires a significant computational burden to solve the three MILP models. To alleviate this issue, we develop a DECOM approach which can be regarded as an extended version of the TPA. The key idea of DECOM is to adopt the *artificial variable* $w^{it}$. A collection of the artificial variable is denoted by $\boldsymbol{w} = \left(w^{it}, \forall i\in\mathcal{I}, t\in\mathcal{T}\right)$. The production capacity constraint (A.3) in $\mathrm{P}_{\mathrm{DET}}$ is reformulated as the following constraints by introducing decision variables $\boldsymbol{w}$:

$$\sum_{j\in\mathcal{J}_D} q_j^{it} \le s^{it} w^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{38}$$

$$\sum_{j\in\mathcal{J}_F} q_j^{it} \le s^{it}\left(1 - w^{it}\right), \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{39}$$

$$w^{it} \ge 0, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{40}$$

There are two advantages to using variables $\boldsymbol{w}$. First, given $\boldsymbol{w}$, the feasible region for variables $q_j^{it}, \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}$ can be reduced. Second, $\mathrm{P}_{\mathrm{TPA}-\nu(\gamma)}$ and $\mathrm{P}_{\mathrm{STATIC}}$ can be solved separately for a retailer's supply chain and the supply chain of 3PP. Consequently, these two advantages could significantly reduce the computational burden, and experimental results will be presented in Section 6.

### 5.1. Phase 1 of DECOM

Phase 1 of DECOM aims to determine the binary decision $\delta$, which is similar to Phase 1 of the TPA. Of special note, we also determine the artificial variable $\boldsymbol{w}$ in Phase 1. We use the $\boldsymbol{\delta}_D, \boldsymbol{\pi}_D$, and $\boldsymbol{\mu}_D$ to denote a collection of variables for the retailer's supply chain and the $\boldsymbol{\delta}_F, \boldsymbol{\pi}_F$, and $\boldsymbol{\mu}_F$ for the supply chain of 3PP as follows:

$$\boldsymbol{\delta}_D = \left(\delta_j^{it}, \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T}\right), \quad \boldsymbol{\delta}_F = \left(\delta_j^{it}, \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}\right),$$

$$\boldsymbol{\pi}_D = \left(q_j^{it}, x_j^{it}, y_k^{it}, u_{jj'}^{it}, v_{jk}^{it},\ \forall i\in\mathcal{I}, j\in\mathcal{J}_D, j'\in\mathcal{J}_D, k\in\mathcal{K}_O, t\in\mathcal{T}\right), \quad \boldsymbol{\pi}_F = \left(q_j^{it}, x_j^{it},\ \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}\right),$$

$$\boldsymbol{\mu}_D = \left(\rho_k^{it}, g_{kk'}^{it}, r_{jk'}^{it}, z_{k''}^{it}, \ \forall i \in \mathcal{I}, j \in \mathcal{J}_D, k \in \mathcal{K}_O, k' \in \mathcal{K}_D, k'' \in \mathcal{K}^-, t \in \mathcal{T}\right), \ \boldsymbol{\mu}_F = \left(\eta_j^{it}, z_{K+1}^{it}, \ \forall i \in \mathcal{I}, j \in \mathcal{J}_F, t \in \mathcal{T}\right).$$

Given $\boldsymbol{\delta}_D, \boldsymbol{\delta}_F, \boldsymbol{\pi}_D, \boldsymbol{\pi}_F, \boldsymbol{\mu}_D$, and $\boldsymbol{\mu}_F$, the total cost for retailer's supply chain is defined as

$$\boldsymbol{\Gamma}_D^{\dagger}\left(\boldsymbol{\delta}_D, \boldsymbol{\pi}_D, \boldsymbol{\mu}_D\right) = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\sum_{j\in\mathcal{J}_D} S_j^{it}\delta_j^{it} + \sum_{j\in\mathcal{J}_D} oc_{ij}^t q_j^{it} + \sum_{j\in\mathcal{J}_D} lh_j^{it} x_j^{i,t+1} + \sum_{k\in\mathcal{K}_O} oh_k^{it} y_k^{i,t+1} + \sum_{k\in\mathcal{K}^-} p_k^{it} z_k^{it} + \sum_{k\in\mathcal{K}_O}\sum_{k'\in\mathcal{K}_D} of_{kk'}^{it} g_{kk'}^{it} \right.$$
$$\left. + \sum_{j\in\mathcal{J}_D}\sum_{j'\in\mathcal{J}_D} tc_{jj'}^{it} u_{jj'}^{it} + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_O} ac_{jk}^{it} v_{jk}^{it} + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_D} ef_{jk}^{it} r_{jk}^{it} + \sum_{k\in\mathcal{K}_O} bf_k^{it}\rho_k^{it}\right),$$

and the total cost for the 3PP supply chain is defined as

$$\boldsymbol{\Gamma}_F^{\dagger}\left(\boldsymbol{\delta}_F, \boldsymbol{\pi}_F, \boldsymbol{\mu}_F\right) = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\sum_{j\in\mathcal{J}_F} S_j^{it}\delta_j^{it} + \sum_{j\in\mathcal{J}_F} oc_{ij}^t q_j^{it} + \sum_{j\in\mathcal{J}_F} lh_j^{it} x_j^{i,t+1} + p_{K+1}^{it} z_{K+1}^{it} + \sum_{j\in\mathcal{J}_F} af_j^{it}\eta_j^{it}\right).$$

In Phase 1, we first solve the following MILP problem to determine $\boldsymbol{w}$:

$(\mathrm{P}_{\mathrm{DECOM}-\nu(1)})$

$$\begin{aligned}
\nu(1) = \min\ & \boldsymbol{\Gamma}_D^{\dagger}\left(\boldsymbol{\delta}_D, \boldsymbol{\pi}_D, \boldsymbol{\mu}_D\right) + \boldsymbol{\Gamma}_F^{\dagger}\left(\boldsymbol{\delta}_F, \boldsymbol{\pi}_F, \boldsymbol{\mu}_F\right) \\
\text{s.t.}\ & \sum_{j\in\mathcal{J}_D} q_j^{it} \le s^{it} w^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \sum_{j\in\mathcal{J}_F} q_j^{it} \le s^{it}\left(1 - w^{it}\right), \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \sum_{j\in\mathcal{J}_D} r_{jk}^{it} + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it} + z_k^{it} \ge \hat{d}_k^{it} + \bar{\zeta}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D \\
& \rho_k^{it} + z_k^{it} \ge \hat{d}_k^{it} + \bar{\zeta}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O \\
& \sum_{j\in\mathcal{J}_F} \eta_j^{it} + z_{K+1}^{it} \ge \hat{d}_{K+1}^{it} + \bar{\zeta}_{K+1}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& w^{it} \ge 0, \qquad i \in \mathcal{I}, t \in \mathcal{T}
\end{aligned}$$

Constraints (A.2), (A.4)–(A.7), (A.11)–(A.23)

Let $\bar{\boldsymbol{w}}$ denote the optimal solution of $\boldsymbol{w}$. We use $\mathrm{P}_{\mathrm{DECOM}-\nu(1)}$ to determine $\boldsymbol{w}$ because of the following two reasons. First, we utilize $\mathrm{P}_{\mathrm{DECOM}-\nu(1)}$ to obtain the robust solution of $\boldsymbol{w}$. Because $\mathrm{P}_{\mathrm{DECOM}-\nu(1)}$ considers the WSU with $\gamma = 1$, it is obvious that the robust solution of $\boldsymbol{w}$ could be obtained. Second, because the optimal value $\nu(1)$ of $\mathrm{P}_{\mathrm{DECOM}-\nu(1)}$ is used to get the cost target for applying the TRO approach, it is not mandatory to implement another unnecessary scheme to determine $\boldsymbol{w}$, which could save computational time. Note that the $\boldsymbol{w}$ is not used for actual decisions (i.e., replenishment, transshipment, allocation, and fulfillment). The $\boldsymbol{w}$ is only used to decompose the proposed problem and reduce computational times.

Let $\bar{\boldsymbol{\delta}}_D^1, \bar{\boldsymbol{\pi}}_D^1, \bar{\boldsymbol{\mu}}_D^1, \bar{\boldsymbol{\delta}}_F^1, \bar{\boldsymbol{\pi}}_F^1$, and, $\bar{\boldsymbol{\mu}}_F^1$ are optimal solutions of Problem $\mathrm{P}_{\mathrm{DECOM}-\nu(1)}$. We define $\nu_D(1) = \boldsymbol{\Gamma}_D^{\dagger}\left(\bar{\boldsymbol{\delta}}_D^1, \bar{\boldsymbol{\pi}}_D^1, \bar{\boldsymbol{\mu}}_D^1\right)$ and $\nu_F(1) = \boldsymbol{\Gamma}_F^{\dagger}\left(\bar{\boldsymbol{\delta}}_F^1, \bar{\boldsymbol{\pi}}_F^1, \bar{\boldsymbol{\mu}}_F^1\right)$, and the sum of $\nu_D(1)$ and $\nu_F(1)$ is equal to $\nu(1)$. Then, we solve the following problem to get value $\nu(0)$ with fixed value $\bar{\boldsymbol{w}}$:

$(\mathrm{P}_{\mathrm{DECOM}-\nu(0)})$

$$\begin{aligned}
\nu(0) = \min\ & \boldsymbol{\Gamma}_D^{\dagger}\left(\boldsymbol{\delta}_D, \boldsymbol{\pi}_D, \boldsymbol{\mu}_D\right) + \boldsymbol{\Gamma}_F^{\dagger}\left(\boldsymbol{\delta}_F, \boldsymbol{\pi}_F, \boldsymbol{\mu}_F\right) \\
\text{s.t.}\ & \sum_{j\in\mathcal{J}_D} q_j^{it} \le s^{it} \bar{w}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \sum_{j\in\mathcal{J}_F} q_j^{it} \le s^{it}\left(1 - \bar{w}^{it}\right), \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \sum_{j\in\mathcal{J}_D} r_{jk}^{it} + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it} + z_k^{it} \ge \hat{d}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D \\
& \rho_k^{it} + z_k^{it} \ge \hat{d}_k^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O \\
& \sum_{j\in\mathcal{J}_F} \eta_j^{it} + z_{K+1}^{it} \ge \hat{d}_{K+1}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}
\end{aligned}$$

Constraints (A.2), (A.4)–(A.7), (A.11)–(A.23)

The first and second constraints use the fixed value $\bar{\boldsymbol{w}}$, which is obtained by solving the $\mathrm{P}_{\mathrm{DECOM}-\nu(1)}$. Let $\bar{\boldsymbol{\delta}}_D^0, \bar{\boldsymbol{\pi}}_D^0, \bar{\boldsymbol{\mu}}_D^0, \bar{\boldsymbol{\delta}}_F^0, \bar{\boldsymbol{\pi}}_F^0$, and, $\bar{\boldsymbol{\mu}}_F^0$ are optimal solutions of Problem $\mathrm{P}_{\mathrm{DECOM}-\nu(0)}$. We have $\nu_D(0) = \boldsymbol{\Gamma}_D^{\dagger}\left(\bar{\boldsymbol{\delta}}_D^0, \bar{\boldsymbol{\pi}}_D^0, \bar{\boldsymbol{\mu}}_D^0\right)$ and $\nu_F(0) = \boldsymbol{\Gamma}_F^{\dagger}\left(\bar{\boldsymbol{\delta}}_F^0, \bar{\boldsymbol{\pi}}_F^0, \bar{\boldsymbol{\mu}}_F^0\right)$, and the sum of $\nu_D(0)$ and $\nu_F(0)$ is equal to $\nu(0)$. In contrast to the procedure of the TPA, DECOM adopts two cost targets, $\psi_D$ and $\psi_F$, one for the retailer and the other for the 3PP. The $\psi_D$ and $\psi_F$ are determined with the following two affine functions of $\phi$, respectively: (1) $\psi_D(\phi) := (1-\phi)\nu_D(1) + \phi\nu_D(0)$ and (2) $\psi_F(\phi) := (1-\phi)\nu_F(1) + \phi\nu_F(0)$.

Given $\bar{\boldsymbol{w}}$, the stochastic optimization model $\mathrm{P_{STOC}}$ can be decomposed into two models, one for the retailer's supply chain and the other for the supply chain of the 3PP. For each model, we could derive two MILP models, $\mathrm{P_{STATIC-D}}$ and $\mathrm{P_{STATIC-F}}$, by applying the TRO approach presented in Section 4.1. $\mathrm{P_{STATIC-D}}$ and $\mathrm{P_{STATIC-F}}$ are formulated for the retailer's and the 3PP supply chains, respectively. In the case of the TPA, the $\gamma^{\dagger}$, which is for maximizing the adjustable uncertainty set, is the same for the retailer's and the 3PP supply chains. On the other hand, in DECOM, we define $\gamma^{\dagger}_D$ for the objective value of $\mathrm{P_{STATIC-D}}$, and $\gamma^{\dagger}_F$ for the objective value of $\mathrm{P_{STATIC-F}}$. $\mathrm{P_{STATIC-D}}$ and $\mathrm{P_{STATIC-F}}$ are presented as follows:

$$
\begin{aligned}
&(\mathrm{P_{STATIC-D}})\\
&\gamma^{\dagger}_D = \max \quad \gamma\\
&\text{s.t.} \quad \boldsymbol{\Gamma}^{\dagger}_D(\boldsymbol{\delta},\boldsymbol{\pi},\boldsymbol{\mu}) \le \psi_D\\
&\quad q^{it}_j \le \bar{q}^i_j \delta^{it}_j, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_D, t \in \mathcal{T}\\
&\quad \sum_{j\in\mathcal{J}_D} q^{it}_j \le s^{it}\bar{w}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}\\
&\quad \sum_{j\in\mathcal{J}_D} r^{it}_{jk} + \sum_{k'\in\mathcal{K}_O} g^{it}_{k'k} + z^{it}_k \ge \hat{d}^{it}_k + \gamma\bar{\zeta}^{it}_k, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_D\\
&\quad \rho^{it}_k + z^{it}_k \ge \hat{d}^{it}_k + \gamma\bar{\zeta}^{it}_k, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}_O\\
&\quad q^{it}_j \ge 0, \delta^{it}_j \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_D, t \in \mathcal{T}\\
&\quad x^{it}_j \ge 0, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_D, t \in \mathcal{T}^+\\
&\quad z^{it}_k \ge 0, \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}^-, t \in \mathcal{T}\\
&\quad \text{Constraints (A.5)–(A.7), (A.11), (A.13), (A.16)–(A.19), (A.22)}\\
&\quad 0 \le \gamma \le 1
\end{aligned}
$$

$$
\begin{aligned}
&(\mathrm{P_{STATIC-F}})\\
&\gamma^{\dagger}_F = \max \quad \gamma\\
&\text{s.t.} \quad \boldsymbol{\Gamma}^{\dagger}_F(\boldsymbol{\delta},\boldsymbol{\pi},\boldsymbol{\mu}) \le \psi_F\\
&\quad q^{it}_j \le \bar{q}^i_j \delta^{it}_j, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_F, t \in \mathcal{T}\\
&\quad \sum_{j\in\mathcal{J}_D} q^{it}_j \le s^{it}(1-\bar{w}^{it}), \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}\\
&\quad \sum_{j\in\mathcal{J}_F} \eta^{it}_j + z^{it}_{K+1} \ge \hat{d}^{it}_{K+1} + \gamma\bar{\zeta}^{it}_{K+1}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}\\
&\quad q^{it}_j \ge 0, \delta^{it}_j \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_F, t \in \mathcal{T}\\
&\quad x^{it}_j \ge 0, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_F, t \in \mathcal{T}^+\\
&\quad z^{it}_{K+1} \ge 0, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}\\
&\quad \text{Constraints (A.4), (A.12), (A.20)}\\
&\quad 0 \le \gamma \le 1
\end{aligned}
$$

Let $\bar{\boldsymbol{\delta}}_D$ and $\bar{\boldsymbol{\delta}}_F$ be optimal solutions for $\boldsymbol{\delta}_D$ and $\boldsymbol{\delta}_F$ obtained by solving the Problems $\mathrm{P_{STATIC-D}}$ and $\mathrm{P_{STATIC-F}}$, respectively. Consequently, the $\bar{\boldsymbol{\delta}}_D$ and $\bar{\boldsymbol{\delta}}_F$ will be used for binary replenishment decisions in Phase 2 of DECOM.

### *5.2. Phase 2 of DECOM*

The goal of Phase 2 of DECOM is to determine the adjustable continuous variables, which is similar to the goal of Phase 2 of the TPA. However, a key difference between these two approaches is that Phase 2 of DECOM utilizes the solution for the artificial variable $\bar{\boldsymbol{w}}$ obtained in Phase 1. In addition, by using the fixed $\bar{\boldsymbol{w}}$, we can decompose the $\mathrm{P_{ARO}}$ into the following two problems $\mathrm{P_{ARO-D}}$ and $\mathrm{P_{ARO-F}}$:

$$
\begin{aligned}
&(\mathrm{P_{ARO-D}})\\
&\min \max_{\mathcal{P}\in\mathcal{F}} \quad \mathbb{E}_{\mathcal{P}}\left[\boldsymbol{\Gamma}_D\left(\boldsymbol{\pi}_D(\tilde{\mathbf{d}}), \boldsymbol{\mu}_D(\tilde{\mathbf{d}})\right)\right]\\
&\text{s.t.} \quad q^{it}_j(\mathbf{d}^{t-1}) \le \bar{q}^i_j \bar{\delta}^{it}_j, \qquad i \in \mathcal{I}, j \in \mathcal{J}_D, t \in \mathcal{T}, \qquad \forall \mathbf{d}^{t-1} \in \mathbf{D}^{t-1}\\
&\quad \sum_{j\in\mathcal{J}_D} q^{it}_j(\mathbf{d}^{t-1}) \le s^{it}\bar{w}^{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, \qquad \forall \mathbf{d}^{t-1} \in \mathbf{D}^{t-1}\\
&\quad q^{it}_j(\tilde{\mathbf{d}}^{t-1}) \ge 0, q^{it}_j(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_D, t \in \mathcal{T}\\
&\quad x^{it}_j(\tilde{\mathbf{d}}^{t-1}) \ge 0, x^{it}_j(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}_D, t \in \mathcal{T}^+\\
&\quad z^{it}_k(\tilde{\mathbf{d}}^{t}) \ge 0, z^{it}_k(\tilde{\mathbf{d}}^{t}) \in \mathcal{R}^{t}, \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}^-, t \in \mathcal{T}\\
&\quad \text{Constraints (5)–(9), (11), (13), (16)–(19), (22)} \qquad \forall \mathbf{d}^{t} \in \mathbf{D}^{t}
\end{aligned}
$$

$$
\begin{aligned}
&(\mathrm{P}_{\mathrm{ARO-F}})\\
&\min \max_{\mathcal{P}\in\mathcal{F}} \quad \mathbb{E}_{\mathcal{P}}\left[\boldsymbol{\Gamma}_F\left(\boldsymbol{\pi}_F(\tilde{\mathbf{d}}),\boldsymbol{\mu}_F(\tilde{\mathbf{d}})\right)\right]\\
&\quad \text{s.t.} \quad q_j^{it}(\mathbf{d}^{t-1}) \le \bar{q}_j^i\bar{\delta}_j^{it}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}, \qquad \forall \mathbf{d}^{t-1}\in\mathbf{D}^{t-1}\\
&\qquad\quad \sum_{j\in\mathcal{J}_F} q_j^{it}(\mathbf{d}^{t-1}) \le s^{it}(1-\bar{w}^{it}), \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, \qquad \forall \mathbf{d}^{t-1}\in\mathbf{D}^{t-1}\\
&\qquad\quad q_j^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, q_j^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}\\
&\qquad\quad x_j^{it}(\tilde{\mathbf{d}}^{t-1}) \ge 0, x_j^{it}(\tilde{\mathbf{d}}^{t-1}) \in \mathcal{R}^{t-1}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}^+\\
&\qquad\quad z_{K+1}^{it}(\tilde{\mathbf{d}}^{t}) \ge 0, z_{K+1}^{it}(\tilde{\mathbf{d}}^{t}) \in \mathcal{R}^{t}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}\\
&\qquad\quad \text{Constraints (4), (10), (12), (20)} \qquad \forall \mathbf{d}^t\in\mathbf{D}^t
\end{aligned}
$$

where given $\mathbf{d}$

$$
\begin{aligned}
&\boldsymbol{\Gamma}_D\left(\boldsymbol{\pi}_D(\mathbf{d}),\boldsymbol{\mu}_D(\mathbf{d})\right) =\\
&\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\Bigg(\sum_{j\in\mathcal{J}_D} oc_{ij}^t q_j^{it}(\mathbf{d}^{t-1}) + \sum_{j\in\mathcal{J}_D} lh_j^{it} x_j^{i,t+1}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}_O} oh_k^{it} y_k^{i,t+1}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}^-} p_k^{it} z_k^{it}(\mathbf{d}^t) + \sum_{j\in\mathcal{J}_D}\sum_{j'\in\mathcal{J}_D} tc_{jj'}^{it} u_{jj'}^{it}(\mathbf{d}^{t-1})\\
&+\sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_O} ac_{jk}^{it} v_{jk}^{it}(\mathbf{d}^{t-1}) + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_D} ef_{jk}^{it} r_{jk}^{it}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}_O}\sum_{k'\in\mathcal{K}_D} of_{kk'}^{it} g_{kk'}^{it}(\mathbf{d}^t) + \sum_{k\in\mathcal{K}_O} bf_k^{it}\rho_k^{it}(\mathbf{d}^t)\Bigg),\\
&\boldsymbol{\Gamma}_F\left(\boldsymbol{\pi}_F(\mathbf{d}),\boldsymbol{\mu}_F(\mathbf{d})\right) =\\
&\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\sum_{j\in\mathcal{J}_F} oc_{ij}^t q_j^{it}(\mathbf{d}^{t-1}) + \sum_{j\in\mathcal{J}_F} lh_j^{it} x_j^{i,t+1}(\mathbf{d}^t) + p_{K+1}^{it} z_{K+1}^{it}(\mathbf{d}^t) + \sum_{j\in\mathcal{J}_F} af_j^{it}\eta_j^{it}(\mathbf{d}^t)\right).
\end{aligned}
$$

In order to restrict feasible space to linear functions, we also utilize the LDR for each adjustable continuous variable. The LDR for a retailer's supply chain is defined as

$$
\begin{aligned}
&q_j^{it}\left(\mathbf{d}^{t-1}\right) = q_j^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t-1} q_j^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T}\\
&x_j^{it}\left(\mathbf{d}^{t-1}\right) = x_j^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t-1} x_j^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T}^+\\
&y_k^{it}\left(\mathbf{d}^{t-1}\right) = y_k^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t-1} y_k^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T}^+\\
&u_{jj'}^{it}\left(\mathbf{d}^{t-1}\right) = u_{jj'}^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t-1} u_{jj'}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, j'\in\mathcal{J}_D, t\in\mathcal{T}\\
&v_{jk}^{it}\left(\mathbf{d}^{t-1}\right) = v_{jk}^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t-1} v_{jk}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, k\in\mathcal{K}_O, t\in\mathcal{T}\\
&\rho_k^{it}\left(\mathbf{d}^{t}\right) = \rho_k^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t} \rho_k^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T}\\
&g_{kk'}^{it}\left(\mathbf{d}^{t}\right) = g_{kk'}^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t} g_{kk'}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, k'\in\mathcal{K}_D, t\in\mathcal{T}\\
&r_{jk}^{it}\left(\mathbf{d}^{t}\right) = r_{jk}^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t} r_{jk}^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, k\in\mathcal{K}_D, t\in\mathcal{T}\\
&z_k^{it}\left(\mathbf{d}^{t}\right) = z_k^{it,0} + \sum_{\sigma\in\mathcal{K}^-}\sum_{\tau=1}^{t} z_k^{it,\sigma\tau} d_\sigma^{i\tau}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}^-, t\in\mathcal{T},
\end{aligned}
$$

and for the 3PP supply chain is defined as

$$
\begin{aligned}
&q_j^{it}\left(\mathbf{d}^{t-1}\right) = q_j^{it,0} + \sum_{\tau=1}^{t-1} q_j^{it,K+1,\tau} d_{K+1}^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}\\
&x_j^{it}\left(\mathbf{d}^{t-1}\right) = x_j^{it,0} + \sum_{\tau=1}^{t-1} x_j^{it,K+1,\tau} d_{K+1}^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}^+\\
&\eta_j^{it}\left(\mathbf{d}^{t}\right) = \eta_j^{it,0} + \sum_{\tau=1}^{t} \eta_j^{it,K+1,\tau} d_{K+1}^{i\tau}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T}
\end{aligned}
$$

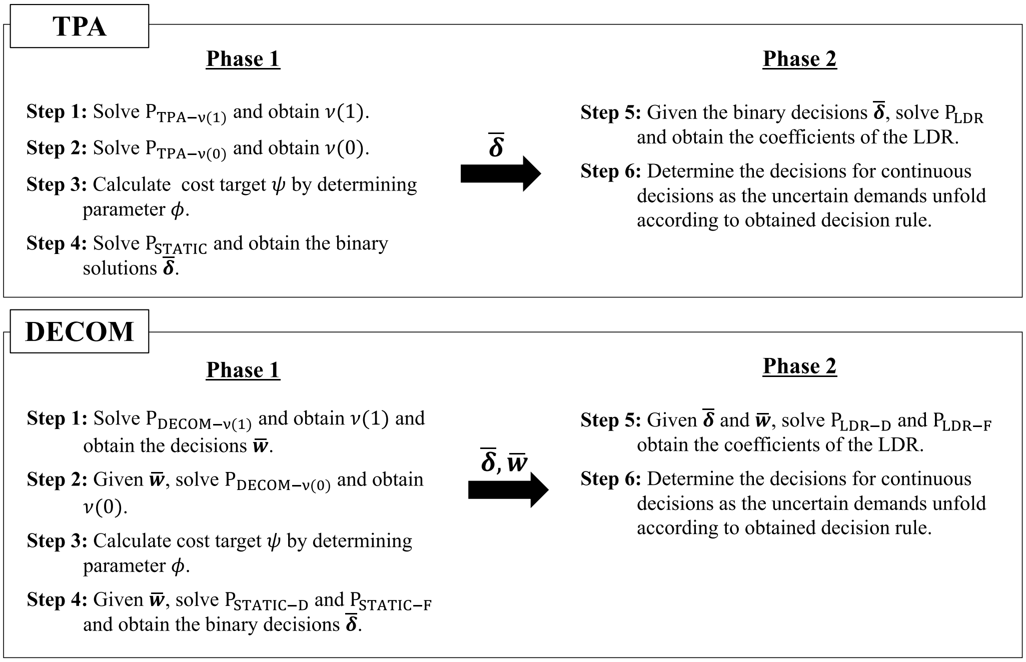


**Fig. 2.** Frameworks of TPA and DECOM.

$$z^{it}_{K+1}\left(\mathbf{d}^t\right) = z^{it,0}_{K+1} + \sum_{\tau=1}^{t} z^{it,K+1,\tau}_{K+1} d^{i\tau}_{K+1}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}.$$

Based on the above LDR, we present $P_{LDR-D}$ for the retailer's supply chain and $P_{LDR-F}$ for the 3PP supply chain to obtain the coefficient of LDR. $P_{LDR-D}$ and $P_{LDR-F}$ are presented in the Online Appendix C of supplementary material.

By following the same logic outlined in Section 4.2 and in the Online Appendix B, $P_{LDR-D}$ and $P_{LDR-F}$ also can be reformulated to the linear deterministic model using the duality theory. In summary, we must solve four MILP models (i.e., $P_{DECOM-\nu(1)}$, $P_{DECOM-\nu(0)}$, $P_{STATIC-D}$, and $P_{STATIC-F}$) and two LP models (i.e., $P_{LDR-D}$ and $P_{LDR-F}$) to implement the DECOM approach. Fig. 2 presents frameworks of the TPA and DECOM.

## 6. Computational experiments

In this section, we conducted extensive experiments to answer the research questions presented in Section 1. Research question 1 is addressed by the results in Sections 6.1 and 6.2.1. Research question 2 is answered by the results in Sections 6.2.2 and 6.3.1. The experimental results in Section 6.4 address Research question 3.

We compare our developed approach with the two benchmark algorithms: TPA and an alternative two-phase approach (DTPA). The DTPA determines the adjustable binary variables $\delta(\tilde{\mathbf{d}})$ with the static rule by solving the *expected value problem*, i.e., the deterministic model $P_{DET}$ with mean demands (Lim et al., 2021). On the other hand, the adjustable continuous variables are determined by applying Phase 2 of the TPA. A PC with an AMD Ryzen 2700X 7-Core CP, 3.60 GHz processor, and 16 GB of RAM with a Windows 10 64-bit system was utilized to conduct every experiment. In addition, every test instance is generated using Python 3.8 with the libraries *SciPy* and *Numpy*. The DTPA, TPA, and DECOM were developed with FICO Xpress 8.6, and we solved every model by utilizing the Xpress-Optimizer with its default parameter settings. In addition, we set the integrality gap tolerance in Xpress to one percent by following the setting of Lim et al. (2021).

We determine the constant parameters in the mathematical model by referring to the parameter setting of Jiu (2022). Parameters are generated randomly by following the uniform distributions in Table B.7. The replenishment lead time $L^i_j$ is generated by the discrete uniform distribution. The continuous uniform distribution is used to determine the rest of the parameters. The locations of logistics centers and offline stores are distributed uniformly over the 50 × 50 XY plane. We determine $oc^t_{ij}, tc^{it}_{jj'}, ac^{it}_{jk}$, and $ef^{it}_{jk}$ based on the Euclidean distance between each location. Even though we consider the offline fulfillment cost in the mathematical models for the sake of generality, we set $bf^{it}_k = 0$ because offline purchases by walk-in customers do not incur any fulfillment cost. In order to accommodate a property that the costs $S^{it}_j, lh^{it}_j$, and $oc^t_{ij}$ for the 3PP channel are more expensive than they are for the retailer, we multiply $\lambda^s$ and $\lambda^h$ to the lower and upper bound of parameters $S^{it}_j$ and $lh^{it}_j$, $\forall j \in \mathcal{J}_F$, respectively. Also, we determine the $oc^t_{ij}$, $\forall j \in \mathcal{J}_F$ by multiplying $\lambda^o$ on the distance between supplier $i$ and FC $j$. We set $\lambda^s = 1.5, \lambda^h = 1.5$, and $\lambda^o = 1.0$ for all experiments, except for in Section 6.4.2.

**Table 1**
Experimental results on symmetric demand distributions.

| | | *Beta*(0.3, 0.3) | | | *Beta*(1, 1) | | | *Beta*(4, 4) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 4$ | $T = 7$ | $T = 10$ | $T = 4$ | $T = 7$ | $T = 10$ | $T = 4$ | $T = 7$ | $T = 10$ |
| DTPA | LDR($\times 10^2$) | 56.97 | 90.03 | 141.82 | 54.09 | 115.22 | 138.23 | 52.37 | 127.06 | 146.25 |
| | SIM($\times 10^2$) | 57.18 | 89.84 | 141.94 | 53.88 | 115.37 | 138.12 | 52.27 | 127.04 | 146.36 |
| | Gap(%) | 31.67 | 28.66 | 20.76 | 33.20 | 28.27 | 13.94 | 20.24 | 21.30 | 27.33 |
| | Std($\times 10^2$) | 3.52 | 6.68 | 7.43 | 3.78 | 5.44 | 3.94 | 1.39 | 1.40 | 2.92 |
| | CPU(s) | 1.49 | 11.79 | 24.43 | 1.86 | 8.68 | 29.22 | 0.88 | 8.93 | 29.63 |
| TPA | LDR($\times 10^2$) | 49.19 | 77.79 | 128.29 | 46.13 | 99.00 | 131.75 | 47.88 | 114.21 | 126.09 |
| | SIM($\times 10^2$) | 49.19 | 77.81 | 128.29 | 46.13 | 99.02 | 131.74 | 47.89 | 114.17 | 126.06 |
| | Gap(%) | 13.27 | 11.43 | 9.14 | 14.05 | 10.09 | 8.67 | 10.15 | 9.01 | 9.68 |
| | Std($\times 10^2$) | 0.69 | 0.83 | 1.13 | 0.36 | 0.84 | 0.93 | 0.31 | 0.67 | 0.42 |
| | CPU(s) | 1.37 | 9.90 | 35.53 | 1.14 | 8.86 | 24.10 | 0.96 | 9.19 | 55.44 |
| | $\phi^*$ | 0.0∼0.4 | 0.0 | 0.2 | 0.2∼0.6 | 0.2 | 0.2∼0.4 | 0.0∼0.8 | 0.2∼0.4 | 0.2 |
| DECOM | LDR($\times 10^2$) | 49.19 | 77.72 | 128.56 | 46.02 | 99.29 | 132.33 | 47.88 | 114.58 | 126.14 |
| | SIM($\times 10^2$) | 49.19 | 77.74 | 128.55 | 46.01 | 99.31 | 132.32 | 47.89 | 114.55 | 126.12 |
| | Gap(%) | 13.27 | 11.32 | 9.37 | 13.76 | 10.42 | 9.15 | 10.15 | 9.37 | 9.72 |
| | Std($\times 10^2$) | 0.70 | 0.83 | 1.14 | 0.36 | 0.87 | 0.93 | 0.31 | 0.65 | 0.43 |
| | CPU(s) | 0.92 | 8.46 | 10.84 | 0.82 | 4.86 | 13.13 | 0.78 | 1.05 | 13.98 |
| | $\phi^*$ | 0.0∼0.4 | 0.0 | 1.0 | 0.0 | 0.2∼1.0 | 0.0∼0.4 | 0.0∼1.0 | 0.6 | 0.0 |
| EVPI | ($\times 10^2$) | 43.43 | 69.83 | 117.54 | 40.45 | 89.95 | 121.23 | 43.48 | 104.74 | 114.94 |

We assume that the sum of every demand for product $i$ at period $t$ falls in $[80, 120]$, i.e., $\sum_{k\in\mathcal{K}} d_k^{it} \in [80, 120]$, $\forall i \in \mathcal{I}, t \in \mathcal{T}$, except for the robustness analysis in Section 6.3.1. To determine each demand $k \in \mathcal{K}$, we define the share of each distribution channel for $\sum_{k\in\mathcal{K}} d_k^{it}$ as: (1) $\alpha_1$ for the retailer's offline channel, (2) $\alpha_2$ for the retailer's online channel, and (3) $\alpha_3$ for the 3PP channel. We set $\alpha_1 = 0.2, \alpha_2 = 0.3$, and $\alpha_3 = 0.5$. Each channel's demand is generated by the assumed demand distributions, which fall in the corresponding support set represented in Table B.8. The mean demand $\hat{d}_k^{it}$ is determined according to the assumed demand distribution. Even though we assume the demand distribution to generate random demand, every algorithm is implemented without any knowledge about the demand distribution.

In order to analyze the effects of the production capacity constraint, we define the following affine function of $\xi$ to determine the $s^{it}$:

$$s^{it}(\xi) := \xi \times \sum_{k\in\mathcal{K}} \hat{d}_k^{it} + (1-\xi) \times 2 \sum_{k\in\mathcal{K}} \bar{d}_k^{it}.$$

where $0 \le \xi \le 1$. According to the above affine function, the production capacity becomes insufficient as the $\xi$ is close to one. Otherwise, there is sufficient production capacity when the $\xi$ is close to zero. The value of $\xi$ will be set as zero in most experiments in the following sections. However, we will evaluate the performance of developed approaches and implement cost analysis by varying the $\xi$ in Sections 6.2.2 and 6.3.2, respectively.

### *6.1. Performance analysis in small problems*

In this section, we compare DECOM with benchmark algorithms in small problems. In Section 6.1.1, we validate DECOM under symmetric and asymmetric distributions. In Section 6.1.2, we evaluate the effectiveness of adopting $P_{DECOM-\nu(1)}$ in determining the artificial variable $\boldsymbol{w}$.

#### *6.1.1. Experiments under symmetric and asymmetric distributions*

We have conducted various experiments for the two purposes. First, we validate the obtained decision rule through Monte Carlo (MC) simulation. Every MC simulation is implemented with 500 samples. Second, we evaluate our approach for symmetric and asymmetric demand distributions. We utilize the beta distribution by referring to Jiu (2022). In this section, we set $I = 3$, $K_O = 3$, $K_D = 3$, $J_D = 2$, $J_F = 2$. We have tested on this setup with three different planning horizons: $T = 4$, 7, and 10. Furthermore, we define the set of candidate parameters $\Phi$ to find the best cost target. We use the notation $\phi^*$ to denote the target coefficient, which shows the best performance. We consider six candidate values for $\phi$ as $\Phi = \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$. Note that we use notation '$a \sim b$' to indicate that multiple values of $\phi^* \in \Phi$ between $a$ and $b$ show the same best performances (i.e., $a \le \phi^* \le b$).

First, we have conducted experiments on three types of symmetric distribution, *Beta*(0.3, 0.3), *Beta*(1, 1), and *Beta*(4, 4), and experimental results are reported in Table 1. We provide shapes of symmetric and asymmetric distributions in the Online Appendix D of the supplementary material. In Table 1, "LDR" means the objective value of $P_{LDR}$ with the fixed order cost $S_j^{it}\delta_j^{it}$ for the TPA, and the sum of objective values of $P_{LDR-D}$ and $P_{LDR-F}$ with the fixed order cost for DECOM. The "SIM" indicates the expected total cost implemented by MC simulation utilizing the obtained decision rule, and the "Std" is the standard deviation of the total cost for 500 samples. The "CPU(s)" means the computation times in seconds. We adopt the *expected value of perfect information* (EVPI) to evaluate the solution quality of each algorithm. To derive the EVPI, we solve the deterministic model $P_{DET}$ under the

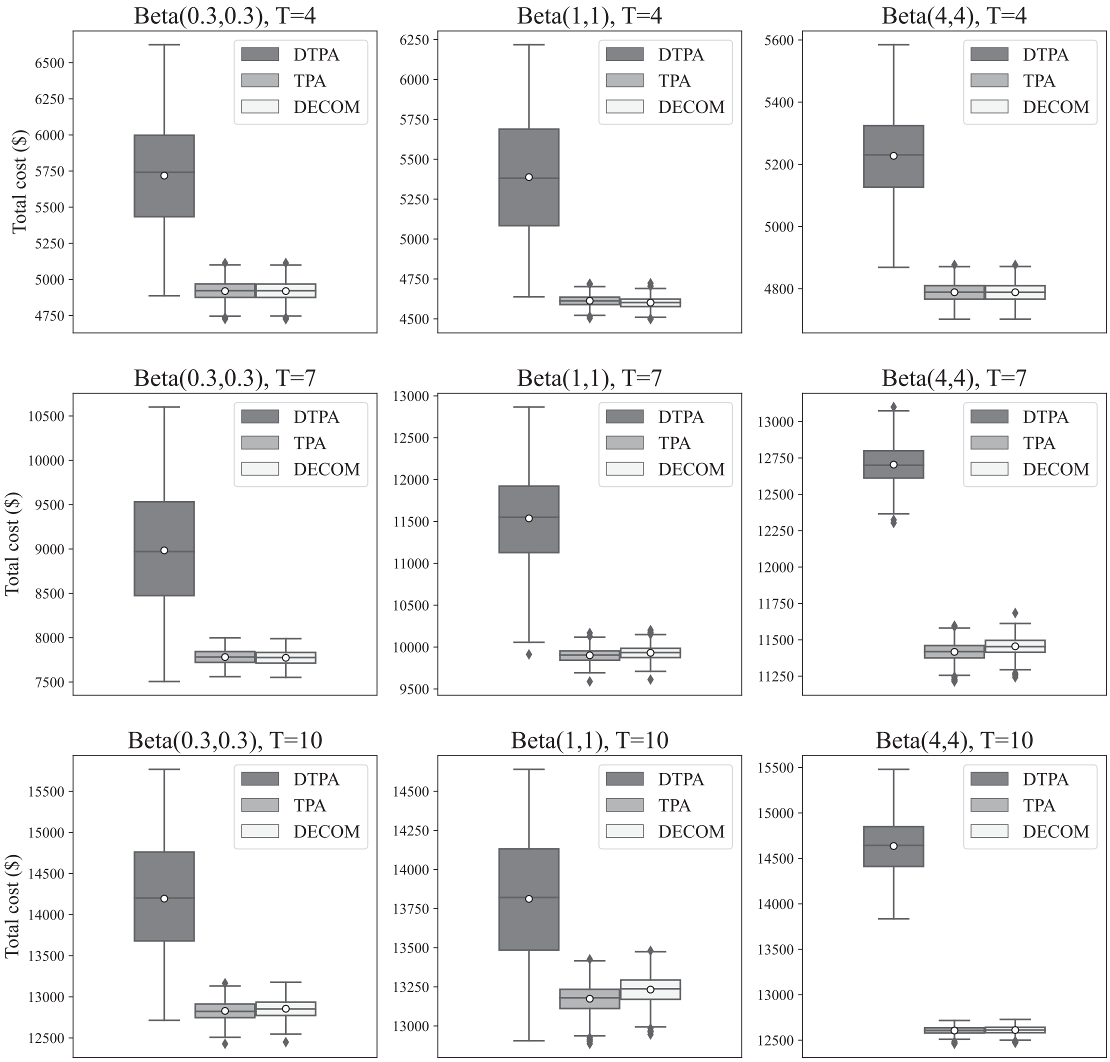


**Fig. 3.** Box plots for the total cost for 500 samples for every algorithm.

perfect information setting (i.e., the deterministic demand setting). We use the "Gap(%)" to measure the solution quality, which is calculated by (SIM − EVPI) × 100/EVPI.

Every experimental result of the TPA and DECOM was reported by adopting the best target coefficient $\phi^*$. The values of LDR and SIM were indifferent, which meant that the obtained decision rule achieved our goal (i.e., minimizing the expected total cost). In terms of solution quality, the Gaps of the TPA and DECOM were around 10 percent. However, the Gap of the DTPA was bigger than 20 percent, except for a result for $Beta(1,1)$ with $T = 10$. As shown in Fig. 3, the total cost of the TPA and DECOM was similar and significantly lower than the total cost of the DTPA. Also, the standard deviation of the TPA and DECOM was relatively small compared to that of the DTPA. For symmetric distributions, there is a tendency for the best solutions of the TPA and DECOM to be derived when the value of $\phi^*$ is small. This tendency meant that conservative binary decisions were necessary when the demand distribution was symmetric.

Second, we have conducted experiments on four types of asymmetric distribution, $Beta(2,5)$, $Beta(5,2)$, $Beta(1,6)$, and $Beta(6,1)$, and the experimental results were reported in Table C.9. As in the case of symmetric distributions, the values of LDR and SIM were indifferent when the demand distributions were asymmetric. However, when the beta distributions were skewed to the right ($Beta(a,b),\ a < b$), the Gap was bigger compared to the beta distributions skewed to the left ($Beta(a,b),\ b < a$). The binary decisions with the static rule $\delta$ could be too conservative for the beta distribution with $a < b$ because the realized demand was usually smaller than the mean value. Also, because the realized demand was relatively small, the $\phi^*$ value was high compared to the symmetric distributions. On the other hand, when the beta distributions were skewed to the left ($Beta(a,b),\ b < a$), the Gap was smaller than 10 percent. Because the realized demand was usually bigger than the mean value, there was no doubt that robust solutions were necessary; thus, the $\phi^*$ value was small.

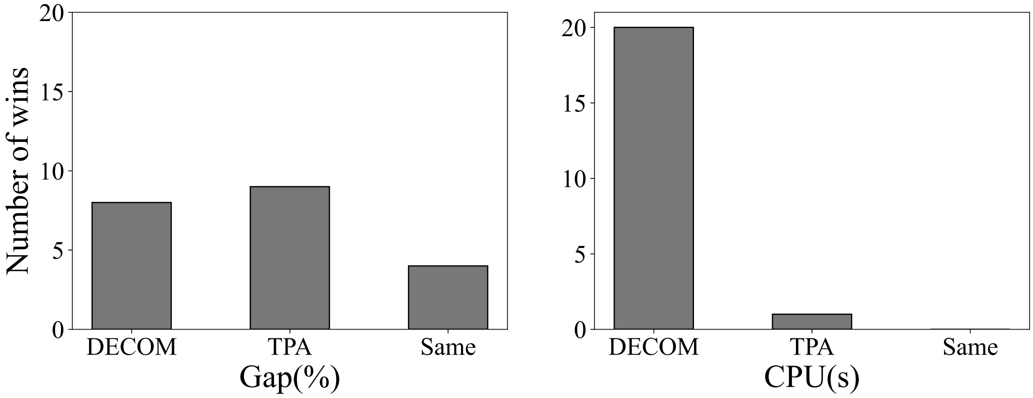


**Fig. 4.** Comparison of performance between TPA and DECOM in terms of Gap and CPU(s).

As shown in Fig. 4, the performance of TPA and DECOM were compared in terms of solution quality (Gap) and computational efficiency (CPU(s)). Among 21 results of experiments (9 for the symmetric distribution and 12 for the asymmetric distribution), the number of wins of DECOM and TPA was indifferent regarding the Gap (i.e., DECOM → 8, TPA → 9, and the same performance → 4). However, for CPU(s), DECOM outperformed TPA except for one result (i.e., DECOM → 20 and TPA → 1).

#### 6.1.2. Impact of the artificial variable $w$ on the performance of DECOM

As indicated in Section 5.1, DECOM determines the artificial variable $\boldsymbol{w}$ with the optimal solution $\boldsymbol{w}$ obtained by solving Problem $\text{P}_{\text{DECOM}-\nu(1)}$. Then, Problem $\text{P}_{\text{DECOM}-\nu(0)}$ is solved with the fixed value $\bar{\boldsymbol{w}}$. In order to evaluate this scheme, we validate the performance of determining $\boldsymbol{w}$ with $\text{P}_{\text{DECOM}-\nu(1)}$ by comparing with the following two alternative methods. The first method determines $\boldsymbol{w}$ with the optimal solution $\boldsymbol{w}$ obtained by solving Problem $\text{P}_{\text{DECOM}-\nu(0)}$. Thereafter, Problem $\text{P}_{\text{DECOM}-\nu(1)}$ was solved with the fixed value $\bar{\boldsymbol{w}}$, which is the opposite procedure of the proposed approach. The second method utilizes the information of mean demand $\hat{d}_k^{it}$ to determine $\boldsymbol{w}$ as follows:

$$w^{it} = \frac{\sum_{k\in\mathcal{K}_O\cup\mathcal{K}_D} \hat{d}_k^{it}}{\sum_{k\in\mathcal{K}} \hat{d}_k^{it}}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}.$$

This simple method could be effective because $\boldsymbol{w}$ can be interpreted as the predetermined ratio dividing the production capacity $s^{it}$ for the retailer's supply chain and the supply chain of the 3PP, respectively (referring to Constraints (38)–(40)).

In Table 2, we compared the above three methods determining $\boldsymbol{w}$. We utilized the problem instances presented in Table 1, and $\phi$ was set as zero. In the first column, "$W\text{P}_{\nu(1)}$" is the proposed method in this research, as illustrated in Section 5.1. "$W\text{P}_{\nu(0)}$" is the first alternative method, and "RATIO" is the second alternative method. In terms of solution quality, we could observe that $W\text{P}_{\nu(1)}$ outperformed other methods for all experiments. $W\text{P}_{\nu(0)}$ had poor solution quality because this method provided the aggressive decision to the uncertainty. In terms of computation times, we found insignificant differences between every method.

To investigate the impact of $\boldsymbol{w}$, we conducted a sensitivity analysis on $\boldsymbol{w}$. For ease of exposition, we use the expression "$\boldsymbol{w} = \beta$" to denote $w^{it} = \beta, \forall i \in \mathcal{I}, t \in \mathcal{T}$. Table 3 shows the solution quality (Gap) derived from each value of $w^{it}, \forall i \in \mathcal{I}, t \in \mathcal{T}$, and we highlighted the smallest value of Gap within $\beta = 0.1, \dots, 0.9$ in boldface at each experiment. The solution obtained with $\boldsymbol{w} = 0.5$ showed the best performance, except for $T = 4$. Also, as the value of $\beta$ deviated from 0.5, the solution quality became poor. The solution with the setting of $\boldsymbol{w} = 0.5$ yielded a smaller gap compared to RATIO. However, $W\text{P}_{\nu(1)}$ outperformed the solution with $\boldsymbol{w} = 0.5$ in all experiments, except for in the case of $Beta(4,4)$ with $T = 7$. For experiments with $T = 10$, we reported the values of $w^{it}, \forall i, t$ returned by $W\text{P}_{\nu(1)}$ in Table C.10. Table C.10 shows that $W\text{P}_{\nu(1)}$ returned the different value of $w^{it}$ depending on the values of $i$ and $t$.

### 6.2. Computational efficiency of DECOM

In the previous section, we observed that DECOM could alleviate the computational burden in small problems. Therefore, this section aims to validate the computational efficiency of DECOM in detail for various test environments. In this section, we set $\phi = 0.0$ for DECOM and TPA, based on the results of Section 6.1.1. We validate the computational efficiency of DECOM for large-scale problems. In addition, we vary the production capacity and compare the DECOM and TPA in detail.

#### 6.2.1. Experiments in large-scale problems

In this section, we have conducted several experiments to examine the computational efficiency of DECOM in large-scale problems. For every experiment, we fix the value of $T$, $K_O$, and $K_D$ as 7, 5, and 5, respectively. In addition, we vary with the value of $I, J_D$, and $J_F$ to change the problem scales, in which the $M$ $(= |\mathcal{I}|, |\mathcal{J}_D|, |\mathcal{J}_F|)$ varies from 3 to 10. We assume that the

**Table 2**
Experimental results on three methods for the artificial variable $w$.

| | | $Beta(0.3, 0.3)$ | | | $Beta(1, 1)$ | | | $Beta(4, 4)$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 4$ | $T = 7$ | $T = 10$ | $T = 4$ | $T = 7$ | $T = 10$ | $T = 4$ | $T = 7$ | $T = 10$ |
| $WP_{v(1)}$ | LDR($\times 10^2$) | 49.19 | 77.72 | 128.58 | 46.02 | 99.34 | 132.33 | 47.88 | 115.42 | 126.14 |
| | SIM($\times 10^2$) | 49.19 | 77.74 | 128.57 | 46.01 | 99.36 | 132.32 | 47.89 | 115.40 | 126.12 |
| | Gap(%) | 13.27 | 11.32 | 9.39 | 13.76 | 10.47 | 9.15 | 10.15 | 10.18 | 9.72 |
| | Std($\times 10^2$) | 0.70 | 0.83 | 1.12 | 0.36 | 0.86 | 0.93 | 0.31 | 0.65 | 0.43 |
| | CPU(s) | 0.91 | 8.65 | 11.01 | 0.82 | 4.36 | 12.93 | 0.78 | 8.01 | 14.93 |
| $WP_{v(0)}$ | LDR($\times 10^2$) | 86.55 | 139.77 | 230.29 | 83.09 | 164.27 | 221.10 | 93.34 | 193.40 | 219.03 |
| | SIM($\times 10^2$) | 87.12 | 140.27 | 231.10 | 82.38 | 164.89 | 220.83 | 93.53 | 193.29 | 219.09 |
| | Gap(%) | 100.60 | 100.86 | 96.62 | 103.66 | 83.32 | 82.16 | 115.14 | 84.55 | 90.61 |
| | Std($\times 10^2$) | 12.06 | 16.86 | 9.83 | 8.75 | 12.08 | 14.03 | 5.01 | 6.95 | 8.29 |
| | CPU(s) | 1.78 | 10.15 | 13.16 | 1.93 | 5.41 | 14.78 | 1.08 | 11.18 | 17.22 |
| RATIO | LDR($\times 10^2$) | 50.32 | 78.64 | 131.18 | 46.95 | 101.36 | 135.55 | 49.07 | 116.00 | 129.95 |
| | SIM($\times 10^2$) | 50.34 | 78.68 | 131.17 | 46.94 | 101.38 | 135.54 | 49.07 | 115.96 | 129.93 |
| | Gap(%) | 15.92 | 12.67 | 11.60 | 16.06 | 12.71 | 11.81 | 12.87 | 10.72 | 13.04 |
| | Std($\times 10^2$) | 0.91 | 1.05 | 1.16 | 0.43 | 0.87 | 1.00 | 0.32 | 0.68 | 0.46 |
| | CPU(s) | 0.78 | 8.14 | 10.12 | 0.98 | 4.80 | 13.90 | 0.88 | 7.67 | 15.35 |
| EVPI | ($\times 10^2$) | 43.43 | 69.83 | 117.54 | 40.45 | 89.95 | 121.23 | 43.48 | 104.74 | 114.94 |

**Table 3**
Experimental results on the solution quality (Gap (%)) by varying the value of $\boldsymbol{w}$.

| | | $Beta(0.3, 0.3)$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\boldsymbol{w} = 0.1$ | $\boldsymbol{w} = 0.2$ | $\boldsymbol{w} = 0.3$ | $\boldsymbol{w} = 0.4$ | $\boldsymbol{w} = 0.5$ | $\boldsymbol{w} = 0.6$ | $\boldsymbol{w} = 0.7$ | $\boldsymbol{w} = 0.8$ | $\boldsymbol{w} = 0.9$ |
| Gap (%) | $T = 4$ | 233.45 | 198.05 | **14.33** | 15.86 | 14.58 | 18.03 | 19.04 | 27.09 | 198.05 |
| | $T = 7$ | 343.12 | 56.10 | 15.10 | 12.62 | **11.96** | 14.45 | 19.25 | 66.10 | 360.15 |
| | $T = 10$ | 325.82 | 62.48 | 13.63 | 11.61 | **11.04** | 12.83 | 16.63 | 67.40 | 314.89 |
| | | $Beta(1, 1)$ | | | | | | | | |
| | | $\boldsymbol{w} = 0.1$ | $\boldsymbol{w} = 0.2$ | $\boldsymbol{w} = 0.3$ | $\boldsymbol{w} = 0.4$ | $\boldsymbol{w} = 0.5$ | $\boldsymbol{w} = 0.6$ | $\boldsymbol{w} = 0.7$ | $\boldsymbol{w} = 0.8$ | $\boldsymbol{w} = 0.9$ |
| Gap (%) | $T = 4$ | 226.62 | 20.50 | 15.16 | 16.08 | 15.78 | **14.79** | 19.17 | 26.92 | 184.62 |
| | $T = 7$ | 259.77 | 45.50 | 14.83 | 12.69 | **12.28** | 14.81 | 15.45 | 47.32 | 280.37 |
| | $T = 10$ | 314.96 | 63.73 | 13.79 | 11.81 | **11.58** | 13.90 | 17.08 | 59.96 | 306.17 |
| | | $Beta(4, 4)$ | | | | | | | | |
| | | $\boldsymbol{w} = 0.1$ | $\boldsymbol{w} = 0.2$ | $\boldsymbol{w} = 0.3$ | $\boldsymbol{w} = 0.4$ | $\boldsymbol{w} = 0.5$ | $\boldsymbol{w} = 0.6$ | $\boldsymbol{w} = 0.7$ | $\boldsymbol{w} = 0.8$ | $\boldsymbol{w} = 0.9$ |
| Gap (%) | $T = 4$ | 198.57 | 31.84 | 14.07 | 12.86 | **11.70** | 13.14 | 15.74 | 26.60 | 201.11 |
| | $T = 7$ | 219.92 | 40.00 | 11.97 | 10.76 | **9.03** | 12.52 | 14.22 | 26.60 | 205.42 |
| | $T = 10$ | 347.75 | 75.10 | 15.14 | 13.06 | **11.55** | 14.16 | 17.19 | 70.42 | 334.41 |

demand distribution follows $Beta(1, 1)$. Based on the experimental result for $Beta(1, 1)$ in Table 1. When solving the MILP models, we terminate the commercial solver if the time limit is reached and output the feasible solution obtained so far (i.e., 3600 s).

We present experimental results for large-scale problems in Table C.11, which presents Gap, Std, CPU(s), and EVPI. We keep in mind that $P_{DET}$ is a MILP model; thus, significant computational power is necessary to solve it 500 times to obtain the EVPI in large-scale problems. Therefore, we utilize the "alternative" EVPI. In the alternative EVPI, we first obtain the optimal binary solution $\bar{\boldsymbol{\delta}}$ by using Phase 1 of DECOM. Then, we fix the binary variable with the value $\bar{\boldsymbol{\delta}}$ to make $P_{DET}$ as an LP model. Therefore, $P_{DET}$ can be solved 500 times with perfect information within a reasonable time. To avoid confusion, the obtained value from the alternative EVPI is also indicated by the term "EVPI" in Tables C.11, C.12, and C.13. The DTPA had the largest value for Gap and Std compared to other approaches, which meant the solution quality of the DTPA was poor. DECOM and TPA had similar values for Std, but DECOM showed the best performance regarding solution quality (Gap). In addition, it required less computation time to implement the DECOM compared to the TPA (CPU(s)).

We analyze the computational efficiency in detail with the following five types of CPU(s): "Phase 1", "Phase 2", "$P_{v(1)}$", "$P_{v(0)}$", and "$P_{STATIC}$". The meaning of these five types of CPU(s) is presented in Online Appendix E of supplementary material. Fig. 5 presents the CPU(s) of Phase 1 and Phase 2 for three approaches. The DTPA could finish Phase 1 within a relatively short computation time compared to the TPA and DECOM. The DTPA and TPA required similar computation times to conduct Phase 2. However, DECOM required less of a computational burden compared to the DTPA and TPA to conduct Phase 2.

Fig. 6 depicts the CPU(s) of $P_{v(1)}$, $P_{v(0)}$, and $P_{STATIC}$ for TPA and DECOM. For $P_{v(1)}$, the performance of the TPA and DECOM was indifferent. However, for $P_{v(0)}$, DECOM required a much shorter time to solve the problem than TPA. The DECOM could finish the procedure for $P_{v(0)}$ within a short time because the feasible region was substantially reduced by fixing the value for $\bar{\boldsymbol{w}}$. When $M \geq 4$, TPA could not solve the Problem $P_{STATIC}$ until the time limit (1 h). On the other hand, when $M \geq 6$, DECOM could not solve Problem $P_{STATIC-D}$ within the time limit; but, Problem $P_{STATIC-F}$ could be solved in less than a second. In addition, DECOM

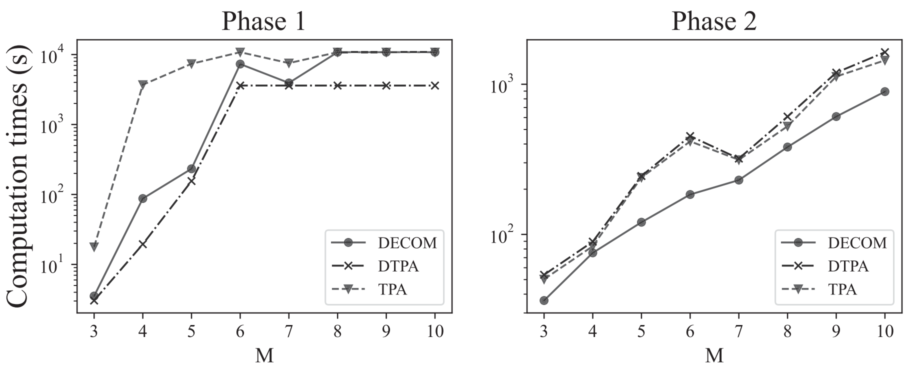


**Fig. 5.** Computation times of Phase 1 and Phase 2 for three approaches in large-scale problems.

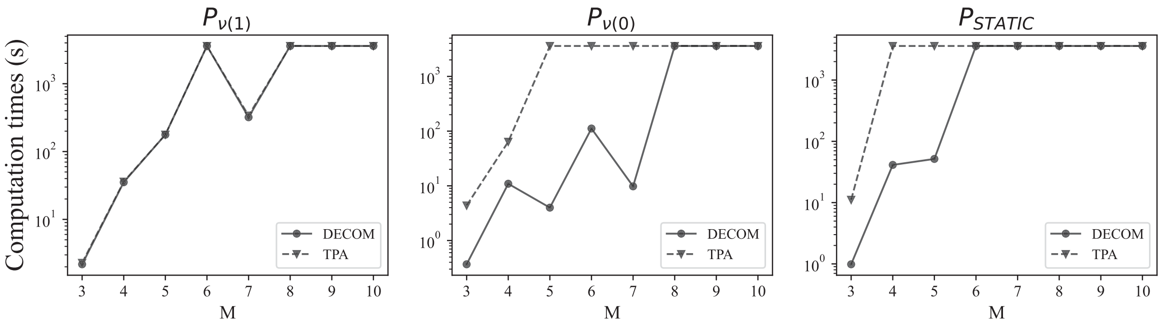


**Fig. 6.** Computation times of $P_{\nu(1)}$, $P_{\nu(0)}$, and $P_{STATIC}$ for DECOM and TPA in large-scale problems.

could provide high solution quality compared to TPA, although both approaches could not finish the procedure for $P_{STATIC}$ within the time limit.

In real-world cases, retailers have to handle various types of products in their supply chain. Therefore, we have examined the performance of DECOM on different product numbers based on the problem instances presented in Table 1. In Table C.12, we compared DTPA, TPA, and DECOM by varying the product numbers from 50 to 200. Similar to previous experiments, DTPA provided poor solution quality. In contrast, DECOM and TPA could provide high solution quality, in which the Gaps were about 10 percent for all results. Furthermore, we could observe that DECOM outperformed TPA regarding both Gap and CPU(s).

### *6.2.2. Performance analysis by varying the production capacity*

In this section, we have conducted experiments to analyze the performance of DECOM by varying the production capacity with different values of $\xi \in \{0.0, 0.1, \ldots, 0.8, 0.9\}$. We set $T = 7, I = 5, K_O = 3, K_D = 3, J_D = 4$, and $J_F = 4$. Also, demand distribution follows $Beta(1,1)$ as in the setting of Section 6.2.1. Table C.13 shows the experimental results with different production capacities. We excluded the DTPA in this experiment because of poor solution quality when the production capacity was insufficient. In order to show the computational efficiency of DECOM, we set the commercial solver's time limit as 10,800 s when solving the Problem $P_{STATIC}$.

As shown in Table C.13, TPA required more than 3 h in all experiments, except for the $\xi = 0.3$. However, DECOM required less than 5 min to solve the same problems, except for in the case of $\xi = 0.9$. Furthermore, because TPA could not solve problems within the time limit, the Gap of DECOM was smaller than it was for TPA for all experiments. Fig. 7 depicts the CPU(s) of Phases 1 and 2, and Fig. 8 shows the CPU(s) of $P_{\nu(1)}$, $P_{\nu(0)}$, and $P_{STATIC}$. Interestingly, DECOM had a clear advantage over solving $P_{STATIC}$, compared to TPA. Except for $\xi = 0.3$, TPA could not finish the procedure for $P_{STATIC}$ within 3 h for all experiments. On the contrary, DECOM could find optimal solutions for $P_{STATIC}$ within 10 s, except for a result for $\xi = 0.0$ (80 s).

### *6.3. Robustness analysis and cost analysis*

In Section 6.3.1, we validate the robustness of DECOM over the demand uncertainty set and a family of demand distributions. Section 6.3.2 examines the cost components by varying the production capacity and target coefficient $\phi$.

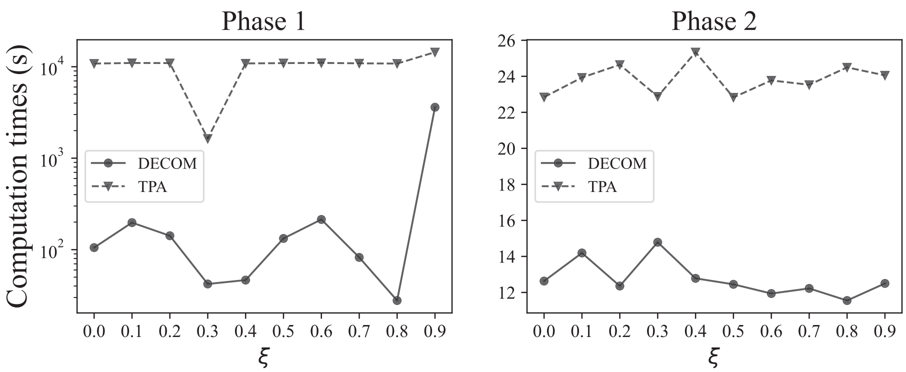


**Fig. 7.** Computation times of Phase 1 and Phase 2 for DECOM and TPA with different production capacity.

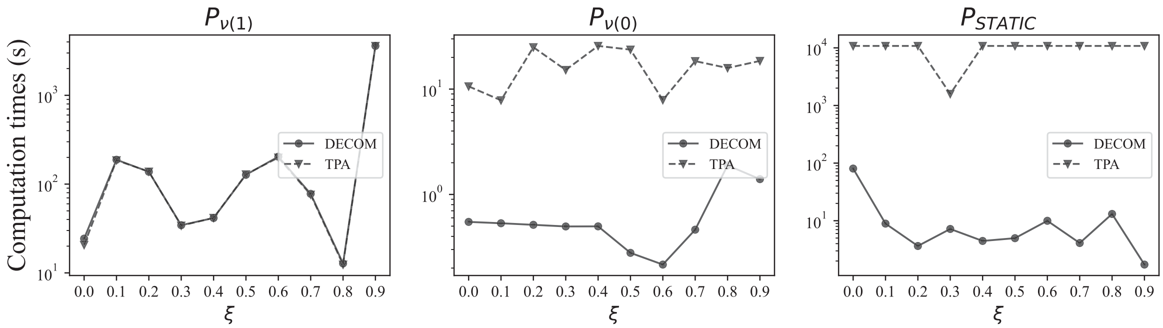


**Fig. 8.** Computation times of $P_{\nu(1)}$, $P_{\nu(0)}$, and $P_{STATIC}$ for DECOM and TPA with different production capacity.

**Table 4**
Robustness analysis by varying the demand uncertainty set.

| | | $\sum_{k\in\mathcal{K}} d_k^{it}$, $\forall i \in \mathcal{I}, t \in \mathcal{T}$ uncertainty set in the simulation | | | | |
|---|---|---|---|---|---|---|
| | | [50,150] | [60,140] | [70,130] | [80,120] | [90,110] |
| DTPA | SIM($\times 10^2$) | 171.98 | 172.08 | 172.18 | 172.29 | 172.39 |
| | Std($\times 10^2$) | 10.73 | 8.58 | 6.44 | 4.29 | **2.15** |
| DECOM | SIM($\times 10^2$) | 114.40 | 114.39 | 114.39 | 114.39 | 114.39 |
| | Std($\times 10^2$) | **1.56** | 1.25 | 0.94 | 0.63 | 0.31 |

*6.3.1. Robustness analysis*

We implemented robustness analysis of DECOM under the condition of $T = 7, I = 4, K_O = 5, K_D = 5, J_D = 4, J_F = 4$ and $\phi = 0.0$. First, we conducted experiments to validate the robustness of DECOM over the demand uncertainty. We implemented DECOM and DTPA under the following assumptions: (1) $\sum_{k\in\mathcal{K}} \hat{d}_k^{it} = 100$ and (2) $\sum_{k\in\mathcal{K}} d_k^{it} \in [50, 150]$, $\forall i \in \mathcal{I}, t \in \mathcal{T}$. Even though we assume the specified demand uncertainty set, the real demand can fall into a narrower uncertainty set. To fulfill this, we derived the solutions of DECOM and DTPA (i.e., the decision rule) under the uncertainty set $[50, 150]$. Then, we conducted simulations with the decision rules obtained under the uncertainty set $[50, 150]$ by varying the demand uncertainty set from $[50, 150]$ to $[90, 110]$ as shown in Table 4. The demands in the simulations are generated by following the $Beta(1, 1)$ with predetermined uncertainty sets.

Table 4 shows that DECOM and DTPA derived similar expected total costs (i.e., SIM), respectively, even if the demand uncertainty set varied. However, DECOM provided a much smaller standard deviation of the total cost (i.e., Std) compared to DTPA. As the range of the demand uncertainty set decreased, the Std of DTPA decreased significantly. However, there were no significant changes in the case of DECOM, compared to DTPA. Furthermore, as indicated in boldface, the Std of DECOM under the most extensive uncertainty set (i.e., [50,150]) was smaller than the Std of DTPA under the narrowest uncertainty set (i.e., [90,110]).

Next, we utilized the decision rules obtained in the experiment of Table 4 to check the robustness under a family of distributions. Because of the definition of a family of distribution in Section 4.2, the sample demand in the simulation has to be generated under the probability distribution, which has the same mean demand (i.e., $\sum_{k\in\mathcal{K}} \hat{d}_k^{it} = 100$). Therefore, as shown in Table 5, we utilized several types of symmetric beta distributions for a simulation. Table 5 represents that the decision rules obtained from the DECOM and DTPA could provide almost the same total expected cost under different distributions. Similar to the results of Table 4, the Std of DECOM was smaller than the Std of DTPA for all demand distributions. In particular, the Std of DECOM in $Beta(0.1, 0.1)$

**Table 5**
Robustness analysis under a family of demand distributions.

| | | Generated demand in the simulation | | | | |
|---|---|---|---|---|---|---|
| | | $Beta(0.1, 0.1)$ | $Beta(0.3, 0.3)$ | $Beta(2, 2)$ | $Beta(3, 3)$ | $Beta(5, 5)$ |
| DTPA | SIM($\times 10^2$) | 172.95 | 172.08 | 172.28 | 172.12 | 172.57 |
| | Std($\times 10^2$) | 16.67 | 14.55 | 8.04 | 6.67 | **5.60** |
| DECOM | SIM($\times 10^2$) | 114.34 | 114.48 | 114.33 | 114.37 | 114.40 |
| | Std($\times 10^2$) | **2.42** | 2.19 | 1.19 | 0.98 | 0.82 |

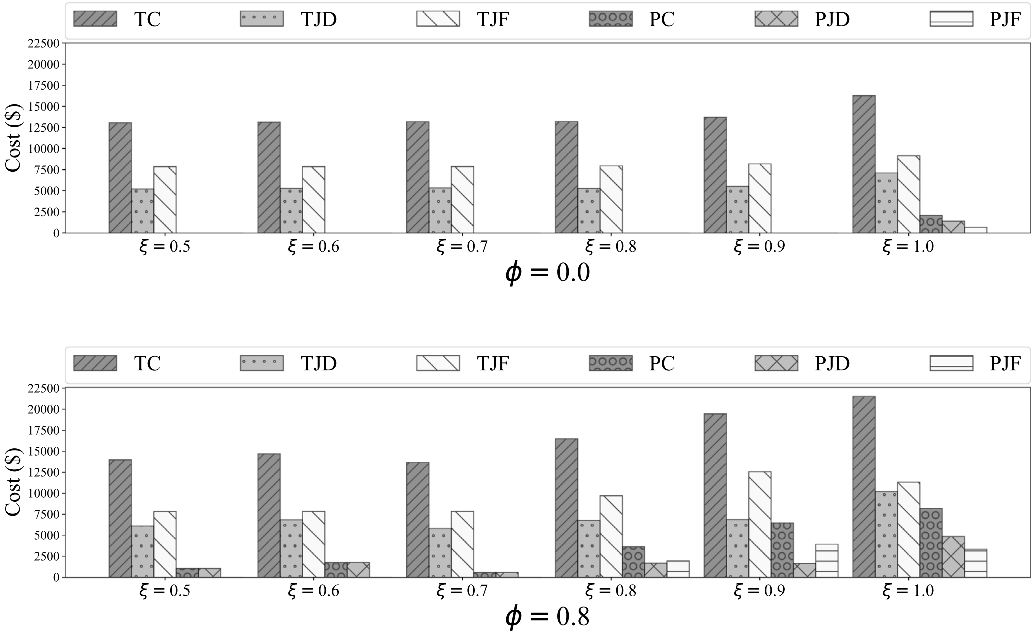


**Fig. 9.** Cost analysis by varying the production capacity.

was smaller than the Std of DTPA in $Beta(5, 5)$, as shown in boldface. These experimental results indicate that the decision rules of DECOM are robust to distributional ambiguity.

### 6.3.2. Cost analysis

We implement cost analysis by varying the production capacities (i.e., $0.5 \leq \xi \leq 1.0$). Fig. 9 presents bar plots for six cost components: (1) the total cost for the whole supply chain (TC), (2) the total cost for the retailer's supply chain (TJD), (3) the total cost for the 3PP supply chain (TJF), (4) the stockout cost for the whole supply chain (PC), (5) the stockout cost for the retailer's supply chain (PJD), and (6) the stockout cost for the supply chain of the 3PP (PJF). We conducted experiments for two decision rules; one was obtained from the DECOM with $\phi = 0.0$, and the other was obtained from the DECOM with $\phi = 0.8$. Because the DECOM with $\phi = 0.0$ could derive the conservative decision rule to the uncertainty, the stockout only occurred when the $\xi = 1.0$, which was the case in which suppliers had the smallest production capacities. However, because the DECOM with $\phi = 0.8$ output the aggressive decision rule to the uncertainty, the stockout occurred when $\xi \geq 0.5$. These results suggest that a conservative decision rule obtained from the DECOM with $\phi = 0.0$ could save costs in the supply chain if the market imbalance between supply and demand is expected.

## 6.4. Sensitivity analysis

In this section, we have conducted three types of experiments to explore the effects of omnichannel retail operations and the 3PP channel by varying several cost parameters. Because the proposed problem involves too many cost parameters, we chose the following cost parameters, which showed the apparent tendency, for sensitivity analysis: (1) fulfillment costs associated with online demand; (2) the additional costs associated with using the 3PP; and (3) the lost sales cost. We employed the problem instances presented in Section 6.2.2, and $\phi$ was set as zero. We employed the DECOM under the condition in which the demand uncertainty set is $[50, 150]$. We measured the cost-saving effect of adopting omnichannel retailing and the 3PP channel as follows:

$$\text{Cost-saving (\%)} = \frac{\text{Expected total cost of No-omni/No-3PP} - \text{Expected total cost of Omni/With-3PP}}{\text{Expected total cost of No-omni/No-3PP}} \times 100.$$

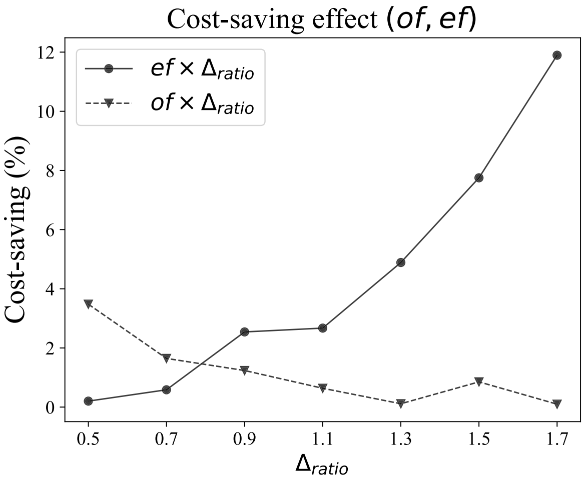


**Fig. 10.** Cost-saving effect of omnichannel operations varying the parameters $of^{it}_{kk'}$ and $ef^{it}_{jk}$.

### 6.4.1. Effects of using omnichannel retail operations

In the first experiment, we investigated the impact of fulfillment cost parameters $of^{it}_{kk'}$ and $ef^{it}_{jk}$ on the advantages of implementing omnichannel retail operations. We employed the parameter $\Delta_{ratio}$ to vary the cost parameters $of^{it}_{kk'}$ and $ef^{it}_{jk}$. We changed the value of cost parameters by multiplying the $\Delta_{ratio}$ on the original value, and the $\Delta_{ratio}$ ranges from 0.5 to 1.7 (i.e., $of^{it}_{kk'}, ef^{it}_{jk} \times \Delta_{ratio}$). We validated the benefits of omnichannel retail operations (i.e., *Omni*), specifically ship-from-store $g^{it}_{kk'}$, by comparing them with the retailer operations without properties of the omnichannel setup (i.e., *No-omni*). The total costs of Omni and No-omni were derived by solving the proposed model with the DECOM. In order to exclude the ship-from-store, we set the $g^{it}_{kk'}$ as zero for No-omni. Because only the retailer's supply chain has omnichannel properties, the total cost and cost-saving effect of the retailer's supply chain was reported.

Fig. 10 presents the cost-saving effect by varying fulfillment cost parameters from the DC to the online demand zone, $ef^{it}_{jk}$, and for ship-from-store for online demands, $of^{it}_{kk'}$. The cost savings by omnichannel operations decreased as the value of $of^{it}_{kk'}$ increased and $ef^{it}_{jk}$ decreased. In particular, the cost savings showed more rapid changes when varying the $ef^{it}_{jk}$ rather than the $of^{it}_{kk'}$. In the case of $of^{it}_{kk'}$, the cost savings was about 4 percent when the $\Delta_{ratio}$ was 0.5. On the other hand, for $ef^{it}_{jk}$, the cost savings was 17 percent when the $\Delta_{ratio}$ was set as 1.7.

### 6.4.2. Effects of introducing the 3PP channel

In the second experiment, we evaluated the advantages of utilizing the 3PP channel by varying the $p^{it}_{K+1}$ and $\lambda^s, \lambda^h$, and $\lambda^o$ employed to reflect the additional costs of using the 3PP channel. In particular, we compared the retail operations using both the retailer channel and the 3PP channel (i.e., *With-3PP*) with using the retailer channel only (i.e., *No-3PP*). To implement No-3PP, we addressed the aggregate demand of the 3PP channel as lost sales by setting the $\delta^{it}_j$, $\forall j \in \mathcal{J}_F$ as zero. Then, because products were replenished only for the retailer's channel, the 3PP channel was not used in the retail operations. The With-3PP yields cost savings because the No-3PP fulfills less demand than does the With-3PP. However, because the impact on cost savings is different for each cost parameter, we compared the With-3PP and the No-3PP and examined the cost-saving effect by implementing a sensitivity analysis on $p^{it}_{K+1}$, $\lambda^s$, $\lambda^h$, and $\lambda^o$.

Fig. 11 shows the cost savings of introducing the 3PP channel by comparing No-3PP and With-3PP. As shown in the left subfigure, we investigated the impact of the lost sales cost parameter for the 3PP channel, $p^{it}_{K+1}$. We varied the value of $p^{it}_{K+1}$ from 4 to 10 with the steplength 0.5. The cost-saving effect of using the 3PP channel was insignificant when $p^{it}_{K+1} \leq 5.5$. Then, when $p^{it}_{K+1}$ was between 5.5 and 6.5, the cost-saving effect increased slightly as the value of $p^{it}_{K+1}$ increased. After that, when $p^{it}_{K+1} \geq 6.5$, the cost-saving effect increased linearly with increasing $p^{it}_{K+1}$.

We employed the parameter $\Delta_\lambda$ to analyze the impact that parameters $\lambda^s, \lambda^h$, and $\lambda^o$ had on cost savings incurred by using the 3PP channel. The $\Delta_\lambda$ ranges from 1.0 to 4.0 with steplength 0.5. The right subfigure shows the cost savings brought about by varying the additional costs using the 3PP channel for the following four cases: (1) fixed participation cost $\lambda^s = \Delta_\lambda$, orange line; (2) inventory holding cost $\lambda^h = \Delta_\lambda$, green line; (3) warehousing cost $\lambda^o = \Delta_\lambda$, red line; and (4) $\lambda^s = \lambda^h = \lambda^o = \Delta_\lambda$, blue line. As the $\lambda^o$ increased, the cost-saving effect of using the 3PP channel decreased significantly compared to $\lambda^s$ and $\lambda^h$. Conversely, the changes of $\lambda^h$ had a minimal effect on the cost savings of utilizing the 3PP channel. As presented in the blue line, if the value of parameters $\lambda^s, \lambda^h$, and $\lambda^o$ were bigger than 4.0, it was insignificant to utilize the 3PP channel in terms of the cost-saving effect.

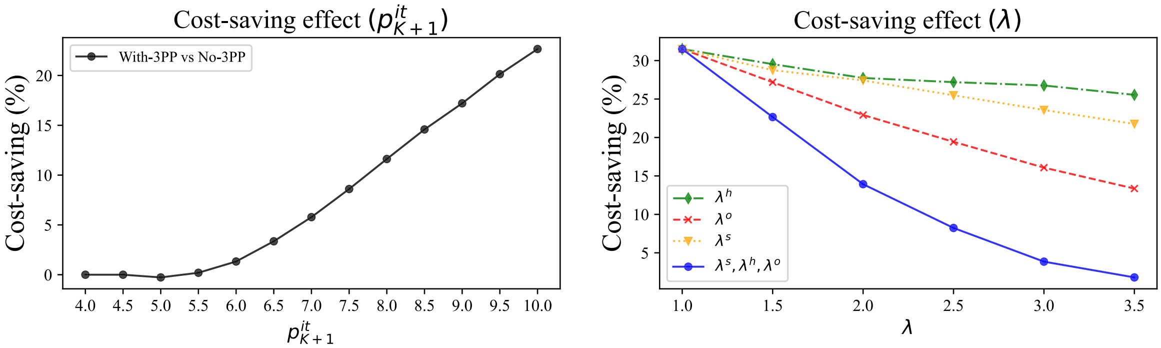


**Fig. 11.** Cost-saving effect of using the 3PP channel varying the parameters $p^{t}_{K+1}$, $\lambda^s, \lambda^h$, and $\lambda^o$.

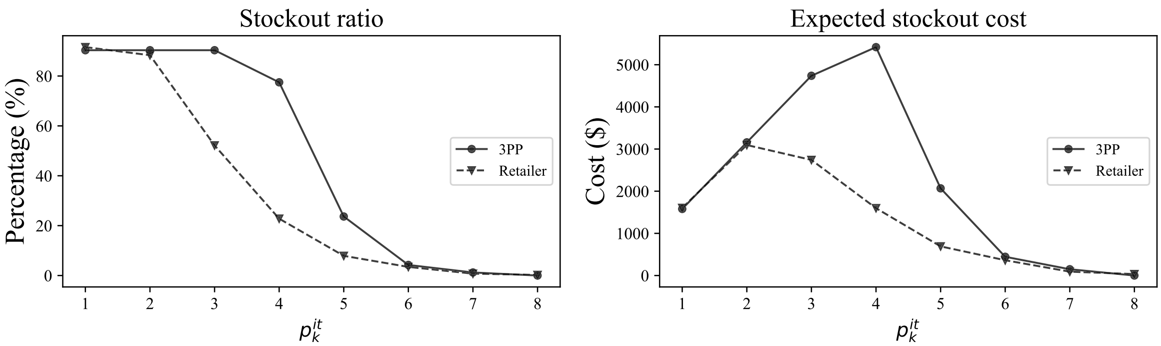


**Fig. 12.** Stockout ratio and expected stockout cost of supply chains of retailer and 3PP.

#### 6.4.3. *Impact of lost sales cost on the retailer and the 3PP supply chains*

In the third experiment, we examined the stockout ratio and the expected stockout cost of the retailer and the 3PP supply chains. The stockout ratio of each supply chain was obtained by dividing the total demand by the total amount of the stockout. We implemented a sensitivity analysis on the lost sales cost parameter for both the retailer channel and the 3PP channels, $p^{it}_k$, $\forall k \in \mathcal{K}$. We varied the value of $p^{it}_k$ from 1 to 8 with the steplength 1. Fig. 12 shows that the stockout ratio of both supply chains decreased as the $p^{it}_k$ increased. However, the stockout ratio of the 3PP was higher than the retailer's supply chain when $p^{it}_k \leq 6$. When $p^{it}_k$ was between 4 and 5, the stockout ratio of the 3PP decreased rapidly. Furthermore, the expected stockout cost of the retailer began to decrease when $p^{it}_k > 2$, and in the case of the 3PP, it decreased when $p^{it}_k > 4$. These results suggest that the stockout ratio and cost of the 3PP supply chain are sensitive to the value of $p^{it}_k$ compared to the retailer's supply chain.

### 6.5. *Managerial insights*

We present several managerial insights that could be instructive to practitioners who are concerned about both omnichannel retailing and the 3PP channel. We underpin the proposed managerial insights by considering the experimental results.

- Based on the experimental results of Section 6.4.1, we could observe that omnichannel retailing can cut total costs compared to retail operations without the ship-from-store option when fulfillment costs from DCs to online demand zones (i.e., $ef^{it}_{jk}$) increase and ship-from-store costs decrease (i.e., $of^{it}_{kk'}$). In particular, the changes in cost parameters regarding fulfillment from DCs lead to a rapid increase or decrease in the cost-saving effect. Therefore, even if the cost of ship-from-store operations is high, it is still beneficial to embrace omnichannel retail operations if the cost of satisfying online demand zones from DC is considerable (e.g., when the DCs are far from the online demand zones).
- The experimental results of the first experiment in Section 6.4.2 indicate that the increase in the warehousing cost leads to a rapid decrease of the cost-saving effect when introducing the 3PP channel in omnichannel retailing among the three types of additional costs associated with the 3PP channel, $\lambda^s, \lambda^h$, and $\lambda^o$. In contrast, the increase in inventory holding costs has a minor impact on the cost savings of employing the 3PP channel. If the three additional costs are too expensive, there is no advantage in using both the retailer and 3PP channels in comparison to operating the retail channel only. Therefore, omnichannel retailers should take into account these additional costs, particularly warehousing costs, when introducing the 3PP channel in their channels.

- On the basis of the results of the second experiment in Section 6.4.2, we could observe that there are no benefits to employing the 3PP channel if the lost sales cost for the 3PP channel demand is low. Otherwise, the cost-saving effect of using the 3PP channel increases as the lost sales cost increases. The retailer usually determines or estimates the lost sales cost parameter. If the lost sales cost parameter is estimated to be smaller than the true value, the retailer could miss the cost-saving effect by using the 3PP. On the other hand, an excess usage of the 3PP channel will be the outcome if the lost sales cost is estimated to be larger. Therefore, retailers should precisely estimate the lost sales parameter before applying our suggested approach.
- Table C.9 shows that it is necessary to derive a conservative solution by setting a large value for the cost target if the demand distribution is skewed to the right. Otherwise, if the distribution is skewed to the left, the cost target should be set with a small value to obtain an aggressive solution to demand uncertainty. Furthermore, the degree of production capacity is also an essential factor in determining the appropriate value of the cost target. Therefore, users of our proposed approach should determine the cost target value by considering their production capacities and by identifying how the demand distribution is roughly shaped (e.g., by examining the skewness or variance of demand distributions).

## 7. Conclusions

We studied the optimization problem considering demand uncertainty in a setting where the omnichannel retailer determined to utilize the 3PP channel in advance. In the proposed problem, the retailer's online and offline channels were operated by the retailer's supply chain, and the 3PP channel was operated by the supply chain of the 3PP. Moreover, we considered joint replenishment, allocation, transshipment, and fulfillment decisions over a multi-period planning horizon. To minimize the expected total cost, we presented the stochastic optimization model from the perspective of a retailer. Furthermore, we accommodated the distinct advantages and properties of adopting the 3PP channel in the proposed model.

However, there were four challenges in our problem. First, the adjustable binary decisions for replenishment should be considered, which incurs a fixed order cost. Second, we should integrate anticipative and reactive decisions when solving the problem. Third, the existence of the 3PP channel increased the problem size because the retailer's supply chain and the supply chain of the 3PP should be considered simultaneously. Fourth, the production capacity constraint made the problem more intractable. Even though the TPA developed by Lim et al. (2021) could mitigate the first and second challenges, TPA often required a high computational burden to solve the proposed problem because of the third and fourth challenges. As a way to overcome these challenges, we proposed a DECOM approach by utilizing artificial variables, and it can solve the problem separately according to the retailer's supply chain and the 3PP supply chain.

Experimental results show beneficial contributions of this research from both academic and managerial perspectives. First, we observed that DECOM and TPA provided solutions with similar quality in various demand distributions. However, DECOM outperformed TPA in terms of computational efficiency. In particular, DECOM was scalable to large-scale problems while maintaining its high solution quality. In addition, the robustness analysis showed that DECOM could provide robust and stable solutions against changes in uncertainty sets and demand distributions. Second, we explored the cost-saving effects of employing omnichannel retailing and introducing the 3PP channel, respectively. We observed that omnichannel retailing leads to significant potential cost savings compared to retailing without omnichannel as the fulfillment cost from DCs to online demand zones increases. As the lost sales cost increases, introducing the 3PP channel in omnichannel retail operations also leads to cost savings compared to utilizing the retailer's channel only, which is because the retailer can absorb additional demand by using the 3PP channel. Therefore, we emphasized the importance of accurately estimating the stockout cost parameter before participating in the 3PP service, since the retailer usually determines the lost sales cost.

Considering the limitations of our study, we conclude by discussing directions for further research. First, our model assumes that the information regarding the storage capacity of FCs is complete. Although the storage capacity can often be observed or estimated, it can be uncertain and vary depending on the time period, because other users of the 3PP channel can also store their products in the FCs. It will be interesting to investigate how uncertainty for the storage capacity of FCs affects the utilization of the 3PP channel. Second, our model does not consider return policies regarding online products. One of the advantages of omnichannel retailing is the return policies. Usually, omnichannel retailing allows customers to return products through all available channels. Therefore, future studies should target developing the model with return policies and then examine how introducing the 3PP channel in omnichannel retailing influences the return flow of products and the profitability of the retailer.

## CRediT authorship contribution statement

**Junhyeok Lee:** Conceptualization, Data curation, Formal analysis, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing. **Ilkyeong Moon:** Funding acquisition, Investigation, Project administration, Resources, Supervision, Validation, Writing – review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.


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## Appendix A. Notations and the deterministic model ($\mathbf{P_{DET}}$)

See Table A.6.

**Table A.6**
Indices, sets, parameters, and decision variables for the mathematical model.

| | |
|---|---|
| Indices and sets | |
| $\mathcal{T}$ | Set of time periods, $t \in \mathcal{T} = \{1, 2, \ldots, T\}$ |
| $\mathcal{T}^+$ | $t \in \mathcal{T}^+ = \{1, 2, \ldots, T+1\}$ |
| $\mathcal{I}$ | Set of products (=suppliers), $i \in \mathcal{I} = \{1, 2, \ldots, I\}$ |
| $\mathcal{K}_O$ | Set of offline demand zones (= offline stores), $k \in \mathcal{K}_O = \{1, 2, \ldots, K_O\}$ |
| $\mathcal{K}_D$ | Set of online demand zones for the retailer's supply chain, $k \in \mathcal{K}_D = \{K_O + 1, \ldots, K_O + K_D\}$ |
| $\mathcal{K}^-$ | Set of online and offline demand zones for DCs, $k \in \mathcal{K}^- = \{1, \ldots, K\}$ $(K = K_O + K_D)$ |
| $\mathcal{K}$ | Set of demand zones for DCs and FCs, $k \in \mathcal{K} = \{1, \ldots, K+1\}$ |
| $\mathcal{J}_D$ | Set of capacitated DCs, $j \in \mathcal{J}_D = \{1, 2, \ldots, J_D\}$ |
| $\mathcal{J}_F$ | Set of capacitated FCs, $j \in \mathcal{J}_F = \{J_D + 1, \ldots, J_D + J_F\}$ |
| $\mathcal{J}$ | $j \in \mathcal{J} = \{1, \ldots, J\}$ $(J = J_D + J_F)$ |
| Parameters | |
| $S_j^{it}$ | Fixed cost to order product $i$ for the logistics center $j$ from supplier $i$ at period $t$ |
| $lh_j^{it}$ | Unit inventory holding cost for the logistics center $j$ per product $i$ at period $t$ |
| $oh_k^{it}$ | Unit inventory holding cost for the offline store $k$ per product $i$ at period $t$ |
| $oc_{ij}^t$ | Unit replenishment cost between the supplier $i$ and the logistics center $j$ per product $i$ at period $t$ |
| $tc_{jj'}^{it}$ | Transshipment cost between the DC $j$ and the other DC $j'$ per product $i$ at period $t$ |
| $ac_{jk}^{it}$ | Allocation cost between the DC $j$ and the offline store $k$ per product $i$ at period $t$ |
| $ef_{jk}^{it}$ | Fulfillment cost from the DC $j$ to the online demand zone $k$ per product $i$ at period $t$ |
| $of_{kk'}^{it}$ | Fulfillment cost from the offline store $k$ to the online demand zone $k'$ per product $i$ at period $t$ |
| $bf_k^{it}$ | Fulfillment cost for the offline demand zone $k$ per product $i$ at period $t$ |
| $af_j^{it}$ | Fulfillment cost for the aggregate demand for FC $j$ per product $i$ at period $t$ |
| $p_k^{it}$ | Lost sales cost for demand type $k$ per product $i$ at period $t$ |
| $s^{it}$ | Production capacity of supplier $i$ at period $t$ |
| $L_j^i$ | Lead time of product $i$ replenished from supplier $i$ to the logistics center $j$ |
| $\bar{q}_j^i$ | Capacity for the replenishment from the supplier $i$ to the logistics center $j$ |
| $\bar{x}_j$ | Storage capacity of the logistics center $j$ |
| $\bar{y}_k$ | Storage capacity of the offline store $k$ |
| $d_k^{it}$ | Realized value of demand type $k$ for product $i$ at period $t$ |
| Adjustable decision variables | |
| $\delta_j^{it}(\tilde{\mathbf{d}}^{t-1})$ | 1 if product $i$ is replenished from supplier $i$ to the logistics center $j$ at the start of period $t$, 0 otherwise |
| $q_j^{it}(\tilde{\mathbf{d}}^{t-1})$ | Quantity of the product $i$ replenished from supplier $i$ to the logistics center $j$ at the start of the period $t$ |
| $x_j^{it}(\tilde{\mathbf{d}}^{t-1})$ | On-hand inventory of product $i$ in the logistics center $j$ at the start of period $t$ |
| $y_k^{it}(\tilde{\mathbf{d}}^{t-1})$ | On-hand inventory of product $i$ in the offline store $k$ at the start of period $t$ |
| $u_{jj'}^{it}(\tilde{\mathbf{d}}^{t-1})$ | Quantity of the product $i$ transshipped from the DC $j$ to the other DC $j'$ at the start of the period $t$ |
| $v_{jk}^{it}(\tilde{\mathbf{d}}^{t-1})$ | Quantity of the product $i$ allocated from the DC $j$ to the offline store $k$ at the start of period $t$ |
| $\rho_k^{it}(\tilde{\mathbf{d}}^{t})$ | Quantity of the product $i$ fulfilled to satisfy the offline demand zone $k$ at the end of period $t$ |
| $\eta_j^{it}(\tilde{\mathbf{d}}^{t})$ | Quantity of the product $i$ fulfilled from the FC $j$ to satisfy the aggregate demand for FCs at the end of period $t$ |
| $g_{kk'}^{it}(\tilde{\mathbf{d}}^{t})$ | Quantity of the product $i$ fulfilled from the offline store $k$ to satisfy the online demand zone $k'$ at the end of period $t$ (i.e., ship-from-store for online demands) |
| $r_{jk}^{it}(\tilde{\mathbf{d}}^{t})$ | Quantity of the product $i$ from the DC $j$ fulfilled to satisfy the online demand zone $k$ at the end of period $t$ |
| $z_k^{it}(\tilde{\mathbf{d}}^{t})$ | Lost sales of product $i$ for the demand type $k$ at the end of period $t$ |

$(\mathrm{P_{DET}})$

$$\min \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\sum_{j\in\mathcal{J}} S_j^{it}\delta_j^{it} + \sum_{j\in\mathcal{J}} oc_{ij}^{t} q_j^{it} + \sum_{j\in\mathcal{J}} lh_j^{it} x_j^{i,t+1} + \sum_{k\in\mathcal{K}_O} oh_k^{it} y_k^{i,t+1} + \sum_{k\in\mathcal{K}} p_k^{it} z_k^{it} + \sum_{k\in\mathcal{K}_O}\sum_{k'\in\mathcal{K}_D} of_{kk'}^{it} g_{kk'}^{it} \right. \tag{A.1}$$

$$\left. + \sum_{j\in\mathcal{J}_D}\sum_{j'\in\mathcal{J}_D} tc_{jj'}^{it} u_{jj'}^{it} + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_O} ac_{jk}^{it} v_{jk}^{it} + \sum_{j\in\mathcal{J}_D}\sum_{k\in\mathcal{K}_D} ef_{jk}^{it} r_{jk}^{it} + \sum_{k\in\mathcal{K}_O} bf_k^{it}\rho_k^{it} + \sum_{j\in\mathcal{J}_F} af_j^{it}\eta_j^{it}\right)$$

$$\text{s.t.}\quad q_j^{it} \le \bar{q}_j^{i}\delta_j^{it}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T} \tag{A.2}$$

$$\sum_{j\in\mathcal{J}} q_j^{it} \le s^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{A.3}$$

$$\sum_{i\in\mathcal{I}}\left(x_j^{it} + q_j^{i,t-L_j^i}\right) \le \bar{x}_j, \qquad \forall j\in\mathcal{J}_F, t\in\mathcal{T} \tag{A.4}$$

$$\sum_{i\in\mathcal{I}}\left(x_j^{it} + q_j^{i,t-L_j^i} + \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{j'j}^{it} - \sum_{k\in\mathcal{K}_O} v_{jk}^{it} - \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{jj'}^{it}\right) \le \bar{x}_j, \qquad \forall j\in\mathcal{J}_D, t\in\mathcal{T} \tag{A.5}$$

$$x_j^{it} + q_j^{i,t-L_j^i} \ge \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{jj'}^{it}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T} \tag{A.6}$$

$$\sum_{i\in\mathcal{I}}\left(y_k^{it} + \sum_{j\in\mathcal{J}_D} v_{jk}^{it}\right) \le \bar{y}_k, \qquad \forall k\in\mathcal{K}_O, t\in\mathcal{T} \tag{A.7}$$

$$\sum_{j\in\mathcal{J}_D} r_{jk}^{it} + \sum_{k'\in\mathcal{K}_O} g_{k'k}^{it} + z_k^{it} = d_k^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, k\in\mathcal{K}_D \tag{A.8}$$

$$\rho_k^{it} + z_k^{it} = d_k^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, k\in\mathcal{K}_O \tag{A.9}$$

$$\sum_{j\in\mathcal{J}_F} \eta_j^{it} + z_{K+1}^{it} = d_{K+1}^{it}, \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{A.10}$$

$$x_j^{i,t+1} = x_j^{it} + q_j^{i,t-L_j^i} + \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{j'j}^{it} - \sum_{j'\in\mathcal{J}_D\setminus\{j\}} u_{jj'}^{it} - \sum_{k\in\mathcal{K}_O} v_{jk}^{it} - \sum_{k\in\mathcal{K}_D} r_{jk}^{it}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, t\in\mathcal{T} \tag{A.11}$$

$$x_j^{i,t+1} = x_j^{it} + q_j^{i,t-L_j^i} - \eta_j^{it}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T} \tag{A.12}$$

$$y_k^{i,t+1} = y_k^{it} + \sum_{j\in\mathcal{J}_D} v_{jk}^{it} - \sum_{k'\in\mathcal{K}_D} g_{kk'}^{it} - \rho_k^{it}, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T} \tag{A.13}$$

$$q_j^{it} \ge 0, \delta_j^{it} \in \{0,1\}, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T} \tag{A.14}$$

$$x_j^{it} \ge 0, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}^+ \tag{A.15}$$

$$y_k^{it} \ge 0, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T}^+ \tag{A.16}$$

$$u_{jj'}^{it} \ge 0, \qquad \forall j\in\mathcal{J}_D, j'\in\mathcal{J}_D, i\in\mathcal{I}, t\in\mathcal{T} \tag{A.17}$$

$$v_{jk}^{it} \ge 0, \qquad \forall j\in\mathcal{J}_D, k\in\mathcal{K}_O, i\in\mathcal{I}, t\in\mathcal{T} \tag{A.18}$$

$$\rho_k^{it} \ge 0, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, t\in\mathcal{T} \tag{A.19}$$

$$\eta_j^{it} \ge 0, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_F, t\in\mathcal{T} \tag{A.20}$$

$$g_{kk'}^{it} \ge 0, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}_O, k'\in\mathcal{K}_D, t\in\mathcal{T} \tag{A.21}$$

$$r_{jk}^{it} \ge 0, \qquad \forall i\in\mathcal{I}, j\in\mathcal{J}_D, k\in\mathcal{K}_D, t\in\mathcal{T} \tag{A.22}$$

$$z_k^{it} \ge 0, \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}, t\in\mathcal{T} \tag{A.23}$$

## Appendix B. Parameter information

See Tables B.7 and B.8.

## Appendix C. Detailed experimental results

See Tables C.9–C.13.

**Table B.7**
Ranges of the parameters.

| $S_j^{it},\ j \in \mathcal{J}_D$ | $S_j^{it},\ j \in \mathcal{J}_F$ | $lh_j^{it},\ j \in \mathcal{J}_D$ | $lh_j^{it},\ j \in \mathcal{J}_F$ | $oh_k^{it}$ | $p_k^{it}$ | $af_j^{it}$ | $L_j^i$ |
|---|---|---|---|---|---|---|---|
| $\mathcal{U}(50, 80)$ | $\mathcal{U}(50 \times \lambda^s, 80 \times \lambda^s)$ | $\mathcal{U}(0.2, 0.5)$ | $\mathcal{U}\left(0.2 \times \lambda^h, 0.5 \times \lambda^h\right)$ | $\mathcal{U}(0.3, 0.6)$ | $\mathcal{U}(60, 80)$ | $\mathcal{U}(2, 3)$ | $\mathcal{U}\{0, 1\}$ |

**Table B.8**
Support set of each channel for product $i$ and period $t$.

| Retailer's offline channel ($d_k^{it},\ \forall k \in K_O$) | Retailer's online channel ($d_k^{it},\ \forall k \in K_D$) | 3PP's online channel ($d_{K+1}^{it}$) |
|---|---|---|
| $\left[\frac{\alpha_1}{K_O} \times \sum_{k\in\mathcal{K}} \underline{d_k^{it}},\ \ \frac{\alpha_1}{K_O} \times \sum_{k\in\mathcal{K}} \bar{d_k^{it}}\right]$ | $\left[\frac{\alpha_2}{K_D} \times \sum_{k\in\mathcal{K}} \underline{d_k^{it}},\ \ \frac{\alpha_2}{K_D} \times \sum_{k\in\mathcal{K}} \bar{d_k^{it}}\right]$ | $\left[\alpha_3 \times \sum_{k\in\mathcal{K}} \underline{d_k^{it}},\ \ \alpha_3 \times \sum_{k\in\mathcal{K}} \bar{d_k^{it}}\right]$ |

**Table C.9**
Experimental results on asymmetric demand distributions.

| | | $Beta(2,5)$ | | | $Beta(5,2)$ | | | $Beta(1,6)$ | | | $Beta(6,1)$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $T=4$ | $T=7$ | $T=10$ | $T=4$ | $T=7$ | $T=10$ | $T=4$ | $T=7$ | $T=10$ | $T=4$ | $T=7$ | $T=10$ |
| DTPA | LDR($\times 10^2$) | 43.34 | 114.93 | 152.14 | 50.77 | 92.81 | 143.59 | 50.44 | 75.05 | 128.33 | 55.26 | 95.04 | 150.38 |
| | SIM($\times 10^2$) | 43.32 | 114.91 | 152.24 | 50.86 | 92.80 | 143.64 | 50.48 | 75.04 | 128.17 | 55.29 | 95.04 | 150.42 |
| | Gap(%) | 27.28 | 58.40 | 37.51 | 23.93 | 5.44 | 29.28 | 40.84 | 20.94 | 30.73 | 17.09 | 2.45 | 7.67 |
| | Std($\times 10^2$) | 0.74 | 5.03 | 4.38 | 1.25 | 0.39 | 1.78 | 3.31 | 0.47 | 3.62 | 0.60 | 0.28 | 0.84 |
| | CPU(s) | 1.71 | 9.42 | 23.08 | 2.01 | 8.46 | 16.96 | 1.01 | 7.02 | 69.39 | 1.42 | 6.62 | 24.65 |
| TPA | LDR($\times 10^2$) | 40.62 | 89.09 | 127.44 | 44.32 | 92.13 | 116.18 | 47.16 | 75.05 | 125.53 | 49.04 | 94.80 | 142.86 |
| | SIM($\times 10^2$) | 40.62 | 89.09 | 127.41 | 44.33 | 92.13 | 116.17 | 47.16 | 75.04 | 125.68 | 49.04 | 94.80 | 142.86 |
| | Gap(%) | 19.34 | 22.81 | 15.09 | 8.01 | 4.67 | 4.55 | 31.58 | 20.94 | 28.19 | 3.86 | 2.19 | 2.26 |
| | Std($\times 10^2$) | 0.24 | 0.05 | 0.84 | 0.24 | 0.39 | 0.38 | 0.65 | 0.47 | 2.32 | 0.17 | 0.30 | 0.36 |
| | CPU(s) | 1.95 | 6.38 | 61.58 | 1.73 | 9.51 | 44.85 | 1.01 | 8.86 | 72.31 | 1.81 | 9.97 | 31.07 |
| | $\phi^*$ | 0.6 | 0.4 | 0.6 | 0.2~0.6 | 0.0~1.0 | 0.0~0.2 | 0.6 | 1.0 | 0.6 | 0.0~0.2 | 1.0 | 0.0~0.4 |
| DECOM | LDR($\times 10^2$) | 41.09 | 84.27 | 126.36 | 44.35 | 91.79 | 116.18 | 46.43 | 74.49 | 116.72 | 49.04 | 94.81 | 143.09 |
| | SIM($\times 10^2$) | 41.09 | 84.27 | 126.34 | 44.36 | 91.79 | 116.17 | 46.44 | 74.48 | 116.73 | 49.04 | 94.81 | 143.09 |
| | Gap(%) | 20.71 | 16.17 | 14.12 | 8.08 | 4.28 | 4.55 | 29.58 | 20.03 | 19.06 | 3.86 | 2.20 | 2.43 |
| | Std($\times 10^2$) | 0.23 | 0.41 | 0.67 | 0.24 | 0.43 | 0.38 | 0.73 | 0.45 | 0.54 | 0.17 | 0.28 | 0.35 |
| | CPU(s) | 1.25 | 5.90 | 15.57 | 1.18 | 6.53 | 11.04 | 1.63 | 4.33 | 13.82 | 1.17 | 5.51 | 16.32 |
| | $\phi^*$ | 0.6~0.8 | 0.6~1.0 | 0.8 | 0.0~0.6 | 0.0 | 0.0~0.8 | 0.8 | 0.8 | 0.8 | 0.0~0.4 | 0.0~0.8 | 0.0~0.8 |
| EVPI | ($\times 10^2$) | 34.04 | 72.54 | 110.71 | 41.04 | 88.02 | 111.11 | 35.84 | 62.05 | 98.04 | 47.22 | 92.77 | 139.71 |

**Table C.10**
Values of $\boldsymbol{w}$ obtained from $WP_{v(1)}$.

| | | $Beta(0.3,0.3)$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $t=1$ | $t=2$ | $t=3$ | $t=4$ | $t=5$ | $t=6$ | $t=7$ | $t=8$ | $t=9$ | $t=10$ |
| $w^{it}$ | $i=1$ | 0.67 | 1.00 | 0.25 | 1.00 | 1.00 | 0.50 | 1.00 | 0.50 | 1.00 | 1.00 |
| | $i=2$ | 0.67 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.25 | 1.00 | 1.00 | 0.00 |
| | $i=3$ | 0.75 | 0.31 | 1.00 | 1.00 | 0.25 | 1.00 | 1.00 | 0.50 | 1.00 | 1.00 |
| | | $Beta(1,1)$ | | | | | | | | | |
| | | $t=1$ | $t=2$ | $t=3$ | $t=4$ | $t=5$ | $t=6$ | $t=7$ | $t=8$ | $t=9$ | $t=10$ |
| $w^{it}$ | $i=1$ | 0.67 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.25 | 1.00 | 1.00 | 0.00 |
| | $i=2$ | 1.00 | 0.56 | 0.50 | 1.00 | 1.00 | 0.25 | 1.00 | 1.00 | 0.50 | 1.00 |
| | $i=3$ | 1.00 | 0.81 | 0.75 | 0.25 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | $Beta(4,4)$ | | | | | | | | | |
| | | $t=1$ | $t=2$ | $t=3$ | $t=4$ | $t=5$ | $t=6$ | $t=7$ | $t=8$ | $t=9$ | $t=10$ |
| $w^{it}$ | $i=1$ | 0.46 | 1.00 | 1.00 | 0.50 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.75 |
| | $i=2$ | 0.92 | 0.00 | 1.00 | 1.00 | 1.00 | 0.50 | 1.00 | 0.50 | 1.00 | 0.00 |
| | $i=3$ | 0.56 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 0.75 | 0.50 | 1.00 | 0.00 |

**Table C.11**
Experimental results on large-scale problems.

| | | Set size $M$ $(= \|\mathcal{I}\|, \|\mathcal{J}_D\|, \|\mathcal{J}_F\|)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $M = 3$ | $M = 4$ | $M = 5$ | $M = 6$ | $M = 7$ | $M = 8$ | $M = 9$ | $M = 10$ |
| DTPA | LDR($\times 10^2$) | 121.14 | 125.21 | 133.91 | 168.23 | 190.42 | 248.98 | 246.12 | 282.11 |
| | SIM($\times 10^2$) | 121.58 | 124.94 | 133.89 | 168.26 | 190.35 | 248.84 | 245.55 | 282.00 |
| | Gap(%) | 41.27 | 37.90 | 11.52 | 10.59 | 19.84 | 25.21 | 18.98 | 26.64 |
| | Std($\times 10^2$) | 6.79 | 4.76 | 2.09 | 1.19 | 4.48 | 7.76 | 6.52 | 5.28 |
| | CPU(s) | 56.90 | 108.95 | 400.43 | 4057.07 | 3922.53 | 4210.85 | 4796.60 | 5235.13 |
| TPA | LDR($\times 10^2$) | 93.75 | 98.79 | 131.15 | 165.88 | 171.94 | 218.30 | 226.32 | 245.68 |
| | SIM($\times 10^2$) | 93.79 | 98.81 | 131.15 | 165.91 | 171.89 | 218.34 | 226.22 | 245.69 |
| | Gap(%) | 8.98 | 9.06 | 9.24 | 9.04 | 8.22 | 9.87 | 9.61 | 10.33 |
| | Std($\times 10^2$) | 0.79 | 0.69 | 0.78 | 0.87 | 0.93 | 1.07 | 1.11 | 1.05 |
| | CPU(s) | 68.05 | 3782.58 | 7615.99 | 11,217.25 | 7852.77 | 11,326.39 | 11,926.40 | 12,245.01 |
| DECOM | LDR($\times 10^2$) | 93.38 | 98.39 | 130.36 | 164.17 | 170.68 | 214.09 | 222.14 | 240.00 |
| | SIM($\times 10^2$) | 93.43 | 98.40 | 130.34 | 164.20 | 170.65 | 214.12 | 222.04 | 240.00 |
| | Gap(%) | 8.56 | 8.61 | 8.56 | 7.91 | 7.44 | 7.75 | 7.59 | 7.78 |
| | Std($\times 10^2$) | 0.79 | 0.69 | 0.77 | 0.88 | 0.93 | 1.08 | 1.12 | 1.06 |
| | CPU(s) | 39.82 | 162.85 | 353.33 | 7494.96 | 4159.56 | 11,183.06 | 11,410.72 | 11,693.43 |
| EVPI | ($\times 10^2$) | 86.06 | 90.60 | 120.06 | 152.16 | 158.83 | 198.73 | 206.38 | 222.69 |

**Table C.12**
Experimental results on different combinations of product numbers.

| | | $T = 4$ | | | $T = 7$ | | |
|---|---|---|---|---|---|---|---|
| | | $I = 50$ | $I = 100$ | $I = 200$ | $I = 50$ | $I = 100$ | $I = 200$ |
| DTPA | LDR($\times 10^2$) | 1029.74 | 2458.36 | 3996.06 | 1782.72 | 3452.43 | 7351.93 |
| | SIM($\times 10^2$) | 1030.46 | 2456.20 | 3993.89 | 1783.23 | 3453.15 | 7350.41 |
| | Gap(%) | 45.49 | 35.99 | 43.83 | 35.21 | 27.95 | 35.48 |
| | Std($\times 10^2$) | 18.08 | 26.00 | 38.25 | 21.83 | 27.29 | 43.13 |
| | CPU(s) | 117.33 | 3726.66 | 4813.41 | 408.05 | 1104.82 | 5449.16 |
| TPA | LDR($\times 10^2$) | 793.25 | 1995.45 | 3116.34 | 1454.29 | 2977.03 | 6040.39 |
| | SIM($\times 10^2$) | 793.39 | 1995.36 | 3116.24 | 1454.54 | 2977.18 | 6040.43 |
| | Gap(%) | 12.02 | 10.48 | 12.23 | 10.29 | 10.31 | 11.33 |
| | Std($\times 10^2$) | 0.88 | 3.27 | 4.49 | 3.21 | 4.82 | 6.33 |
| | CPU(s) | 3718.07 | 3789.26 | 5146.47 | 3991.88 | 4490.21 | 12,694.83 |
| DECOM | LDR($\times 10^2$) | 787.80 | 1987.14 | 3089.80 | 1432.64 | 2943.06 | 5893.83 |
| | SIM($\times 10^2$) | 787.93 | 1987.09 | 3089.48 | 1432.79 | 2943.08 | 5893.97 |
| | Gap(%) | 11.25 | 10.02 | 11.26 | 8.64 | 9.05 | 8.63 |
| | Std($\times 10^2$) | 1.89 | 3.13 | 4.14 | 3.06 | 4.13 | 5.88 |
| | CPU(s) | 3675.77 | 3670.18 | 3742.76 | 3977.86 | 3910.71 | 8299.79 |
| EVPI | ($\times 10^2$) | 708.25 | 1806.11 | 2776.75 | 1318.86 | 2698.88 | 5425.50 |

**Table C.13**
Experimental results on different production capacities.

| | | Production capacity $s^{it}(\xi)$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\xi = 0.0$ | $\xi = 0.1$ | $\xi = 0.2$ | $\xi = 0.3$ | $\xi = 0.4$ | $\xi = 0.5$ | $\xi = 0.6$ | $\xi = 0.7$ | $\xi = 0.8$ | $\xi = 0.9$ |
| TPA | LDR($\times 10^2$) | 128.18 | 129.93 | 130.13 | 130.06 | 130.15 | 131.04 | 132.49 | 132.18 | 133.25 | 138.08 |
| | SIM($\times 10^2$) | 128.16 | 129.92 | 130.11 | 130.05 | 130.14 | 131.02 | 132.46 | 132.16 | 133.23 | 138.05 |
| | Gap(%) | 7.77 | 8.48 | 8.45 | 8.18 | 8.00 | 7.62 | 7.64 | 6.87 | 6.29 | 5.92 |
| | Std($\times 10^2$) | 0.85 | 0.88 | 0.92 | 0.83 | 0.86 | 0.90 | 0.91 | 0.92 | 0.96 | 1.25 |
| | CPU(s) | 10,854.03 | 11,016.63 | 10,987.28 | 1658.89 | 10,892.53 | 10,975.34 | 11,034.56 | 10,918.95 | 10,852.73 | 14,443.41 |
| DECOM | LDR($\times 10^2$) | 128.56 | 129.12 | 129.44 | 129.86 | 129.73 | 130.49 | 131.26 | 131.82 | 132.12 | 137.09 |
| | SIM($\times 10^2$) | 128.55 | 129.11 | 129.43 | 129.84 | 129.72 | 130.48 | 131.24 | 131.79 | 132.09 | 137.06 |
| | Gap(%) | 8.09 | 7.80 | 7.89 | 8.01 | 7.65 | 7.18 | 6.65 | 6.58 | 5.38 | 5.16 |
| | Std($\times 10^2$) | 0.81 | 0.85 | 0.86 | 0.86 | 0.86 | 0.92 | 0.92 | 0.92 | 0.97 | 1.03 |
| | CPU(s) | 117.92 | 211.76 | 154.26 | 56.94 | 59.26 | 145.09 | 225.84 | 94.51 | 39.34 | 3617.10 |
| EVPI | ($\times 10^2$) | 118.92 | 119.77 | 119.96 | 120.21 | 120.50 | 121.73 | 123.05 | 123.66 | 125.35 | 130.34 |

## Appendix D. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.tre.2024.103466.

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