Taylor & Francis
Taylor & Francis Group

International Journal of Production Research

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/tprs20

Universal point scheme with a platform and multiple retailers

Xuehao Feng, Baolin Wen, Wenming Shi & Ilkyeong Moon

To cite this article: Xuehao Feng, Baolin Wen, Wenming Shi & Ilkyeong Moon (2024) Universal point scheme with a platform and multiple retailers, International Journal of Production Research, 62:8, 2943-2962, DOI: 10.1080/00207543.2022.2026004

To link to this article: https://doi.org/10.1080/00207543.2022.2026004

Published online: 27 Jan 2022.

INTERNATIONAL JOURNAL OF PRODUCTION RESEARCH
2024, VOL. 62, NO. 8, 2943–2962
https://doi.org/10.1080/00207543.2022.2026004

Universal point scheme with a platform and multiple retailers

Xuehao Feng[a], Baolin Wen[a], Wenming Shi [b] and Ilkyeong Moon [c]

[a]Ocean College, Zhejiang University, Zhoushan, People's Republic of China; [b]Australian Maritime College, University of Tasmania, Newnham, Australia; [c]Department of Industrial Engineering and Institute for Industrial Systems Innovation, Seoul National University, Seoul, Korea

ABSTRACT

This paper studies the universal point scheme with multiple retailers and a platform that implements the points within a channel. First, we analysed the equilibriums of the channel under the decentralised control and the mode of lateral cost-sharing among retailers. We found that retailers are willing to set the lowest point conversion ratios under the lateral cost-sharing mode compared with the decentralised and centralised control modes. The optimal conversion ratio of a retailer under the decentralised control is greater than the one under the centralised control when the cost spillover and double marginalisation are not significant. Next, we extended the model by considering the participation of new retailer. In this dynamic channel, the impact of the cost spillover phenomenon on the channel members' preferences is discussed. Third, we showed that the wholesale price contract for point management used in the real world has a limitation in terms of profit split. Following this, we proposed a buyback contract and showed the optimal contract parameters under which the channel's profit could be maximised and arbitrarily split. The managerial insights we obtained shed light on how to design a universal point scheme and select channel members to achieve an all-win scenario.

ARTICLE HISTORY
Received 16 April 2021
Accepted 29 December 2021

KEYWORDS
Universal point; third-party platform; supply chain management; retail supply chain; coordinating contract; retailer selection

1. Introduction

Customer reward programmes (CRPs) have been widely implemented in modern marketing to promote sales and enhance customer loyalty. According to a survey by Sun and Zhang (2019), 55 of the top 100 retailers in the U.S. implement CRPs. Different types of CRPs can be observed within the practice, such as consumption points, coupons, special discounts for membership, and free gifts. The consumption point programme is one of the most popular CRPs, under which retailers send points to customers' accounts according to the payment from customers and point generation ratios. Those points can be accumulated in customer accounts and be redeemed for free products or discounts on future purchases.

In the real world, several platforms are running universal point schemes (UPSs) that are extended versions of consumption point programmes. Under a UPS, a company acts as a platform and sells the universal points to the retailers in the channel at wholesale prices. Retailers then send the points to customers, who can then redeem the universal points at any retailer in the channel. In South Korea, for example, CJONE company's universal point platform serves more than twenty retailers from different retailing industries, including food, cosmetics, movies, English training programmes, and so on.[1] Customers can accumulate the *CJONE points* in their own accounts when purchasing products from the retailers in the channel. The amount of obtained points from each purchase depends on the payment made to the retailer and that retailer's point conversion ratio. Customers can redeem their universal points at any of the retailers under the CJONE's UPS.

At first glance, the advantage of the UPS in promoting sales would appear straightforward. For one thing, many retailers set thresholds of point redemption. For example, Starbucks' 'My Starbucks Rewards Card' program sends a certain number of 'stars' to one customer when the customer purchases one cup of coffee. The customer can redeem the stars for a free drink when the number of stars reaches a threshold, such as 12. Another type of threshold can be set with finite reward expiration terms (Sun and Zhang 2019). These thresholds may reduce the likelihood of redeeming the points, in particular for occasional customers. Moreover, the impact of the finite expiry period could make the decision-making more challenging with the competition of multiple retailers (e.g. Bazargan, Karray, and Zolfaghari 2017). Under the UPS, however, customers can accumulate points from multiple retailers and

CONTACT Ilkyeong Moon ikmoon@snu.ac.kr Department of Industrial Engineering and Institute for Industrial Systems Innovation, Seoul National University, Seoul 08826, Korea

thereby reach the redemption threshold more quickly. In such cases, the consumption points may have a greater impact on sales promotion than under the traditional isolated point programmes. Next, the UPS enables customers to enjoy a greater flexibility by allowing them to redeem their points at a wide range of retailers. This means that customers may find a wide menu of products for point redemption. This flexibility becomes even more significant when the number of retailers in the channel is increased. Third, the point inventory at retailers is not involved in the procurement process because the current information system technologies can ensure a smooth transfer of points from the platform to the retailers. This means that retailers can get universal points from the platform immediately and send them instantaneously to customers.

However, it must be noted that the UPS is still not ubiquitous in the real world, and many retailers are running their own isolated point programmes. This implies that challenges exist for an easy implementation of the UPS. To benefit from operating the UPS, a platform usually acts as the leader under the UPS by inviting retailers to join the channel and deciding the prices of the universal points. Prices will influence the unit cost of point generation at retailers, so decisions of the platform and retailers are jointly influenced. Moreover, the phenomenon of cost spillover could result in new characteristics of the equilibrium under the UPS. As a consequence, it may not be straightforward for practitioners to see how to make good decisions with the UPS, or what their profits under the UPS will be. In addition, the objectives of the platform and retailers may conflict and the channel's overall profit may not be achieved. In this case, it would become even more complicated when the platform has the option to invite new retailers to join the UPS. Thus, it is important to explore how the platform makes decisions and the equilibrium between the platform and retailers. This paper attempts to discuss the following three main questions.

(1) how does the platform make the pricing decision and how does the decision influence the decisions of the retailers?
(2) is it always better for all channel members to involve more retailers in the UPS?
(3) how can contracts be developed that coordinate the channel to promote the implementation of the UPS?

The contribution of this paper is threefold. First, we uncover the platform's optimal pricing decision and the corresponding equilibriums under two point-operation modes (i.e. the decentralised control and the mode when retailers laterally share point costs). We compare the equilibriums under the UPS that involves point cost spillover and double marginalisation simultaneously. The theoretical findings in this study show that the various decisions made under the UPS have novel phenomena compared with the pure point-sharing policy (Moon et al. 2020) and with advertising for supply chains. The analyses may provide the platform and retailers with a tool of decision-making and shed light on how to select a good mode to improve their own profits.

Second, we discuss performance in light of involving new retailers under different modes, and we look at the impact of channel parameters on performance. The centralised control is analysed as a benchmark to reveal potential profit improvement under the UPS. According to our study, the platform could be capable of inducing appropriate retailers for the UPS to increase all members' profits under different point-operation modes. Existing and potential retailers also can learn from this study how to estimate their profits under a dynamic channel.

Third, we propose a buyback contract and discuss how to determine the contract's parameters to coordinate the channel and arbitrarily split the total profit. The comparison between our optimal solution and the current practice of one platform in the real world showed the advantage of our buyback contract. Our results will help practitioners design more efficient UPSs in order to achieve all-win scenarios in the retail space.

The rest of this paper is organised as follows. We review relevant literature with the comparison with the current study in the next section. In Section 3, we analyse the optimal decisions of the channel members with multiple retailers under the decentralised control and lateral cost-sharing mode. The case of creating a dynamic channel is discussed in the same section. Section 4 develops the wholesale price contract and buyback contract and discusses the optimal contract parameter setting to coordinate the channel. Section 5 summarises managerial insights and offers directions for future study.

2. Literature review

2.1. Customer reward programme and point sharing

As far as we know, the UPS with a platform and multiple retailers has not been well explored in the existing literature. Most of the studies within the stream of customer reward programmes focused on isolated programmes of a single company. Shin and Sudhir (2010) identified the conditions in which a firm can benefit from rewarding its own customers with price discrimination by considering the competitors' customers. Kopalle et al. (2012) summarised some real examples of frequency reward and customer tier, and emphasised that these two components should be jointly considered to develop a model of customer loyalty programmes. Rossi (2018) explained some reasons for the success of CRPs by investigating customers' purchasing behaviour in the travel industry. Guo et al. (2019) analysed the performance of a reward advertising programme for apps, under which customers can obtain rewards by viewing advertisements from app developers when they are using apps. That study uncovered the necessary balance between encouraging advertisement viewing and accelerated satiation for premium content. Sun and Zhang (2019) developed a model of CRP by considering short and long expiration terms, and found the conditions when a firm's profit can be increased under the CRP with a finite expiration term. Bueker, Kim, and Kim (2020) considered an $M/G/1$ queue model wherein the service supplier may provide the waiting customers with a reward to incentive them to stay until the service is finished. Gu et al. (2021) investigated the retailer's optimal decision on the order quantity of a seasonal product and investment in the customers reward programme. In that model, customers can select to buy the product in the first period with a full price and a reward, or to buy the product in the second period with a lower price but no reward.

In addition to the studies on isolated programmes, the UPS and point-sharing scheme have also been discussed from different aspects. Several studies attempted to obtain the optimal operation of points generated by a third-party platform (Cao, Nsakanda, and Diaby 2012; Cao, Nsakanda, and Diaby 2015; Cao et al. 2015). In those studies, one third-party platform sells its points to multiple retailers, who send those points to customers under a reward policy. Customers can then redeem the points at the platform by receiving free gifts from a limited menu. Those studies focused on the platform's optimal supply and inventory plan of the gifts, without considering the point sharing among retailers. In contrast, our UPS allows customers to redeem points across retailers, which results in a cost spillover phenomenon. Hence, retailers and the platform have quite different decision frameworks under the UPS than they do under those schemes in most existing studies. Moon et al. (2020) studied the point-sharing policy of two retailers who allow customers to redeem points at any partner, and proposed a target rebate contract to coordinate the retailers. That study showed that point sharing may increase the profits of both retailers, but that the cost spillover phenomenon might be a bottleneck for maximising the overall profit. The current paper differs from that study from several aspects. First, this paper analyses the novel characteristics of the UPS under three different modes when the platform makes decisions on the wholesale price of the universal points. Second, the dynamic channel with the participation of potential retailers is discussed in this paper, while Moon et al. (2020) focused on a static channel with two retailers. Third, according to the existence of the platform, we discussed the limitation of the wholesale price contract and developed a tailored buyback contract to coordinate the channel.

2.2. Cooperative sales promotion

Existing studies have discussed various efforts on cooperative sales promotion. Many studies focused on horizontal cooperation between retailers. For example, Gou et al. (2014) discussed a model composed of two firms that sell their own products. In that model, the firms can cooperate by jointly producing a new product and engaging in advertising. Machowska (2019) analysed horizontal cooperative advertising between two firms by considering dynamic goodwill with delayed effects. Yu et al. (2021) proposed a horizontal advertising programme between two retailers with the consideration of advertising threshold effects. In the programme, two retailers can share the advertising cost or share a common brand name. There is also a large body of literature on vertical cooperation between retailers and manufacturers. He, Prasad, and Sethi (2009) developed a dynamic model with a manufacturer and a retailer, in which the former contributes a percentage of the retailer's advertising expenditure. He et al. (2012) provided a generalisation of the model of cooperative advertising between a manufacturer and a retailer, taking into account dynamic retail oligopolies. Chutani and Sethi (2018) discussed the decisions of multiple retailers and multiple manufacturers on the advertising efforts and the subsidy rates to support these efforts. Cao and Ke (2019) designed optimal cooperative advertising policies from the perspective of a single manufacturer, who shared the search advertising cost of multiple retailers. Lu et al. (2019) formulated a game model of a manufacturer and a retailer who decides the advertising level by considering myopic and farsighted strategies of the supply chain members. That study showed that the members' best selections of the strategies are influenced by the marginal contribution of advertising efforts. Gou et al. (2020) analysed the cooperative advertising programme of a supply chain that used a local media company for advertising. That study found that a manufacturer may have a different optimal strategy when a local media company is involved, as compared against a classic two-tier supply chain setting. Xiao et al. (2019) proposed a hybrid game model for a supply chain with horizontal and vertical cooperative advertising. In that model, a single manufacturer invested the brand advertising and multiple non-competing retailers made local advertising efforts. Yang et al. (2019) conducted an empirical study on the performance of cooperative promotion among multiple retailers. Their results stated that customers could evaluate a featured product more positively in an independent sales programme than in a cooperative programme. Li, Zhang, and Dan (2021) discussed the design of cooperative advertising contract between an online retailer and an offline showroom who decides the retail price and the advertising effort, respectively. In that contract, the offline showroom benefit from the commission paid from the retailer for every order through the showroom. Yan et al. (2021a) developed a game-theoretic model to analyse the equilibrium of two cross-market retailers who used 3D printing in their joint sales promotion schemes. Readers may refer to Aust and Buscher (2014) for a detailed review of cooperative advertising models.

Many studies in this stream focused on the spillover of the positive impact of sales promotion, such as the free riding of advertisements (e.g. Liu, Liu, and Chintagunta 2017; Zhou, Liu, and Cai 2019). Cost sharing has been a key approach of achieving channel coordination in sales promotion (e.g. He, Prasad, and Sethi 2009; Cao and Ke 2019; Wang et al. 2020). Yan et al. (2021b), which analysed the cooperative sales promotion with two non-competing retailers in a shopping mall, could be the most closed to our study. In their model, the shopping mall provides customers purchasing at one retailer with coupons which can be used for discount during the purchasing at the other retailer. Yan et al. (2021b) uncovered the optimal decisions of retailers and explored the performance of the horizontal joint promotion. In that study, the denomination of coupons is modelled as a parameter and the profit of the shopping mall is not considered. Unlike that and many other literatures, our model considers the platform's decision on the wholesale prices of the universal point as important decision variables, and we analyse how the platform's pursuit of profits influence the equilibrium of the channel.

2.3. Channel operation with third-party platforms

In the UPS, the platform plays an important role by setting the selling price of universal points. With the popularity of the third-party platform in channel management, many researchers focused on channel management of different types of platforms, such as online sales platforms, matching platforms, sharing platforms, and logistics platforms (Liu et al. 2020). Abhishek, Jerath, and Zhang (2016) studied electronic platform retailing and revealed when retailers should use agency selling instead of traditional reselling. Chu and Manchanda (2016) analysed the impact of networks on consumer-to-consumer platforms and suggested that the cross-network effect may significantly improve the platforms' growth. Aseri et al. (2018) discussed the optimal procurement policies for mobile-promotion platforms who enable advertisers to launch their advertisements on mobile applications. However, the platform's pricing decisions for its advertisement products has not been considered in that study. Tian et al. (2018) showed the impacts of order fulfilment costs and competition intensity on the selection of retailing modes among reselling, platform selling, and hybrid modes. Choi (2019) explored the selling platform for luxury supply chains under blockchain technology. Hagiu and Wright (2020) developed a two-period model with a platform and multiple customers and retailers who may operate new or established products. Haviv, Huang, and Li (2020) showed that sellers on video game platforms may generate a positive intertemporal spillover effect which expanded the demand for other sellers on the platform. In that study, an identification strategy has been proposed to leverage the exogenous variation in the release timing of games. Liu et al. (2020) analysed how market size and data-driven marketing influence the platform's decision on sales mode (i.e. agency selling or reselling). Zhang et al. (2020) conducted a randomised field experiment with Alibaba Group, and their results showed that the long-term effects of price promotions on customers' strategic behaviour may spill over to the sellers on the platform who did not previously offer promotions.

Many studies attempted to find how to coordinate the channel with a platform. Cachon, Daniels, and Lobel (2017) proposed a commission contract for a platform facing service providers when the contract resembles the surge pricing mechanism that has been used by Uber. In the same study, the commission contract was compared with the fixed and dynamic wage structures and dynamic price contracts to show its performance. Bai et al. (2019) discussed the optimal decision of an on-demand service platform by considering impatient customers and service providers. That study revealed how the platform should determine the wage rate and price under different conditions, for example, the number of service providers and waiting costs. Yang et al. (2020) designed a reward mechanism with surge pricing for ride sourcing platforms, under which customers are equipped with reward accounts for peak and off-peak hours. Choi and He (2019) showed that a revenue-sharing contract would be beneficial for the platform of fashion products with peer-to-peer collaborative consumption. Choi et al. (2019) developed a leftover food-sharing platform that covers a single supplier and multiple retailers. Barenji et al. (2019) proposed a logistics platform for better scheduling at e-commerce logistics parks. Compared with the above platforms, the novel managerial challenge of the universal point platform in our study results from the point cost spillover among retailers, which incentives retailer to buy and generate more points in the decentralised control than under some coordination cases. This paper analyses the equilibrium of the channel under three point-operation modes to reveal the platform's preference to these modes. The findings may fill the gap in the literature about the operational of the universal point platform.

3. The universal point scheme under a wholesale price policy

3.1. Problem description

Consider a Stackelberg game model of the UPS with a single platform that operates universal points and N retailers, in which the platform is the leader and retailers are followers. The platform sells the universal points to each retailer with a wholesale price, and retailers use these points to attract more customers. Each of the retailers purchase a unique type of product from the supply market with a unit procurement cost and sell them to customers. Each customer has a point account (e.g. a membership card) and can accumulate some units of universal points in his account when he purchases one unit of the product at a retailer. Customers can use the balance of points in their accounts at any retailer in the channel to obtain the corresponding number of products without payment.

Retail prices set by retailers are important factors that have a joint impact with point conversion on demand. In this study, we assume that retailers do not compete in retail pricing and set retail prices as parameters for the following reasons. First, the UPS in the real world usually covers a single retailer from each of the involved industries to avoid pricing competition among channel members and to achieve a smooth operation of the UPS. In China, for example, one UPS covers a single retailer from each industry among mobile communication companies, airlines, and hotels. Second, the high-price-low-point and low-price-high-point strategies of retailers may influence customer choice under the UPS. In particular, more strategic customers would appear in the channel to utilise the points. Given this, the operation of the UPS would be quite complicated in both practice and research. Hence, the platforms in the real world usually select retailers to avoid this condition. As can be found in the UPS of CJONE, retailers are of similar scales and the retail prices and point conversion ratios are within similar ranges. Third, this assumption helps us to focus on the decisions of the channel members related to point management, and it can also be commonly seen in existing studies on cooperative sales promotion (e.g. Gou et al. 2014; Xiao et al. 2019; Huang and Bai 2021).

We assume that each retailer faces a linear demand function that has been widely used in the literature of related areas of study (e.g. Tian et al. 2018; Choi, Feng, and Li 2020; Moon et al. 2020). Each customer can accumulate $p_i\lambda_i$ units of universal points when he purchases one unit of the product at retailer i. Then, the total amount of universal points converted by retailer i is equal to $D_i(\lambda_i)p_i\lambda_i$. When customers redeem the points at one retailer, they may decide how many points from which retailers will be used. By doing so, the channel can monitor the detailed flow of the points and analyse the behavioural trait of customers. This setting can be achieved by the platform who may tag the points generated by each retailer. If the channel does not track the points by distinguish retailers, then we can obtain $\theta_{1j} = \theta_{2j}, \dots, \theta_{Nj}$ for any j which is a special case. Therefore, we use θ_{ij} to make our model more general. Moreover, we assume for retailer i, at least one θ_{ij} should be non-zero among all $j = 1, 2, \dots, N$ and $j \neq i$. We assume that $(p_i - c_i)b_i > a_i p_i \sum_{j=1}^{N}(\theta_{ij}c_j/p_j)$ to ensure positive optimal λ_i and w_i, $i = 1, 2, \dots, N$. Without loss of generality, we normalise the value of each universal point to be \$1 during the redemption, and then one unit of a point can be redeemed for $1/p_i$ unit of the product at retailer i. We define $1/p_i$ as the unit point redemption cost at retailer i. The notations used in the models are summarised as follows, wherein $i, j = 1, 2, \dots, N$.

3.2. Comparison of three point-operation modes

We consider three point-operation modes, (i.e. decentra lised control, lateral cost-sharing, and centralised control). Under the first two modes, retailer i needs to purchase the universal point from the platform with w_i. We have $\pi_i^{d}(\lambda_i)$ as follows.[2]

$$\pi_i^d(\lambda_i) = (p_i - c_i)D_i(\lambda_i) - \theta_{ii}c_i\lambda_i D_i(\lambda_i) - \sum_{j=1, j\neq i}^{N} \frac{\theta_{ji}p_j\lambda_j D_j(\lambda_j)c_i}{p_i} - w_i p_i \lambda_i D_i(\lambda_i) \quad (1)$$

The first term in Equation (1) presents the gross profit of retailer i that is related to the product operation. The second and third terms show the point cost due to local redemption and flow-in redemption, respectively. The last term is the point purchasing cost of retailer i, $i = 1, 2, \dots, N$. Without loss of generality, we assume that the unit cost of a point at the platform is zero, while an amortised fixed cost of working with each retailer in a period

w_i	Wholesale price of unit universal point for retailer *i* charged by the platform (*a decision variable*)
f	Platform's amortised fixed cost of working with each retailer in a period
λ_i	Conversion ratio of the universal point at retailer *i* (*a decision variable*)
a_i	A positive constant that represents the inherent demand at retailer *i* in a period
b_i	A positive coefficient that represents the effect of λ_i on the demand at retailer *i* in a period
$D_i(\lambda_i)$	Demand at retailer *i* in a period with $D_i(\lambda_i) = a_i + b_i\lambda_i$
$\Delta D_i(\lambda_i)$	$= \Delta b_i\lambda_i$, representing the additional demand at retailer *i* resulting from the participation of the $(N+1)th$ retailer
p_i	Retail price of products at retailer *i*
c_i	Unit procurement cost of retailer *i* related to the product or material supplier
θ_{ij}	Percentage of points converted by retailer *i* that is redeemed at retailer *j*, with $0 \le \theta_{ij} \le 1$ and $\sum_{j=1}^{N} \theta_{ij} = 1$
β_i	Buyback price for retailer *i* charged by the platform (*a decision variable*)
${\pi_i}^d(\lambda_i)$	Profit of retailer *i* under the decentralised control
${\pi_0}^d$	Profit of the platform under the decentralised control
${\pi_i}^l(\lambda_i)$	Profit of retailer *i* under the lateral cost-sharing mode
${\pi_0}^l$	Profit of the platform under the lateral cost-sharing mode
${\pi_{ch}}^c$	Total profit of the channel under the centralised control
$\Delta\pi_i$	Gap between the corresponding profits of retailer *i* after and before a new retailer joins the UPS
${\pi_i}^w(\lambda_i)$	Profit of retailer *i* under the wholesale price contract
${\pi_0}^w$	Profit of the platform under the wholesale price contract
${\pi_i}^b(\lambda_i)$	Profit of retailer *i* under the buyback contract
${\pi_0}^b$	Profit of the platform under the buyback contract

is considered. Then, we have the profit function of the platform, ${\pi_0}^d(w_1, w_2, \ldots, w_N)$, as follows.

$$\pi_0^d(w_1, w_2, \ldots, w_N) = \sum_{i=1}^{N} w_i p_i \lambda_i D_i(\lambda_i) - f \cdot N \quad (2)$$

Let ${\lambda_i}^d$ and ${w_i}^d$ be the equilibrium λ_i and $w_i$under the decentralised control. Because the platform acts as the leader, we first analyse λ_i, which is a function of w_i. Next, we derived the platform's optimal decision on w_i, based on the retailers' decision. We can obtain the equilibrium of the channel as Theorem 3.1 shows.

Theorem 3.1: *Under the decentralised control, there exists a unique pair of ${\lambda_i}^d$ and ${w_i}^d$ ($i = 1, 2, \ldots, N$) that satisfies*

$$w_i^d = \frac{\sqrt[3]{s_i^d + \sqrt{\left(s_i^d\right)^2 + \left(u_i^d\right)^3}} + \sqrt[3]{s_i^d - \sqrt{\left(s_i^d\right)^2 + \left(u_i^d\right)^3}} - c_i\theta_{ii}}{p_i} \quad (3)$$

$$\lambda_i^d = \frac{(p_i - c_i)b_i - (\theta_{ii}c_i + p_i w_i^d)a_i}{2b_i(\theta_{ii}c_i + p_i w_i^d)} \quad (4)$$

wherein $s_i^d = \theta_{ii}c_i(p_i - c_i)^2 b_i^2/a_i^2$ *and* $u_i^d = (p_i - c_i)^2 b_i^2/(3a_i^2)$.

Because the UPS allows customers to redeem their points at any retailer in the channel, the point redemption cost would be undertaken by all retailers. This cost spillover depends on the percentage of switched redemption (i.e. $\sum_{j=1, j\neq i}^{N} \theta_{ij}$). When $\sum_{j=1, j\neq i}^{N} \theta_{ij}$ is increased, retailer *i*'s marginal cost of point conversion will be decreased and the retailer will have more of an incentive to set a higher λ_i. Even though other retailers' points flow to retailer *i*, this cost will not influence the retailer's decision on λ_i. This implies that under the decentralised control, retailers might be willing to set higher point-conversion ratios compared with the optimal one for overall profit maximising of all retailers. This phenomenon is similar with the finding in Moon et al. (2020) for a two-retailer case, but can be more significant under a larger N when customers can redeem their points at more retailers and θ_{ii} is smaller. In this case, the utility of customers can be improved because of higher point-conversion ratios, while retailers may lose profits.

As a part of the marginal cost of generating one point of retailer *i*, w_i has a negative impact on retailer *i*'s incentive of raising $\lambda_i$that is similar to the double marginalisation under the newsvendor problem (e.g. Cachon and Lariviere 2005). The impact of the cost spillover could be neutralised and ${\lambda_i}^d$ might be relatively close to the channel-wide optimal decision than under the pure point-sharing scheme without such a cost (Moon et al. 2020). It means that the double marginalisation here plays a different role from the role it plays in the traditional supply chain, in which it is the reason that the channel's total profit under the decentralised control is lower than the centralised control (e.g. see Moon and Feng 2017). A higher $\theta_{ii}c_i/p_i$ leads to a higher unit cost of point redemption at retailer *i*. In this case, a lower w_i may not result in a much higher point demand of retailer *i*, and vice versa. Then, both the cost spillover and the impact of w_i on λ_i would be less significant. Therefore, the platform could be incentivised to set a higher ${w_i}^d$ when $\theta_{ii}c_i/p_i$ is increased. This may also explain the phenomenon found from the proof of Theorem 3.1 that ${w_i}^d$ is limited within the range of $(0, \theta_{ii}c_i/p_i)$. This means that the upper bound of ${w_i}^d$ is also increased with $\theta_{ii}c_i/p_i$which implies a possibility of a higher ${w_i}^d$. Further, we may conjecture that a lower level of cost spillover might result in a higher wholesale price which enhances the double marginalisation. This joint impact also distinguishes the UPS with a platform from the existing model discussed by Moon et al. (2020).

It would stand to reason that retailers are interested in addressing this challenge of cost spillover in order to achieve higher profits. Next, we discuss the mode under which all retailers laterally share the total point redemption cost to see the performance of a partial coordination of the channel. This mode can be defined as *lateral cost-sharing*.

Remark 3.1: $w_i{}^d$ is limited within the range of $(0, \theta_{ii}c_i/p_i)$.

Definition 1: Under the lateral cost-sharing (LCS) mode, all retailers laterally share the total point redemption cost so that each retailer undertakes all of the cost related to the redemption of the points generated by it.

Under the LCS mode, the retailers need to monitor the flow of points among them to determine the cost to share. As discussed earlier, the platform can help retailers to do so without difficulty by tagging the universal points generated by each retailer. Otherwise, the retailers may use θ_{ij} with $\theta_{1j} = \theta_{2j}, \ldots, \theta_{Nj}$ for any j. We have the profit function of retailer i under the LCS mode, $\pi_i{}^l(\lambda_i)$, as follows.

$$\pi_i^l(\lambda_i) = (p_i - c_i)D_i(\lambda_i) - \theta_{ii}c_i\lambda_i D_i(\lambda_i) - \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}p_i\lambda_i D_i(\lambda_i)c_j}{p_j} - w_i p_i \lambda_i D_i(\lambda_i) \quad (5)$$

Under the LCS mode, the distortion of the decision on λ_i resulting from the cost spillover can be eliminated, because each retailer only needs to take care of all the redemption cost retailed to the points generated by it. Theorem 3.2 shows the equilibrium of the channel under the LCS mode.

Theorem 3.2: *Under the LCS mode, (i) there exists a unique pair of $\lambda_i{}^l$ and $w_i{}^l$ that satisfies*

$$w_i^l = \frac{\sqrt[3]{s_i^l + \sqrt{(s_i^l)^2 + (u_i^l)^3}} + \sqrt[3]{s_i^l - \sqrt{(s_i^l)^2 + (u_i^l)^3}} - p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j}}{p_i} \quad (6)$$

$$\lambda_i^l = \frac{(p_i - c_i)b_i - \left(p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j} + p_i w_i^l\right) a_i}{2b_i \left(p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j} + p_i w_i^l\right)} \quad (7)$$

wherein $s_i^l = p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j}(p_i - c_i)^2 b_i^2/a_i^2$ *and* $u_i^l = (p_i - c_i)^2 b_i^2/(3a_i^2)$.

(ii) each retailer is incentivised to set a higher point conversion ratio under the decentralised control than under the LCS mode (i.e. $\lambda_i{}^d > \lambda_i{}^l$, $i = 1, 2, \ldots, N$).

(iii) the platform obtains a lower profit than under the decentralised control.

Under the LCS mode, one unit of a point generated by retailer i is associated with a unit cost of $(w_i^l + \sum_{j=1}^{N} \theta_{ij}c_j/p_j)$. Similar to the discussion about Theorem 3.1, retailer i would be less sensitive with w_i for a higher marginal cost of point redemption. Under the LCS mode, the platform may select a higher $w_i{}^l$ if $\sum_{j=1}^{N} \theta_{ij}c_j/p_j$ is increased. This may explain why $\sum_{j=1}^{N} \theta_{ij}c_j/p_j$ is the upper bound of $w_i{}^l$ as the proof of Theorem 3.2 shows. When comparing the LCS mode and the decentralised control, we can see that the unit point redemption cost under the latter mode is $\theta_{ii}c_i/p_i$ with $\theta_{ii}c_i/p_i < \sum_{j=1}^{N} \theta_{ij}c_j/p_j$. We may conjecture that the platform is more likely to set a higher w_i under the LCS mode than under the decentralised control which results in $w_i^l + \sum_{j=1}^{N} \theta_{ij}c_j/p_j > w_i^d + \theta_{ii}c_i/p_i$ (see Example 3.1 as an experimental illustration). This means that, because the cost spillover is addressed under the LCS mode, the double marginalisation would be more significant. As a consequence, the LCS mode is associated with a lower λ_i and a lower demand at retailer i than the decentralised control. In addition, the platform's profit is lower than under the decentralised control because the amount of points purchased by retailer i is decreased more significantly. This means that the platform may prefer decentralised control to the LCS mode. However, it is not certain if all retailers can always benefit from the LCS mode compared to the decentralised control. In the latter mode, retailer i could undertake a small portion of the point cost because of a high flow-out percentage (i.e. $\sum_{j=1,j\neq i}^{N} \theta_{ij}$). That retailer may need to undertake more costs under the LCS mode if $\sum_{j=1,j\neq i}^{N} \theta_{ji}$ is small.

Remark 3.2: $w_i{}^l$ is limited within the range of $(0, \sum_{j=1}^{N} \theta_{ij}c_j/p_j)$.

To further investigate the performance of the UPS, we compare the above two modes with the centralised control, under which the profit function of the channel,

$\pi_{ch}{}^{c}(\lambda_1, \lambda_2, \ldots, \lambda_N)$, can be written as follows.

$$\pi_{ch}^{c}(\lambda_1, \lambda_2, \ldots, \lambda_N) = \sum_{i=1}^{N}(p_i - c_i)D_i(\lambda_i) - \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\theta_{ij}p_i\lambda_i D_i(\lambda_i)c_j}{p_j} - f \cdot N \tag{8}$$

Let $\lambda_i{}^c$ be the optimal λ_i that maximises $\pi_{ch}{}^{c}(\lambda_1, \lambda_2, \ldots, \lambda_N)$.

Theorem 3.3: *(i) Retailer i is incentivised to set a higher point conversion ratio under the centralised control than under the LCS mode, i.e.* $\lambda_i{}^c > \lambda_i{}^l$, *i = 1, 2, … , N.*

(ii) Retailer i is incentivised to set a higher point conversion ratio under the decentralised control than under the centralised control when $\frac{\theta_{ii}c_i}{p_i} \le \sum_{j=1,j\neq i}^{N}\frac{\theta_{ij}c_j}{p_j}$.

When considering the centralised control as a benchmark, we can see that the cost spillover reduces the marginal cost from λ_i, while the double marginalisation has an opposite impact on the marginal cost. Hence, $\lambda_i{}^c$is greater than $\lambda_i{}^l$ when the cost spillover is not involved under the LCS mode.

As Theorem 3.3 shows, a comparison between the modes of the decentralised control and centralised control can be more complicated because of the joint impact of the cost spillover and double marginalisation. Consider $\lambda_i{}^c$ as a benchmark value. It is negatively associated with the unit redemption cost of points generated by retailer i which is equal to $(\theta_{ii}c_i/p_i + \sum_{j=1,j\neq i}^{N}\theta_{ij}c_j/p_j)$. We can see that $\lambda_i{}^d$ is influenced by $(w_i{}^d + \theta_{ii}c_i/p_i)$ under the decentralised control. We discussed that a low $\theta_{ii}c_i/p_i$ means a low unit cost of local redemption at retailer i, and it might result in a lower w_i under the decentralised control. If $\theta_{ii}c_i/p_i$ is sufficiently small, then $w_i{}^d$ will also be smaller than $\sum_{j=1,j\neq i}^{N}\theta_{ij}c_j/p_j$. In this case, the impact of the cost spillover is more influential than the double marginalisation which may lead to $\lambda_i{}^d < \lambda_i{}^c$. Hence, the relationship between $\theta_{ii}c_i/p_i$ and $\sum_{j=1,j\neq i}^{N}\theta_{ij}c_j/p_j$ has a crucial impact on $\lambda_i{}^d$. Note that the $\theta_{ii}c_i/p_i \le \sum_{j=1,j\neq i}^{N}\theta_{ij}c_j/p_j$ is a sufficient but not necessary condition because $\theta_{ii}c_i/p_i$ is a rough upper bound on $w_i{}^d$. On the other hand, however, when $\theta_{ii}c_i/p_i$ is much larger than $\sum_{j=1,j\neq i}^{N}\theta_{ij}c_j/p_j$, we may find cases wherein $\lambda_i{}^d < \lambda_i{}^c$. Suppose a special case wherein $\theta_{ii} = 1$, and we have

$$\lambda_i^c = \frac{(p_i - c_i)b_i - a_i c_i}{2b_i c_i} \tag{9}$$

and

$$\lambda_i^d = \frac{(p_i - c_i)b_i - (c_i + p_i w_i^d)a_i}{2b_i(c_i + p_i w_i^d)} \tag{10}$$

The proof of Theorem 3.1 shows that $w_i{}^d > 0$ still holds when $\theta_{ii} = 1$. Then, it can be found that $\lambda_i{}^d < \lambda_i{}^c$. This means that the double marginalisation will be dominant when the cost spillover phenomenon is not significant.

Based on the above theorical analysis, we may discuss Theorem 3.3(ii) from a more intuitive and practical perspective as follows. Retailer i has a lower cost of local redemption than the cost due to flow-out redemption when $\theta_{ii}c_i/p_i < \sum_{j=1,j\neq i}^{N}\theta_{ij}c_j/p_j$. It means that retailer i bears a relatively small cost of the points generated by itself, while other retailers have to undertake a large cost for retailer i in the decentralised control. In this case, the unit cost of point redemption of retailer I under the decentralised control would be smaller than the one under the centralised control. With a lower cost resulting from the point redemption, retailer i will be persuaded to set a $\lambda_i{}^d$ that is higher than $\lambda_i{}^c$, $i = 1, 2, \ldots, N$.

3.3. Discussion of the cases with a dynamic channel

The above discussions show the decisions of the retailers and the platform under a fixed group of retailers. The cases with a dynamic channel would be more complicated, wherein potential retailers would join the UPS and existing retailers might quit. Let retailer i have an additional demand, $\Delta D_i(\lambda_i)$, with the participation of the $(N+1)th$ retailer. Then, $\pi_i{}^d(\lambda_i)$ and the profit of the $(N+1)th$ retailer under the decentralised control can be written as follows, $i = 1, 2, \ldots, N+1$.

$$\pi_i^d(\lambda_i) = (p_i - c_i - c_i\theta_{ii}\lambda_i - w_i p_i\lambda_i)(D_i(\lambda_i) + \Delta D_i(\lambda_i)) - \sum_{j=1,j\neq i}^{N+1}\frac{\theta_{ji}p_j c_i\lambda_j(D_j(\lambda_j) + \Delta D_j(\lambda_j))}{p_i} \tag{11}$$

$$\begin{aligned}\pi_{N+1}^d(\lambda_{N+1}) &= (p_{N+1} - c_{N+1} - c_{N+1}\theta_{N+1,N+1}\lambda_{N+1} \\ &\quad - w_{N+1}p_{N+1}\lambda_{N+1})(D_{N+1}(\lambda_{N+1}) \\ &\quad + \Delta D_{N+1}(\lambda_{N+1}))\end{aligned}$$

$$-\sum_{j=1}^{N}\frac{\theta_{j,N+1}p_j c_{N+1}\lambda_j(D_j(\lambda_j)+\Delta D_j(\lambda_j))}{p_{N+1}} \tag{12^3}$$

More retailers imply more options for the customers' point redemption. If a new retailer is invited to join the UPS, the demand of all the existing N retailers and the new retailer would be increased. Hence, we can conjecture that more retailers involved in the UPS would result in a higher total profit of the channel under the centralised control, if the channel's administration cost of a new retailer is negligible.

In contrast, it may not be acceptable for the channel to do likewise under the decentralised control and the LCS mode. When the $(N+1)th$ retailer joins the UPS, existing Retailers 1 to N may benefit from two aspects. The first benefit is the increased demand because of the higher flexibility of point redemption. The second benefit is that $\sum_{j=1}^{N}\theta_{ij}$ is also lower because some customers can redeem their points at the new retailer, (i.e. $\sum_{j=1}^{N+1}\theta_{ij}=1$). This means that a higher percentage of points flow from retailer i to other retailers. Then, retailer i has a lower unit cost of point redemption at retailer i in the channel of $(N+1)$ retailers than N retailers under the decentralised control. However, the above two advantages do not guarantee higher profits of existing retailers. Suppose an extreme case, wherein most of the points generated by the $(N+1)th$ retailer are redeemed at retailer i, $i=1, 2,\ldots,N$. Then, retailer i's profit could be lower than her profit under the N-retailer case. If a new retailer joins but one existing retailer quits, it is not certain if the platform benefits from this invitation, which may further lead to a penalty of the goodwill of the UPS. We illustrate the impact of the number and scales of retailers with Example 1.

Example 3.1: This example shows the optimal decisions and corresponding profits of retailers and the platform under the decentralised control, LCS mode, and centralised control. Consider Retailer 6 as the potential retailer with a fixed demand before joining the UPS (i.e. a_6). Tables 1 and 2 show the parameter settings containing a_i, b_i, c_i, p_i, f *and* θ_{ij} of each retailer. When Retailer 6 participates in the UPS, the demands of Retailers 1–6 could be increased, and this increment coefficient is defined as Δb_i. Then, we have the increased demands of retailer i as $\Delta D_i(\lambda_i)\quad \Delta b_i\lambda_i$ in this case, $i=1, 2, 3, 4, 5, 6$.

First, we investigated the cases with five existing retailers (i.e. Retailers 1–5). The detailed results and comparison with five retailers under the decentralised control and LCS mode are summarised in Table 3. When retailers laterally share their point redemption costs, they all selected lower λ_i, and the platform was incentivised to set higher wholesale prices of the universal points in this scenario than in the decentralised control scenario. Hence, most of the retailers could obtain higher profits under the LCS mode. However, Retailer 4 may find the decentralised control more beneficial. As Table 2 shows, Retailer 4 has relatively high flow-out and low flow-in percentages of point redemption (i.e. $\sum_{j=1,j\neq 4}^{5}\theta_{4j}=0.6$ and $\sum_{i=1,i\neq 4}^{5}\theta_{i4}=0.33$). This means that under the decentralised control, this retailer only needs to undertake a small part of the point cost. Under the LCS mode, however, Retailer 4 needs to undertake all its point cost. This difference is also associated with the largest gap between ${\lambda_4}^d$ and ${\lambda_4}^l$ among all retailers. In addition, the profits of the platform under the decentralised control and LCS mode are decreased from \$2274 to \$–2054, respectively. The negative profit under the LCS mode results from the gap between low revenue and high fixed costs.

Table 1. Parameter setting of Retailers 1–6.

Retailer	1	2	3	4	5	6
a_i	100	150	160	125	100	130
b_i	2400	2000	1600	850	1200	0
Δb_i	200	50	150	80	100	1600
c_i	80	100	90	150	120	130
p_i	104	130	126	195	156	160
f			1200			

We extended the above case by considering Retailer 6, who has the potential to participate in the UPS. The profits of Retailers 1–5 and the platform without Retailer 6 are used as the benchmark. The profit of Retailer 6 before it joins the UPS can be obtained from $(p_6–c_6)a_6$. Figure 1 shows the impact of cost efficiency with different p_6 on the profits of the channel members. When p_6 is small, the cost efficiency of Retailer 6 is high. However, the total amount of points generated by Retailer 6 is also small because of the small p_6 under the LCS mode and the increased revenue of the platform. Given this, the channel cannot meet the fixed cost of involving the new retailer, for the respective reasons (see Figure 1(a,d)). This shows that the platform should not only pay attention to the cost efficiency of retailers, but should also investigate the scale of points each retailer has.

Retailer 6 would be incentivised to set a higher λ_6 for a higher demand when p_6 is increased, because of a higher cost efficiency. In this case, the total points generated by Retailer 6 would also increase, and Retailers 1–5would need to undertake a higher flow-in point redemption

Table 2. The percentage of point flow-in and flow-out redemption between retailers.

θ_{ij}		j = 1	2	3	4	5	6
i	1	0.8 (0.7)	0.05 (0.05)	0.07 (0.07)	0.03 (0.03)	0.05 (0.05)	0 (0.1)
	2	0.1 (0.1)	0.5 (0.45)	0.15 (0.15)	0.1 (0.1)	0.15 (0.15)	0 (0.05)
	3	0.075 (0.075)	0.075 (0.075)	0.7 (0.55)	0.1 (0.1)	0.050 (0.050)	0 (0.15)
	4	0.1 (0.1)	0.225 (0.125)	0.175 (0.175)	0.4 (0.4)	0.1 (0.1)	0 (0.1)
	5	0.05 (0.05)	0.25 (0.2)	0.1 (0.1)	0.1 (0.1)	0.5 (0.5)	0 (0.05)
	6	0 (0.275)	0 (0.125)	0 (0.05)	0 (0.1)	0 (0.05)	0 (0.4)

Note: The numbers in the brackets illustrate the percentages in the six-retailer case.

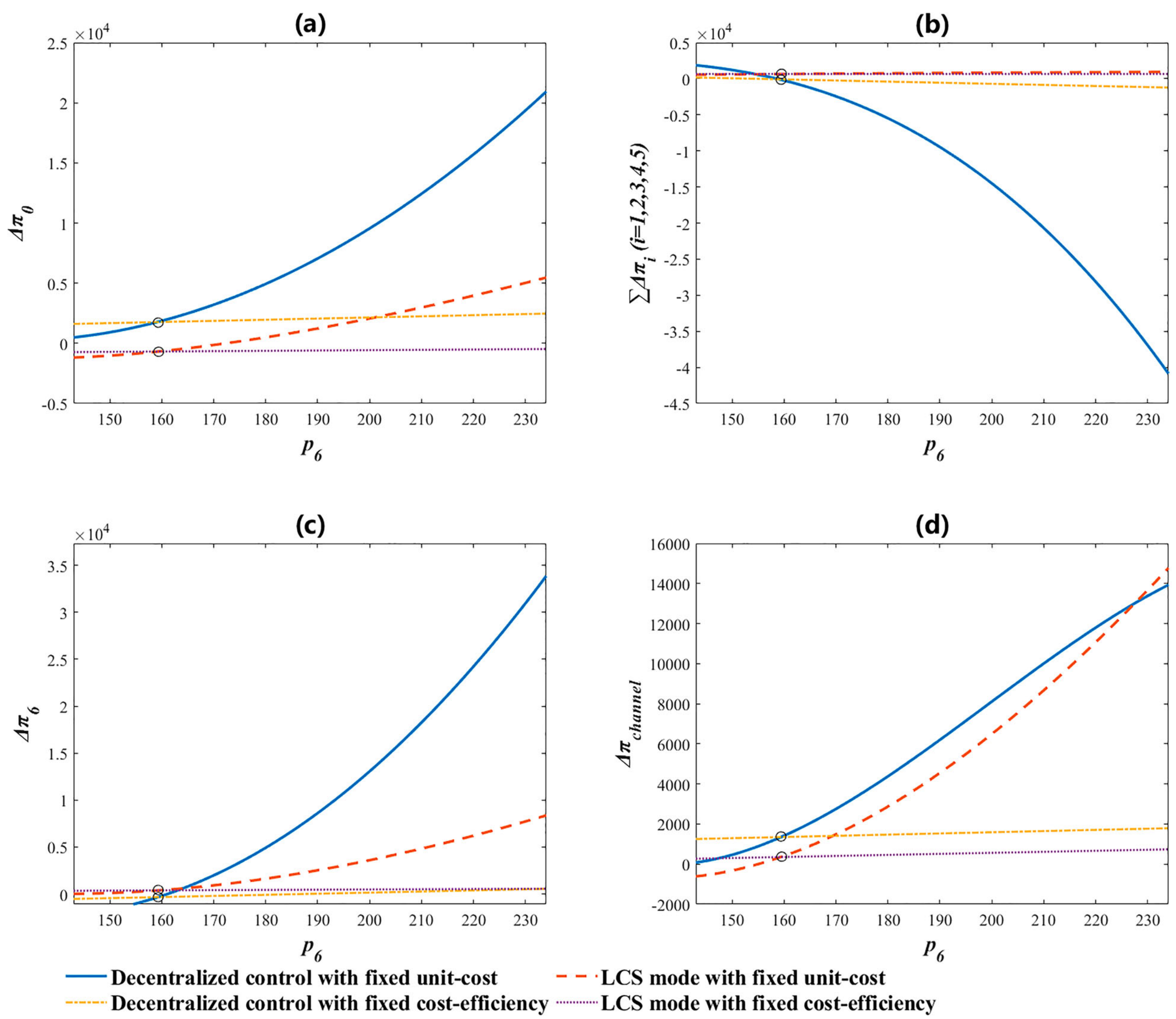

Figure 1. Impact of the cost efficiency of Retailer 6.

Table 3. Results under the decentralised control and LCS mode.

Retailer	λ_i^d	λ_i^l	ω_i^d	ω_i^l	π_i^d	π_i^l
1	0.077	0.060	0.562	0.669	2750	3672
2	0.121	0.052	0.344	0.532	4946	5399
3	0.107	0.064	0.408	0.507	5969	6840
4	0.137	0.036	0.241	0.290	6112	5856
5	0.118	0.050	0.336	0.500	4027	4182

cost under the decentralised control. Even though the unit cost of point redemption at Retailer 6 that is undertaken by Retailers 1–5 is decreased under the higher cost efficiency, the total profits of these five existing retailers are still decreased, because of the higher redemption cost of points from Retailer 6 (see Figure 1(b)). However,

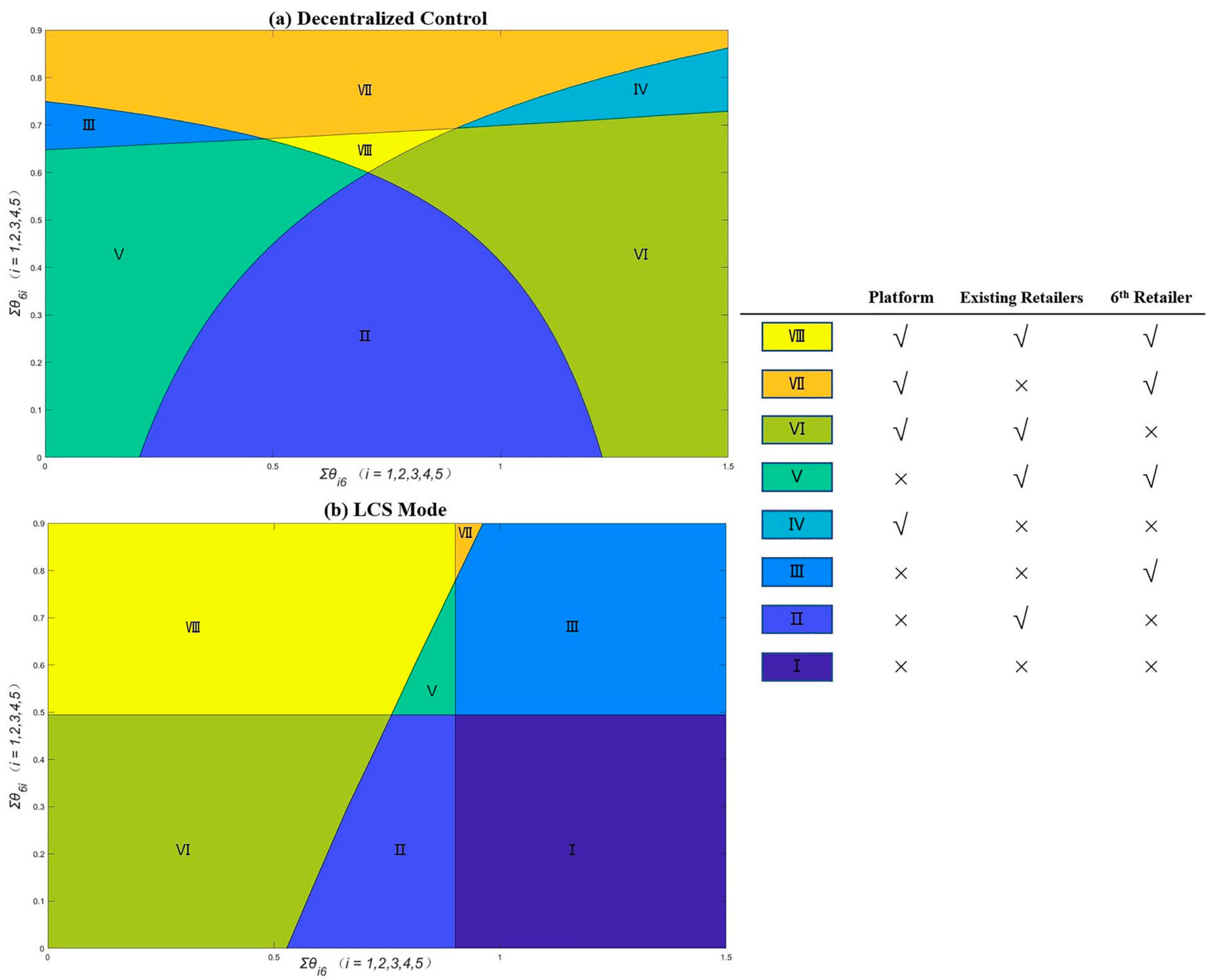

Figure 2. Profit changes of the channel members under the decentralised control and LCS mode.

because those five retailers do not need to undertake such costs under the LCS mode, the change of cost efficiency has a less significant impact on Retailers 1–6 than it does in the decentralised control. In addition, for a higher cost efficiency at Retailer 6, the number of products corresponding to each unit of point redeemed at Retailer 6 is smaller. As a consequence, the unit point redemption costs of Retailers 1–5 are decreased, and Figure 1(b) shows the increasing profits of those retailers with p_6 under the LCS mode. From Figure 1(c), we can see that $\Delta\pi_6$ is more significant under the decentralised control than under the LCS mode, because of the cost spillover. A linear relationship between the fixed p_6/c_6 and the profit of the platform can be seen in Figure 2 under both the decentralised control and the LCS mode. Generally speaking, for a p_6/c_6, the simultaneous change of p_6 and c_6 echoes similar impacts of dynamic p_6/c_6 with a changeable p_6. As can be seen from Figure 2(b), however, the profits of Retailers 1–5 are not influenced by p_6/c_6 under the LCS mode because of the fixed unit point redemption cost at Retailer 6 (see Equation 5).

Note that the above phenomena shown in Figure 1 may not always hold under all parameter settings. Under the decentralised control, the $(N+1)th$ retailer results in extra redemption cost of the points at existing retailer i, equal to $\left(D_{N+1}(\lambda_{N+1})p_{N+1}\lambda_{N+1}\theta_{(N+1)i}c_i/p_i + \sum_{j=1}^{N+1}\Delta D_j(\lambda_j)p_j\theta_{ji}\lambda_j c_i/p_i\right)$. Note that this cost depends on $D_{N+1}(\lambda_{N+1})$, $\Delta D_j(\lambda_j)$, and p_j, which are independent of retailer i, for $j \neq i$. Under the LCS mode, existing retailer i can also benefit from the higher demand, but has an extra redemption cost as $\left(D_i(\lambda_i)p_i\lambda_i\theta_{i(N+1)}c_{N+1}/p_{N+1} + \sum_{j=1}^{N+1}\Delta D_i(\lambda_i)p_i\theta_{ij}\lambda_i c_j/p_j\right)$. This means that there also exists the possibility of a profit decrease for retailer i when the $(N+1)th$ retailer joins if $\theta_{i(N+1)}c_{N+1}/p_{N+1}$ is sufficiently high. Therefore, it is not easy to obtain general conclusions on the conditions under which retailer i can benefit or lose from the participation of retailer $N+1$. In this example, we studied the joint impacts of flow-in and flow-out redemption percentages on the profits to provide more managerial insights into how parameters will influence the performance of the UPS.

Figure 2(a,b) is respectively divided into different regions on the basis of profit change when Retailer 6 joins the UPS. For each region in Figure 2, '$\surd$' represents that the corresponding member(s) do not have smaller profits when Retailer 6 joins than in the case when they are compared against the benchmark, and '$\times$' represents the opposite result. In particular, if any one of Retailers 1–5 has a smaller profit when Retailer 6 joins, then we mark the corresponding case with '$\times$'. From Figure 2(a), we can see that for a $\sum_{i=1}^{5}\theta_{i6}$, the profit of Retailer 6 is non-decreasing with $\sum_{i=1}^{5}\theta_{6i}$. For example, when $\sum_{i=1}^{5}\theta_{i6} = 1.3$, the region types are VI, IV, and VII if $\sum_{i=1}^{5}\theta_{6i}$ is increased from zero to 0.9, respectively. This means that the new retailer can obtain a higher profit under the UPS if a higher percentage of points generated by that retailer is redeemed at existing retailers. Retailers 1–5 would be positive about the involvement of Retailer 6 if $\sum_{i=1}^{5}\theta_{i6}$ is relatively high under the decentralised control. We can conjecture that retailers prefer high flow-out and low flow-in ratios for a low level of point cost. Figure 2(a) also implies the preference of the platform. The regions in that figure with '$\times$' for the platform are Regions II, III, and V, and are associated with low $\sum_{i=1}^{5}\theta_{i6}$ and $\sum_{i=1}^{5}\theta_{6i}$. When customers would like to redeem more points across the channel, the cost spillover would be more significant. Then, retailers would be incentivised to generate more points and the platform might obtain a high revenue that could exceed the fixed cost of inviting Retailer 6. Therefore, the platform would benefit from inviting retailers sharing common customers, for a high level of switched redemption under the decentralised control (i.e. a high θ_{ij}, $i, j = 1, 2, \ldots, 6$, $i \neq j$).

Under the LCS mode, the layout of regions is quite different from the layout of regions under the decentralised control. Because the cost spillover is eliminated by the LCS, the retailers' motivation on generating points is weaker than it is under the decentralised control. As a consequence, generally speaking, the total area of regions with '$\surd$' for the platform in Figure 2(b) is smaller than it is in Figure 2(a). Moreover, one region type might have different positions under these two modes. Under the LCS mode, the profit of a retailer is influenced by the cost efficiencies of all channel members. In our example, c_6/p_6 is equal to 0.8125, which is greater than c_i/p_i, $i = 1, 2, \ldots, 5$. Then, one unit point redeemed at Retailer 6 can result in a higher cost than the redemption of one unit point at Retailers 1–5. This means that, under the LCS mode with a relatively small percentage of flow-out redemption and low-cost efficiency, the new retailer is always worse off by joining the UPS (see Regions I, II, and VI in Figure 2(b)). Under the same reasoning, existing retailers can lose their profits under the LCS mode when more points generated by them are redeemed at the new retailer with low-cost efficiency. For example, Regions VIII$\rightarrow$V$\rightarrow$III in Figure 2(b) show that the profits of Retailers 1–5 could be decreased with a higher $\sum_{i=1}^{5}\theta_{i6}$.

Those above results illustrate that different members may have different preferences regarding the new retailer under different modes. Under the decentralised control, existing retailers may worry about the large number of point redemption flowing from a new large-scale retailer. Retail managers should look into the cost efficiency and percentages of switched redemption in the channel under both the decentralised control and the LCS mode, to avail themselves of a higher profit by joining the UPS. The platform may have an opposing preference because including a large-scale retailer implies high point sales. This difference could be much smaller under the LCS mode without cost spillover. From the view of the platform, it is not easy to tell whether a larger N can result in higher profits to the platform. A platform in the real world needs tailored analyses of its performance under the condition of inviting more retailers, and such analyses would need to consider many characteristics. For example, the retail price and demand scale of a potential retailer influences the number of points that can be purchased by it. This increment in the point revenue of the platform should be higher than the extra fixed cost incurred. The platform also needs to investigate the profit thresholds existing retailers need to maintain to stay in the UPS to avoid their quitting it. The effects of inviting a new retailer to join the platform are different under the decentralised control and LCS modes, and these effects are also important considerations for expanding the channel.

As can be seen from Figure 2, the LCS mode can achieve the all-win scenario under a wider range of parameter settings than is offered under the decentralised control because of the elimination of cost spillover. However, we can see that the LCS mode still fails to achieve the all-win scenario under Regions I to VII. This implies that the UPS may be difficult to implement widely without full coordination when the number of retailers is increased. This may explain why this scheme is not ubiquitous in the real world. In the next section, we propose a buyback contract for point management that can coordinate the whole channel with the highest flexibility of profit split.

4. The buyback contract for point management

Prior to introducing the buyback contract, we analysed the parameter setting of the wholesale price contract, which is analysed in Section 3, to coordinate the channel.

Theorem 4.1: *Under the wholesale price contract, the channel profit can be coordinated when* $w_i = \sum_{j=1,j\neq i}^{N} \theta_{ij}c_j/p_j$ *and the profit split is fixed.*

Theorem 4.1 shows that the wholesale price contract can coordinate the channel by setting w_i as the unit cost of per point under the flow-out redemption. In this case, each retailer needs to undertake all of the redemption cost due to the points generated by that retailer. Then, the cost spillover and the double marginalisation can be eliminated simultaneously. However, the profit split is also fixed under such a contract and the platform's profit can be obtained as

$$\pi_0^w(\lambda_1, \lambda_2, \ldots, \lambda_N) = \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j} p_i\lambda_i D_i(\lambda_i) - N \cdot f \tag{13}$$

Considering that each member has an opportunity cost of joining the UPS, the wholesale price contract may fail to coordinate the channel if the fixed profit split cannot meet all profit requirements of the channel members. Figure 3 illustrates the profits of retailers, the platform, and the whole channel under three different modes. We can see that the channel's profit has been increased by the wholesale price contract compared with the decentralised control and the LCS mode. Under the fixed profit split of the contract, however, Retailers 1–3 obtain higher profits than they do under the decentralised control, while the profits of Retailers 4–5 and the platform are decreased. When using the profits under the LCS as the benchmark, we can find that Retailers 2, 3, and 5 obtain lower profits, respectively. The above results imply that the wholesale price contract cannot ensure an increment in the profit for all of the channel members. When some members, the platform in particular, fail to benefit from the wholesale price contract, such a contract may not be easy to implement.

Under the buyback contract, each retailer pays the platform a price for a unit point generated by that retailer. In addition, the retailer can obtain a buyback price from the platform for a unit point that the customers redeem at this retailer. Note that these redeemed points could be sent from several different retailers in the channel. Let the buyback price for retailer i be β_i and the optimal λ_i under the buyback contract be ${\lambda_i}^b$. Retailer i and the platform's profit functions can be shown as follows.

$$\pi_i^b(\lambda_i) = (p_i - c_i - c_i\theta_{ii}\lambda_i - w_ip_i\lambda_i)D_i(\lambda_i) - \sum_{j=1,j\neq i}^{N} \frac{\theta_{ji}p_jc_i\lambda_jD_j(\lambda_j)}{p_i} + \beta_i\left(p_i\theta_{ii}\lambda_iD_i(\lambda_i) + \sum_{j=1,j\neq i}^{N} \theta_{ji}p_j\lambda_jD_j(\lambda_j)\right) \tag{14}$$

$$\pi_0^b = \sum_{i=1}^{N} w_ip_i\lambda_iD_i(\lambda_i) - \sum_{i=1}^{N}\sum_{j=1}^{N} \beta_i\theta_{ji}p_j\lambda_jD_j(\lambda_j) - N \cdot f \tag{15}$$

Theorem 4.2: *The buyback contract can coordinate the channel and arbitrarily split the total profit when the contract parameters satisfy*

$$w_i - \beta_i\theta_{ii} = \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j} \tag{16}$$

From Equation (16), we can see that the coordinating w_i is positively associated with β_i under the buyback contract. That means that a high unit point purchasing cost of one retailer also implies a high buyback price of the points redeemed at that retailer. Therefore, the double marginalisation and cost spillover can be solved simultaneously if the contract parameters have been set appropriately. As Figure 4 shows, β_1 and the profit of Retailer 1 are both increasing with w_1, while the platform may obtain a lower profit. With any set of $(w_1, \beta_1), (w_2, \beta_2), \ldots, (w_N, \beta_N)$ that satisfy Theorem 4.2 under the contract, retailer i is willing to select ${\lambda_i}^c$, which may maximise the channel's profit, $i = 1, 2, \ldots, N$. Table 4 shows the contract parameters and the profits of Retailers 1–5. Moreover, because ${\pi_i}^b(\lambda_i)$ is always increasing with β_i, the total profit can be arbitrarily split among the retailers and the platform, so that the buyback contract can be workable for a wide range of channels.

The buyback contract has several advantages from the aspect of easy implementation. First, the information flow can be ensured. The existing UPSs in the real world are always equipped with information management systems for a smooth flow of information. Hence, it is easy for the platform and retailers to monitor the point generation and redemption information. Second, the flow of funds is simple and a lateral transfer of money among

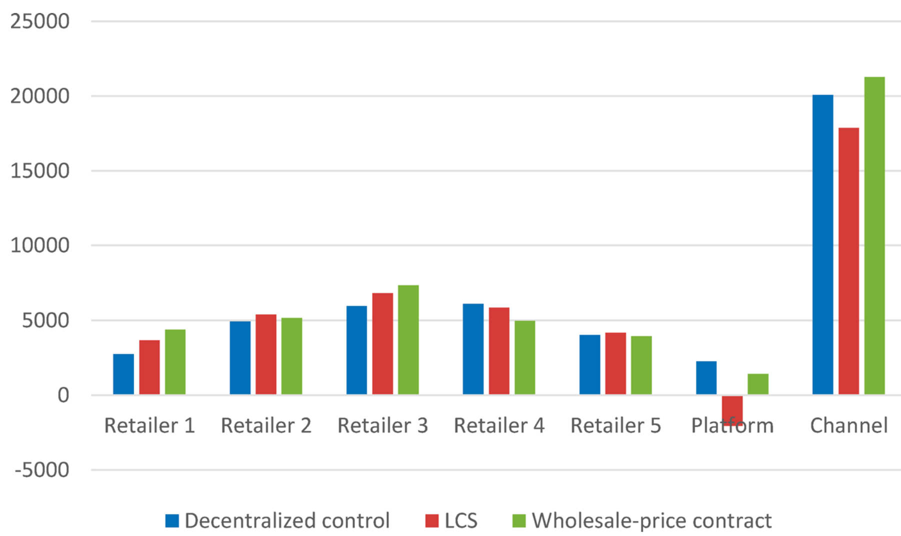

Figure 3. Profits under the three different modes.

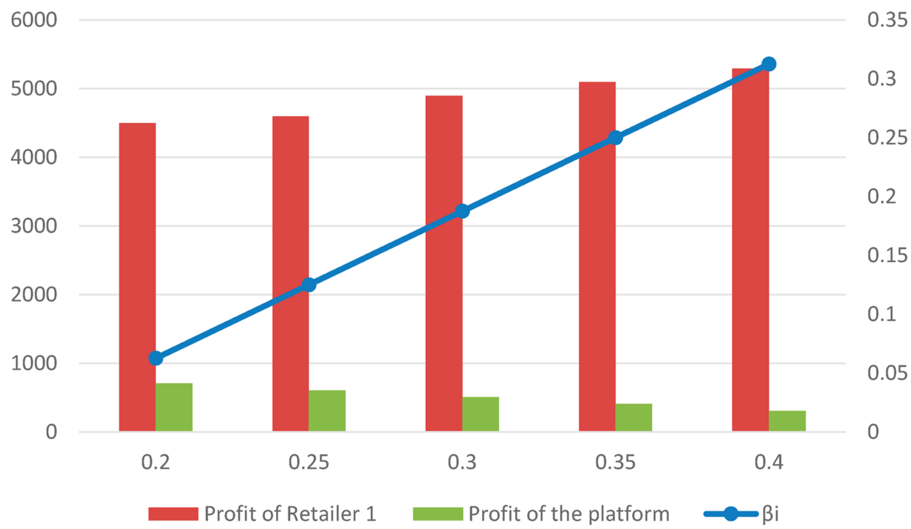

Figure 4. Profits of Retailer 1 and the platform under different values of w_1.

Table 4. Contract parameters and the profits of the retailers under the buyback contract.

i	w_i	β_i	Profit
1	0.2	0.0625	4499.9
2	0.4	0.0472	5298.0
3	0.3	0.0989	7562.4
4	0.5	0.1202	5202.2
5	0.4	0.4176	4021.8

retailers is not involved. Under the buyback contract, each retailer only has a financial relationship with the platform. Compared with the LCS mode, such a one-to-all relationship can significantly simplify the operation of the UPS. Third, the parameters for one retailer will not influence other retailers. This means that some retailers can cooperate with the platform under a determined condition for a long time, which may reduce their risk of operation under the UPS.

In Table 4, we can see that the buyback price is higher than the wholesale price for Retailer 5. Under the buyback contract, retailer i needs to pay for all points generated by it, while receiving buyback revenue for the points redeemed at it. It is true that w_i could be smaller than β_i under some parameter settings of θ_{ii} and $\sum_{j=1,j\neq i}^{N} \theta_{ij}c_j/p_j$. From Equation (14), however, we can see that the gap between the cost incurred by retailer i and the buyback revenue related to λ_i is equal to $(w_i p_i + c_i\theta_{ii} - \beta_i p_i \theta_{ii})\lambda_i D_i(\lambda_i)$. Equation (16) ensures that this difference is greater than zero and increases with λ_i under the buyback contract. Also note that $\beta_i \sum_{j=1,j\neq i}^{N} \theta_{ji} p_j \lambda_j D(\lambda_j)$ does not influence the decision by retailer i on λ_i. Therefore, even though the wholesale price of the universal point is lower than the buyback price in some cases, the retailer cannot benefit from this gap by purchasing and generating an infinite number of points. Managers in the real world may not follow exactly the contract parameter setting established in our manuscript. Hence, it is essential to emphasise that the contract design should avoid the parameter setting on w_i and β_i (such as $\beta_i p_i \theta_{ii} > w_i p_i + c_i \theta_{ii}$) under which retailer i can benefit from the strict application of universal points.

5. Conclusions, managerial insights, and future research

5.1. Concluding remarks

To establish and maintain the equilibrium of a channel with a fixed structure, we showed that the equilibrium under the decentralised control could be complicated with the influence of cost spillover and double marginalisation which have opposite impacts on retailers' decision-making. The wholesale prices of the universal points charged by the platform not only represent the procurement cost of retailers but also neutralises the impact of the cost spillover, which is a novel characteristic of the UPS. Our analysis determines that ${\lambda_i}^d$ is larger than ${\lambda_i}^l$ and finds the necessary condition when ${\lambda_i}^d > {\lambda_i}^c$. This means that the over-generation of points may not always exist under the decentralised control. We also found that ${\lambda_i}^c > {\lambda_i}^l$, which means that the double marginalisation is dominant under the LCS mode. Moreover, the range of optimal wholesale prices within the platform under the two modes is influenced by unit procurement costs, retail prices, and percentages of flow-in and flow-out redemption of points.

For dynamic groups of retailers, the results showed that the percentages of flow-in and flow-out redemption may affect the preference of channel members on inviting new retailers. The profit improvements of the retailers and the platform under the two modes can be quite different. The LCS mode may lead to a wider range of percentages of flow-in and flow-out redemption than the decentralised control, which can result in an all-win scenario in more cases in which a new retailer joins the UPS.

For a coordinating contract, we found that the wholesale price contract can coordinate the channel when $w_i = \sum_{j=1, j\neq i}^{N} \theta_{ij} c_j / p_j$. Cases exist when this profit allocation cannot improve the profits of all the channel members with the decentralised control and LCS modes as benchmarks. A buyback contract can coordinate the channel and arbitrarily split the total profit among retailers and the platform. The profit of each retailer is positively associated with the buyback price under the contract parameter setting that can coordinate the channel. With this flexibility, the platform can easily operate the UPS with a dynamic group of retailers when existing retailers quit, and new retailers join.

5.2. Managerial insights

In this section, we discuss the implications of the UPS and managerial insights from the angles of the platform, the retailers, and channel coordination.

In a static channel, the platform prefers the decentralised control to the LCS mode if it seeks a high profit with existing retailers. This is because retailers address the cost spillover and the phenomenon of over-generating points under the latter mode (see Example 3.1). This means that the platform needs to pay attention to the lateral relationship and potential cooperation among retailers in the channel. For a dynamic channel, when the platform attempts to invite new retailers, it should carefully investigate the influence of the parameters and the current point-operation modes. In this case, it might be beneficial for the platform to invite retailers with a high level of demand and retail price to sell more points to the channel if existing retailers always stay. Moreover, the platform can usually obtain a higher profit under the decentralised control than under the LCS mode by inviting new retailers. However, it is not certain that the platform can always benefit from this strategy if the profit thresholds of existing retailers should be met. Because acquiring a large new retailer results in higher costs for existing retailers because of cost spillover, existing retailers may lose or even quit the channel when such a retailer joins the UPS. Note that the quitting of the platform by retailers could lead to the long-term penalty for the goodwill of the UPS. Considering that most retailers may obtain higher profits under the LCS mode than under the decentralised control, the platform would be better off embracing the LCS mode under which a new retailer's participation could more easily lead to an all-win scenario. This strategy could be helpful for the platform's establishing a stable channel with a wide range of retailers under the UPS in the long run.

Retailers need to understand the benefits and challenge of the UPS under different point-operation modes. Even though the UPS can result in higher demands for retailers in the channel, their profits are negatively influenced by the over-generation of points. In addition to the price and cost, retailers need to pay attention to the flows of point redemption in the channel. Under the decentralised control, retailer i hopes to see a high $\sum_{j=1, j\neq i}^{N} \theta_{ij}$ and a small $\sum_{j=1, j\neq i}^{N} \theta_{ji}$, which are influenced by the customers' preferences for products in the channel.

Under the LCS mode, however, retailer i's total cost might be decreased when more points generated by it are redeemed at the retailer with a high cost efficiency. This means that retailers need to investigate different characteristics of other retailers in the channel under different modes. By using the LCS mode, retailers can eliminate the scale of over-generating points. However, this mode cannot guarantee increments in profits to all retailers with the decentralised control as the benchmark. Retailer i may prefer the decentralised control when that retailer has a high flow-out and low flow-in percentage of points (Table 3 provides an example of this). As a consequence, a retailer needs to evaluate the channel parameters with different approaches under different business modes when it makes decisions on whether to join a UPS.

Because the decentralised control and LCS modes may limit the possibility of establishing a wider range of the channel, the implementation of coordinating contracts would be helpful in obtaining a higher overall profit with the participation of more retailers. The wholesale price contract addresses the cost spillover and double marginalisation simultaneously, by associating the wholesale price with the cost of point flow-out redemption. The other advantage of this contract is the extreme ease of its implementation. The weak point in this setup is that the profit split is fixed. Implementing this contract would be challenging when this profit split cannot improve the profits of all members. The platform may obtain a lower profit under this contract than under the decentralised control, which limits the implementation of the contract. The buyback contract, however, provides the channel with the option for setting contract parameters (i.e. wholesale price and buyback price) to arbitrarily split the total profit. By setting buyback prices and corresponding wholesale prices for some retailers, the platform can allocate the channel profit among itself and other retailers to achieve an all-win scenario. In addition, the platform can invite more retailers if the overall profit can be increased, and the channel can be expanded more easily. The platform should select the appropriate contract according to the expected profit and condition of the channel.

5.3. Limitations and future research

As a new marketing strategy, there must be many novel characteristics of the UPS that remain unexplored. First, this paper considers the point redemption policy in the real world, wherein customers redeem their points by obtaining a product without payment. This means that the point redemption does not lead to new profitable demand. However, there exist several other point redemption policies wherein customers can buy products with the points standing in as money. In such cases, the flow-in redemption to one retailer would not only boost costs but also bring additional sales for this retailer. Hence, it would be interesting to study the UPS under such point redemption policies and multiple periods that result in new demand functions and decision-making frameworks of retailers implementing the UPS. Second, this paper considers retail prices as parameters based on some real examples. However, retailers in some industries may find pricing and point generation as simultaneous decision variables. Even though a higher retail price might have a negative impact on demand, it is also associated with a higher point generation level, which could mitigate that impact. This joint influence is more complicated because of the cost spillover under the UPS with strategic customers. Hence, it would be interesting to study the UPS when retail prices are decision variables, taking into account strategic customers. Third, retailers' decision-making under the UPS is influenced by their unit procurement cost. This means that the product or material suppliers' pricing strategies have new impacts under the UPS. Given this, it would be interesting to analyse the UPS from the view of suppliers and discuss how suppliers can benefit from the UPS with new supply chain strategies.

Notes

1. http://www.cjone.com.
2. Note that $\pi_i(\lambda_i)$ can also be written as $(p_i - c_i)D_i(\lambda_i) - \sum_{j=1}^{N} \theta_{ji} p_j \lambda_j D_j(\lambda_j) c_i / p_i - w_i p_i \lambda_i D_i(\lambda_i)$. We use Equation (1) which can show the sources of point cost more clearly.
3. $\Delta D_{N+1}(\lambda_{N+1})$ includes the demand raised by the UPS.

Acknowledgements

The authors are grateful to the associate editor and three anonymous referees for their valuable comments on earlier versions of the paper.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the National Natural Science Foundation of China [No. 71801191].

Notes on contributors

Dr Feng Xuehao is currently an Associate Professor in Ocean College, Zhejiang University, China. He received his Bachelor and M.S. degrees from Beijing Jiaotong University, China. He received his Ph.D degree from Pusan National University, South Korea, in 2013. His research interests include supply chain management, maritime logistics optimisation, and smart port.

Mr Wen Baolin is currently a Ph.D. candidate in Ocean College, Zhejiang University, China. He received his Bachelor degree from the aforementioned university in 2021. His research interests include smart port and transportation science.

Dr Wenming Shi is a Senior Lecturer at Australian Maritime College, University of Tasmania, Australia. Her main expertise and research interests are in the fields of maritime economics, port development and maritime logistics and management. Currently, Dr Shi is an Administrative Editor of Maritime Policy & Management (MPM) and is mainly responsible for Special Issues.

Dr Ilkyeong Moon is a Professor of Industrial Engineering at Seoul National University in Korea. He received his B.S. and M.S. in Industrial Engineering from Seoul National University, and Ph.D. in Operations Research from Columbia University. His research interests include supply chain management, logistics, and inventory management. He published over 140 papers in international journals. He was a former Editor-in-Chief of *Journal of the Korean Institute of Industrial Engineers* which is a flagship journal of Korean Institute of Industrial Engineers (KIIE). He was a president of KIIE in which he had served from 2019 to 2020. He currently serves as associate editors for several journals including *European Journal of IE* and *Flexible Services and Manufacturing*. He is a fellow of Asia Pacific Industrial Engineering and Management Society and a vice chair of WG 5.7 of IFIP.

ORCID

Wenming Shi http://orcid.org/0000-0001-6551-0499
Ilkyeong Moon http://orcid.org/0000-0002-7072-1351

References

Abhishek, V., K. Jerath, and Z. J. Zhang. 2016. "Agency Selling or Reselling? Channel Structures in Electronic Retailing." *Management Science* 62 (8): 2259–2280.

Aseri, M., M. Dawande, G. Janakiraman, and V. Mookerjee. 2018. "Procurement Policies for Mobile-Promotion Platforms." *Management Science* 64 (10): 4590–4607.

Aust, G., and U. Buscher. 2014. "Cooperative Advertising Models in Supply Chain Management: A Review." *European Journal of Operational Research* 234: 1–14.

Bai, J. R., K. C. So, C. S. Tang, X. Q. Chen, and H. Wang. 2019. "Coordinating Supply and Demand on an On-Demand Service Platform with Impatient Customers." *Manufacturing & Service Operations Management* 21 (3): 556–570.

Barenji, A. V., W. M. Wang, Z. Li, and D. A. Guerra-Zubiaga. 2019. "Intelligent E-Commerce Logistics Platform Using Hybrid Agent Based Approach." *Transpiration Research Part E* 126: 51–31.

Bazargan, A., S. Karray, and S. Zolfaghari. 2017. "Modeling Reward Expiry for Loyalty Programs in a Competitive Market." *International Journal of Production Economics* 193: 352–364.

Bueker, O., B. R. Kim, and J. S. Kim. 2020. "Customer Equilibrium and Optimal Pricing in an M/G/1 Queue with Heterogeneous Rewards and Waiting Cost Rates." *Operations Research Letters* 48: 152–156.

Cachon, G. P., K. M. Daniels, and R. Lobel. 2017. "The Role of Surge Pricing on a Service Platform with Self-Scheduling Capacity." *Manufacturing & Service Operations Management* 19 (3): 368–384.

Cachon, G. P., and M. A. Lariviere. 2005. "Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations." *Management Science* 51 (1): 30–44.

Cao, X. Y., and T. T. Ke. 2019. "Cooperative Search Advertising." *Marketing Science* 38 (1): 44–67.

Cao, Y. H., A. L. Nsakanda, and M. Diaby. 2012. "A Stochastic Linear Programming Modelling and Solution Approach for Planning the Supply of Rewards in Loyalty Reward Programs." *International Journal of Mathematics in Operational Research* 4 (4): 400–421.

Cao, Y. H., A. L. Nsakanda, and M. Diaby. 2015. "Planning the Supply of Rewards with Cooperative Promotion Considerations in Coalition Loyalty Programmes Management." *Journal of Operational Research Society* 66 (7): 1140–1154.

Cao, Y. H., A. L. Nsakanda, M. Diaby, and M. J. Armstrong. 2015. "Rewards-Supply Planning Under Option Contracts in Managing Coalition Loyalty Programmes." *International Journal Production Research* 53 (22): 6772–6786.

Choi, T. M. 2019. "Blockchain-Technology-Supported Platforms for Diamond Authentication and Certification in Luxury Supply Chains." *Transportation Research Part E* 128: 17–29.

Choi, T. M., L. P. Feng, and R. Li. 2020. "Information Disclosure Structure in Supply Chains with Rental Service Platforms in the Blockchain Technology era." *International Journal of Production Economics* 221: 107473.

Choi, T. M., S. Guo, N. Liu, and X. T. Shi. 2019. "Values of Food Leftover Sharing Platforms in the Sharing Economy." *International Journal of Production Economics* 213: 23–31.

Choi, T. M., and Y. Y. He. 2019. "Peer-to-Peer Collaborative Consumption for Fashion Products in the Sharing Economy: Platform Operations." *Transportation Research Part E* 126: 49–65.

Chu, J. H., and P. Manchanda. 2016. "Quantifying Cross and Direct Network Effects in Online Consumer-to-Consumer Platforms." *Marketing Science* 35 (6): 870–893.
Chutani, A., and S. P. Sethi. 2018. "Dynamic Cooperative Advertising Under Manufacturer and Retailer Level Competition." *European Journal of Operational Research* 268: 635–652.
Gou, Q. L., J. Shao, X. Wang, and L. L. Yu. 2020. "Co-Op Supply Chains with a Local Media Company: Models and Analysis." *Transportation Research Part E* 136: 101893.
Gou, Q. L., J. Zhang, L. Liang, Z. M. Huang, and A. Ashley. 2014. "Horizontal Cooperative Programmes and Cooperative Advertising." *International Journal of Production Research* 52 (3): 691–712.
Gu, W., X. T. Luan, Y. N. Song, and J. Shang. 2021. "Impact of Loyalty Program Investment on Firm Performance: Seasonal Products with Strategic Customers." *European Journal of Operational Research*. doi:10.1016/j.ejor.2021.09.046.
Guo, H., X. Y. Zhao, L. Hao, and D. Liu. 2019. "Economic Analysis of Reward Advertising." *Production and Operations Management* 28: 2413–2430.
Hagiu, A., and J. Wright. 2020. "Platforms and the Exploration of New Products." *Management Science* 66 (4): 1527–1543.
Haviv, A., Y. F. Huang, and N. Li. 2020. "Intertemporal Demand Spillover Effects on Video Game Platforms." *Management Science* 66 (10): 4788–4807.
He, X., A. Krishnamoorthy, A. Prasad, and S. P. Sethi. 2012. "Co-Op Advertising in Dynamic Retail Oligopolies." *Decision Sciences* 43 (1): 73–106.
He, X. L., A. Prasad, and S. P. Sethi. 2009. "Cooperative Advertising and Pricing in a Dynamic Stochastic Supply Chain: Feedback Stackelberg Strategies." *Production and Operations Management* 18 (1): 78–94.
Huang, Z. S., and P. J. Bai. 2021. "Dynamic Cooperative Promotion in the Presence of Consumer Reference Effect with Competing Retailers." *International Journal of Retailing and Consumer Services* 60: 102441.
Kopalle, P. K., Y. C. Sun, S. A. Neslin, B. H. Sun, and V. Swaminathan. 2012. "The Joint Sales Impact of Frequency Reward and Customer Tier Components of Loyalty Programs." *Marketing Science* 31 (2): 216–235.
Li, M. L., X. M. Zhang, and B. Dan. 2021. "Cooperative Advertising Contract Design in a Supply Chain with an Offline Showroom Under Asymmetric Information." *Journal of Operational Research Society*. doi:10.1080/01605682.2020.1843974.
Liu, H. J., Q. Liu, and P. K. Chintagunta. 2017. "Promotion Spillovers: Drug Detailing in Combination Therapy." *Marketing Science* 36 (3): 382–401.
Liu, W. H., X. Y. Yan, X. Li, and W. Y. Wei. 2020. "The Impacts of Market Size and Data-Driven Marketing on the Sales Mode Selection in an Internet Platform Based Supply Chain." *Transportation Research Part E* 136: 101914.
Lu, F. X., W. S. Tang, G. W. Liu, and J. X. Zhang. 2019. "Cooperative Advertising: A Way Escaping from the Prisoner's Dilemma in a Supply Chain with Sticky Price." *Omega* 86: 87–106.
Machowska, D. 2019. "Delayed Effects of Cooperative Advertising in Goodwill Dynamics." *Operations Research Letters* 47: 178–184.
Moon, I. K., and X. H. Feng. 2017. "Supply Chain Coordination with a Single Supplier and Multiple Retailers Considering Customer Arrival Times and Route Selection." *Transportation Research Part E* 106: 78–97.
Moon, I. K., J. Xu, X. H. Feng, and X. Ruan. 2020. "Cooperative Sales Promotion with a Point-Sharing Policy: Advantages and Limitations." *Omega* 94: 102038.
Rossi, F. 2018. "Lower Price or Higher Reward? Measuring the Effect of Consumers' Preferences on Reward Programs." *Management Science* 64 (9): 4451–4470.
Shin, J. W., and K. Sudhir. 2010. "A Customer Management Dilemma: When Is It Profitable to Reward One's Own Customers?" *Marketing Science* 29 (4): 671–689.
Sun, Y. C., and D. Zhang. 2019. "A Model of Customer Reward Programs with Finite Expiration Terms." *Management Science* 65 (8): 3889–3903.
Tian, L., A. J. Vakharia, Y. L. Tan, and Y. F. Xu. 2018. "Marketplace, Reseller, or Hybrid: Strategic Analysis of an Emerging E-Commerce Model." *Production and Operations Management* 27 (8): 1595–1610.
Wang, N. N., T. Zhang, X. J. Fan, and X. X. Zhu. 2020. "Game Theoretic Analysis for Advertising Models in Dual-Channel Supply Chains." *International Journal of Production Research* 58 (1): 256–270.
Xiao, D., Y. W. Zhou, Y. G. Zhong, and W. Xie. 2019. "Optimal Cooperative Advertising and Ordering Policies for a Two-Echelon Supply Chain." *Computers & Industrial Engineering* 127: 511–519.
Yan, K., G. W. Hua, and T. C. E. Cheng. 2021a. "Cooperative Promotion of Cross-Market Firms Adopting 3D Printing Technology." *Asia-Pacific Journal of Operational Research*. doi:10.1142/S0217595921400285.
Yan, K., G. W. Hua, T. C. E. Cheng, S. Y. Wang, and J. X. Dong. 2021b. "Joint Promotion of Cross-Market Retailers: Models and Analysis." *International Journal of Production Research*. doi:10.1080/00207543.2021.1922946.
Yang, L., Y. H. Huang, Y. C. Ho, and Z. J. Lin. 2019. "Is Online Multiple-Stores Cooperative Promotion Better Than Single-Store Promotion? Misprediction from Evaluation Mode." *Information & Management* 56: 103148.
Yang, H., C. Y. Shao, H. Wang, and J. P. Ye. 2020. "Integrated Reward Scheme and Surge Pricing in a Ridesourcing Market." *Transportation Research Part B* 134: 126–142.
Yu, L. L., X. L. He, J. Zhang, and C. Y. Xu. 2021. "Horizontal Cooperative Advertising with Advertising Threshold Effects." *Omega* 98: 102104.
Zhang, D. J., H. C. Dai, L. X. Dong, F. F. Qi, N. N. Zhang, X. F. Liu, Z. Y. Liu, and J. Yang. 2020. "The Long-Term and Spillover Effects of Price Promotions on Retailing Platforms: Evidence from a Large Randomized Experiment on Alibaba." *Management Science* 66 (6): 2589–2609.
Zhou, Y. J., T. M. Liu, and G. S. Cai. 2019. "Impact of in-Store Promotion and Spillover Effect on Private Label Introduction." *Service Science* 11 (2): 96–112.

Appendix

Proof of Theorem 3.1: First, we analyse the retailers' decisions which are functions of the wholesale prices charged by the platform. According to Equation (1), we can obtain that for a

w_i,

$$\frac{d\pi_i^d(\lambda_i)}{d\lambda_i} = (p_i - c_i - (c_i\theta_{ii} + p_i w_i)\lambda_i)\frac{dD_i(\lambda_i)}{d\lambda_i} - (c_i\theta_{ii} + p_i w_i)D_i(\lambda_i) \quad \text{(A1)}$$

and

$$\frac{d\pi_i^{d^2}(\lambda_i)}{d(\lambda_i)^2} = -2(c_i\theta_{ii} + p_i w_i)b_i < 0 \quad \text{(A2)}$$

Therefore, $\lambda_i{}^d$, which satisfies Equation (4) can maximise the profit of retailer i and it is a function of w_i. By substituting $\lambda_i{}^d$ under Equation (4) into the platform's profit function, we can rewrite $\pi_0{}^d\,(w_1, w_2, \ldots, w_N)$ as

$$\pi_0^d(w_1, w_2, \ldots, w_N) = \sum_{i=1}^{N} w_i \frac{(p_i - c_i)^2 b_i{}^2 - a_i{}^2(c_i\theta_{ii} + p_i w_i)^2}{4b_i(c_i\theta_{ii} + p_i w_i)^2} p_i - f \cdot N \quad \text{(A3)}$$

Let $\frac{\partial\pi_0^d(w_1,w_2,\ldots,w_N)}{\partial w_i} = 0$ and we have

$$\frac{-a_i{}^2(c_i\theta_{ii} + p_i w_i)^3 - (p_i - c_i)^2 b_i{}^2(c_i\theta_{ii} + p_i w_i) + 2c_i\theta_{ii}(p_i - c_i)^2 b_i{}^2}{4b(c_i\theta_{ii} + p_i w_i)^3} = 0 \quad \text{(A4)}$$

In addition, we have

$$\frac{\partial\pi_0^{d^2}(w_1, w_2, \ldots, w_N)}{\partial(w_i)^2} = \frac{(p_i - c_i)^2 b_i{}^2(p_i w_i - 2c_i\theta_{ii})}{4b(c_i\theta_{ii} + p_i w_i)^4} \quad \text{(A5)}$$

When $w_i = 0$, we have $\frac{\partial\pi_0^d(w_1,w_2,\ldots,w_N)}{\partial w_i} = \frac{c_i\theta_{ii}[(p_i-c_i)b_i + a_i c_i\theta_{ii}][(p_i-c_i)b_i - a_i c_i\theta_{ii}]}{4b(c_i\theta_{ii})^3} > 0$. When $w_i = \theta_{ii}c_i/p_i$, we have $\frac{\partial\pi_0^d(w_1,w_2,\ldots,w_N)}{\partial w_i} = \frac{-a_i^2}{4b} < 0$. There exists a $w_i{}^d$ within the range of $(0, \theta_{ii}c_i/p_i)$, which satisfies Equation (A4) because $\frac{\partial\pi_0^{d^2}(w_1^d,w_2^d,\ldots,w_N^d)}{\partial(w_i^d)^2} < 0$. According to the characteristic of the simple cubic equation, we can obtain the $w_i{}^d$ is the single real root of Equation (A4) and it satisfies

$$c_i\theta_{ii} + p_i w_i = \sqrt[3]{\frac{c_i\theta_{ii}(p_i - c_i)^2 b_i{}^2}{a_i{}^2} + \sqrt{\left(\frac{c_i\theta_{ii}(p_i-c_i)^2 b_i{}^2}{a_i{}^2}\right)^2 + \left(\frac{(p_i-c_i)^2 b_i{}^2}{3a_i{}^2}\right)^3}} + \sqrt[3]{\frac{c_i\theta_{ii}(p_i - c_i)^2 b_i{}^2}{a_i{}^2} - \sqrt{\left(\frac{c_i\theta_{ii}(p_i-c_i)^2 b_i{}^2}{a_i{}^2}\right)^2 + \left(\frac{(p_i-c_i)^2 b_i{}^2}{3a_i{}^2}\right)^3}} \quad \text{(A6)}$$

In addition, we have the Hessian Matrix of $\pi_0{}^d(w_1{}^d, w_2{}^d, \ldots, w_N{}^d)$ as

$$H(w_1{}^d, w_2{}^d, \ldots, w_N{}^d) = \begin{pmatrix} \frac{\partial\pi_0^{d^2}(w_1^d,w_2^d,\ldots,w_N^d)}{\partial(w_1^d)^2} & & & \\ & \frac{\partial\pi_0^{d^2}(w_1^d,w_2^d,\ldots,w_N^d)}{\partial(w_2^d)^2} & & \\ & & \cdots & \\ & & & \frac{\partial\pi_0^{d^2}(w_1^d,w_2^d,\ldots,w_N^d)}{\partial(w_N^d)^2} \end{pmatrix} \quad \text{(A7)}$$

By calculating the kth order principal minor of H, we can find that the principal minor is negative when k is odd, and it is positive when k is even. Then, we can obtain that the Hessian Matrix is a negative definite matrix. Hence, this $w_1{}^d, w_2{}^d, \ldots, w_N{}^d$ is the optimal solution of $\pi_0{}^d(w_1, w_2, \ldots, w_N)$. ■

Proof of Theorem 3.2: The proof of (i) is similar with the one of Theorem 3.1, and we omit it here;

(ii) as the proof of Theorem 3.1 shows, $w_i{}^d$ should satisfy Equation (3), and $w_i{}^l$ should satisfy Equation (6). Let $t_i{}^d = p_i w_i{}^d + \theta_{ii}c_i$ and $t_i^h = p_i w_i^h + \sum_{j=1}^{N} \frac{p_i\theta_{ij}c_j}{p_j}$. Then, Equations (3) and (6) can be rewritten as

$$a_i^2(t_i^d)^3 + (p_i - c_i)^2 b_i^2 t_i^d - 2\theta_{ii}c_i(p_i - c_i)^2 b_i^2 = 0 \quad \text{(A8)}$$

$$a_i^2(t_i^l)^3 + (p_i - c_i)^2 b_i^2 t_i^l - 2p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j}(p_i - c_i)^2 b_i^2 = 0 \quad \text{(A9)}$$

Considering that $p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j} > \theta_{ii}c_i$, we can obtain $t_i{}^d < t_i{}^l$ and $p_i w_i{}^d + \theta_{ii}c_i < p_i w_i^l + \sum_{j=1}^{N} \frac{p_i\theta_{ij}c_j}{p_j}$. According to Equations (4) and (7), it can be found that $\lambda_i{}^d > \lambda_i{}^l$, $i = 1, 2, \ldots, N$.

(iii) According to the proof of Theorem 3.1, for $\lambda_i{}^l$, we can rewrite $\pi_0{}^l(w_1, w_2, \ldots, w_N)$ as

$$\pi_0^l(w_1, w_2, \ldots, w_N) = \sum_{i=1}^{N} w_i \frac{(p_i - c_i)^2 b_i{}^2 - a_i{}^2\left(p_i \sum_{j=1}^{N} \theta_{ij}c_j/p_j + p_i w_i\right)^2}{4b_i\left(p_i \sum_{j=1}^{N} \theta_{ij}c_j/p_j + p_i w_i\right)^2} p_i - f \cdot N \quad \text{(A10)}$$

By comparing Equations (A3) and (A10), we can find that for any $(w_1, w_2, \ldots, w_N)$, $\pi_0{}^d(w_1, w_2, \ldots, w_N)$ is always greater than $\pi_0{}^l(w_1, w_2, \ldots, w_N)$ because $p_i \sum_{j=1}^{N} \theta_{ij}c_j/p_j > \theta_{ii}c_i$ for all $i = 1, 2, \ldots, N$. Therefore, the maximum value of $\pi_0{}^d(w_1, w_2, \ldots, w_N)$ is greater than the one of $\pi_0{}^l(w_1, w_2, \ldots, w_N)$. This means that the platform can obtain a lower profit under the LCS mode than under the decentralised control. ■

Proof of Theorem 3.3: (i) It is easy to prove that $\lambda_i{}^c$ should satisfy

$$\lambda_i^c = \frac{(p_i - c_i)b_i - a_i p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j}}{2b_i p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j}} \quad \text{(A11)}$$

Considering that ${w_i}^l$ is within the range of $(0, \sum_{j=1}^{N} p_i\theta_{ij}c_j/p_j)$, we have ${\lambda_i}^c > {\lambda_i}^l$ with Equation (7).

(ii) By comparing Equations (4) and (A10), we can see that

$$\lambda_i^c - \lambda_i^d = \frac{(p_i - c_i)}{2p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j}} - \frac{(p_i - c_i)}{2(\theta_{ii}c_i + p_i w_i^d)} \tag{A12}$$

Then, it can be obtained that $p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j} - \theta_{ii}c_i - p_i w_i^d = p_i \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j} - p_i w_i^d > p_i \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j} - \theta_{ii}c_i$, because ${w_i}^d < \theta_{ii}c_i/p_i$. As a consequence, ${\lambda_i}^d > {\lambda_i}^c$ when $\sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j} > \frac{\theta_{ii}c_i}{p_i}$, $i = 1, 2, \ldots, N$. ■

Proof of Theorem 4.1: By using $w_i = \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j}$, we can obtain the profit of retailer i under the wholesale price contract from Equation (1) as

$$\pi_i^w(\lambda_i) = (p_i - c_i)D_i(\lambda_i) - c_i\theta_{ii}\lambda_i D_i(\lambda_i) - \sum_{j=1,j\neq i}^{N} \frac{\theta_{ji}p_j c_i \lambda_j D_j(\lambda_j)}{p_i} - \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j} p_i \lambda_i D_i(\lambda_i) \tag{A13}$$

It is easy to show that the optimal λ_i that maximises $\pi_i^w(\lambda_i)$ is the same with ${\lambda_i}^c$ and the maximum $\pi_i^w(\lambda_i)$ is fixed. ■

Proof of Theorem 4.2: According to the proof of Theorem 3.3, we can see that ${\lambda_i}^c$ should satisfy

$$\left(p_i - c_i - p_i \sum_{j=1}^{N} \frac{\theta_{ij}c_j\lambda_i}{p_j}\right) \frac{dD_i(\lambda_i)}{d\lambda_i} = p_i D_i(\lambda_i) \sum_{j=1}^{N} \frac{\theta_{ij}c_j}{p_j} \tag{A14}$$

From Equation (13) we have

$$\frac{d\pi_i^b(\lambda_i)}{d\lambda_i} = (p_i - c_i - c_i\theta_{ii}\lambda_i - w_i p_i \lambda_i + \beta_i p_i \theta_{ii}\lambda_i)\frac{dD_i(\lambda_i)}{d\lambda_i} - (c_i\theta_{ii} + w_i p_i - \beta_i p_i \theta_{ii})D_i(\lambda_i) \tag{A15}$$

$$\frac{d\pi_i^{b^2}(\lambda_i)}{d(\lambda_i)^2} = (p_i - c_i - c_i\theta_{ii}\lambda_i - w_i p_i \lambda_i + \beta_i p_i \theta_{ii}\lambda_i)\frac{d^2 D_i(\lambda_i)}{d(\lambda_i)^2} - 2(c_i\theta_{ii} + w_i p_i - \beta_i p_i \theta_{ii})\frac{dD_i(\lambda_i)}{d\lambda_i} < 0 \tag{A16}$$

Therefore, ${\lambda_i}^b = {\lambda_i}^c$ when $w_i - \beta_i\theta_{ii} = \sum_{j=1,j\neq 1}^{N} \frac{\theta_{ij}c_j}{p_j}$. Then, Equation (14) can be written as

$$\pi_i^b(\lambda_i) = \left(p_i - c_i - c_i\theta_{ii}\lambda_i - p_i\lambda_i \sum_{j=1,j\neq i}^{N} \frac{\theta_{ij}c_j}{p_j}\right) D_i(\lambda_i) - \left(\frac{c_i}{p_i} - \beta_i\right) \sum_{j=1,j\neq i}^{N} \theta_{ji}p_j\lambda_j D_j(\lambda_j) \tag{A17}$$

Given the overall profit of the channel, the profit can be arbitrarily allocated among the members because ${\pi_i}^b(\lambda_i)$ is always increasing with β_i under the buyback contract when the contract parameters satisfy Equation (16). ■