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Dual channel closed-loop supply chain coordination with a reward-driven remanufacturing policy
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This paper investigates a reward-driven policy, employed in a closed-loop supply chain (CLSC), for acquiring used products earmarked for remanufacture. Under the examined model, a single manufacturer sells products through a retailer as well as directly to end users in a forward supply chain. In the reverse supply chain, three different modes of collection are employed to capture used products for remanufacture: they are through a third party, directly by the manufacturer and from the retailer. Mathematical models for both non-cooperative and centralized scenarios are developed to characterise the pricing decisions and remanufacturing strategies that indicate individual and overall supply chain performance. Optimality of all the proposed models is examined with theory. To coordinate and achieve a win–win outcome for channel members, we proposed a three-way discount mechanism for the manufacturer. Extended numerical investigation provides insights on ways to manage an efficient reward-driven CLSC in a dual-channel environment.

Keywords: closed-loop supply chain; remanufacturing; dual-channel; coordination

1. Introduction
In the recent times, due to the economic and environmental benefits of product remanufacturing, closed-loop supply chain (CLSC) management has received an enormous amount of attention from marketing and supply chain management as well as from researchers (Yuan and Gao 2009; Kannan, Sasikumar, and Devika 2010; Zhang, Jiang, and Pan 2012; Huang et al. 2013; Özceylan and Paksoy 2013). A simple CLSC consists of three main types of channel participants: retailer, manufacturer and third-party collector (Savaskan, Bhattacharya, and VanWassenhove 2004). The forward supply chain involves the movement of products from the upstream manufacturer to the consumers, while the reverse supply chain involves the movement of used products for remanufacturing from customers to upstream members. Firms such as Caterpillar, GE, IBM, HP, Ford, Sony and others, have established cost-effective remanufacturing systems either by themselves or via outsourcing to a third party (Karakayali, Emir-Farinas, and Akcal 2007). An extensive collection of literature on CLSCs can be found in articles by Srivastava (2007), Atasu, Sarvary, and VanWassenhove (2008), Guide and VanWassenhove (2009), Jayant, Gupta, and Garg (2012), Chen and Chang (2013), Amin and Zhang (2013), Choi, Li, and Xu (2013), Khalili, Tavana, and Najmodin (2015) and Yoo, Kim, and Park (2015), but most of the studies reflect the assumption that customers simply want to eliminate their used products without expectation of compensation and thus consider the collection rate as a function of the investment. This premise was inspired by observations of reward-driven return policies executed in several industries; for example, almost every car manufacturer sets a high residual value to increase the return volume and to sell new products. In the computer and electronic industries, manufacturers such as Staples, Microsoft, Verizon, Sony and Hewlett Packard have successfully implemented reward-driven return policies. In the furniture industry, Rachel’s Antique Emporium provides reward value for old furniture. ReCellular, one of the oldest and most well-known remanufacturing companies in consumer electronics, purchases used phones directly from consumers. BuyMyTronics and reBuy pay up to $350 for a used, working, Apple iPad 4th Generation. All these examples have motivated us to analyse the performance of members in dual channel CLSCs operating under commonly observed reward-driven remanufacturing policies.

In today’s competitive business environment, especially in light of the explosion of different channel possibilities, several companies have promoted multichannel distribution systems. Many firms set up two or more marketing channels to promote products, one of which is direct; that is the product is directly sold to the end user. According to one survey, approximately 42% of top suppliers in a variety of industries use an online–offline model to promote their products (Dan, Xu, and Liu 2012). The physical stores give customers the ability to interact physically with a product, which builds trust in ways technology-driven
channels cannot imitate. However, direct sales via technology save the manufacturer on costs, increase revenue and expand new market segments. Chen, Zhang, and Sun (2012) pointed out that many customers like to purchase products online to save transportation cost and time. As a result of these findings, the performances of the dual-channel supply chain under several constraints have evoked considerable interest from both the academic community and industry management (Yu, Zeng, and Zhao 2009; Jiang, Xu, and Sheng 2010; Khouja, Park, and Cai 2010; Li, Zhou, and Wang 2013; Lei et al. 2014; Li et al. 2014; Saha 2014; Xu et al. 2014; Shang and Yang 2015; Zhao et al. 2015). In this study, we explore the characteristics of dual channel CLSC members operating in cooperative and non-cooperative environments. Many researchers have given considerable attention to the issue of channel coordination, and they have studied the effectiveness and performance of various contracts under different supply chain configurations to enhance profits of the entire channel and each individual member. They have investigated multiple versions of commonly used contract mechanisms, including those described as quantity discount, two-part tariffs, buy-back, quantity flexibility, revenue sharing and mail-in rebate, to name a few (Cachon 2003; Sarmah, Acharya, and Goyal 2006; Simchi-Levi, Kaminsky, and Simchi-Levi 2008; Datta and Christopher 2011; Zhang, Xiong, and Xiong 2014). To the best of our knowledge, however, few have analysed the coordination issues affecting dual channels in CLSCs operating through reward-driven manufacturing policies. Therefore, we propose a three-way price discount mechanism for the manufacturer to coordinate the CLSC. The manufacturer provides all unit-price discounts to the retailer, the third-party collector and all the consumers in the direct channel. Results show that the mechanism not only coordinates the dual-channel supply chain, but also outperforms non-cooperative CLSCs.

For this study, we merge the analysis of three separate issues: remanufacturing, the CLSC and coordination of the dual-channel supply chain. Specifically, we consider here three different modes of collection in a non-cooperative environment: through a third party, by the manufacturer directly and from the retailer. We use the centralised model as the benchmark with which to compare our findings with the results obtained in other non-cooperative situations. Further, we examined whether our three-way price discount mechanism can coordinate the channel, and we found that it not only coordinates the channel, but also provides win-win outcomes for every member of it. To the best of our knowledge, pricing and coordination in dual channel CLSCs with reward-driven collection policies have not been studied. The rest of the paper is organised as follows. The model is developed and results are compared for non-cooperative and centralised scenarios in Section 2. The tests of concavity are conducted for each model, and the coordination of the CLSC is analysed; the analytical findings managerial implications are presented in Section 3. We also conduct sensitivity analysis of the model in Section 3. Finally, conclusions and extensions of the model are discussed in Section 4.

2. Development of the model

In this paper, a single-period remanufacturing policy is considered where a manufacturer produces products and supplies to an independent retailer, who sells the product to the customers. The manufacturer also maintains a direct channel in addition to the retail channel. The following assumptions and notation are considered in the development of the model. Additional notation and assumptions are listed wherever they are needed.

(i) The demand functions under the direct channel \( D_d \) and retail channel \( D_r \) with self- and cross-price effects (Huang and Swaminathan 2009) are assumed, respectively, as:
\[
D_d = \alpha_2 a - \beta_2 p_m - \delta(p_m - p_r),
\]
\[
D_r = \alpha_1 a - \beta_1 p_r + \delta(p_m - p_r),
\]
Here, \( p_r \) and \( p_m \) are, respectively, the selling price of the product to consumers in the retail and direct channel. \( a \) is the market potential of the product. \( \delta(>0) \) represents the number of customer switches from the retail channel to the direct channel per unit increase in the price difference between \( p_r \) and \( p_m \); \( \alpha_1 a, (0 < \alpha_1 < 1) \) represents the number of customers preferring the retail channel, while \( \alpha_2 a, (0 < \alpha_2 < 1) \) captures the number of customers preferring the direct channel \( (\alpha_1 + \alpha_2 \leq 1) \). \( \beta_1 \) and \( \beta_2 \), respectively, represent the relative portion of price sensitivity in the retail and direct channel.

(ii) The manufacturer collects the used product from the customers directly or through the retailer or through third party, and remanufacture the used products. If the manufacturer is not directly collecting the used products from the customer, then the manufacturer takes back the used product from the third party or the retailer at a price \( b \). The pictorial representation of three collection modes are given in Figure 1(a)–(c).

(iii) \( c_m \) and \( c_r \) represent the unit cost and unit remanufacturing cost of the used product \( (c_r < c_m) \) of the manufacturer, respectively. The wholesale price of the manufacturer to the retailer is \( w \).

(iv) There is no difference between the remanufactured products and new products (Savaskan, Bhattacharya, and Van-Wassenhove 2004).

(v) We have assumed reward-driven return policy and the return rate of used products is considered as:
\[
f(r) = (1 - \frac{r_0}{r}),
\]
where \( r \) represents the amount of reward received by the customer for returning the used products, \( r_0 \) is a minimum reward threshold at which customers start to return and \( \gamma \) represents the relative portion of price sensitivity. Note that
Figure 1. (a) Structure of dual channel CLSC under third-party collection. (b) Structure of dual channel CLSC with manufacturer collection. (c) Structure of dual channel CLSC under retailer collection.

as $r \rightarrow \infty$, then $f(r) = 1$ and if $r \rightarrow r_0$, then $f(r) = (1 - \gamma)$ with the special case where at $\gamma = 1$, and $f(r) = 0$, i.e. no return (Zeng 2013).

(vi) Although, the merchandising costs associated with selling and recycling the products sometime depends on product categories, but for analytical simplicity and to obtain closed form solution, we have considered the cost as zero (Hua, Wang, and Cheng 2010). The information among the channel member is symmetric.

In the next subsection, first we derive the expressions of all the decision variables under third-party collection mode under non-cooperative environment.

2.1 Third-party collection mode under non-cooperative environment

We model the decision process as a non-cooperative game where the third party decides the reward value $r$ to encourage return, the retailer decides the retail price $p_r$ and the manufacturer decides the wholesale price $w$ for the retailer, the retail price of the direct channel $p_m$ and collection price $b$ to collect the used product from third party. In this scenario, profit
functions of the retailer ($\pi_r$), the third party ($\pi_3$) and the manufacturer ($\pi_m$) are as follows:

$$\pi_r(p_r) = (p_r - w)(\alpha_1 a - \beta_1 p_r + \delta(p_m - p_r))$$

(1)

$$\pi_3(r) = (b - r) \left(1 - \frac{y_0}{r}\right) \left((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m\right)$$

(2)

$$\pi_m(p_m, b, w) = (\alpha_1 a - \beta_1 p_r + \delta(p_m - p_r))w + (\alpha_2 a - \beta_2 p_m - \delta(p_m - p_r))p_m - c_m((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m)$$

$$+ (c_m - c_r - b) \left(1 - \frac{y_0}{r_d}\right) \left((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m\right)$$

(3)

The solution of the above problem is a sequential game consisting of two Stackelberg games. When making decisions, the manufacturer, acts as a Stackelberg leader, considers retailer’s and the third party’s best responses to its decisions. As follows, the retailer and the third party make decisions after observing the manufacturer’s decision. We solve this two-stage sequential game using backward induction moving from the second stage, the retailer and the third-party decision, to the manufacturer decision problem in the first stage. In the second stage, retailer maximises its profit from selling new products through retail channel only as

$$\text{Max} \quad \pi_r(p_r) = (p_r - w)(\alpha_1 a - \beta_1 p_r + \delta(p_m - p_r))$$

From the first-order conditions of optimisation $\frac{\partial \pi_r}{\partial p_r} = 0$, the retailer sets the retail price as

$$p_r = \frac{\alpha_1 a + \delta p_m + w(\beta_1 + \delta)}{2\beta_1 + \delta}$$

(4)

Since $d^2 \pi_r(p_r)/dp_r^2 = -2(\beta_1 + \delta) < 0$, i.e. profit function of the retailer is concave. Similarly, in the second stage, the third party determines the reward value $r$ by optimising profit obtained in Equation (2). From the first-order conditions of optimisation $\frac{\partial \pi_3}{\partial \alpha} = 0$, the third-party firm sets the reward value as

$$r = \sqrt{\frac{b}{2}(\beta_1 + \delta)}$$

(5)

Since $d^2 \pi_3(r)/dr^2 = -\frac{2b(\beta_1 + \delta)}{r^2} < 0$, hence, the profit function of the third-party firm is also concave. Finally, in the first stage, the manufacturer solves the problem to maximise its total profit. The profit function of the manufacturer can be represented as

$$\text{Max} \quad \pi_m(w, p_m, b) = (\alpha_1 a - \beta_1 p_r + \delta(p_m - p_r))w + (\alpha_2 a - \beta_2 p_m - \delta(p_m - p_r))p_m - c_m((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m)$$

$$+ (c_m - c_r - b) \left(1 - \frac{y_0}{r_d}\right) \left((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m\right)$$

(6)

In the above profit function, the first two terms represent the revenue earn by the manufacturer from selling retail and direct channel, the third term represents the total unit cost and the last term represents the profit earn from remanufactured products. It is straightforward to find solution that maximises profit of the manufacturer and is obtained by solving $\frac{\partial \pi_m}{\partial w} = 0$, $\frac{\partial \pi_m}{\partial p_m} = 0$ and $\frac{\partial \pi_m}{\partial b} = 0$. After simplification, we have obtained the retail price of the products in the direct channel $p_m = p_{md}$, the wholesale price of the manufacturer in retail channel $w = w_d$ and collection price $b$, respectively, as follows:

$$p_{md} = \frac{a(\alpha_2(\beta_1 + \delta) + \alpha_1 \delta) + c_m - (c_m - c_r - b) \left(1 - \frac{y_0}{r_d}\right)}{2(\beta_1 + \delta + \alpha_1 \delta)}$$

(7)

$$w_d = \frac{\alpha(\alpha_2 \delta + \alpha_1 (\beta_2 + \delta))}{2(\beta_1 + \delta + \alpha_1 \delta)} + \frac{c_m - (c_m - c_r - b) \left(1 - \frac{y_0}{r_d}\right)}{2}$$

(8)

$$b^3 - \frac{y_0 b^2}{4} - \frac{y_0 (c_m - c_r) b}{2} - \frac{y_0 (c_m - c_r)^2}{4} = 0$$

(9)

It needs to be mentioned that the retailer will participate in dual channel if $p_{md} > w_d$, i.e. by comparing Equations (7) and (8), we have obtained the condition $\beta_1 \alpha_2 > \beta_2 \alpha_1$. By using Cardan’s method, one may find the solution of Equation (9) as

$$b = 0.08 y_0 + \frac{0.63 y_0 (c_m - c_r) + 0.03 y_0^2}{\left(N + \sqrt{M}\right)^{1/3}} + 0.26 \left(N + \sqrt{M}\right)^{1/3}$$

(10)

where $N = y_0(6.75(c_m - c_r)^2 + 1.13r_0 y_0 (c_m - c_r) + 0.03r_0^2 y_2^2) + 0.06 r_0^3$. From Equation (9), one may easily observe that the collection price of the used product of the manufacturer $b$ is independent of the demand parameter. It is only the function of unit cost and remanufacturing cost. An important point must be mentioned here,
the optimal reward value $r_d$ provided by third party obtained in Equation (5) is also independent of the demand parameters. Now to guarantee the profit function of the manufacturer is concave and has a unique maxima, we compute the Hessian matrix of the manufacturer profit function as follows:

$$H_m = \begin{pmatrix}
\frac{\partial^2 \pi_m}{\partial \alpha^2} & \frac{\partial^2 \pi_m}{\partial \alpha \partial \beta} & \frac{\partial^2 \pi_m}{\partial \beta^2} \\
\frac{\partial^2 \pi_m}{\partial \alpha \partial \beta} & \frac{\partial^2 \pi_m}{\partial \beta^2} & \frac{\partial^2 \pi_m}{\partial \beta^2} \\
\frac{\partial^2 \pi_m}{\partial \alpha \partial \beta} & \frac{\partial^2 \pi_m}{\partial \beta^2} & \frac{\partial^2 \pi_m}{\partial \beta^2}
\end{pmatrix} = \begin{pmatrix}
-(\beta_1 + \delta) & \delta & -2\beta_2 - \frac{2\beta_1 + \delta^2}{(\beta_1 + \delta)} \\
\delta & -\frac{4\beta_1 + \delta^2}{(\beta_1 + \delta)} & 0 \\
-2\beta_2 - \frac{2\beta_1 + \delta^2}{(\beta_1 + \delta)} & 0 & L
\end{pmatrix}
$$

where $L = -\sqrt{\gamma_0 \left(\frac{3(\gamma_0 - \gamma_1)}{\alpha_0 \beta_0} + \frac{1}{4\alpha_0^2}\right)} (a(\alpha_1 + \alpha_2) - \beta_1 p_r - \beta_2 p_m)$. Since values of principle minors of the Hessian matrix, $\Delta_1 = -(\beta_1 + \delta) < 0$, $\Delta_2 = 2(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta) > 0$ and $\Delta_3 = -2(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta)\sqrt{\gamma_0 \left(\frac{3(\gamma_0 - \gamma_1)}{\alpha_0 \beta_0} + \frac{1}{4\alpha_0^2}\right)} (a(\alpha_1 + \alpha_2) - \beta_1 p_r - \beta_2 p_m) < 0$ are alternative in sign, i.e. $H_m$ is negative definite. It assures $\pi_m$ is concave. Substituting the optimal values obtained in Equations (5), (7) and (8), we have the optimal selling price of the retail channel $p_r = p_{rd}$, profit functions of the channel member $\pi_r = \pi_{rd}$, $\pi_3 = \pi_{3pd}$, $\pi_m = \pi_{md}$ and sales volume in non-cooperative channel $Q = Q_d$ as

$$p_{rd} = \frac{a(2\alpha_2 \beta_1 (\beta_1 + \delta) + \alpha_1 (3\beta_1 (\beta_2 + \delta) + \delta (3\beta_2 + 2\delta))) + c_m \beta_1 + 2\delta}{4(\beta_1 + \delta)} \left(1 - \frac{\gamma_0}{r_d}\right) + \frac{\beta_1^2}{4(\beta_1 + \delta)} \left(c_m - c_m - c_r - b\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right)
$$

$$\pi_{3pd} = \left[1 - \frac{\gamma_0}{r_d}\right] \left[\frac{a(\alpha_1 + \alpha_2)}{4(\beta_1 + \delta)} + \frac{a(\alpha_1 \delta + \alpha_2 (\beta_1 + 4\delta))}{4(\beta_1 + \delta)} \left(c_m - c_m - c_r - b\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right)
$$

$$\pi_{md} = \frac{\beta_1 a^2 (\alpha_2 (\beta_1 + \delta) + \beta_2 (\beta_1 + \delta) + \alpha_1 \delta)^2}{8(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta)} + \frac{\beta_2 a^2 (\alpha_2 (\beta_1 + \delta) + \alpha_1 \delta)^2}{8(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta)} + \frac{\delta a^2 (\alpha_2 \beta_2 - \alpha_2 \beta_1)^2}{8(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta)} - \frac{a(\alpha_1 + \alpha_2)}{4(\beta_1 + \delta)} \left(c_m - c_m - c_r - b\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right)
$$

$$Q_d = \frac{a(\alpha_1 + \beta_2)}{4(\beta_1 + \delta)} + \frac{a(\alpha_1 \delta + \alpha_2 (\beta_1 + \delta))}{4(\beta_1 + \delta)} - \frac{\beta_1 + 2\beta_2}{4(\beta_1 + \delta)} \left(\beta_1 + 2\beta_2 + \delta \beta_2 + \beta_1 \delta\right) \left(c_m - c_m - c_r - b\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right) \left(1 - \frac{\gamma_0}{r_d}\right)
$$

Based on the above results, the following proposition is proposed:

**Proposition 1** Profit functions of each channel member in non-cooperative scenario under third party collection mode are concave and the retailer will participate in dual channel iff $\beta_1 \alpha_2 > \beta_2 \alpha_1$.

Now, $\frac{\partial Q_d}{\partial \alpha_1} = \frac{(\beta_1 + 2\alpha_2)}{4(\beta_1 + \delta)} < \frac{\partial Q_d}{\partial \alpha_1} = \frac{(\beta_1 + 2\alpha_2)}{4(\beta_1 + \delta)}$, i.e. the sales volume in non-cooperative channel is highly sensitive with respect to $\alpha_2$ compared to $\alpha_1$. Again, $\frac{\partial Q_d}{\partial \beta_1} = \frac{(\beta_1 + 2\alpha_2)}{4(\beta_1 + \delta)} \left(\beta_1 + 2\beta_2 + \delta \beta_2 + 2\beta_1 \delta\right) > \frac{2\alpha_2}{4(\beta_1 + \delta)} = \frac{\partial Q_d}{\partial \beta_1}$, i.e. the retail price of the product increases more with respect to $\alpha_1$ compared to $\alpha_2$. Similarly, $\frac{\partial Q_d}{\partial \alpha_2} = \frac{(\beta_1 + 2\alpha_2)}{4(\beta_1 + \delta)} > \frac{\partial Q_d}{\partial \alpha_2}$, i.e. the direct price of the product increases more with respect to $\alpha_2$ compared to $\alpha_1$. The results are fairly reasonable. As $\alpha_1$ (or $\alpha_2$) increases, the market share of the retailer (or manufacturer) also increases and as a result, the power of retailer (or manufacturer) enhances to set higher retail price. Next, we derived the expressions when the manufacturer directly collects the used products from the customer.

### 2.2 Direct collection by the manufacturer under non-cooperative environment

When the manufacturer collects directly, the retailer decides the retail price $p_r$ and the manufacturer decides the wholesale price $w$ for the retailer, the retail price of the direct channel $p_m$ and reward value $r$ for the customer to encourage return of used product directly to the manufacturer. The profit functions of the retailer ($\pi_r$) and the manufacturer ($\pi_m$) under this mode of collection are as follows:
When making decisions, the manufacturer acts as a Stackelberg leader considering retailer’s best responses to its decisions. Similar to the previous subsection, the retailer maximises its profits from selling new products through retail channel and the retailer sets the retail price \( p_r = \frac{\alpha_1 a + \delta p_m + \omega(\beta_1 + \delta)}{2(\beta_1 + \delta)} \) as

\[
\pi_{m1}(p_m, w) = (\alpha_1 a - \beta_1 p_r + \delta (p_m - p_r)) w + (\alpha_2 a - \beta_2 p_m - \delta (p_m - p_r)) p_m - cm((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m)
\] 
\[
+ (cm - cr - r) \left( 1 - \frac{\gamma r_0}{r} \right) ((\alpha_1 + \alpha_2) a - \beta_1 p_r - \beta_2 p_m)
\]

Similarly, the manufacturer maximises its profits from remanufacturing, it must be greater than the reservation price, \( \gamma r_0 < (c_m - c_r) \) for the customer to encourage return.

\[
\gamma r_0 < (c_m - c_r)
\]

Since \( c_m - c_r \) represents gain of the manufacturer due to remanufacturing, it must be greater than the reservation price, otherwise remanufacturing is not profitable for the manufacturer. Substituting the optimal values obtained in Equations (19)–(21), we have the optimal selling price of the retail channel \( p_r \), profit of the retailer \( \pi_r \), the manufacturer \( \pi_m \) and sales volume in non-cooperative channel \( Q = Q_1 \) as follows:

\[
p_r = \frac{a(2\alpha_2 \delta(\beta_1 + \delta) + \alpha_1(3\beta_1(\beta_2 + \delta) + \delta(3\beta_2 + 2\delta)))}{4(\beta_1 + \delta)(\beta_1 \beta_2 + \delta \beta_1 + \delta \beta_2)} + \frac{cm \beta_1 + 2\delta}{4} - \frac{\beta_1 + 2\delta}{4} (c_m - c_r - r_1) \left( 1 - \frac{\gamma r_0}{r_1} \right)
\]

\[
\pi_r = \frac{a^2 \alpha_1^2}{16(\beta_1 + \gamma)} - \frac{\alpha_1 \beta_1 a}{8(\beta_1 + \gamma)} (c_m - (c_m - c_r - r_1)(1 - \frac{\gamma r_0}{r_1})) + \frac{\beta_1^2}{16(\beta_1 + \delta)} (c_m - (c_m - c_r - r_1)(1 - \frac{\gamma r_0}{r_1}) \right)^2
\]

\[
\pi_m = \frac{\beta_1^2 a(2\alpha_2(\beta_1 + \delta) + \alpha_1 \delta)^2}{8(\beta_1 \beta_2 + \delta \beta_1 + \delta \beta_2)^2} + \frac{\beta_2^2 a(2\alpha_2(\beta_1 + \delta) + \alpha_1 \delta)^2}{8(\beta_1 \beta_2 + \delta \beta_1 + \delta \beta_2)^2} + \frac{\delta a(2(\alpha_1 \beta_2 - \alpha_2 \beta_1))^2}{8(\beta_1 \beta_2 + \delta \beta_1 + \delta \beta_2)^2} + \frac{a^2(\alpha_2(\beta_1 + \delta) + \alpha_1 \delta)^2}{8(\beta_1 \beta_2 + \delta \beta_1 + \delta \beta_2)} - \left( \frac{4(\alpha_1 + \alpha_2)}{4} + \frac{4(\alpha_2(\beta_1 + \delta) + \alpha_1 \delta)}{4} \right) (c_m - (c_m - c_r - r_1)(1 - \frac{\gamma r_0}{r_1}) \right)^2
\]

\[
Q_1 = \frac{a(\alpha_1 + \alpha_2)}{4} + \frac{a(\alpha_1 \delta + \alpha_2(\beta_1 + \delta))}{4(\beta_1 + \delta)} - \left( \frac{\beta_1 + \beta_2}{4} + \frac{\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta}{4(\beta_1 + \delta)} \right) (c_m - (c_m - c_r - r_1)(1 - \frac{\gamma r_0}{r_1}) \right)^2
\]

Next, we have derived the expressions of decision variables where collection is done by the retailer.

### 2.3 Retailer collection mode under non-cooperative environment

In this mode of collection, the retailer decides the retail price \( p_r \) and the reward value \( r \) for the customer to encourage return, and the manufacturer decides the wholesale price \( w \) for the retailer, the retail price of the direct channel \( p_m \) and collection price \( b \) to collect the used product from the retailer. In this mode, profit functions of the retailer (\( \pi_r \)) and the manufacturer (\( \pi_m \)) are as follows:
\[
\pi_2(p_r, r) = (p_r - w)(\alpha_1 a - \beta_1 p_r + \delta(p_m - p_r)) + (b - r)(1 - \frac{\gamma r_0}{r})(\alpha_1 + \alpha_2)a - \beta_1 p_r - \beta_2 p_m \tag{27}
\]
\[
\pi_{m2}(p_m, b, w) = (\alpha_1 a - \beta_1 p_r + \delta(p_m - p_r))w + (\alpha_2 a - \beta_2 p_m - \delta(p_m - p_r))p_m - c_m((\alpha_1 + \alpha_2)a - \beta_1 p_r - \beta_2 p_m) + (c_m - c_r - b)(1 - \frac{\gamma r_0}{r})(\alpha_1 + \alpha_1)a - \beta_1 p_r - \beta_2 p_m \tag{28}
\]

Similar to the previous subsection, the optimal values of the retailer’s decision variables, the selling price of the retail channel \(p_r = p_{r2}\) and collection price of the used products from the customer \(r = r_2\) are obtained as follows:

\[
p_{r2} = \frac{\alpha_1 a + \delta p_m + w(\beta_1 + \delta)}{2(\beta_1 + \delta)} \tag{29}
\]
\[
r_2 = \sqrt{\frac{\gamma r_0}{b}} \tag{30}
\]

Finally, the optimal values of the manufacturer’s decision variables, the optimal collection price from the retailer \(b = b_2\), the retail price of the direct channel \(p_m = p_{m2}\) and collection price in non-cooperative channel \(c_c = Q_2\) are obtained as

\[
b_2 = 0.08\gamma r_0 + \frac{0.63\gamma r_0(c_m - c_r) + 0.03\gamma^2 r_0^2}{(N + \sqrt{M})^{1/3}} + 0.26(N + \sqrt{M})^{1/3} \tag{31}
\]

Solving Equation (31), we have

\[
b_2 = \frac{\gamma r_0 b_2^2 - \gamma r_0(c_m - c_r)b_2 - \gamma r_0(c_m - c_r)^2}{4} = 0 \tag{31}
\]
\[
p_{m2} = \frac{\alpha(\alpha_2(\beta_1 + \delta) + \alpha_1(3\beta_1(\beta_2 + \delta) + \delta(3\beta_2 + 2\delta)))}{2(\beta_1 + \delta)(\beta_1 + 2\delta)} + \frac{c_m}{2} \tag{32}
\]
\[
w_2 = \frac{\alpha(\alpha_2(\beta_1 + \delta) + \alpha_1(\beta_2 + \delta))}{2(\beta_1 + 2\delta)} + \frac{c_m}{2} \tag{33}
\]

Substituting the optimal values obtained in Equations (32)–(34), we have the optimal selling price of the direct channel \(p_{r2}\), profit of the retailer \(\pi_{r2}\), the manufacturer \(\pi_{m2}\) and sales volume in non-cooperative channel \(Q = Q_2\) as follows:

\[
p_{r2} = \frac{\alpha(2\alpha_2(\beta_1 + \delta) + \alpha_1(3\beta_1(\beta_2 + \delta) + \delta(3\beta_2 + 2\delta)))}{4(\beta_1 + \delta)(\beta_1 + 2\delta)(\beta_1 + \delta_2)} + \frac{c_m}{2} \tag{35}
\]
\[
\pi_{r2} = \frac{\alpha^2(\alpha_1^2 + \alpha_1^2)}{16(\beta_1 + \gamma)} - \frac{\alpha_1^2 a(\beta_1 + \delta)}{8(\beta_1 + \gamma)(c_m - (c_m - c_r - b_2)(1 - \frac{\gamma r_0}{r_2}))} + \frac{\beta_1^2}{16(\beta_1 + \gamma)}(c_m - (c_m - c_r - b_2)(1 - \frac{\gamma r_0}{r_2}))^2 + \frac{b_2 - r_2}{r_2}(1 - \frac{\gamma r_0}{r_2}) \tag{35}
\]
\[
\pi_{m2} = \frac{\beta_1^2(\alpha_1^2 + \beta_1^2)}{8(\beta_1 + \gamma)(\beta_1 + 2\delta)(\beta_1 + \delta_2)} + \frac{\beta_2^2(\alpha_2(\beta_1 + \delta) + \alpha_1^2)}{8(\beta_1 + 2\delta)(\beta_1 + \delta_2)^2} + \frac{\delta(a_2(\beta_1 + \delta) + \beta_1^2) + a_2^2(\beta_1 + \delta) + \beta_1^2}{8(\beta_1 + 2\delta)(\beta_1 + \delta_2)^2} \tag{36}
\]
\[
Q_2 = \frac{\alpha_1^2}{4} + \frac{\alpha_2^2}{4} + \frac{\alpha_1^2}{4} \tag{37}
\]

Note that Equations (9) and (31) as well as Equations (5) and (30) are identical in nature, i.e. one may easily conclude that the reward value of customer remains same for the third party or the retailer collection mode and corresponding recollection price of the manufacturer from third party and retailer also remain same. From Equation (31), if we consider \(f(b_2) = b_2^2 - \gamma r_0 b_2^2 - \gamma r_0(c_m - c_r)b_2 - \gamma r_0(c_m - c_r)^2\), then \(f(0) = -\frac{\gamma r_0(c_m - c_r)^2}{4} < 0\). Again, using the condition that \(\gamma r_0 < (c_m - c_r)\), we obtain \(f(c_m - c_r) = (c_m - c_r)^2(c_m - c_r - \gamma r_0) > 0\). So, optimal value of \(b_2\) obtained by solving Equation (31), must satisfy \(b_2 \in (0, c_m - c_r)\). More precisely, \(f(r_1) = -\gamma r_0(c_m - c_r)(\sqrt{(c_m - c_r)} - \sqrt{r_0})^2 < 0\), as a consequence, one may conclude that optimal value of \(b_2\) obtained by solving Equation (31), must satisfy \(b_2 \in (r_1, c_m - c_r)\), i.e. collection price of the manufacturer from third party or retailer is always less than \(c_m - c_r\). The result is quite justified,
otherwise remanufacturing is not profitable for the manufacturer. Since \( b_2 \in (r_1, c_m - c_r) \), one can easily conclude that \( b_2 > r_1 \), i.e., the manufacturer has to pay more if the manufacturer collects the products through the retailer or the third party. Again by comparing Equations (5), (19) and (30), one may easily observe that \( r_d = r_2 < r_1 \), i.e. the end customer gets maximum reward under the manufacturer collection mode. Based on the above results, we proposed the following proposition:

**Proposition 2**

(a) The manufacturer gets maximum benefit if manufacturer collects the used products directly from the customer. Profit of the manufacturer remains unchanged under the retailer or the third-party collection mode (by comparing Equations (14), (25) and (37)).

(b) The retailer gets maximum benefit under her own collection mode (by comparing Equations (12), (24) and (36)).

(c) Among the three collection modes, the customer gets maximum benefit when the manufacturer collects directly from the retailer.

(d) Under non-cooperative environment, channel profit in the manufacturer direct collection mode is always greater compared to channel profit in the retailer or the third-party collection mode.

From the above proposition, one can also conclude that if the manufacturer collects the products directly then the end customer may be benefited more. Since in the third-party or the retailer collection mode more marketing units compete for revenue for product return, obviously customer benefits decrease. It is also observed that the profit obtained by each channel member under different collection modes are not identical. In the third-party and the retailer collection mode, all the values of decision variables remain same but the additional profit of the third party goes to the retailer. From the profit structure of channel member under three collection modes, one may easily observe that three collection modes can be integrated to a single centralised model and in the next subsection, we have derived the expressions of decision variables of centralised model.

### 2.4 Centralised model

In the centralised CLSC, channel members are vertically integrated and there exists a central decision-maker who determines the retail price of the direct as well as retail channel, and determines the amount \( r \). In this situation, the wholesale price \( w \) is fixed. In the centralised CLSC, channel members are vertically integrated to a single centralised model and in the next subsection, we have derived the expressions of decision variables of centralised model.

The centralised model optimises the following problem:

\[
\pi_c(p_r, p_m, r) = p_r(\alpha_1 a - \beta_1 p_r + \delta (p_m - p_r)) + (\alpha_2 a - \beta_2 p_m - \delta (p_m - p_r))p_m - c_m((\alpha_1 + \alpha_2)a - \beta_1 p_r - \beta_2 p_m) + (c_m - c_r - r)(1 - \frac{\gamma r_0}{r})((\alpha_1 + \alpha_2)a - \beta_1 p_r - \beta_2 p_m)
\]

Comparing Equations (41) and (42), we have \( \frac{\partial \pi_c}{\partial p_r} = \frac{a(\alpha_1 \beta_2 + \delta \alpha_2)}{2(\beta_1 \beta_2 + \delta \beta_1 + \delta \beta_2)} + \frac{c_m}{2} - \frac{(c_m - c_r - r_c)}{2} \left(1 - \frac{\gamma r_0}{r_c}\right) \left(1 - \frac{\gamma r_0}{r_c}\right) \frac{r}{r_c} > 0 \) (using Proposition 1), i.e. retail price in the direct channel is always greater compared to retail channel. In order to guarantee the profit function in centralised channel is concave and has a unique maxima, we compute the Hessian matrix of the centralised channel as follows:

\[
H_c = \begin{pmatrix}
\frac{\partial^2 \pi_c}{\partial p_r^2} & \frac{\partial^2 \pi_c}{\partial p_r \partial p_m} & \frac{\partial^2 \pi_c}{\partial p_r \partial r} \\
\frac{\partial^2 \pi_c}{\partial p_m \partial p_r} & \frac{\partial^2 \pi_c}{\partial p_m^2} & \frac{\partial^2 \pi_c}{\partial p_m \partial r} \\
\frac{\partial^2 \pi_c}{\partial r \partial p_r} & \frac{\partial^2 \pi_c}{\partial r \partial p_m} & \frac{\partial^2 \pi_c}{\partial r^2}
\end{pmatrix}
= \begin{pmatrix}
-2(\beta_1 + \delta) & 2\delta & 0 \\
2\delta & -2(\beta_2 + \delta) & 0 \\
0 & 0 & -2\gamma r_0(c_m - c_r)((\alpha_1 + \alpha_2)a - \beta_1 p_r - \beta_2 p_m)/r_c^3
\end{pmatrix}
\]

Since the value of principle minors of the Hessian matrix, \( \Delta_1 = -2(\beta_1 + \delta) < 0 \), \( \Delta_2 = 4(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta) > 0 \) and \( \Delta_3 = -8\gamma r_0(c_m - c_r)((\alpha_1 + \alpha_2)a - \beta_1 p_r - \beta_2 p_m)/r_c^3(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta) < 0 \) are alternative in sign, i.e. \( H_c \) is negative definite. It assures
\( \pi_c \) is concave. Substituting the optimal values obtained in Equations (40)–(42), we have the optimal channel profit \( \pi_c \) and sales volume \( Q = Q_c \) as

\[
\pi_c = \frac{a^2(\alpha_1^2\beta_2 + \alpha_2^2\beta_1 + (\alpha_1 + \alpha_2)\delta)}{4(\beta_1\beta_2 + \delta\beta_1 + \delta\beta_2)} + \frac{(\beta_1 + \beta_2)}{4}\left(c_m - \sqrt{c_m - c_r - \sqrt{\gamma_0}}\right)^2
\]

\[
Q_c = \frac{a(\alpha_1 + \alpha_2 - (\beta_1 + \beta_2)c_m)}{2} + \frac{(\beta_1 + \beta_2)}{2}\left(c_m - c_r - \sqrt{\gamma_0}\right)^2
\]  

(43)

(44)

Now, \( \frac{\partial Q_c}{\partial Q_c} = \frac{\partial Q_c}{\partial Q_c} = \frac{a}{2} \), i.e., increment of sales volume remain same for both \( \alpha_1 \) and \( \alpha_2 \). Again, \( \frac{\partial \pi_c}{\partial \alpha_1} - \frac{\partial \pi_c}{\partial \alpha_2} = \frac{2a^2(\alpha_1\beta_1 - \alpha_2\beta_2)}{4(\beta_1\beta_2 + \delta\beta_1 + \delta\beta_2)} > 0 \), i.e. \( \pi_c \) is more sensitive to \( \alpha_2 \) compared to \( \alpha_1 \) due to the fact that \( p_{nc} \) > \( p_{rc} \). Now, by comparing the optimal value of retail price in centralised scenario and wholesale price for the retailer in third-party collection mode, we have \( w_d - p_{rc} = \frac{1}{2}\left(b - r_c + \frac{\gamma_0}{r_d}(c_m - c_r - b) - \frac{\gamma_0}{r_c}(c_m - c_r - r_d)\right) \). Since \( r_d < r_c \Rightarrow w_d - p_{rc} > \frac{1}{2}\left(b - r_c + \frac{\gamma_0}{r_d}(c_m - c_r - b) - \frac{\gamma_0}{r_d}(c_m - c_r - r_d)\right) = \frac{1}{2}(b - r_c)\left(1 - \frac{\gamma_0}{r_d}\right) > 0 \), i.e. \( w_d > p_{rc} \). From analysis of the above subsection, we proposed the following propositions:

**Proposition 3**

(a) Profit function of centralised channel is concave.

(b) The reward value of used products in centralised mode and the manufacturer collection mode are identical.

(c) The wholesale price of the manufacturer in third party collection mode in non-cooperative scenario is greater compared to selling price of retail channel in centralised scenario.

From the above propositions, one may conclude that the optimal solution of the centralised model exists uniquely. It is also observed that in centralised scenario the selling price of the product is less compared to that in non-cooperative scenario, but the reward value of the customer for returning the product is also higher. From these one may conclude that the customer gets benefits from purchasing as well as returning the products under centralised model.

So far, we have discussed characteristics of three collection modes under non-cooperative scenario as well as in the centralised scenario. Now, the coordination issue of non-cooperative supply chain is examined. For this purpose, first, we have determined the optimal value of discounts that the central planner may provide on non-cooperative retail price in the retail channel as well as direct channel under third-party collection mode such that the channel profit is maximum when the reward value of consumer for used product remains \( r_c \). From Proposition 2, it is observed that the manufacturer can directly collect used products from the customer at a price \( r_1 = r_c \). If \( r_1 \) and \( r_2 \) amount of unit discount are provided, respectively, on selling price of the retail and direct channel simultaneously, to entice the end customer, then the selling prices of the products are reduced to \( p_{rd} - R_1 \) and \( p_{md} - R_2 \). As a consequence, the sales volume in retail as well as direct channel increase to \( (\alpha_1a - \beta_1(p_{rd} - R_1) - \delta(p_{rd} - p_{md} - R_1 + R_2)) \) and \( (\alpha_2a - \beta_2(p_{md} - R_2) + \delta(p_{rd} - p_{md} - R_1 + R_2)) \), respectively, where \( p_{rd} \) and \( p_{md} \) are obtained from Equations (11) and (7), respectively. In this situation, the channel profit function \( \pi_{cc}(R_1, R_2) \) becomes

\[
\pi_{cc}(R_1, R_2) = (\alpha_1a - \beta_1(p_{rd} - R_1) + \delta(p_{md} - p_{rd} - R_1 + R_2)) + (\alpha_2a - \beta_2(p_{md} - R_2) + \delta(p_{rd} - p_{md} - R_1 + R_2)) - c_m(\alpha_1 + \alpha_2) - \beta_1(p_{rd} - R_1) - \beta_2(p_{md} - R_2) + (c_m - c_r - r_c)\left(1 - \frac{\gamma_0}{r_c}\right)((\alpha_1 + \alpha_2)\alpha - \beta_1(p_{rd} - R_1) - \beta_2(p_{md} - R_2))
\]

(45)

The optimal solutions for the above problem can be obtained by solving \( \frac{\partial \pi_{cc}}{\partial R_1} = 0 \) and \( \frac{\partial \pi_{cc}}{\partial R_2} = 0 \), and after simplification, solutions are obtained as follows:

\[
p_{rd} - R_1 = \frac{a(\alpha_1(\beta_2 + \delta) + \alpha_2\delta)}{2(\beta_1\beta_2 + \delta\beta_1 + \delta\beta_2)} + \frac{c_m - c_r}{2}\left(1 - \frac{\gamma_0}{r_c}\right)
\]

\[
p_{md} - R_2 = \frac{a(\alpha_1\delta + \alpha_2(\beta_1 + \delta))}{2(\beta_1\beta_2 + \delta\beta_1 + \delta\beta_2)} + \frac{c_m - c_r}{2}\left(1 - \frac{\gamma_0}{r_c}\right)
\]

(46)

(47)

Substituting the values of \( p_{rd} \) and \( p_{md} \) in Equation (46) and (47), we have

\[
R_1 = \frac{1}{4(\beta_1 + \delta)}\left(\alpha_1 - (c_m - (c_m - c_r - b))(1 - \frac{\gamma_0}{r_d})\right) + \frac{1}{2}\left(c_m - c_r - r_c\right)\left(1 - \frac{\gamma_0}{r_c}\right) - (c_m - c_r - b)(1 - \frac{\gamma_0}{r_d})
\]

(48)

\[
R_2 = \frac{1}{2}\left(c_m - c_r - r_c\right)\left(1 - \frac{\gamma_0}{r_c}\right) - (c_m - c_r - b)(1 - \frac{\gamma_0}{r_d})
\]

(49)
As $\frac{\partial^2 \pi_c}{\partial R_1^2} = -2(\beta_1 + \delta) < 0$ and $\frac{\partial^2 \pi_c}{\partial R_2^2} = \left(\frac{\partial \pi_c}{\partial R_2}\right)^2 = 4(\beta_1 \beta_2 + \delta \beta_2 + \beta_1 \delta) > 0$, it assures $\pi_c$ is concave. After rearrangement of Equation (49), we have $R_2 = \frac{1}{\beta_1} \left(\frac{c_m - c_r - r_c}{p_m - p_r} \left(\frac{\gamma_0}{\delta R_2} - \frac{\gamma_0}{r_c} + (b - r_c) \left(1 - \frac{\gamma_0}{r_d}\right)\right) + \frac{1}{\beta_1} \left(\frac{c_m - c_r - r_c}{p_m - p_r} \left(\frac{\gamma_0}{\delta R_2} - \frac{\gamma_0}{r_c} + (b - r_c) \left(1 - \frac{\gamma_0}{r_d}\right)\right)\right)\right) > 0$ as it is found previously that $b > r_c > r_d$. Hence, $R_1$ and $R_2$ obtained from (48) and (49), both are non-negative. From Equations (48) and (49), it is also observed that $R_1 > R_2$. Substituting the values of $R_1$ and $R_2$ in Equation (45) and after simplification, we have

$$\pi_{cc} = \pi_c = (\pi_{rd} + \pi_{pd} + \pi_{md}) + \beta_1 R_1^2 + \beta_2 R_2^2 + \delta (R_1 - R_2)^2 + Q_d \left(\frac{r_c - r_d}{r_d}\right)$$

From Equation (50), one may easily conclude that if $R_1$ and $R_2$ amount of discount are provided, respectively, on non-cooperative retail prices of retail as well as direct channel, then the system will be coordinated. It is also observed that $\pi_c \geq (\pi_{rd} + \pi_{pd} + \pi_{md})$, i.e. channel profit in centralised scenario is greater compared to channel profit in non-cooperative scenario under third-party collection mode. Again, $Q_c > Q_d = \beta_1 R_1 + \beta_2 R_2 > 0$, i.e. sales volume in centralised scenario is also greater compared to sales volume in non-cooperative scenario under third-party collection mode. From the analysis of this subsection, we propose the following proposition:

**Proposition 4**

(a) *If the centralised planner provides $R_1$ and $R_2$ amount of discount on retail prices of retail channel as well as direct channel in non-cooperative third-party collection mode, then the system will be coordinated.***

(b) *The channel profit and sales volume in centralised scenario is always greater compared to channel profit and sales volume in non-cooperative scenario under third-party collection mode.***

(c) *Retail price of the product in retail channel is always greater in non-cooperative scenario compared to centralised scenario ($p_{rd} > w_d > p_{rc}$).***

It follows from the Proposition 4, that, in non-cooperative system under third-party collection mode, channel profit and sales volume are less as compared to channel profit and sales volume in the centralised system and the result is well recognised in supply chain literature. It is also observed that the system will be coordinated if $R_1$ and $R_2$ amount of discounts are provided from the system, but conflict arises among the channel members that who will provide the $R_1$ and $R_2$ amount of discount on the prices of retail as well as direct channel. To reduce channel conflict and to obtain channel coordination, in the next section, we propose a coordination mechanism for non-cooperative third-party collection mode.

3. Manufacturer three-way discount mechanism for third-party collection mode

In the preceding section, it is established that, if $R_1$ and $R_2$ amount of discounts are provided on non-cooperative prices under third-party collection mode, then retail price of the product in the retail and the direct channel, respectively, reduces to $p_{rd} - R_1 = p_{rc}$ and $p_{md} - R_2 = p_{mc}$ and the non-cooperative channel converts into centralised channel. For this purpose, we have proposed a three-way discount mechanism. In this mechanism, the manufacturer provides $x$ amount of discount on wholesale price to entice the retailer to provide $R_1$ amount of discount to the end customer. The manufacturer exclusively provides $R_2$ amount of discount on the price of the direct channel to enhance the flow of products in direct channel. To encourage third party, the manufacturer provides $y$ amount of discount so that the third party pays the reward value $r_c$ to end consumer, and hence overall flow of the product. Our objective here is to verify whether such a mechanism has the potential to coordinate the supply chain and lead to a win–win situation for all the channel members. The profit functions of the retailer (πrc1), the third party (π3pc1) and the manufacturer (πmc1) under three-way discount mechanism can be expressed as follows:

$$\pi_{rc1} = (a_1 a - \beta_1 p_{rc} + \delta (p_{mc} - p_{rc}))(p_{rc} - w_d + x)$$

$$\pi_{3pc1} = (b + y - r_c) \left(1 - \frac{\gamma_0}{r_c}\right) (a_1 + a_2) a - \beta_1 p_{rc} - \beta_2 p_{mc})$$

$$\pi_{mc1} = (a_1 a - \beta_1 p_{rc} + \delta (p_{mc} - p_{rc}))(w_d - x) - c_m (a_1 + a_2) a - \beta_1 p_{rc} - \beta_2 p_{mc})$$

$$+ (a_2 a - \delta (p_{mc} - p_{rc}) - \beta_2 p_{mc}) p_{mc} + (c_m - c_r - (b + y)) (1 - \frac{\gamma_0}{r_c}) ((a_1 + a_2) a - \beta_1 p_{rc} - \beta_2 p_{mc})$$

From (51)–(53), it is clear that $\pi_{rc1} + \pi_{3pc1} + \pi_{mc1} = \pi_c$. Now, the win–win outcomes of the system will be achieved only when all the members of the supply chain achieve higher profit than what they achieve in non-cooperative scenario. For the win–win outcome of all the channel members, we must have $\pi_{rc1} \geq \pi_{rd}$, $\pi_{3pc1} \geq \pi_{3pd}$ and $\pi_{mc1} \geq \pi_{md}$. Simplifying the
inequalities, we have

\[
\begin{align*}
  h_1 x &\geq z_1 \quad (54) \\
  h_2 y &\geq z_2 \quad (55) \\
  c_{22} &\geq h_2 y + h_1 x \quad (56)
\end{align*}
\]

where \( h_1 = (a_1 a - b_1 p_{rc} + \delta (p_{mc} - p_{rc})) \); \( h_2 = (1 - \frac{yQ}{r_c})((a_1 + \alpha_2)a - b_1 p_{rc} - \beta_2 p_{mc}) \); \( z_1 = (p_{rd} - w_d)(a_1 a - b_1 p_{rd} + \delta (p_{md} - p_{rd})) - (p_{rd} - w_d)(a_1 a - b_1 p_{rc} + \delta (p_{mc} - p_{rc})) \); \( z_2 = (b - r_d) \left(1 - \frac{yQ}{r_d}\right)((a_1 + \alpha_2)a - b_1 p_{rd} - \beta_2 p_{md}) - (b - r_d) \left(1 - \frac{yQ}{r_d}\right)((a_1 + \alpha_2)a - b_1 p_{rc} - \beta_2 p_{mc}) \).

and \( c_{22} = w_d(b_1 R_1 + \delta (R_1 - R_2)) + p_{mc}(a_2 a - \beta_2 p_{mc} - \delta (p_{mc} - p_{rc})) - p_{md}(a_2 a - \beta_2 p_{md} - \delta (p_{md} - p_{rd})) - c_m(b_1 R_1 + b_2 R_2) + (c_m - c_r - b) \left(1 - \frac{yQ}{r_c}\right)((a_1 + \alpha_2)a - b_1 p_{rc} - \beta_2 p_{mc}) - (c_m - c_r - b) \left(1 - \frac{yQ}{r_c}\right)((a_1 + \alpha_2)a - b_1 p_{rd} - \beta_2 p_{md}) \).

Note that the above set of constraints will represent the feasible region if \( c_{22} > z_1 \) and \( c_{22} > z_2 \). On simplification, we have

\[
\begin{align*}
  c_{22} - z_2 &= \frac{R_1}{2} \left((a_1 a - b_1 c_m) + b_1 (c_m - c_r - r_c) \left(1 - \frac{yQ}{r_c}\right) + \frac{\beta_1 R_1 + \beta_2 R_2}{2} (b - r_c) \left(1 - \frac{yQ}{r_c}\right) + Q_d \left(\frac{r_c - r_d}{r_d}\right) > 0 \right) \\
  c_{22} - z_1 &= \beta_1 R_1 \left(\frac{(a_1 a - b_1 c_m) + b_1 (c_m - c_r - r_c) \left(1 - \frac{yQ}{r_c}\right) + 2r_c Q_d \left(\frac{r_c - r_d}{r_d}\right)}{4(\beta_1 + \delta)} \right) + Q_d \left(\frac{r_c - r_d}{2r_c r_d}\right) + Q_d \left(\frac{r_c - r_d}{2r_c r_d}\right) > 0
\end{align*}
\]

From here, one can infer that the proposed three-way discount mechanism coordinates the system always. The feasible region generated by the above three constraints are given in Figure 2.

One can easily observe from Figure 2 that the extreme points of the feasible region are \( A \left(\frac{Z_1}{h_1}, \frac{Z_2}{h_2}\right) \); \( B \left(\frac{c_{22} - Z_2}{h_1}, \frac{Z_2}{h_2}\right) \); and \( C \left(\frac{Z_1}{h_1}, \frac{c_{22} - Z_1}{h_2}\right) \), respectively. Thus, any value of the discounts, \( x \) and \( y \), satisfying the above region coordinates the channel and provides a win–win situation. Looking at the profit structure of the retailer, one can easily observe that as \( x \) increases, the profit of the retailer increases, i.e. the profit gain of the retailer will be maximum when \( x \) is maximum and the profit of the retailer will be maximum at the point B; similarly, the profit of the manufacturer will be maximum when \( x \) and \( y \) will be minimum, i.e. at the point A the manufacturer achieves maximum profit. Similarly, the profit of third party will be maximum at the point C. Now to check the flexibility of the proposed mechanism, we compute the maximum profit gain of each channel member under proposed coordination mechanism compared to their respective non-cooperative profit. One may easily observe that maximum profit gaining opportunity (MPGO) of each channel member becomes, \( \pi_{rc_{max}} \) \( - \pi_{rd} = \pi_{3pc_{max}} \) \( - \pi_{pmd} = \pi_{mc_{max}} \) \( - \pi_{md} = \beta_1 R_1^2 + \beta_2 R_2^2 + \delta (R_1 - R_2)^2 + Q_d \left(\frac{r_c - r_d}{r_d}\right)^2 \), i.e. each channel member has equal opportunity to gain extra profit where \( \pi_{rc_{max}}, \pi_{3pc_{max}}, \pi_{mc_{max}} \) represent maximum profit of the retailer, third party and the manufacturer under coordinated scenario obtained at the extreme point B, C and A, respectively. From the above discussion, we propose the following proposition:
Table 1. Optimal prices, collection price, reward value, $R_1$, $R_2$, profits and sales volume under various settings.

<table>
<thead>
<tr>
<th></th>
<th>$p_r$</th>
<th>$p_m$</th>
<th>$w$</th>
<th>$r$</th>
<th>$\beta$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$\pi_r$</th>
<th>$\pi_{3p}$</th>
<th>$\pi_m$</th>
<th>$\pi_c$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3PC</td>
<td>166.83</td>
<td>176.34</td>
<td>135.01</td>
<td>5.54</td>
<td>15.39</td>
<td>–</td>
<td>–</td>
<td>536.68</td>
<td>206.04</td>
<td>3899.64</td>
<td>4696.38</td>
<td>41.31</td>
</tr>
<tr>
<td>MC</td>
<td>163.29</td>
<td>169.63</td>
<td>128.31</td>
<td>11.83</td>
<td>–</td>
<td>–</td>
<td>648.68</td>
<td>–</td>
<td>4474.61</td>
<td>5123.29</td>
<td>44.42</td>
<td></td>
</tr>
<tr>
<td>RC</td>
<td>166.83</td>
<td>176.34</td>
<td>135.02</td>
<td>5.54</td>
<td>15.39</td>
<td>–</td>
<td>–</td>
<td>742.73</td>
<td>–</td>
<td>3899.64</td>
<td>4696.38</td>
<td>41.31</td>
</tr>
<tr>
<td>C</td>
<td>128.31</td>
<td>169.63</td>
<td>–</td>
<td>11.83</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5771.98</td>
<td>61.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. The graphical representation of the manufacturer (blue), the third party (green) and the retailer (orange) profit under cooperative scenario (see http://dx.doi.org/10.1080/00207543.2015.1090031 for color version).

Proposition 5

- Any arbitrary values of discount sharing fractions satisfying (54)–(56) coordinates the system perfectly and leads to acceptable outcomes for all the channel members.

We now illustrate the above analytical findings numerically. The following parameters are used for illustration throughout the paper: $\alpha_1 = 0.6$, $\alpha_2 = 0.3$, $a = 200$, $\beta_1 = 0.5$, $\beta_2 = 0.3$, $c_m = 100$, $c_r = 30$, $\delta = .03$, $r_0 = 10$ and $\gamma = 0.5$. The corresponding optimal solutions are given in Table 1.

In Table 1, 3PC, MC and RC represent the third-party, manufacturer and retailer collection mode under non-cooperative scenario, and C and CR, respectively, represent centralised and centralised discount model. The above numerical results justify all the analytical findings as well as propositions. Channel profit obtained in all non-cooperative scenarios is less compared to centralised scenario. It is also observed that decision variables obtained in third-party collection mode and the retailer collection mode are identical. One may also observe that if $R_1$ and $R_2$ amount of discounts are provided from the system on retail prices under non-cooperative third-party collection mode, then the system remains coordinated. For the above data, the graphical representation of profit function of the manufacturer, the third party and the retailer under three-way discount are shown in Figure 3.

From Figure 3, we can easily observe that as $x$ increases, the profit of the retailer also increases (orange colour). Again, as $y$ increases, the profit of the third party (green colour) also increases. But, the profit of the manufacturer (blue colour) decreases as both $x$ and $y$ increase. The profit structures obtained in Equations (51)–(53) also justify the analytical findings. The results of sensitivity analysis of the profit functions of each member under non-cooperative scenario under third-party collection mode, the centralised profit and MPGO under proposed coordination mechanism is given in Table 2.

From Table 2, it can be observed that as $a$ increases, profit of each individual member as well as centralised profit and MPGO also increases. Since $a$ is the market potential, as $a$ increases, demand of the product increases and as a consequence, overall profit increases. Similarly, as $\beta_1$ and $\beta_2$ increase, the profit of each channel member as well as MPGO decreases. Since $\beta_1$ and $\beta_2$ are the price sensitive parameters, as $\beta_1$ and $\beta_2$ increase, demand of the product decreases, and as a consequence,
Table 2. Sensitivity analysis of the profit functions of the manufacturer, the third party and the retailer under non-cooperative third-party collection model, centralised model and MPGO.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\delta$</th>
<th>$c_m$</th>
<th>$c_r$</th>
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<tbody>
<tr>
<td>$\pi_{rd}$</td>
<td>↑</td>
<td>↓</td>
<td>↔</td>
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<td>↓</td>
<td>↓</td>
<td>↑</td>
</tr>
<tr>
<td>$\pi_{3p}$</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
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<td>↓</td>
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<tr>
<td>$\pi_{md}$</td>
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<td>↓</td>
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<td>↓</td>
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<tr>
<td>$\pi_c$</td>
<td>↑</td>
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<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
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<tr>
<td>MPGO</td>
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<td>↓</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
</tr>
</tbody>
</table>

Notes: ↑↑ Highly Sensitive (>30% change); ↑ moderately sensitive (10%–30% change) ↓↓ Low sensitive (<10% change); ↔ Constant

profit also decreases. Note that as profit function of the retailer is independent of $\beta_2$, so profit of the retailer remains unchanged with respect to $\beta_2$. Again, as $\delta$ increases, profit of the manufacturer increases but profit of the retailer decreases. Further as $c_r$ increases, overall remanufacturing cost increases and profit of every member, channel profit and MPGO decreases. Similarly, as $r_0$ increases, reservation price of the customer increases and as a result, profit of every member decreases.

Although, we have explored the characteristics of the proposed three-way coordination mechanism for third-party collection mode, it can also be applied to evaluate the coordination for other two collection mode. Similar to third-party collection mode, by evaluating the values of discount on selling price of direct and retail channel on retailer and manufacturer collection mode, one may simply verify the supremacy of the proposed mechanism. But the value of MPGO corresponding to the manufacturer collection mode will be less as we observed numerically as well as analytically that the channel profit in non-cooperative manufacturer collection mode is higher compared to third-party mode. Since MPGO in manufacturer-direct collection mode is less compared to third-party collection mode, the retailer with higher bargaining power may also want to participate in third-party collection mode instead of direct collection mode of the manufacturer.

3.1 Managerial implications

In this paper, we have discussed a reward-driven acquisition policy employed to secure used products for remanufacturing in a CLSC. The analytical derivations of this paper reveal the three different collection modes influence important decisions, such as those regarding product and transfer prices as well as the reward value for returning used products and profits. One can observe that in a non-cooperative environment, neither the retailer nor the manufacturer benefit from delegating the task of used product collection to a third party. As a consequence, the results of this paper serve as a reference for those looking to employ reward-driven recycling method. In addition, we found that if customers are sensitive to reward value then the manufacturer should conduct recycling instead of giving the task to a retailer or third party. Numerical results indicate that as $\delta$ increases, the profit of the manufacturer increases but the profit of the retailer decreases. In this case, $\delta$ represents the number of customer switches due to the differences between the price in direct and retail channels. The finding reveals that the manufacturer can and will use inconsistent and differentiated prices to maximise channel profit share. The coordination mechanism analysed in this study is easy to implement and will likely fit the needs of many different industries.

4. Summary and concluding remarks

This study was developed in light of three major areas in the supply chain literature, namely reward-driven profitable remanufacturing policies, CLSC in dual-channel environments and contract-based mechanisms for supply chain coordination. To explore the characteristics of remanufacturing, we considered a supply chain structure consisting of, a manufacturer, a third-party collector and a retailer. We further assumed that the manufacturer adopts direct and retail sales to enhance the sale of the product. We also analysed characteristics of three modes of collection in non-cooperative environments and corresponding centralised model and we observed that the remanufacturing rate is maximised when the used product is
procured directly from the manufacturer. To promote collaboration between the channel members, a three-way coordination mechanism was proposed that not only coordinates the non-cooperative channel in which third-party collectors are used, but also provides win–win outcomes for all the channel members. This study makes two key contributions to the literature. First, observations of coordination of CLSC members in dual-channel supply chain are shown. Second, in contrast to existing literature, the characteristics of CLSCs under reward-driven return policies are analysed. We believe that insights drawn from the analytical as well as numerical results will be useful for managers who set the parameters for a reward-driven return policy.

The model presented in this paper has limitations. For example, we assumed that the return rate depends on reward value; we advocated an investigation of model that features return rate as a function of both sales and reward value. Another interesting study would examine a model in which all the members are assumed capable of collecting used products for remanufacture. Finally, we assumed that reward value of the used product is uniform irrespective of product condition. In the future, researchers may want to determine the effect of the model when product condition affects reward value as well as look at the impact of recycling costs.

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