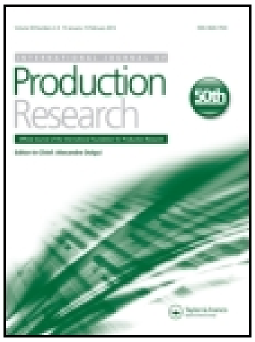

Taylor & Francis
Taylor & Francis Group

International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: http://www.tandfonline.com/loi/tprs20

A time-varying lot sizes approach for the economic lot scheduling problem with returns

Heechul Bae, Ilkyeong Moon & Wonyoung Yun

To cite this article: Heechul Bae, Ilkyeong Moon & Wonyoung Yun (2016) A time-varying lot sizes approach for the economic lot scheduling problem with returns, International Journal of Production Research, 54:11, 3380-3396, DOI: 10.1080/00207543.2015.1110633

To link to this article: http://dx.doi.org/10.1080/00207543.2015.1110633

Published online: 13 Nov 2015.

International Journal of Production Research, 2016
Vol. 54, No. 11, 3380–3396, http://dx.doi.org/10.1080/00207543.2015.1110633

A time-varying lot sizes approach for the economic lot scheduling problem with returns

Heechul Bae[a], Ilkyeong Moon[b]* and Wonyoung Yun[c]

[a] *IT Convergence Technology Research Laboratory, ETRI, Daejeon, Korea;* [b] *Department of Industrial Engineering, Seoul National University, Seoul, Korea;* [c] *Department of Industrial Engineering, Pusan National University, Busan, Korea*

(*Received 2 August 2014; accepted 13 October 2015*)

We consider the economic lot scheduling problem with returns by assuming that each item is returned by a constant rate of demand. The goal is to find production frequencies, production sequences, production times, as well as idle times for several items subject to returns at a single facility. We propose a heu ristic algorithm based on a time-varying (TV) lot sizes approach. The problem is decomposed into two distinct portions: in the first, we find a combinatorial part (production frequencies and sequences) and in the second, we determine a continuous part (production and idle times) in a specific production sequence. We report computational results that show that, in many cases, the proposed TV lot sizes approach with consideration of returns yields a relatively minor error.

Keywords: economic lot and scheduling problem; returns; time-varying lot sizes approach

1. Introduction

The economic lot scheduling problem with returns (ELSPR) describes the challenge of finding a feasible cyclic schedule for producing several items subject to returns at a single facility. If returns are ignored (i.e. demand is fully satisfied from manufactured items), the traditional economic lot scheduling problem (ELSP) applies. However, if returns have economic value, recovered items can be used as substitutes for manufactured items. The ELSPR reflects production situations in manufacturing facilities that make, for example, single-use cameras, pallets and containers or service parts for cars or computers (Tang and Teunter 2006). Uncoordinated manufacturing and remanufacturing lot sizes result in additional inventories and associated set-up costs for the returned items. Therefore, by using the ELSPR to coordinate manufacturing and remanufacturing production scheduling, firms try to find feasible, simultaneous, production schedules with the best trade-off between inventory and set-up costs related to manufacturing and remanufacturing operations. The determination of recovery strategies for return items is described in Hosseini-Nasab and Dehghanbaghi (2015).

Under the ELSPR model, a single facility is dedicated to the production of manufacturing and remanufacturing operations that can be used only to produce one item at a time. Demand is met by manufactured and remanufactured items without stockouts in each production cycle. Figure 1 represents the system components involved in the ELSPR model and the main relationships among them (Zanoni et al. 2012). Because manufacturing and remanufacturing operations are performed at the same production facility, two kinds of inventory are utilized to meet the demand. The serviceable inventory is used for manufactured and remanufactured items that meet the demand. The recoverable inventory is comprised of returned items used until remanufacturing is initiated in the production schedule.

Because of its nonlinearity, combinatorial characteristics and complexity, the ELSP is generally known as an NP-hard problem (Hsu 1983; Gallego and Shaw 1997). Because the ELSP is a special case of the ELSP with returns, the ELSPR is also NP-hard. The ELSPR is more complex than the ELSP for two reasons (Tang and Teunter 2006): first, manufacturing and remanufacturing lot sizes for the same product are linked by the return and demand ratio, and therefore, their values are restricted; second, the sequencing of lots within a production cycle influences the inventory holding cost.

Different coordination models can be used for the ELSP. The production environments in these models are similar to the one-machine situation. However, the models tend to be more complicated than the ELSP variants mainly because of the additional coordination needed for supply, delivery and returns. The economic lot and supply scheduling problem (ELSSP) is a coordination model that regards supply of raw or input materials. Gallego and Joneja (1994) first extended the traditional ELSP by considering the holding and ordering issues for raw materials. Kuhn and Liske (2011) was the first example of an ELSSP considering raw material supply routing and they Kuhn and Liske (2014) presented a basic period (BP) approach for the ELSSP using a power-of-two policy. Bae, Moon, and Yun (2014) developed a heuristic algorithm through which

*Corresponding author. Email: ikmoon@snu.ac.kr

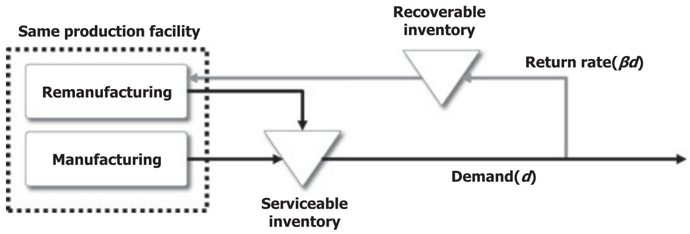

Figure 1. ELSPR system.

Table 1. Comparison between the researches of different ELSP coordination models.

	ELSPR (This paper)	ELSSP (Bae, Moon, and Yun 2014)
Coordination	Manufacturing and remanufacturing, Consideration of return items	Supply and manufacturing, Consideration of raw materials
Environment	Multi-item, Single-production system for manufacturing and remanufacturing	Multiple raw materials and multi-item, Single-production system for manufacturing
Condition	Constant return rate	A raw material belongs to one item
Total cost factors	Holding costs for serviceable and recoverable inventory, Set-up costs for manufacturing and remanufacturing	Holding costs for raw materials and serviceable inventory, Set-up costs for manufacturing, Routing costs for raw materials
Decisions	Cycle length, Idle times, Production sequence for manufacturing and remanufacturing, Production times for manufacturing and remanufacturing	Cycle length, Idle times, Production sequence, Production times, Transportation route and quantity for raw materials
Methodology	Nonlinear program, Modified LB, Common cycle approach, TV lot sizes approach	Nonlinear program, Modified LB, TV lot sizes approach

they applied the existing lower bound (LB) and time-varying (TV) lot sizes approach to solve the ELSSP by considering three issues: production, inventory and inbound transportation routing in an integrated way. The differences and similarities between Bae, Moon, and Yun (2014) and this paper are summarised in Table 1. A production management system for capacity utilisation and minimisation of work-in-process inventory is described in Shin, Ryu, and Ryu (2014). The economic lot and delivery scheduling problem (ELDSP) addresses coordination between production schedules on a single machine at a supplier and direct transportation schedules to an assembly facility. The ELDSP concept with a single item was first introduced by Hahm and Yano (1992). They (Hahm and Yano 1995) extended the ELDSP for multiple items with the common cycle (CC) approach. Osman and Demirli (2012) recently proposed a new methodology for three stages consisting of two suppliers and a single assembly facility. In this method, they presented a hybrid algorithm of combining the outer approximation decomposition method with the Benders decomposition approach. While coordinating between production and delivery decisions with the ELDSP, theses researchers have been only interested in the direct delivery costs through multiple stages. Another coordination model, the economic lot and scheduling problem with returns (ELSPR), was first introduced by Tang and Teunter (2006), who presented a CC approach for the ELSPR using a mixed integer programming (MIP) model. They only solved a problem consisting of five items. They considered that a serviceable inventory is jointly managed (JM) with manufacturing and remanufacturing operations; the inventory levels of manufacturing and remanufacturing operations for each item are not separated but are managed together. They also proposed a LB for the ELSPR, but their suggestion reflected, not a LB, but a minimum cycle length based on the inventory and set-up costs for the ELSPR. Using the CC approach, Teunter, Kaparis, and Tang (2008) developed a MIP for the ELSPR. They considered separate production lines for manufacturing and remanufacturing operations. Teunter, Tang, and Kaparis (2009) suggested a simple heuristic to generate manufacturing and remanufacturing production sequences under the CC approach. They also assumed use of a JM serviceable inventory.

Table 2. Summary of literature review by solution methods.

Model	CC	BP	TV
JM	Teunter, Tang, and Kaparis (2009), Teunter, Kaparis, and Tang (2008)[a], Tang and Teunter (2006)	X	X
IM	This paper	Zanoni et al. (2012)	This paper

[a]Separate production lines for manufacturing and remanufacturing.

In the ELSP, the CC approach always allows one to find a feasible schedule using a very simple procedure. The objective value obtained from this method serves as the upper bound on the general ELSP (Moon, Silver, and Choi 2002). Jones and Inman (1989) provided a detailed analysis of conditions under which the CC approach provides optimal or near-optimal solutions. The feasibility of the CC solution in the ELSPR JM model, however, is not guaranteed unless the manufacturing and remanufacturing production sequences are determined within a row-per-cycle manufacturing and remanufacturing processes for each items. To find the feasible manufacturing and remanufacturing production sequence under the CC approach, Teunter, Tang, and Kaparis (2009) developed a simple heuristic for the ELSPR and Teunter, Kaparis, and Tang (2008) considered separate production lines for manufacturing and remanufacturing operations. Recently, Zanoni et al. (2012) developed a BP approach for the ELSPR. They considered that serviceable inventory levels of manufacturing and remanufacturing operations for each item are separated and independently managed (IM). This disconnected management process increases the serviceable inventory, but IM makes the problem simple to find various feasible production frequencies. Table 2 presents a description of the papers about the ELSPR that we reviewed. In the table, X means that we found no existing approaches to the specific problem.

Generally, research on the ELSP has focused on cyclic schedules. Moreover, almost all researchers have restricted their attention to cyclic schedules that satisfy the zero switch rule (ZSR) which states that a production run for any particular product can be started only if its physical inventory is zero (Moon, Silver, and Choi 2002). Counterexamples to the optimality of this rule have been found but are rare (Maxwell 1964; Delporte and Thomas 1977).

At the present time, the most dominant and widely studied approach for the ELSP is the TV lot sizes approach. It was first presented by Dobson (1987). Gallego and Shaw (1997) showed that the ELSP is strongly NP-hard under the TV lot sizes approach with or without the ZSR restriction, giving theoretical justification to the development of heuristics (Moon, Silver, and Choi 2002). In this paper, we propose a TV lot sizes approach for the ELSPR in an IM inventory model.

The contributions of this paper to ELSPR research include a proposal of (1) a TV lot sizes approach to the ELSPR that produces a feasible cyclic schedule that accounts for return items; (2) a simple idle-times insertion method for determination of the continuous part of a specific production sequence; (3) a modified LB on the ELSP that can be solved by the well-known independent solution (IS) approach such that the synchronisation constraint is ignored. Furthermore, the study shows, through an extensive computational study, that in many cases, the TV lot sizes approach with respect to returns yields a small error (gap) compared with the LB in the objective values.

This paper is organised as follows. We explain the basic assumptions and notation and present a LB for the ELSPR model in Section 2. In Section 3, we first present the ELSPR JM model with the CC approach and then propose the ELSPR IM model with the TV lot sizes approach. The numerical results of the proposed algorithm as well as comparisons with other methods are reported in Section 4. Finally, in Section 5, we present our conclusions.

2. A LB for the ELSPR model

We based the research on the following assumptions about the ELSPR:

(1) Manufacturing of several items at a single facility results in item-specific competition because only one item can be manufactured or remanufactured at a time.
(2) Demand rates, production rates, as well as set-up costs and times for all items are known constants.
(3) Demand is met without stockouts.
(4) Demand can be satisfied with two methods: manufacturing and remanufacturing.
(5) There are two kinds of inventory: serviceable and recoverable.
(6) In a serviceable inventory, manufactured and remanufactured items are stocked through the IM model.

(7) Each item is returned based on a constant rate of demand.
(8) A recoverable inventory is used for collection of returned items.

The following notation is used in the models:

- m number of items
- j index for items, $j = 1, 2, \ldots, m$
- k index for position, $k = 1, 2, \ldots, K$
- p_j^M constant production rate of item j for manufacturing (units/unit time)
- p_j^R constant production rate of item j for remanufacturing (units/unit time)
- d_j constant demand rate of item j (units/unit time)
- h_j^s known inventory holding cost of item j for serviceable inventory (\$/unit/unit time)
- h_j^r known inventory holding cost of item j for recoverable inventory (\$/unit/unit time)
- A_j^M known set-up cost of item j for manufacturing (\$)
- A_j^R known set-up cost of item j for remanufacturing (\$)
- s_j^M known set-up time of item j for manufacturing (unit time)
- s_j^R known set-up time of item j for remanufacturing (unit time)
- T_j cycle length for item j (unit time)
- β_j constant return rate of item j, $0 \le \beta_j < 1$

We first find a LB on the minimum average cost of the ELSPR which will be used for a TV lot sizes heuristic in Section 3. A LB can be easily obtained by considering each item in isolation and calculating economic production quantities. In this IS approach, the capacity challenges caused by the use of a single machine to make several products are ignored. A tight LB for the ELSP has been implicitly suggested by Bomberger (1966) and restated in several different ways by other researchers (Dobson 1987; Gallego and Moon 1992). We apply the existing LB on the ELSP, which can be solved by the well-known IS approach, to our ELSPR. The nonlinear program that we propose provides a tight LB of the average total cost (TC) per unit time for the ELSPR.

ELSPR-LB

$$\text{Min}_{T_1,\ldots,T_m} \sum_{j=1}^{m} \left[\frac{A_j^M + A_j^R}{T_j} + H_j^s T_j + H_j^r T_j \right] \tag{1}$$

subject to

$$\sum_{j=1}^{m} \frac{(s_j^M + s_j^R)}{T_j} \le \kappa, \tag{2}$$

$$T_j \ge 0, \quad j = 1, \ldots, m. \tag{3}$$

The objective function is a LB on the average cost of producing m different items on a single machine. The constraint is the proportion of time available for set-ups. The objective function and the constraint set are convex in T_j's. Therefore, the optimal points of the LB model satisfy the Karush-Kuhn-Tucker (KKT) conditions as follows:

$$A_j^M + A_j^R - (H_j^s + H_j^r)T_j^2 + \lambda(s_j^M + s_j^R) = 0, \quad j = 1, 2, \ldots, m, \tag{4}$$

$$\lambda \left[\kappa - \sum_{j=1}^{m} \frac{(s_j^M + s_j^R)}{T_j} \right] = 0, \tag{5}$$

$\lambda \ge 0$ complementary slackness with $\sum_{j=1}^{m} \left[(s_j^M + s_j^R)/T_j \right] \le \kappa$, where $H_j^s = h_j^s d_j \Big[(1-\beta_j)(1-(1-\beta_j)d_j/p_j^M) + \beta_j(1-\beta_j d_j/p_j^R) \Big]/2$ and $H_j^r = h_j^r \beta_j d_j (1-\beta_j d_j/p_j^R)/2$; $j = 1, 2, \ldots, m$.

The above conditions are derived by assuming that T_j's are non-trivial. Equation (4) yields:

$$T_j = \sqrt{\frac{A_j^M + A_j^R + \lambda(s_j^M + s_j^R)}{H_j^s + H_j^r}}, \quad \forall j. \tag{6}$$

We can use the following procedure to find the optimal T_j:
Algorithm for Lower Bound

(Step 1) Check if $\lambda = 0$ gives an optimal solution.
Set $\lambda = 0$ and find T_js ($j = 1, \ldots, m$) from Equation (6).
(Step 2) If $\sum_{j=1}^{m}[(s_j^M + s_j^R)/T_j] \le \kappa$, then the T_js are an optimal solution.
Otherwise, go to Step 3.
(Step 3) Start with an arbitrary $\lambda > 0$.
(Step 4) Compute T_js ($j = 1, \ldots, m$) from Equation (6).
(Step 5) If $\sum_{j=1}^{m}[(s_j^M + s_j^R)/T_j] < \kappa$, reduce λ. Go to Step 4.
If $\sum_{j=1}^{m}[(s_j^M + s_j^R)/T_j] > \kappa$, increase λ. Go to Step 4.
If $\sum_{j=1}^{m}[(s_j^M + s_j^R)/T_j] = \kappa$, stop. The T_js are optimal.

3. The ELSPR model

The ELSPR inventory is classified into two models: JM serviceable inventory and IM serviceable inventory. The JM model requires that one item is manufactured, and the inventory is reduced to zero and then the remanufacturing of the same item is started. The same constant demand rate applies for the manufactured and remanufactured items such that the inventory is jointly managed. In this case, the CC approach, which specifies that the items cycle lengths must be equal, does not always allow for a feasible schedule through a simple procedure. Thus, an additional method is needed to find or check the feasibility of the solution. The IM model involves that one item is manufactured, and the remanufacturing of the same item is started before the inventory of the manufactured item is reduced to zero. For each item, demand of manufactured and remanufactured items is met separately based on the return rate, β, so that inventory is managed independently and demand rate is reduced differentially for the manufactured and remanufactured items. In this case, the CC approach always allows one to find a feasible schedule using a simple procedure. Thus, the ELSPR under IM model does not require manufacturing and remanufacturing sequences to be placed in a cyclic schedule. Because the IM model reflects a disconnection of manufacturing and remanufacturing operations, the serviceable inventory cost is increased. The error in the IM model was at a maximum when the return rate is 50% (Zanoni et al. 2012). Generally speaking, in the IM model, the average TC per unit time, but especially the serviceable inventory holding cost, is increased; however, this IM model allowed us to obtain a closed form expression for a scheduling problem by adopting various production frequencies related to the implementation of the BP policy (Zanoni et al. 2012). Figure 2 depicts a comparison with inventory levels and cycle lengths for the IM and JM models. The ELSPR IM model involving N items is a special case of the ELSP featuring $2N$ items. Therefore, any solution algorithm for the ELSP can be used to solve the ELSPR IM model.

3.1 *ELSPR JM model with the CC approach*

The inventory holding cost of the serviceable inventory per cycle is reflected as the sum of the area under the inventory triangles of the manufacturing and remanufacturing operations (Figure 2(a)). The inventory holding cost of the recoverable inventory per cycle is also reflected by the area under the inventory triangle of returned items (Figure 2(b)).

The average TC which consists of the set-up costs for manufacturing and remanufacturing and inventory holding costs for serviceable and recoverable inventory per unit time, is therefore, stated by:

$$\mathrm{TC} = \sum_{j=1}^{m}\left[\frac{A_j^M + A_j^R}{T_j} + H_j^s T_j + H_j^r T_j\right], \tag{7}$$

where $H_j^s = h_j^s d_j\left[(1-\beta_j)^2(1-d_j/p_j^M) + \beta_j^2(1-d_j/p_j^R)\right]/2$ and $H_j^r = h_j^r d_j \beta_j (1-\beta_j d_j/p_j^R)/2$; $j = 1, 2, \ldots, m$.

In the CC approach, we have $T_1 = T_2 = \cdots = T_m = T$ (say). Therefore, the average TC function above can be stated as

$$\mathrm{TC} = \frac{(A^M + A^R)}{T} + (H^s + H^r)T, \tag{8}$$

where $A^M = \sum_{j=1}^{m} A_j^M$, $A^R = \sum_{j=1}^{m} A_j^R$, $H^s = \sum_{j=1}^{m} H_j^s$ and $H^r = \sum_{j=1}^{m} H_j^r$.

TC is minimised by T^* which satisfies Equation (9).

$$T^* = \sqrt{\frac{A^M + A^R}{H^s + H^r}}. \tag{9}$$

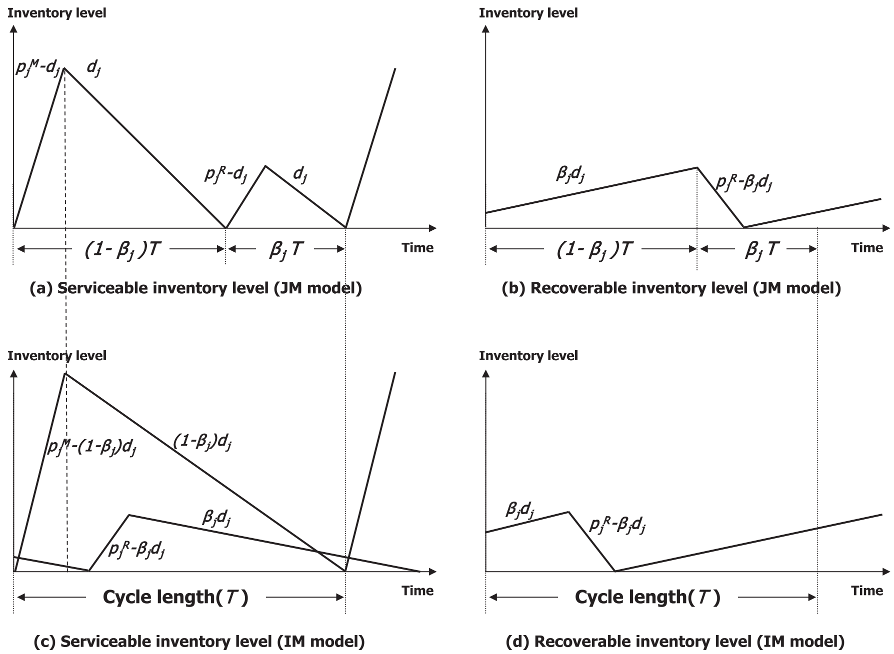

Figure 2. Comparison with inventory level for the JM and IM models.

Moreover, before we get T^* as the minimum cycle length in the interval, we should consider the available time for set-ups within the cycle. Because the sum of total set-up and production time per cycle should be less than the cycle length, we impose the following constraint on T:

$$\sum_{j=1}^{m} \left[s_j^M + s_j^R + d_j(1-\beta_j)T/p_j^M + d_j\beta_j T/p_j^R \right] \leq T \tag{10}$$

$$\text{or,} \quad T \geq \frac{\sum_{j=1}^{m} (s_j^M + s_j^R)}{\kappa} \equiv T_{\min} \text{ (say)}, \tag{11}$$

where $\kappa = 1 - \sum_{j=1}^{m} \{d_j(1-\beta_j)/p_j^M + \beta_j d_j/p_j^R\}$, the long-run proportion of time available for set-ups. TC(T) is convex in T; therefore, the minimum cycle length should be taken as the maximum of T^* and $T_{\min}$; that is, $T = \max\{T^*, T_{\min}\}$. The remanufacturing operation is followed by a manufacturing operation based on the time that the serviceable inventory level of the manufacturing operation reaches zero. In this CC approach of the JM model, the feasibility of the solution is not guaranteed. Thus, the CC approach of the ELSPR JM model must be subjected to additional procedures to find production sequences of manufacturing and remanufacturing in a feasible schedule.

3.2 *ELSPR IM model with the CC approach*

Because each inventory from production operations continues to be maintained within the same cycle length T_j, the inventory holding cost of the serviceable inventory per cycle is reflected by the sum of the area under the inventory triangles of manufacturing and remanufacturing operations (Figure 2(c)). The average inventory holding cost of the recoverable inventory per cycle is also the same as that of the JM model (Figure 2(d)).

The average TC function per unit time of the ELSPR IM model also can be stated as:

$$\text{TC} = \sum_{j=1}^{m} \left[\frac{A_j^M + A_j^R}{T_j} + H_j^s T_j + H_j^r T_j \right], \tag{12}$$

where $H_j^s = h_j^s d_j \left[(1-\beta_j)(1-(1-\beta_j)d_j/p_j^M) + \beta_j(1-\beta_j d_j/p_j^R) \right]/2$ and $H_j^r = h_j^r \beta_j d_j (1-\beta_j d_j/p_j^R)/2$; $j = 1, 2, \ldots, m$.

The average TC per unit time consists of the set-up costs for manufacturing and remanufacturing and the inventory holding costs for serviceable and recoverable inventory per unit time.

In the CC approach, we have $T_1 = T_2 = \cdots = T_m = T$ (say). Therefore, the average TC function above can be stated as

$$\mathrm{TC} = \frac{(A^M + A^R)}{T} + (H^s + H^r)T, \tag{13}$$

where $A^M = \sum_{j=1}^m A_j^M$, $A^R = \sum_{j=1}^m A_j^R$, $H^s = \sum_{j=1}^m H_j^s$ and $H^r = \sum_{j=1}^m H_j^r$.

TC is easily minimised by

$$T^* = \sqrt{\frac{A^M + A^R}{H^s + H^r}}. \tag{14}$$

Moreover, before we get T^* as the minimum cycle length in the interval, we should consider the available time for set-ups within the cycle. Because the sum of total set-up and production time per cycle should be less than the cycle length, we impose the following constraint on T :

$$\sum_{j=1}^{m} \left[s_j^M + s_j^R + d_j(1-\beta_j)T/p_j^M + d_j\beta_j T/p_j^R \right] \le T \tag{15}$$

$$\text{or,} \quad T \ge \frac{\sum_{j=1}^m (s_j^M + s_j^R)}{\kappa} \equiv T_{\min} \text{ (say)}, \tag{16}$$

where $\kappa = 1 - \sum_{j=1}^m \{d_j(1-\beta_j)/p_j^M + \beta_j d_j/p_j^R\}$, the long-run proportion of time available for machine set-ups. TC(T) is convex in T; therefore, the minimum cycle length should be taken the maximum of T^* and $T_{\min}$; that is, $T = \max\{T^*, T_{\min}\}$.

The CC approach with the ELSPR IM model makes cycle lengths of all items equal. Thus, it always yields a feasible schedule, which makes it advantageous over the CC approach with the ELSPR JM model. However, the CC solution generally works as an upper bound and sometimes it is quite far from the LB.

3.3 *ELSPR IM model with the TV lot sizes approach*

We first represent Dobson's (1987) original formulation of the ELSP for the ELSPR. To formulate this problem some additional notation is needed. The subscript j and superscript k are the indexes of item j and the item produced at the kth position in the sequence, respectively. The definition of $\mathcal{F}$ is the set of all possible finite sequences for items. Here K_j is the set of positions under a given production sequence of item j; that is, $K_j = \{k \mid f^k = j\}$. Let L_l represent the set of positions in a given sequence from l to the previous position of the sequence where the item f^l has repeated itself. The ELSPR can be formulated as:

$$\begin{aligned}
&\inf_{f\in\mathcal{F}} \ \min\nolimits_{t\ge0\, u\ge0 \quad T>0} \\
&\frac{1}{T}\left[\sum_{k=1}^{K}\left(\frac{1}{2}h_s^k(p_M^k - (1-\beta^k)d^k)\left(\frac{p_M^k}{(1-\beta^k)d^k}\right)(t_M^k)^2\right)\right] \\
&\quad + \frac{1}{T}\left[\sum_{k=1}^{K}\left(\frac{1}{2}h_s^k(p_R^k - \beta^k d^k)\left(\frac{p_R^k}{\beta^k d^k}\right)(t_R^k)^2\right)\right] \\
&\quad + \frac{1}{T}\left[\sum_{k=1}^{K} A_M^k + \sum_{k=1}^{K} A_R^k + \sum_{k=1}^{K}\left(\frac{1}{2}h_r^k(p_R^k - \beta^k d^k)\left(\frac{p_R^k}{\beta^k d^k}\right)(t_R^k)^2\right)\right]
\end{aligned} \tag{17}$$

subject to

$$\sum_{k\in K_j}(p_j^M t_M^k + p_j^R t_R^k) = d_j T, \qquad j = 1,\ldots,m, \tag{18}$$

$$\sum_{k\in L_l}(t_M^k + s_M^k + u_M^k + t_R^k + s_R^k + u_R^k) = (p_M^l/d^l)t_M^l + (p_R^l/d^l)t_R^l,$$

$$l = 1,\ldots,L, \tag{19}$$

$$\sum_{k=1}^{K}(t_M^k + s_M^k + u_M^k + t_R^k + s_R^k + u_R^k) = T. \tag{20}$$

The objective function is the sum of the inventory holding costs for serviceable and recoverable inventory and the set-up costs for manufacturing and remanufacturing per unit time. Constraint (18) means that enough production time for manufacturing and remanufacturing process to each item j is allocated to the production of item j to meet its demand over the cycle. Constraint (19) ensures that we must produce enough of the item in position l to meet its demand for the next time till the item is produced again. Constraint (20) implies that the cycle length T must include times with respect to production, set-up and idle times of manufacturing and remanufacturing operations for all the produced items in the cycle.

In this paper, the TV lot sizes approach proposed by Dobson (1987) is considered to solve the ELSPR. The proposed heuristic first selects the relative production frequencies by solving the **ELSPR-LB** model. We then find the production frequencies rounded to power-of-two integers using an algorithm as suggested by Roundy (1989). Finally, a production sequence comes out of the bin-packing heuristic with respect to frequencies and average loads. Based on these given production frequencies and sequence, we can compute the continuous variables that are composed of production times **t**, idle times **u** and the cycle length T. The procedure that allows one to find a feasible production schedule can be described in the following steps:

Step 1 We define the relative production frequencies by solving the **ELSPR-LB** model. x_j is defined as the relative production frequency for item j:

$$x_j = \frac{\mathrm{Max}_j\ \{T_j^*\}}{T_j^*}, \qquad j = 1,2,\ldots,m,$$

where T_j^* represents the optimal cycle length for item j in program **ELSPR-LB**. We present a one-dimensional line search algorithm in the previous section to find a KKT point of the **ELSPR-LB** model (the detailed procedure of this problem can be found in Section 2).

Step 2 We round the relative production frequencies solved in Step 1 to power-of-two integers. Roundy (1989) has shown that the cost increase cannot exceed 6% when the intervals are rounded off to the power-of-two integers. We define y_j as the production frequencies comes from the power-of-two integer of item j. Then y_j is calculated:

$$y_j = 2^p \quad \text{if} \quad x_j \in \left[\frac{1}{\sqrt{2}}\,2^p, \sqrt{2}\,2^p\right], \qquad p = 0,1,\ldots.$$

Step 3 We determine an efficient production sequence **f** via the bin-packing heuristic proposed by Dobson (1987). The items should be spread out as uniformly as possible in b bins where $b = \max_j\{y_j\}$. When we find an assignment of the items to the bins, a variation of the longest processing time (LPT) heuristic can be used as (y_j, v_j). The items are first ranked in decreasing order of the production frequencies, y_j. The items with identical frequencies y_j are assigned in decreasing order of the estimated production time duration, v_j where $v_j = s_j^M + (d_j(1-\beta_j)T/p_j^M y_j) + s_j^R + (d_j\beta_j T/p_j^R y_j)$. The heuristic can handle a production sequence **f** efficiently through minimising the maximum height of the bins (see Dobson 1987 for more details).

Step 4 For a given production sequence **f**, we solve for production times **t** and idle times **u**. The idle times at the manufacturing lots are assumed to be identical, except that the idle time for the first position of sequence is zero. The available idle times easily can be spaced out evenly by Equation (21). Let T_{CC} be the CC length and K be the total number of positions in the production sequence; u^k is equal to zero in the first position of the sequence. Otherwise,

$$u^k = \left[T_{CC} - \sum_{j=1}^{m}\left(s_j^M + s_j^R + d_j T_{CC}/p_j\right)\right]/(K-1), \tag{21}$$

where $p_j = p_j^M p_j^R / \left[(1-\beta_j)p_j^R + \beta_j p_j^M\right]$; $j = 1,2,\ldots,m$.

After the production times have been determined using Equation (22) under a given **f** and **u**, we can find the average TC per unit time of the schedule in the objective function. If idle times are assumed to be zero (i.e. $\mathbf{u} = \mathbf{0}$), it can be easily seen that we can find the production times **t** using Equation (22) with a given **f** (see Dobson 1987 for more details). This approximation fits very well at a highly loaded facility. Although, given a fixed sequence, the nonlinear programming problem that determines the variables **t** and **u** at the same time is a rather complex, an optimal solution can be solved by a parametric quadratic program (see Zipkin 1991).

$$t = (I - P^{-1}L)^{-1}P^{-1}L(s + u), \tag{22}$$

where P is a diagonal matrix with $P_{kk} = p_M^k p_R^k / \left[(1 - \beta^k)d^k p_R^k + \beta^k d^k p_M^k\right]$.

Step 5 We can improve the average TC function per unit time by modifying idle times under the TV lot sizes approach. The average TC function per unit time (17) can be written as:

$$\mathrm{TC} = \frac{1}{T}\left[\sum_{k=1}^{K} A_M^k + \sum_{k=1}^{K} A_R^k + \sum_{k=1}^{K} H_s^k (t^k p^k / d^k)^2 + \sum_{k=1}^{K} H_r^k (t^k p^k / d^k)^2\right], \tag{23}$$

where $H_s^k = h_s^k d^k \left[(1 - \beta^k)(1 - (1 - \beta^k)d^k / p_M^k) + \beta^k (1 - \beta^k d^k / p_R^k)\right]/2$, $H_r^k = h_r^k \beta^k d^k (1 - \beta^k d^k / p_R^k)/2$ and $p^k = p_M^k p_R^k / \left[(1 - \beta^k)p_R^k + \beta^k p_M^k\right]$; $j = 1, 2, \ldots, m$.

The values for the inventory holding costs for serviceable and recoverable inventory of items have been changed to those shown in Equation (23) as applied to the CC length for the idle times. Thus, this step intends to find balancing between modified inventory holding and set-up costs. To modify the cycle length T, we set T to αT in Equation (23) such that

$$\mathrm{TC} = \frac{1}{\alpha T}\left[\sum_{k=1}^{K}\left(A_M^k + A_R^k\right) + \alpha^2 \sum_{k=1}^{K}\{H_s^k (t^k p^k / d^k)^2 + H_r^k (t^k p^k / d^k)^2\}\right]. \tag{24}$$

To find the value of α that minimises TC(α), we set TC$'(\alpha)$ equal to zero. This yields:

$$\alpha = \sqrt{\frac{\sum_{k=1}^{K}\left(A_M^k + A_R^k\right)}{\sum_{k=1}^{K}\left[H_s^k (t^k p^k / d^k)^2 + H_r^k (t^k p^k / d^k)^2\right]}}. \tag{25}$$

We can find the updated idle times **u** using the modified T_{CC} in Equation (21) as follows: $T_{CC} = \max\{\alpha T_{CC}, T_{\min}\}$. After calculating the modified idle times **u**, we can get the updated feasible production times and the average TC per unit time of the modified schedule. This procedure of idle times calculation is repeated until no additional idle times are added compared with the previous idle times.

4. Computational experiments

In this section, we present computational results for the ELSPR. Our computational experiments were performed on three sets: Tang and Teunter's (2006) 5-item problem, Teunter, Tang, and Kaparis's (2009) 10-item problem and a randomly generated 10-item problem. Specifically, we compare the performances of our algorithm with the CC approach of the JM model and the modified BP approach of the IM model as proposed by Tang and Teunter (2006) and Zanoni et al. (2012).

Example I Modified Tang and Teunter's (T&T) 5-item problem

We solved Tang and Teunter's (2006) 5-item problem. The modified data related to the items in Tang and Teunter's problem are shown in Table 3. To get a highly loaded facility example, we changed the production rate for manufacturing, p_j^m from 80 to 60. We verified the result by computing $1 - \kappa = \sum_{j=1}^{m}\{d_j(1 - \beta_j)/p_j^M + d_j(\beta_j)/p_j^R\} = 0.95$. The 95% represents the facility utilisation to meet the demand and it may be possible to use the rest of the utilisation for set-up and idle times. The following steps detail the heuristic:

Step 1 Compute a LB and the optimal cycle length for item j in program **ELSPR-LB**.

$$T_1 = 71.90, \quad T_2 = 58.65, \quad T_3 = 50.85, \quad T_4 = 28.57, \quad T_5 = 69.56,$$

with a LB \$8.06.

Step 2 Round the relative production frequencies to power-of-two integers.

$$y_1 = 1, \quad y_2 = 1, \quad y_3 = 1, \quad y_4 = 2, \quad y_5 = 1.$$

Table 3. Data for the modified Tang and Teunter's problem ($\kappa = 0.05$ case).

	Manufacturing			Remanufacturing			Holding cost			
j	A_j^m	s_j^m	p_j^m	A_j^r	s_j^r	p_j^r	h_j^s	h_j^r	d_j	β_j
1	20	0.25	60	20	0.25	80	0.00175	0.00088	9	0.2
2	20	0.25	60	20	0.25	80	0.00263	0.00132	9	0.2
3	20	0.25	60	20	0.25	80	0.00350	0.00175	9	0.2
4	20	0.25	60	20	0.25	80	0.00438	0.00219	30	0.2
5	20	0.25	60	20	0.25	80	0.00525	0.00263	3	0.2

Table 4. Results for the modified Tang and Teunter's problem ($\kappa = 0.05$ case).

IM model			JM model	
LB	TV	CC	LB	CC
8.06 ($T = 71.90$)	8.17 ($T = 60.00$)	8.68 ($T = 50.00$)	6.48 ($T = 86.57$)	6.85[a] ($T = 58.40$)

[a]Infeasible solution (Tang and Teunter 2006).

Step 3 Generate an efficient production sequence using the bin-packing heuristic; we used the following lexicographic order to spread items out as evenly as possible in bins.

$$(y_4, v_4) = (2, 17.58) \geq^L (y_1, v_1) = (1, 10.75) \geq^L (y_2, v_2) = (1, 10.75)$$
$$\geq^L (y_3, v_3) = (1, 10.75) \geq^L (y_5, v_5) = (1, 3.92).$$

The production sequence is

$$\mathbf{f} = (4, 1, 3, 4, 2, 5).$$

Step 4 Compute idle times $\mathbf{u}$ and production times $\mathbf{t}$ using Equations (21) and (22), respectively, for the sequence and the average TC per unit time using Equation (17).

$$\mathbf{u} = (0, 0, 0, 0, 0, 0) \text{ and}$$
$$\mathbf{t} = (16.83, 8.55, 8.55, 11.67, 8.55, 2.85)$$

with the average TC per unit time \$8.17.

Step 5 Compute α and αT_{CC} using Equation (25). We update idle times $\mathbf{u}$, production times $\mathbf{t}$ and the average TC per unit time.

$$\alpha = 0.9797, \ \mathbf{u} = (0, 0, 0, 0, 0, 0) \text{ and}$$
$$\mathbf{t} = (16.83, 8.55, 8.55, 11.67, 8.55, 2.85)$$

with the average TC per unit time \$8.17.

In some situations, the IM model does not yield a better objective value than the JM model, primarily because the IM model requires more serviceable inventory within the same cycle length.

However, the disconnection (IM serviceable inventory) makes finding a feasible schedule relatively simple. This disconnection increases the serviceable inventory holding costs, but using this assumption, we can obtain a closed form expression for the scheduling problem by adopting various production frequencies (Zanoni et al. 2012). The feasibility of the solution is not guaranteed even when the CC approach of the JM model is used. Therefore, the JM model is more complicated because it must be subjected to additional procedures to find feasible manufacturing and remanufacturing production sequences.

We compare the models and summarise the results in Table 4. A feasible solution under the CC approach of the ELSPR IM model is illustrated in Figure 3. A feasible solution under the TV lot sizes approach of the ELSPR IM model is shown in Figure 4.

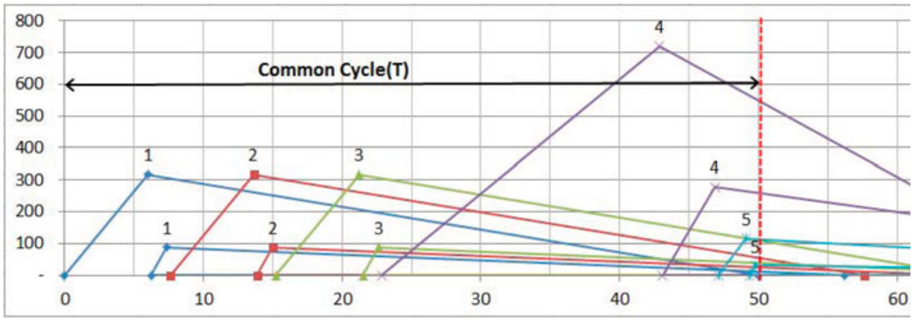

Figure 3. The CC approach of the ELSPR IM model ($T = 50.00$).

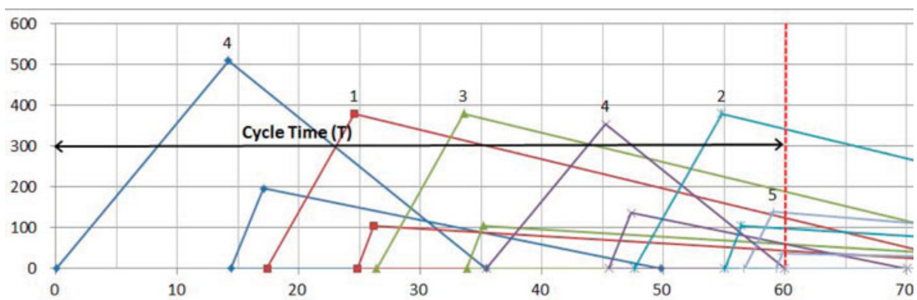

Figure 4. The TV lot sizes approach of the ELSPR IM model ($T = 60.00$).

Example II Teunter et al.'s 10-item problem

Teunter, Tang, and Kaparis (2009) proposed heuristics to obtain a feasible schedule with the JM model and the CC approach. To present their results, they provided 120 sets of data (instances) generated from the well-known Bomberger (1966) data. We conducted experiments on the Teunter, Tang, and Kaparis's (2009) 10-item problem with the aim of evaluating the performance of the proposed methods in the preceding sections. Teunter, Tang, and Kaparis (2009) grouped the 120 sets of data into four groups based on the utilisation. The following data sets were used:

- Examples 1–30: utilisation is between 12 and 47%, average 35%;
- Examples 31–60: utilisation is between 50 and 75%, average 61%;
- Examples 61–85: utilisation is between 75 and 95%, average 85%;
- Examples 86–120: cycle length T_{CC} is smaller than $T_{\min}$ and utilisation is between 87% and 99%, average 96%.

We regrouped examples by utilisation instead of 30 instances in a group of Teunter, Tang, and Kaparis (2009) who used the same uniform distributions of manufacturing and remanufacturing intervals. The returns holding cost should be smaller than the serviceable holding cost because remanufacturing adds value to an item (Teunter, VanderLaan, and Inderfurth 2000; Teunter, Tang, and Kaparis 2009). In the 120 examples of 10-item production, however, the recoverable holding costs of 160 among 1,200 items were larger than the serviceable holding costs (i.e. Item 1 for Example 1 in Table 1 of Teunter, Tang, and Kaparis (2009)).

In Table 5, we compare the performance of our algorithm with another algorithm proposed by Tang and Teunter (2006) for 10-item problem with low utilisation ($\kappa > 0.5$). The results of the proposed algorithm for four groups of test problems are summarised in Table 6. In these tables, the percentages improvement between the CC approach of Tang and Teunter (2006) and the proposed heuristic of the IM model using the TV lot sizes approach are shown under the heading T&T − TV, whereas CC − TV represents the percentages improvement between the CC approach of the IM model and the TV lot sizes approach. Teunter, Tang, and Kaparis (2009) showed the exact algorithm result for the 120 sets of their data. We refer to the result as T&T. We calculate the percentages improvement from T&T − TV = (T&T − TV)/T&T × 100, CC − TV = (CC − TV)/CC × 100. The headings CC − LB refer to the percentage gap between the CC approach of the IM model and the one using a LB, whereas the percentage gap between the proposed heuristic of the IM model under the TV lot sizes approach and the one using a LB is shown under the heading TV − LB. We calculate the LB gap from CC − LB = (CC − LB)/LB × 100,

Table 5. Computational results for examples 1–30 with low utilisation ($\kappa > 0.5$ case).

No.	$1-\kappa$	Average TC ($)				Improvement (%)		LB gap (%)	
		T&T	CC	TV	LB	T&T − TV	CC − TV	CC − LB	TV − LB
1	0.12	12.43	15.27	13.16	12.92	−5.90	13.79	18.17	1.87
2	0.20	27.88	35.82	27.71	27.12	0.61	22.63	32.05	2.16
3	0.22	20.17	25.43	14.18	14.09	29.68	44.23	80.50	0.67
4	0.24	19.33	21.74	18.62	18.45	3.66	14.32	17.82	0.95
5	0.25	22.68	27.64	23.26	22.75	−2.56	15.84	21.47	2.23
6	0.26	14.14	16.90	13.79	13.39	2.48	18.41	26.20	2.97
7	0.26	18.95	23.34	13.65	13.37	27.99	41.53	74.53	2.04
8	0.29	45.90	51.79	34.15	32.97	25.60	34.06	57.08	3.58
9	0.30	21.75	27.77	16.91	16.32	22.26	39.12	70.20	3.62
10	0.33	35.46	43.08	26.81	26.39	24.38	37.76	63.26	1.62
11	0.33	20.50	26.54	20.30	19.66	0.96	23.50	35.00	3.28
12	0.34	44.89	53.36	33.81	32.65	24.69	36.65	63.42	3.53
13	0.34	23.27	26.72	22.34	21.34	3.98	16.39	25.21	4.69
14	0.34	15.61	17.97	11.69	11.58	25.08	34.92	55.15	0.97
15	0.35	14.49	17.30	13.84	13.55	4.47	20.00	27.69	2.16
16	0.37	31.01	32.52	19.52	18.69	37.06	39.99	73.99	4.41
17	0.38	11.40	15.47	14.85	14.67	−30.29	4.01	5.50	1.27
18	0.38	25.83	32.57	23.34	22.96	9.63	28.32	41.86	1.68
19	0.40	23.60	29.24	24.20	23.49	−2.53	17.25	24.46	3.00
20	0.40	21.77	27.84	23.94	23.64	−9.99	13.99	17.76	1.28
21	0.40	28.64	34.81	17.59	17.44	38.59	49.47	99.63	0.87
22	0.41	35.96	46.56	33.04	31.56	8.11	29.04	47.52	4.68
23	0.41	35.34	45.79	40.57	40.05	−14.79	11.40	14.32	1.29
24	0.42	30.54	39.42	29.24	28.10	4.24	25.81	40.28	4.08
25	0.43	14.54	15.78	10.58	10.45	27.21	32.91	51.01	1.30
26	0.43	31.29	36.58	24.96	23.40	20.22	31.75	56.34	6.70
27	0.43	24.84	26.28	18.15	15.50	26.91	30.92	69.56	17.14
28	0.46	23.76	25.00	19.42	19.07	18.27	22.34	31.12	1.83
29	0.47	47.80	53.93	40.02	32.45	16.27	25.78	66.17	23.33
30	0.47	24.98	31.85	24.03	23.28	3.80	24.56	36.83	3.23
Max.						38.59	49.47	99.63	23.33
Ave.						11.34	26.69	44.80	3.75
Min.						−30.29	4.01	5.50	0.67

Table 6. Computational results for test problems.

Examples	Improvement (%)						LB gap (%)					
	T&T − TV			CC − TV			CC − LB			TV − LB		
	Max.	Ave.	Min.	Max.	Ave.	Min.	Max.	Ave.	Min.	Max.	Ave.	Min.
1–30	38.59	11.34	−30.29	49.47	26.69	4.01	99.63	44.80	5.50	23.33	3.75	0.67
31–60	42.04	5.73	−23.29	46.38	24.52	0.79	111.81	47.70	11.13	71.15	10.33	1.55
61–85	32.17	0.68	−51.58	36.87	19.92	−28.65	72.44	39.75	7.86	56.47	10.14	1.17
86–120	21.75	−131.18	−804.68	35.26	−66.51	−511.72	710.89	233.22	28.54	2,508.98	519.29	17.53
1–85	42.04	6.22	−51.58	49.47	23.93	−28.65	111.81	44.34	5.50	71.15	7.95	0.67
1–120	42.04	−33.85	−804.68	49.47	−2.45	−511.72	710.89	99.43	5.50	2,508.98	157.09	0.67

Table 7. A comparative study of computational results.

Examples	$1-\kappa$	Improvement (%)					
		T&T − modified BP IM			T&T − TV		
		Max.	Ave.	Improved instances	Max.	Ave.	Improved instances
1–30	$0.12 \leq 1-\kappa \leq 0.47$	52.0	15.9	83	38.6	11.3	80
31–60	$0.50 \leq 1-\kappa \leq 0.75$	43.4	13.6	87	42.0	5.7	60

Table 8. Randomly generated uniform distributions of data for test problems.

Parameters	Set
Production rate of item for manufacturing and remanufacturing (units/unit time)	[100, 200]
Demand rate of item (units/unit time)	[5, 20]
Holding cost of item for serviceable and recoverable inventory ($/unit/unit time)	[0.00006, 0.002]
Set-up cost of item for manufacturing and remanufacturing ($)	[50, 100]
Set-up time of item for manufacturing and remanufacturing (unit time)	[0.1, 3]
Return rate of item	[0, 1]

TV − LB = (TV − LB)/LB × 100. As shown in these tables, the proposed heuristic using the TV lot sizes approach outperforms the other previously published heuristics in terms of improvement and LB gap. The proposed heuristic improves T&T's heuristic in 24 out of 30 problems through the application of the IM model in Table 5. Table 6 summarises the results of the proposed algorithm for four groups of test problems that feature different utilisation of items.

We compare the performance of our algorithm with the modified BP approach under the IM model as proposed by Zanoni et al. (2012), namely, the modified BP IM. Table 7 presents the comparative results between the T&T and the modified BP IM or the proposed algorithm (TV). We only consider the instances of 1–60 in this paper because the cycle length T_{CC} is smaller than the $T_{\min}$ for most other instances. We show the maximum and average improvement rates and the number of improved instances compared with the CC approach of Tang and Teunter (2006). Because each result of the modified BP IM is not available in Zanoni et al.'s paper, we cannot directly compare the modified BP IM with the proposed algorithm. For these test problems, the modified BP IM outperforms the proposed algorithm due to additional refinement procedures based on the BP approach.

In the case of heterogeneous products and high utilisation, the solution under the TV lot sizes approach is generally better than the solutions under the BP and the CC approaches. However, the CC approach can perform well in cases of homogeneous products in which the item's independent cycle times are not too different between items (Gallego and Shaw 1997).

Example III Randomly generated 10-item problem

Additional computational experiments are conducted for comparison of the proposed heuristics of the IM model with other models. We obtained the data-set over randomly generated uniform distributions in Table 8. For two test problems with different utilisations, we randomly generated 50 instances for each test problem. The ELSP is more meaningful and difficult to find feasible solutions when κ is small (Dobson 1987). Thus, we generated only problems with $\kappa \leq 0.1$.

We used the same uniform distributions of the intervals for manufacturing and remanufacturing. The returns holding cost is smaller than the serviceable holding cost, just as in the problems of Teunter, Tang, and Kaparis (2009). The comparison of the algorithms for 10-item problem with high utilisation are shown in Tables 9 and 10. As shown in these tables, the proposed TV lot sizes approach outperforms the CC approach at highly loaded facility in terms of improvement and LB gap. Because of the non-linearity and combinatorial properties of the ELSP, most researchers have focused on the development of heuristic algorithms to find a near-optimal solution, which is commonly compared against a LB (Moon, Silver, and Choi 2002). In these tables, the LB gap of TV solutions over CC solutions is smaller. The results of the proposed algorithm are summarised in Table 11. For higher utilisation, the improvement of TV solutions over CC solutions is slightly bigger. The results clearly show that the TV solutions are better than the CC solutions at the highly loaded facility.

Table 9. Computational results for examples 1–50 with high utilisation ($0.05 < \kappa \leq 0.1$ case).

No.	$1-\kappa$	Average TC ($)			Improvement (%)	LB gap (%)	
		CC	TV	LB	CC – TV	CC – LB	TV – LB
1	0.94	60.78	50.46	47.89	16.99	26.93	5.36
2	0.91	38.36	34.78	33.79	9.33	13.52	2.93
3	0.93	26.09	22.93	21.71	12.11	20.20	5.64
4	0.90	29.98	27.50	26.95	8.27	11.24	2.04
5	0.94	27.33	26.23	25.69	4.01	6.37	2.11
6	0.94	43.35	41.01	39.08	5.39	10.91	4.93
7	0.94	48.79	43.64	41.33	10.55	18.04	5.59
8	0.94	58.52	52.23	50.12	10.75	16.77	4.22
9	0.90	28.36	25.95	25.03	8.50	13.30	3.67
10	0.90	29.18	26.14	25.42	10.42	14.79	2.83
11	0.94	25.47	22.32	21.56	12.37	18.15	3.53
12	0.92	48.65	43.93	43.22	9.70	12.56	1.64
13	0.94	42.15	41.14	40.13	2.38	5.02	2.52
14	0.93	21.11	19.35	18.26	8.33	15.60	5.96
15	0.92	37.26	33.22	31.49	10.84	18.32	5.49
16	0.95	34.83	29.44	27.99	15.49	24.45	5.17
17	0.94	32.97	26.66	25.72	19.14	28.21	3.66
18	0.93	34.02	28.47	27.34	16.33	24.44	4.13
19	0.91	18.17	17.65	17.28	2.84	5.09	2.11
20	0.92	26.95	25.49	24.25	5.42	11.11	5.09
21	0.92	30.00	28.72	28.00	4.27	7.12	2.55
22	0.92	29.80	27.99	26.92	6.08	10.68	3.95
23	0.91	26.70	24.78	23.73	7.20	12.53	4.43
24	0.92	37.33	34.44	33.13	7.73	12.68	3.97
25	0.90	29.14	27.99	27.25	3.92	6.93	2.73
26	0.90	27.28	25.60	24.39	6.16	11.88	4.99
27	0.91	27.04	24.35	23.68	9.95	14.20	2.83
28	0.93	42.43	39.80	38.00	6.21	11.67	4.74
29	0.94	49.12	39.88	36.38	18.81	35.01	9.61
30	0.93	45.66	38.58	37.72	15.50	21.03	2.28
31	0.91	23.28	19.84	18.74	14.77	24.19	5.85
32	0.91	24.44	24.28	23.74	0.64	2.94	2.27
33	0.95	41.77	37.66	36.67	9.83	13.90	2.71
34	0.91	32.52	28.76	27.79	11.57	17.05	3.50
35	0.91	27.34	26.56	25.45	2.83	7.40	4.36
36	0.92	29.97	28.12	27.63	6.17	8.47	1.77
37	0.94	59.85	57.01	54.16	4.75	10.52	5.27
38	0.94	41.33	41.29	40.08	0.10	3.12	3.02
39	0.93	38.07	34.79	33.68	8.63	13.03	3.28
40	0.91	32.69	28.95	27.92	11.44	17.09	3.70
41	0.93	33.79	30.70	29.15	9.14	15.93	5.34
42	0.92	41.05	39.97	37.43	2.65	9.67	6.77
43	0.92	39.81	34.40	32.66	13.59	21.92	5.34
44	0.92	32.68	31.72	30.22	2.94	8.14	4.96
45	0.93	26.53	21.73	20.78	18.11	27.69	4.57
46	0.92	27.95	26.38	25.38	5.62	10.11	3.92
47	0.92	25.96	22.90	22.25	11.77	16.68	2.95
48	0.92	28.06	22.12	21.08	21.15	33.07	4.92
49	0.90	21.20	20.89	20.16	1.48	5.16	3.61
50	0.91	30.12	28.39	28.01	5.73	7.52	1.36
	Max.				21.15	35.01	9.61
	Ave.				8.96	14.65	4.00
	Min.				0.10	2.94	1.36

Table 10. Computational results for examples 51–100 with high utilisation ($0 < \kappa \leq 0.05$ case).

No.	$1-\kappa$	Average TC ($)			Improvement (%)	LB gap (%)	
		CC	TV	LB	CC − TV	CC − LB	TV − LB
51	0.98	158.83	130.71	122.49	17.70	29.67	6.71
52	0.96	57.34	49.09	47.20	14.40	21.49	4.00
53	0.98	92.19	83.08	78.27	9.88	17.78	6.14
54	0.98	81.95	76.87	72.25	6.20	13.42	6.40
55	0.95	63.52	56.54	54.93	10.98	15.63	2.94
56	0.97	80.10	67.53	64.16	15.70	24.86	5.25
57	0.96	61.38	58.71	54.73	4.35	12.14	7.27
58	0.97	89.47	79.65	76.01	10.97	17.71	4.79
59	0.96	50.87	47.68	45.95	6.27	10.70	3.76
60	0.99	197.12	182.25	173.68	7.54	13.49	4.93
61	0.98	134.95	109.85	104.38	18.60	29.29	5.24
62	0.98	150.04	126.75	118.93	15.52	26.16	6.57
63	0.97	100.58	77.39	74.25	23.06	35.46	4.22
64	0.96	100.97	95.55	85.97	5.37	17.45	11.14
65	0.96	77.33	74.66	73.33	3.45	5.45	1.81
66	0.99	184.79	160.52	152.60	13.13	21.10	5.19
67	0.96	69.39	62.97	59.71	9.25	16.22	5.47
68	0.97	106.75	102.43	99.79	4.05	6.97	2.64
69	0.96	40.20	36.87	35.95	8.27	11.81	2.56
70	0.95	64.21	61.74	59.47	3.85	7.98	3.82
71	0.97	139.56	134.21	128.09	3.83	8.95	4.78
72	0.97	76.35	57.12	54.18	25.18	40.91	5.43
73	0.99	560.82	524.07	501.34	6.55	11.86	4.54
74	0.99	186.61	167.51	163.57	10.24	14.08	2.40
75	0.99	187.91	170.78	164.35	9.12	14.33	3.91
76	0.98	188.99	161.42	153.87	14.59	22.83	4.91
77	0.97	97.89	81.09	78.89	17.16	24.08	2.79
78	0.98	133.75	115.78	107.48	13.43	24.44	7.72
79	0.99	225.46	188.15	175.71	16.55	28.31	7.08
80	0.99	337.57	297.30	281.41	11.93	19.96	5.65
81	0.99	200.59	195.90	179.95	2.34	11.47	8.86
82	0.97	65.57	60.15	58.04	8.27	12.97	3.63
83	0.95	67.64	64.25	61.17	5.01	10.58	5.04
84	0.99	504.10	450.45	427.52	10.64	17.91	5.36
85	0.95	51.72	41.86	40.56	19.07	27.53	3.21
86	0.97	80.34	78.07	72.33	2.83	11.07	7.93
87	0.95	59.37	56.66	54.37	4.56	9.19	4.21
88	0.96	68.79	62.10	60.11	9.73	14.45	3.31
89	0.97	112.55	99.42	95.32	11.67	18.07	4.30
90	0.96	63.52	55.89	53.57	12.01	18.57	4.33
91	0.99	180.50	167.85	160.63	7.01	12.37	4.50
92	0.96	58.18	49.39	47.14	15.10	23.41	4.78
93	0.98	96.72	88.18	85.56	8.83	13.04	3.06
94	0.95	48.55	42.63	39.24	12.17	23.71	8.65
95	0.97	89.87	86.41	84.00	3.85	6.99	2.87
96	0.97	72.57	60.49	58.77	16.65	23.48	2.91
97	0.95	62.95	59.33	55.16	5.75	14.11	7.55
98	0.95	35.59	33.35	30.22	6.30	17.79	10.37
99	0.96	63.23	56.02	54.81	11.40	15.37	2.21
100	0.97	81.30	66.19	62.83	18.59	29.41	5.35
Max.					25.18	40.91	11.14
Ave.					10.58	17.92	5.05
Min.					2.34	5.45	1.81

Table 11. Computational results for test problems.

Examples	$1-\kappa$	Improvement (%)			LB gap (%)					
		CC − TV			CC − LB			TV − LB		
		Max.	Ave.	Min.	Max.	Ave.	Min.	Max.	Ave.	Min.
1–50	$0.90 \le 1-\kappa < 0.95$	21.15	8.96	0.10	35.01	14.65	2.94	9.61	4.00	1.36
51–100	$0.95 \le 1-\kappa < 1.00$	25.18	10.58	2.34	40.91	17.92	5.45	11.14	5.05	1.81
1–100	$0.90 \le 1-\kappa < 1.00$	25.18	9.77	0.10	40.91	16.28	2.94	11.14	4.53	1.36

5. Conclusions

In this paper, we have considered the case of the ELSPR. We have assumed that each item is returned at a constant rate of demand and remanufactured items substitute for manufactured items. The goal of our approach was to obtain a feasible production schedule in situations in which returned items are allowed. We have proposed a TV lot sizes algorithm to solve the ELSPR. We have also shown that a simple insertion method of idle times can improve the cyclic schedule in specific production sequences of the ELSPR. We have also successfully solved the ELSPR by modifying the existing LB model in the ELSP with an IS approach in which the synchronisation constraint is ignored. The results of the computational experiments show that in many cases the proposed TV lot sizes approach with respect to returns yields a small error (gap) with the LB in the objective values.

Even in the CC approach of the JM model, the feasibility of the solution is not guaranteed. To find feasible manufacturing and remanufacturing production sequences, one must subject the JM model to additional procedures. However, the IM model does not require manufacturing and remanufacturing sequences to be placed together in a cyclic schedule because demand of each manufactured and remanufactured item is met separately based on the return rate. Moreover, in the IM model, the serviceable inventory holding cost is higher than in the JM model because serviceable inventory levels of manufacturing and remanufacturing operations for each item are separated and IM. To handle feasibility issues of the JM model and a relatively high average TC of the IM model, we have developed a more efficient heuristic algorithm for the IM model, based on a TV lot sizes approach that allows various lot sizes for any products during a cyclic schedule.

A hybrid meta-heuristic algorithm, especially the genetic algorithm, may have tremendous potential on decisions regarding production scheduling with returns. Genetic algorithms can be applied to find combinatorial parts such as production frequencies and sequences. For instance, Moon, Cha, and Bae (2006) and Moon, Silver, and Choi (2002) already developed a hybrid genetic algorithm in the ELSP and the GT-ELSP, respectively. They demonstrated the effectiveness of the hybrid genetic algorithm in the combinatorial problems. Finally, retail ordering coordination is another extension. For example, Kusukawa (2014) showed the benefit of retail ordering coordination with production system in supply chain. Coordination of order batches and production schedule is a future research topic.

Acknowledgements
The authors are grateful for the useful comments from three anonymous referees.

Disclosure statement
No potential conflict of interest was reported by the authors.

Funding
This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) [grant number 2012R1A2A2A01012355]; the ICT R&D program of MSIP/IITP ['B0364-15-1008', 'Development of Open FaaS IoT Service Platform for Mass Personalization'].

References

Bae, H., I. Moon, and W. Yun. 2014. "Economic Lot and Supply Scheduling Problem: A Time-varying Lot Sizes Approach." *International Journal of Production Research* 52 (8): 2422–2435.

Bomberger, E. 1966. “A Dynamic Programming Approach to a Lot Size Scheduling Problem.” *Management Science* 12: 778–784.

Delporte, C., and L. Thomas. 1977. “Lot Sizing and Sequencing for N Products on One Facility.” *Management Science* 23: 1070–1079.

Dobson, G. 1987. “The Economic Lot-scheduling Problem: Achieving Feasibility Using Time-varying Lot Sizes.” *Operations Research* 35: 764–771.

Gallego, G., and D. Joneja. 1994. “Economic Lot Scheduling Problem with Raw Material Considerations.” *Operations Research* 42: 92–101.

Gallego, G., and I. Moon. 1992. “The Effect of Externalizing Setups in the Economic Lot Scheduling Problem.” *Operations Research* 40: 614–619.

Gallego, G., and D. Shaw. 1997. “Complexity of the ELSP with General Cyclic Schedules.” *IIE Transactions* 29: 109–113.

Hahm, J., and C. Yano. 1992. “The Economic Lot and Delivery Scheduling Problem: The Single Item Case.” *International Journal of Production Economics* 28 (2): 235–252.

Hahm, J., and C. Yano. 1995. “The Economic Lot and Delivery Scheduling Problem: The Common Cycle Case.” *IIE Transactions* 27: 113–125.

Hosseini-Nasab, H., and M. Dehghanbaghi. 2015. “A Fuzzy Expert System for Supporting Returned Products Strategies.” *International Journal of Industrial Engineering: Theory, Applications and Practice* 22 (2): 277–291.

Hsu, W. 1983. “On the General Feasibility Test for Scheduling Lot Sizes for Several Products on One Machine.” *Management Science* 29: 93–105.

Jones, P., and R. Inman. 1989. “When is the Economic Lot Scheduling Problem Easy?” *IIE Transactions* 21: 11–20.

Kuhn, H., and T. Liske. 2011. “Simultaneous Supply and Production Planning.” *International Journal of Production Research* 49 (13): 3795–3813.

Kuhn, H., and T. Liske. 2014. “An Exact Algorithm for Solving the Economic Lot and Supply Scheduling Problem Using a Power-of-two Policy.” *Computers and Operations Research* 51: 30–40.

Kusukawa, E. 2014. “Supply Chain Coordination in 2-Stage-ordering-production System with Update of Demand Information.” *Industrial Engineering and Management Systems* 13: 304–318.

Maxwell, W. 1964. “The Scheduling of Economic Lot Sizes.” *Naval Research Logistics Quarterly* 11: 89–124.

Moon, I., B. Cha, and H. Bae. 2006. “Hybrid Genetic Algorithm for Group Technology Economic Lot Scheduling Problem.” *International Journal of Production Research* 44 (21): 4551–4568.

Moon, I., E. Silver, and S. Choi. 2002. “Hybrid Genetic Algorithm for the Economic Lot-scheduling Problem.” *International Journal of Production Research* 40 (4): 809–824.

Osman, H., and K. Demirli. 2012. “Economic Lot and Delivery Scheduling Problem for Multi-stage Supply Chains.” *International Journal of Production Economics* 136: 275–286.

Roundy, R. 1989. “Rounding Off to Powers of Two in Continuous Relaxations of Capacitated Lot Sizing Problems.” *Management Science* 35: 1433–1442.

Shin, M., T. Ryu, and K. Ryu. 2014. “Agent-based Production Simulation of TFT-LCD Fab Driven by Material Handling Requests.” *International Journal of Industrial Engineering: Theory, Applications and Practice* 21: 396–407.

Tang, O., and R. Teunter. 2006. “Economic Lot Scheduling Problem with Returns.” *Production and Operations Management* 15 (4): 488–497.

Teunter, R., K. Kaparis, and O. Tang. 2008. “Multi-product Economic Lot Scheduling Problem with Separate Production Lines for Manufacturing and Remanufacturing.” *European Journal of Operational Research* 191: 1241–1253.

Teunter, R., O. Tang, and K. Kaparis. 2009. “Heuristics for the Economic Lot Scheduling Problem with Returns.” *International Journal of Production Economics* 118 (1): 323–330.

Teunter, R., E. VanderLaan, and K. Inderfurth. 2000. “On How to Set the Holding Cost Rates in Inventory Models with Returned Items that can be Remanufactured.” *Omega* 28 (4): 409–415.

Zanoni, S., A. Segerstedt, O. Tang, and L. Mazzoldi. 2012. “Multi-product Economic Lot Scheduling Problem with Manufacturing and Remanufacturing Using a Basic Period Policy.” *Computers and Industrial Engineering* 62: 1025–1033.

Zipkin, P. 1991. “Computing Optimal Lot Sizes in the Economic Lot Scheduling Problem.” *Operations Research* 39: 56–63.