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Channel coordination for multi-stage supply chains with revenue-sharing contracts under budget constraints

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Real-life situations show that revenue-sharing (RS) contracts used in multi-stage supply chains have more complex structures than those that have been studied in recent research. In this paper, we study RS contracts in multi-stage supply chains where some members work with more than one upstream member. This general supply chain structure closely resembles those in actual practice under RS contracts. The literature on supply chain contracts has not adequately addressed contract design for supply chains with members who face budget constraints. We show that the RS contract could fail to coordinate supply chains when members are under particular budget constraints. In response, we propose a revenue-sharing with budget constraints (RSB) contract that adds no administrative cost. A properly designed RSB contract can be used to achieve supply chain coordination and to arbitrarily allocate profits in multi-stage supply chains. Our numerical results provide insights into ways supply chain coordination can be achieved under budget constraints through the RSB contract.

Keywords: revenue sharing; coordination; multi-stage supply chain; budget constraints

1. Introduction

Supply chains are playing an increasingly important role in the modern marketplace. They employ concepts not only used in logistics management but also in strategic-level collaboration. In the traditional market setting, the retailer sends an order to the supplier and wholesale payment per unit. The assumption of ‘deep pockets’, under which each supply chain member has an infinite budget, is a common premise used in recent studies of supply chain contracts. It forms the basis of the well-known practice undertaken in a traditional market in which, because of double marginalisation, retailers place fewer orders for newsvendor-type products than the optimal quantity accommodated by an integrated system (Spengler 1950).

Members use mechanisms to achieve supply chain coordination even as each member attempts to maximise profits. For instance, they engage in a contract if it can simultaneously improve the profit of each member who signs it. A higher total profit for the supply chain means that each member is likely to obtain a higher profit. The revenue-sharing (RS) contract is a popular means used to improve profits in a two-stage supply chain. Under it, the supplier charges the retailer a low wholesale price for each unit purchased and the retailer shares the after-sales revenue with the supplier.

This study differs from previous work on supply chain coordination in two regards. First, we study it a general structure. Most products are made up of different components, and members of a supply chain typically manage these various parts and composition materials. In these cases, the supply chain looks like a tree that we refer to as a *multi-stage supply chain*. We observe that in multi-stage supply chains, the order quantity is decided by the retailer, who faces an uncertain demand. When the retailer decides the order quantity based on demand and cost, the other members accept it. Only the coordination of the retailer and the adjacent upstream supplier apparently must be achieved such that the other members accept the order quantity. However, we show that the total profit of the supply chain cannot be maximised unless all of the members sign a coordinating contract. It is more difficult to coordinate these multi-stage supply chains because any member could create a bottleneck. For example, from August to December 2011, customers encountered much higher prices for computers or hard discs because of the major floods during the 2011 monsoon season in Thailand where some digital production plants are located.

Real-life situations show that RS contracts have been used in multi-stage supply chains. In the construction industries in China, for example, RS contracts have been implemented to coordinate multi-echelon supply chains. In such a

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supply chain, a single real estate developer acts as the sole retailer and purchases the construction service from one building company, who needs to buy or rent building equipment, such as excavators and cranes, from the distributor. The distributor purchases or rents the equipment from suppliers. Under an RS contract, the equipment suppliers, the distributor and the building company charge low prices for their products. Then, the retailer shares the after-sales revenue with all the contract members.

In some construction supply chains under RS contracts, the distributor acts as a third party who organises the supply chain members to negotiate RS agreements even though some suppliers directly provide equipment for the building company. However, others may spearhead the RS contract agreements. For example, in the video rental industry, a third-party firm can join the supply chain and help to negotiate the RS contracts of small rental firms and retailers (Mortimer 2002).

Another example of RS contract application can be found in the alloy supply chains in China. Under the RS contract, one distributor purchases several types of materials from different mining companies and supplies them to the alloy manufacturer who subsequently shares revenue with the distributor and the mining companies who had signed the RS contact. These structures are more complex than those of serial supply chains. Recently, more retailers and manufacturers have paid attention to upstream suppliers. Some companies directly specify preferred suppliers. For example, in the Aston Martin supply chain, the automobile manufacturer purchases some parts while requiring that key materials come from specified material suppliers. In some construction supply chains, the real estate developers specify the cement supplier for the building company. This practice guarantees good quality materials and close business relationships. As a result of close interactions, retailers can easily share revenues with other supply chain members in the multi-stage supply chain. However, van der Rhee et al. (2010) noted that contracts created between pairs of adjacent members create major drawbacks from an implementation standpoint. Thus, a single contract covering all of the members simultaneously may be more advantageous. We discuss an RS contract that can coordinate multi-stage supply chains and arbitrarily allocate total profit.

Second, we consider the budget constraints of firms in the supply chain. In a realistic setting, budget constraints could be one reason that members order fewer products than the channel-wide optimal-order quantity. For example, if the wholesale price charged by the supplier is \$2 and the budget of the retailer is \$80, then the retailer can purchase up to 40 units of the product from the supplier. However, if the channel-wide optimal-order quantity is more than 40 units, the supply chain will lose the opportunity to obtain maximum profit. In some industries, budget constraints may create more common obstacles than double marginalisation and be particularly important to firms because of current financial crises.

In previous work, researchers have shown that some supply chain firms contract with members of the financial market so they can borrow money. If the members are credit-worthy borrowers and if the negotiation between the supply chain and financial market firms is not costly, then such an arrangement creates a benefit to signatories. In some cases, however, it may not be feasible or efficient to involve the financial market, especially for those in multi-stage supply chains. First, supply chain members may fail to raise a loan; even some powerful supply chain members experience this problem. For example, JL is a distributor in electronic product supply chains in China. Its annual revenue is approximately \$100,000,000, but it failed to raise a loan due to an insufficient fixed assets-to-mortgage ratio, which is an important evaluation indicator for Chinese banks, nor did it have adequate cash flow to satisfy the banks. Therefore, the manager of JL must make decisions under an absolute budget constraint. See also Caldentey and Haugh (2009) for reasons that a company may face an absolute budget constraint.

Second, the financial market benefits from setting the interest rate based on both the default risk of the members and the expected loss of member default (Dada and Hu 2008; Lee and Rhee 2010). Consequently, all members must agree on the credit ratings assigned to the supplier and retailers by the financial market members. The negotiation process can be time consuming and costly in multi-stage supply chains.

Third, when supply chain contracts are used (e.g. RS contracts), the financial institution must monitor the supply chain's information flow. In some cases, it may need to sign the contracts, leading to higher administrative costs.

Therefore, we need to address the following two issues: (i) conditions under which current contracts are feasible under budget constraints and (ii) a new contract that replaces those that are infeasible under budget constraints. To our knowledge, few studies have focused on these two realistic problems. Based on these considerations, we study the supply coordination of multi-stage supply chains with budget-constrained members to whom the financial market is unavailable. All of the members in a supply chain require a budget to maintain their operations under RS contracts. Therefore, if some of the members fail to secure adequate budgets to maintain their operations under an RS contract, the RS contract is no longer feasible. This phenomenon is demonstrated by the analytical and numerical results of our study. Hence, we propose a new contract – based on the RS type – that addresses budget constraints: (RSB) contract. The RSB contract can achieve supply chain coordination and arbitrarily allocate the total profit under certain budget constraints for cases in which the RS contract is infeasible.

This paper is organised as follows. The next section shows a literature survey on supply chain contracts and coordination with financial market inclusion. Section 3 proposes an RS contract for a multi-stage supply chain. In Section 4, an RSB contract is described in reference to a multi-stage supply chain. A numerical example is discussed in Section 5 and our conclusions are presented in Section 6.

2. Literature review

Many contracts have been studied for supply chain management: buy-back (BB) (Pasternack 1985), quantity–flexibility (QF) (Tsay 1999) and sales–rebate contracts (Taylor 2002). An RS contract is a popular means of coordinating a supply chain. Cachon and Lariviere (2005) proposed and discussed the limitation of an RS contract for a two-stage supply chain that consists of a retailer and supplier. Their study showed that when retailers compete on price and quantity the RS contract is not feasible. Giannoccaro and Pontrandolfo (2004) proposed two objectives for RS contracts: *effectiveness,* which ensures channel coordination, and *desirability,* which ensures that all members obtain higher profits than they would in the traditional market setting.

van der Veen and Venugopal (2005) showed that RS contracts can optimise the video rental supply chain and bring win–win situations to the members. Chauhan and Proth (2005) studied the supply chain partnership under a profit-sharing mechanism. Yao, Leung, and Lai (2008) analysed the performance of the manufacturer in RS contacts under retailer competition. Giannoccaro and Pontrandolfo (2009) proposed RS contract negotiation among those in a two-stage supply chain. Li, Zhu, and Huang (2009) discussed a Nash bargaining model featuring a revenue sharing mechanism within a consignment contract. Hou, Zeng, and Zhao (2009) proposed an extended RS contract by incorporating inventory and lead times. Linh and Hong (2009) studied the RS contract in a two-period newsvendor problem with two buying opportunities. Chan and Chan (2010) conducted a review of coordination studies in the context of supply chain dynamics and pointed out that the RS contract offers an important means to achieve supply chain coordination. Xiao, Yang, and Shen (2011) proposed an RS contract with a quality assurance policy to coordinate a two-stage supply chain.

Kunter (2012) developed a cost-and-revenue sharing contract in a two-stage supply chain in which both the retailer and the manufacturer can simultaneously affect the demand via marketing efforts. Under this contract, the retailer's revenue is shared with the manufacturer based on a revenue-sharing rate. The non-price marketing effort of the retailer (manufacturer) is shared with the manufacturer (retailer) based on a marketing-effort participation rate. Pan et al. (2010) compared the wholesale price contract with the RS contract under a different supply chain channel. They showed that an RS contract is more beneficial for manufacturers under certain conditions. Some composite contracts have also been developed. Xiong, Chen, and Xie (2011) proposed a composite contract based on BB and QF contracts; they showed it was more flexible in terms of profit and risk allocation. Chen (2011) studied returns with a wholesale-price-discount contract in a newsvendor problem. Chen and Bell (2011) proposed a new BB policy that accounts for customer returns.

Most of the previously described models address the problem of coordination in a supply chain with two members: a retailer and supplier. Some researchers have explored the coordination of supply chains with more complex structures. Guardiola, Meca, and Timmer (2007) studied the cooperation and profit allocation in a two-echelon supply chain with a single supplier and multiple retailers. Huang, Chen, and Lin (2013) studied the same type of supply chains by considering capacity allocation. Ding and Chen (2008) proposed flexible return policies to coordinate a three-level supply chain. van der Rhee et al. (2010) extended the RS model to multi-stage supply chains; that is, they looked at those with more than two stages and in which each member manages the same product. Feng, Moon, and Ryu (2014) proposed an RS contract by considering the reliability of supply chain members for a multi-stage supply chain.

Budget constraints commonly influence supply chain coordination. Dada and Hu (2008) studied budget-constrained newsvendors who could borrow funds from a bank. Caldentey and Haugh (2009) studied three finance contracts stipulating that a budget-constrained retailer could hedge the budget constraint by trading in the financial market. Chen and Cai (2011) analysed a supply chain model with a supplier, budget-constrained retailer, bank and third-party logistics firm. In that model, the retailer obtained trade credit from either a bank or third-party logistics firm. In these studies, double marginalisation is not addressed. Lee and Rhee (2010) studied the performance of RS and BB contracts for coordinating supply chains with a budget-constrained retailer and a supplier who can acquire loans from the financial market firms that sign the contract. In contrast to the above studies, we developed our supply chain model under absolute budget constraints; that is, we only consider instances in which the firm does not have access to the financial market to ease budget constraints.

3. A revenue-sharing contract in a multi-stage supply chain

We propose an RS contract in a multi-stage supply chain with N risk-neutral members. One newsvendor-type final product is produced and sold by this supply chain. Different suppliers provide components or materials to the downstream members. As the number of supply chain participants increases, the potential that a member could create a bottleneck, such that the retail-order quantity is less than the channel-wide optimal quantity, also increases. In Figure 1, we show a branching tree to describe this type of multi-stage supply chain.

In this type of tree, each node represents a company in the supply chain. The nodes to the left represent the downstream companies, such as the retailer and manufacturer. The lines between each pair of nodes represent the corresponding material and fund flows. Moreover, no node has more than one direct predecessor, and the nodes without any successor are called *terminal members*. The node without any predecessor is called the *starting member*. Without loss of generality, we refer to the starting member as the retailer (Member 1). Figure 1 shows five members in the supply chain. Member 1 is the retailer, Member 2 is the manufacturer and Members 3–5 are the suppliers. The following assumptions and notation were used to develop the RS contract in a multi-stage supply chain:

(1) There is only one retailer in the supply chain.
(2) The retail price is determined by the market and is fixed for one selling season.
(3) Each supply chain member has a constant per-unit cost of production.

Decision variables

q_1	retailer's order quantity
q_j	member j's production quantity, $j = 2, 3, \ldots, N$
λ_i	proportion of member i's expected profit in the expected supply chain profit, defined over the range of [0,1] and $\sum_{i=1}^{N} \lambda_i = 1$, $i = 1, 2, \ldots, N$

Contract parameters

ω_j	wholesale price that member j charges per unit, $j = 2, 3, \ldots, N$
Φ_i	member i's total share of the retailer's revenue generated from each unit, $i = 1, 2, \ldots, N$

Parameters

p	retail price of the final product
s	salvage value of the final product ($s < p$)
$R(q)$	expected total revenue of the retailer
X	random variable representing the market demand
$F(x)$	cumulative probability function of X
C_i	unit production cost of member i, $i = 1, 2, \ldots, N$
C	$= C_1 + C_2 + \cdots + C_N$
S_i	the set that contains member i and all of upstream members, $i = 1, 2, \ldots, N$
S_i^d	the set that contains the direct upstream members of member i, $i = 1,2, \ldots ,N$
S_i^u	the set that contains the upstream members of member i, $i = 1,2, \ldots ,N$
T	the set that contains all of the terminal members

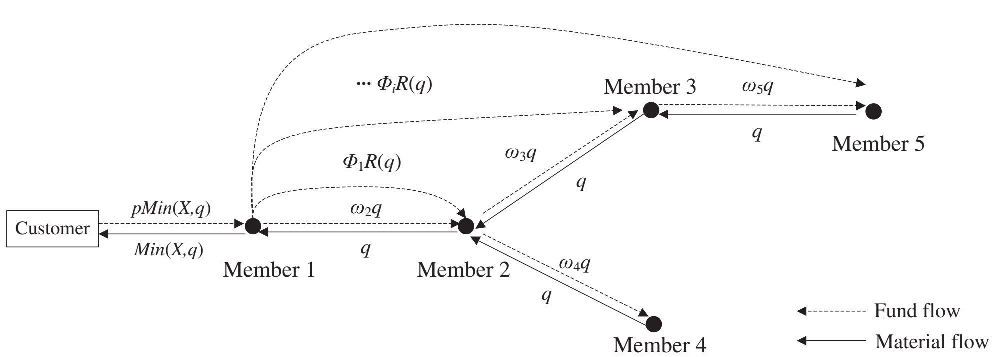

Figure 1. A branching tree for a multi-stage supply chain.

$\Pi_i(\Phi_i,q)$	expected profit of member i, $i = 1,2,\ldots,N$
$\Pi_{SC}(q)$	expected total profit of the supply chain
Π_{M_i}	expected profit of member i under the traditional market setting, $i = 1,2,\ldots,N$
Π_{M_SC}	expected total profit of the supply chain under the traditional market setting
q	quantity of the final product that is produced by the supply chain
q^*	channel-wide optimal quantity of the final product
q_1^M	retailer's optimal order quantity under the traditional market setting
q^t	quantity of the final product produced by the supply chain that is smaller than q^*
Δq	$= q^* - q^t$
Y_i	budget of member i, $i = 1,2,\ldots,N$
Y_i^t	budget of member i that is smaller than Y_i, $i = 1,2,\ldots,N$
$G_i()$	required budget of member i, $i = 1,2,\ldots,N$
$\omega_j^{\min}$	lower bound on the wholesale price that member j charges per unit, $j = 2,3,\ldots,N$
$\omega_j^{\max}$	upper bound on the wholesale price that member j charges per unit, $j = 2,3,\ldots,N$

The retailer faces an uncertain demand and must send the order(s) to the upstream member(s) before the selling season begins. Each member pays the immediate upstream member (member j) ω_j for each unit purchased, $j = 2,3,\ldots,N$. Without loss of generality, we let ω_1 be zero. Member i decides the value of q_i to maximise profit. Obviously, $q_j \le q_i$ wherein member j is directly upstream from member i, $i = 1,2,\ldots,N$ and $i \notin T$. Therefore, $q = \min(q_1,q_2,\ldots,q_N)$. We refer to the traditional market setting as the decentralised setting without any supply chain contract (see Giannoccaro and Pontrandolfo 2004). Under the traditional market setting, the retailer decides q_1 based on the demand, C_1, and $\sum_{i\in S_1^d}\omega_i$, and thus, $q = q_1 = \cdots = q_N$. Using the same approach as van der Rhee et al. (2010), we show that q^*, which maximises the expected supply chain profit, satisfies

$$(p-s)F(q^*) = p - C \tag{1}$$

As shown in Equation (1), q^* leads to the greatest expected supply chain profit; it is equal to the one achieved under the centralised supply chain setting. The retailer's optimal-order quantity under the traditional market setting satisfies

$$(p-s)F(q_1^M) = p - C_1 - \sum_{j\in S_1^d}\omega_j \tag{2}$$

Under an RS contract, member i obtains ω_i per unit from the direct downstream member. At the end of the selling season, the salvage revenue obtained by the retailer is also shared. Member i shares Φ_i of the retailer's revenue and $\sum_1^N \Phi_i = 1$. We obtain the expected profit function of member i under the RS contract as follows ($i = 1,2,\ldots,N$):

If member i is a terminal member,

$$\Pi_i(\Phi_i,q) = \Phi_i R(q) + \omega_i q - C_i q. \tag{3}$$

If member i is not a terminal member,

$$\Pi_i(\Phi_i,q) = \Phi_i R(q) + \omega_i q - C_i q - \sum_{j\in S_i^d}\omega_j q. \tag{4}$$

In Equation (4), member j is the direct upstream member of member i. The expected total profit function of the supply chain is

$$\Pi_{SC}(q) = R(q) - Cq = R(q) - \sum_{i=1}^{N} C_i q. \tag{5}$$

As per Cachon (2003), we define a coordinated multi-stage supply chain as one in which no supply chain member can experience a profitable unilateral deviation from the set of channel-wide optimal actions. Our definition implies that if the supply chain is coordinated, q^* can be accepted by all of the members and $\Pi_{SC}(q^*)$ is achieved. We say that a contract is considered *coordinating* if supply chain coordination can be achieved when all of the members sign the contract. The flexibility of a coordinating contract depends on the range of profit allocation that it can achieve.

Property 1. *The expected total profit of the supply chain is maximised when all of the members sign a coordinating contract.*

All proofs are located in the Appendix. From Property 1, we see that the expected supply chain profit can be maximised only when all of the members sign a coordinating contract. As discussed in the literature review, it is more appropriate to impose a contract that covers all of the members than it is to employ contracts between every pair of adjacent supply chain members. Φ_i and ω_j are parameters of RS contracts. Revenue sharing is not efficient in supply chain coordination under every choice of Φ_i and ω_j. Therefore, we aim to find the values of Φ_i and ω_j that can be used to coordinate the supply chain, $i = 1,2,\ldots,N$, and $j = 2,3,\ldots,N$. We develop an RS contract in which all members simultaneously agree such that the multi-stage supply chain is coordinated through the use of $\Pi_i(\Phi_i,q)$ as an affine transformation of $\Pi_{SC}(q)$.

Theorem 1. *For any profit allocation scenario, $(\lambda_1,\lambda_2,\ldots,\lambda_N)$, consider the set of RS contracts with*

$$\omega_j = \sum_{k\in S_j} C_k - \sum_{k\in S_j} \Phi_k C \tag{6}$$

and $\Phi_i = \lambda_i$, $i = 1,2,\ldots,N$, and $j = 2,3,\ldots,N$. Under these contracts, the expected profit function of member i is

$$\Pi_i(\Phi_i,q) = \Phi_i\Pi_{SC}(q) = \lambda_i\Pi_{SC}(q) \tag{7}$$

In this case, member i can obtain λi of the total profit of the supply chain, and all of the members are willing to choose the quantity that maximises the expected total profit of the supply chain.

Under such a properly designed RS contract, with a predetermined λ_i, member i can obtain the maximum profit when the expected supply chain profit is maximised. Hence, all of the members are willing to accept q^*, which maximises $\Pi_{SC}(q)$. Moreover, Φ_i is equal to λ_i under the RS contract. Therefore, the expected total profit of the supply chain can be arbitrarily allocated under the RS contract if members set Φ_i, $i = 1,2,\ldots,N$. Supply chain members can negotiate $(\lambda_1,\lambda_2,\ldots,\lambda_N)$ and as a result, the profit allocation depends on supply chain members' relative bargaining power. Note that Property 1 and Theorem 1 are special cases of Lemma 1 and Theorem 1 in Feng, Moon, and Ryu (2014), which are based on an assumption that all products are perfect.

To achieve supply chain coordination, an RS contract should improve the profits of all members; a member who cannot earn a higher profit under the RS contract will not sign it. As observed in Theorem 1, the expected profit of member i depends on Φ_i; therefore, it is important for the supply chain to properly set Φ_i. However, this selection could entail significant time and cost in terms of bargaining among supply chain members, creating a limitation of RS contract usefulness. van der Rhee et al. (2010) discussed two easy methods to determine Φ_i ($i = 1,2,\ldots N$) to ensure that all of the supply chain members can obtain higher profits under the RS contract than without it in a traditional market setting. We refer to them as Scenarios Ra and Ab.

Scenario Ra.

In Scenario Ra, member i's expected profit is shown in Equation (8); this feasible method of determining Φ_i ensures the desirability of the RS contract (Giannoccaro and Pontrandolfo 2004). In this scenario, the rates of increase in member and overall supply chain profits will be identical. If Π_{M_i} and Π_{M_SC} are the expected profit of member i and the expected total profit of the supply chain under the traditional market setting ($i = 1,2,\ldots,N$), respectively, and $\Phi_i = \Pi_{M\_i}/\Pi_{M\_SC}$, then, the expected profit of member i under the RS contract is

$$\Pi_i(\Phi_i,q) = \Phi_i\Pi_{SC}(q) = \frac{\Pi_{M\_i}}{\Pi_{M\_SC}}\Pi_{SC}(q). \tag{8}$$

The rate of increase in the expected profit of the supply chain is $\frac{\Pi_{SC}(q)-\Pi_{M\_SC}}{\Pi_{M\_SC}}$. The rate at which the expected profit of member i increases is $\frac{\Phi_i\Pi_{SC}(q)-\Pi_{M\_i}}{\Pi_{M\_i}}$. Because $\Phi_i = \frac{\Pi_{M\_i}}{\Pi_{M\_SC}}$, we obtain $\frac{\Phi_i\Pi_{SC}(q)-\Pi_{M\_i}}{\Pi_{M\_i}} = \frac{\Pi_{SC}(q)-\Pi_{M\_SC}}{\Pi_{M\_SC}}$, $i = 1,2,\ldots,N$.

It is easy to show $\Pi_{SC}(q^*) > \Pi_{M\_SC}$. From Theorem 1, we can confirm that under the RS contract, the expected profit of member i is $\Pi_i(\Phi_i,q^*)$. Considering (8), we can derive that $\Pi_i(\Phi_i,q^*) > \Pi_{M\_i}$. Therefore, all of the members can obtain higher profits under the RS contract than they could in a traditional market setting.

Scenario Ab.

If $\Phi_i = \left(\frac{\Pi_{SC}(q)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}\right)/\Pi_{SC}(q)$, then, the expected profit of member i under the RS contract is

$$\Pi_i(\Phi_i, q) = \Phi_i \Pi_{SC}(q) = \frac{\Pi_{SC}(q) - \Pi_{M\_SC}}{N} + \Pi_{M\_i}. \tag{9}$$

The absolute increase in the profit of the supply chain is $\Pi_{SC}(q) - \Pi_{M\_SC}$. The absolute increase in the profit of member i is $\Phi_i \Pi_{SC}(q) - \Pi_{M\_i} = \frac{\Pi_{SC}(q) - \Pi_{M\_SC}}{N}$. Therefore, we can infer that all of the members obtain the same absolute increase in their profits. By the same approach used in Scenario Ra, we can find that $\Pi_i(\Phi_i, q^*) > \Pi_{M\_i}$, $i = 1,2, \ldots, N$; that is, Scenario Ab ensures the desirability of the RS contract.

In both of the above scenarios, each member obtains higher profits under the RS contract than under the traditional market setting. These options are more easily accepted by a supply chain that lacks a bargaining mechanism. The main administrative cost of running an RS contract is supply chain members' costs of monitoring the retailer's revenue (Cachon and Lariviere 2005). As discussed in many studies, the development of information technology helps supply chain members to acquire complete information (e.g. see Cachon and Lariviere 2005; Kunter 2012). Therefore, we can infer that the administrative cost of running an RS contract in an N-stage supply chain will be decreased in the future due to the development of information technology.

Cash flow is among a company's main concerns, especially during a financial crisis. In many cases, managing the cash flow is a more pressing issue than making a profit. Theorem 1 shows that, under the RS contract, the wholesale price charged by member i is smaller than the unit cost when $\Phi_i > 0$, $i = 2,3, \ldots, N$. This price-cost relationship means that the direct income from sales is reduced for the chain members. The lower wholesale price is compensated by a lower purchasing cost and a share of the retailer revenue, but the revenue comes only after the retailer's sale (i.e. it is postponed). Because of the uncertain demand and retailer risk of bankruptcy, supply chain members face uncertainty, and experience more pressure and cash flow risk. Therefore, companies impose budget constraints on managers to maintain cash flow safety. This phenomenon is also explained by (see Caldentey and Haugh 2009), i.e. some companies restrict their managers by setting a budget constraint to ensure the safety of cash flow.

4. A revenue-sharing contract that addresses member budget constraints

Under the RS contract, the supply chain is coordinated and Φ_i and ω_i are optimised to improve the profit of each member. Furthermore, the supply chain can arbitrarily divide profit by setting Φ_i, $i = 1,2, \ldots, N$ (such that the shares add up to one). However, each member in the supply chain needs a budget to maintain its operation, even under RS contracts. If some members fail to secure an adequate budget to maintain their operations, then the RS contract is no longer feasible. In this section, we show that the RS contract presented in Section 2 may not be feasible when some members face a budget constraint. Therefore, we propose using the RSB contract in multi-stage supply chains so that the budget constraints of the supply chain can be considered.

Under RS contracts, required budgets of supply chain members vary. For example, in a selling season, each member may rely solely on funds from the payments of downstream members and the revenue-sharing mechanism, even while each pays production and purchasing costs. Furthermore, the supply chain cannot be coordinated under an extremely limited total budget. In addition, coordination may be particularly difficult for members of multi-stage supply chains who face budget constraints in a sub-supply chain. Therefore, in this paper, we only consider situations in which no pair of adjacent members faces budget constraints at the same time. In this case, if we suppose that all supply chain members are coordinated by a single entity (central control), then the channel-wide optimal order quantity is q^* as can be obtained with Equation (1). Hereafter, we discuss the case of decentralised control. Let Φ_{B_i} be member i's total share of the revenue generated from each unit under the RS contract, $i = 1,2, \ldots, N$.

Case 1 describes the required budget of member i, who is a terminal member, which involves the purchasing and production cost ($C_i q$). Thus, for the terminal member, the required budget depends only on q. In this case, we cannot address the budget constraint by modifying the RS contract.

Case 2 describes the required budget of member i, who is not a terminal member, which includes production and purchasing costs:

$$C_i q + \sum_{j \in S_i^d} \omega_j q = \sum_{k \in S_i} C_k q - \sum_{k \in S_i^u} \Phi_{B\_k} C q. \tag{10}$$

The required budget of member i can be explained with a general functional form:

$$G_i\left(\sum_{k\in S_i^u}\Phi_{B\_k},q\right)=\sum_{k\in S_i}C_kq-\sum_{k\in S_i^u}\Phi_{B\_k}Cq. \tag{11}$$

For a set of $(\Phi_{B\_1},\Phi_{B\_2},\dots,\Phi_{B\_N})$, the decision framework of member i under the RS contract can be obtained as P1.

$$\begin{aligned}&\text{Max}\quad \Pi(\Phi_{B\_i},q)=\Phi_{B\_i}\Pi_SC(q)\\&\text{s.t.}\quad \sum_{k\in S_i}C_kq-\sum_{k\in S_i^u}\Phi_{B\_k}Cq\le Y_i\end{aligned} \tag{P1}$$

Because $q=\min(q_1,q_2,\dots,q_N)$, member i can influence q by deciding q_i. When $Y_i\ge\sum_{k\in S_i}C_kq^*-\sum_{k\in S_i^u}\Phi_{B\_k}Cq^*$, member i's optimal order quantity is equal to q^* under $(\Phi_{B\_1},\Phi_{B\_2},\dots,\Phi_{B\_N})$. In addition, the optimal q_k of member k with an infinite Y_k are also equal to q^*. In this case, supply chain coordination can be achieved under the RS contract, $i=1,2,\dots,N$ and $i\notin T$. When Y_i decreases to Y_i^t and $Y_i^t<\sum_{k\in S_i}C_kq^*-\sum_{k\in S_i^u}\Phi_{B\_k}Cq^*$, member i can adopt one of two feasible approaches.

Method 1: Decrease the percentage of the retailer's revenue that the budget-constrained member retains and increase the percentage of revenue retained by the upstream members by that amount.

Let Φ_i^t be the lower percentage it retains compared with Φ_{B_i}; that is, $\Phi_{B\_i}-\Delta\Phi_i=\Phi_i^t$ and $\Delta\Phi_i>0$. Member i can transfer $\Delta\Phi_{ui}$ to upstream members and $\Delta\Phi_{di}$ to downstream members; furthermore, $\Delta\Phi_{ui}+\Delta\Phi_{di}=\Delta\Phi_i$. Therefore, from Equation (11), we obtain Equation (12):

$$G_i\left(\sum_{k\in S_i^u}\Phi_{B\_k}+\Delta\Phi_{ui},q^*\right)=\sum_{k\in S_i}C_kq^*-\left(\sum_{k\in S_i^u}\Phi_{B\_k}+\Delta\Phi_{ui}\right)Cq^*=Y_i^t. \tag{12}$$

As shown in Equation (12), $\Delta\Phi_{di}$ cannot reduce the budget requirement. Moreover, the profit of member i will be

$$(\Phi_{B\_k}-\Delta\Phi_{ui}-\Delta\Phi_{di})\Pi_{SC}(q^*)=(\Phi_{B\_k}-\Delta\Phi_i)\Pi_{SC}(q^*). \tag{13}$$

Equation (13) shows that larger values of $\Delta\Phi_{ui}$ and $\Delta\Phi_{di}$ create lower profits for member i. From Equation (12), we can see that only $\Delta\Phi_{ui}$ affects the budget required by member i. Hence, member i will opt to set $\Delta\Phi_{di}$ to zero and transfer all of $\Delta\Phi_i$ to the upstream members.

Method 2: Decrease q.
As discussed above, member i can influence q by deciding q_i because $q=\min(q_1,q_2,\dots,q_N)$. If member i becomes budget constrained and thus supplies fewer than q^* to its downstream member, then the quantity of the final product, q^t is smaller than q^*: $q^t=q^*-\Delta q$ and $\Delta q>0$. In this case, the expected profits of the supply chain and member i will decrease to $\Pi_{SC}(q^t)$ and $\Phi_{B\_i}\Pi_{SC}(q^t)$, respectively. When $(\Phi_{B\_i}-\Delta\Phi_i)\Pi_{SC}(q^*)<\Phi_{B\_i}\Pi_{SC}(q^t)$, member i will choose Method 2, which decreases the total profit of the supply chain, in pursuit of a higher profit for itself.

The above two methods illustrate that, under the RS contract, budget-constrained member i faces a trade-off between Φ_{B_i} and q. If member i's bargaining power is relatively strong, it might be incentivised to reduce q such that the supply chain is not coordinated. In these cases, the RS contract shown in Theorem 1 cannot coordinate the supply chain. This phenomenon is illustrated in Section 5.

For any q and $(\Phi_{B\_1},\Phi_{B\_2},\dots,\Phi_{B\_N})$ with $\sum_{i=1}^N\Phi_{B\_i}=1$ and $\Phi_{B\_i}\in[0,1]$, if $\sum_{k\in S_i}C_kq-\sum_{k\in S_i^u}\Phi_{B\_k}Cq\le Y_i^t$, then, member i is budget constrained. Theorem 2 shows that a properly designed RSB contract can achieve supply chain coordination and arbitrarily allocate the supply chain profit even when a member is working under budget constraints.

Theorem 2. *Consider the RS contracts in Theorem 1. Change the wholesale price that budget-constrained member i pays to the direct upstream member j to*

$$\omega_j=\sum_{k\in S_j}C_k-\left(\Delta\Phi_{ij}+\sum_{k\in S_j}\Phi_{B\_k}\right)C, \tag{14}$$

where $\sum_{j\in S_i^d}\Delta\Phi_{ij}=\Delta\Phi_i$, $j=2,3,\dots N$. *Note that* $\Delta\Phi_i>0$, $\Delta\Phi_{ij}>0$, $\sum_{i=1}^N\Phi_{B\_i}=1$ *and* $\Phi_{B\_i}\in[0,1]$. *Let* $\Delta\Phi_{pij1}$ *and* $\Delta\Phi_{pij2}$ *be penalty factors such that* $\Delta\Phi_{pij1}-\Delta\Phi_{pij2}=\Delta\Phi_{ij}$. *Let the percentage of the retailers' revenue that member i* *retains be* $\Phi_{B\_i} - \sum_{j\in S_i^d} \Delta\Phi_{pij1} + \sum_{j\in S_i^d} \Delta\Phi_{pij2}\frac{Cq}{R(q)}$*, and let the percentage of the retailers' revenue that member j retains be* $\Phi_{B\_j} + \Delta\Phi_{pij1} - \Delta\Phi_{pij2}\frac{Cq}{R(q)}$*. The shares and wholesale prices of other members who have sufficient budgets remain as in Theorem 1. Under this type of RSB contract, the expected profits of members i and j are* $\left(\Phi_{B\_i} - \sum_{j\in S_i^d} \Delta\Phi_{pij1}\right)\Pi_{SC}(q)$ *and* $\left(\Phi_{B\_j} + \Delta\Phi_{pij1}\right)\Pi_{SC}(q)$*, respectively.*

The RSB contract can be implemented in the following steps:

- Before the selling season, the members decide the values of Φ_{B_i} which originally sets the profit allocation scenario.
- The members report the budgets (Y_i^t) and decide the values of q, $i = 1,2,\ldots,N$ and $i \notin T$. If $\sum_{k\in S_i} C_k q - \sum_{k\in S_i^u} \Phi_{B\_k} Cq \le Y_i^t$, then they determine ω_j and the value of $\Delta\Phi_{ij}$ using Equation (14) that satisfies $C_i q + \sum_{j\in S_i^d} \omega_j q \le Y_i^t$. Otherwise, they determine ω_j with Equation (6) by letting $\Phi_k = \Phi_{B\_k}$, $j \in S_i^d$ and $k \in S_j$.
- Members i and j decide the values of $\Delta\Phi_{pij1}$ and $\Delta\Phi_{pij2}$ using Theorem 2 based on $\Delta\Phi_{ij}$, which adjusts the original profit allocation. For other members (e.g. member l), $\Delta\Phi_{plk1} = \Delta\Phi_{plk2} = 0$, $l \notin T$ and $k \in S_i^d$.
- Member j obtains $\omega_j q$ from the direct downstream member and starts production, $j = N, N-1, \ldots, 2$. All the final products are delivered to the retailer, who sells them during the selling season.
- At the end of the selling season, the unsold products are salvaged. The retailer shares the revenue with other members based on Φ_{B_i}, $\Delta\Phi_{pij1}$ and $\Delta\Phi_{pij2}$.

Under a set of ($\Phi_{B\_1},\Phi_{B\_2},\ldots,\Phi_{B\_N}$), budget-constrained member i can be found with q and Y_i^t. By using Theorem 2, we obtain the decision framework of budget-constrained member i as P2.

$$\text{Max} \quad \Pi_i\left(\Phi_{B\_i}, \Delta\Phi_{pij1}, q\right) = \left(\Phi_{B\_i} - \sum_{j\in S_i^d} \Delta\Phi_{pij1}\right)\Pi_{SC}(q)$$

s.t.

$$\begin{cases} \sum\limits_{j\in S_i} C_j q - \left(\Delta\Phi_i + \sum\limits_{j\in S_i^u} \Phi_{B\_j}\right) Cq \le Y_i^t \\ \Delta\Phi_{pij1} - \Delta\Phi_{pij2} = \Delta\Phi_{ij} \quad \forall\, j \in S_i^d \\ \sum\limits_{j\in S_i^d} \Delta\Phi_{ij} = \Delta\Phi_i \\ \Phi_{B\_i} = \lambda_i \end{cases} \tag{P2}$$

Let $\Delta\Phi_{pi1} = \sum_{j\in S_i^d} \Delta\Phi_{pij1}$ and $\Delta\Phi_{pi2} = \sum_{j\in S_i^d} \Delta\Phi_{pij2}$. For any λ_i and λ_j, we can always find a ($\Phi_{B\_i},\Phi_{B\_j},\Delta\Phi_{pij1},\Delta\Phi_{pij2}$) that satisfies $\Phi_{B\_i} - \Delta\Phi_{pi1} = \lambda_i$ and $\Phi_{B\_j} + \Delta\Phi_{pij1} = \lambda_j$ as well as the constraints in (P2) by setting $\Delta\Phi_{pij1} = 0$, $j \in S_i^d$. Then, member i's expected profit function is $\lambda_i \Pi_{SC}(q)$ and its optimal-order quantity in (P2) is equal to q^*. That is, under the RSB contract, supply chain coordination can be achieved and the supply chain profit can be arbitrarily allocated under budget constraints. Because $\Pi_{SC}(q^*) > \Pi_{SC}(q^t)$ and $\Pi_{SC}(q^*)$ can be arbitrarily allocated under the RSB contract, all of the supply chain members can obtain higher profits under an RSB contract than under an RS contract that cannot coordinate the supply chain with budget constraints. In addition, when $\Delta\Phi_{pij1} = \Delta\Phi_{pij2} = \Delta\Phi_{ij} = 0$, the RSB contract is the same as the RS contract; that is, the RS contract is a special case of the RSB contract.

Theorem 2 shows that the RSB contract can arbitrarily allocate profits when one member faces a budget constraint and asks for a lower wholesale price from the upstream members. To compensate the upstream members, that budget-constrained member (e.g. member i) transfers part of the company's shared revenue to the upstream members with $\Delta\Phi_{pi1}$. Under this type of RSB contract, the expected profit of budget-constrained member i is $(\Phi_{B\_i} - \Delta\Phi_{pi1})\Pi_{SC}(q)$ and its optimal-order quantity is still q^*. When q and Φ_{B_i} are fixed, a larger $\Delta\Phi_{pi1}$ results in a lower expected profit of member i, which is the reason it is termed a '*penalty factor*'. We can propose some profit allocation adjustment approaches that link profit allocation with the budgets of supply chain members. An original profit allocation (Φ_{B_i}) is determined and adjusted by consideration of the members' budgets ($\Delta\Phi_{pi1}$). This idea has been widely used in many supply chains in which members cooperate and share the supply chain profit, such as the truck brake supply chain in China. If the member still obtains a higher profit than under other choices, then the total profit of the supply chain remains optimal and supply chain coordination is achieved.

Remark 1. When the RSB contract penalty factors are set appropriately, members cannot benefit from hiding their budgets.

However, member i must obtain a higher profit under the RSB contract than under Method 1, Method 2 or the traditional market setting. This finding means that

$$\left(\Phi_{B\_i} - \Delta\Phi_{pi1}\right)\Pi_{SC}\left(q^*\right) \geq \max\left[(\Phi_{B\_i} - \Delta\Phi_i)\Pi_{SC}\left(q^*\right), \Phi_{B\_i}\Pi_{SC}(q^t), \Pi_{M\_i}\right]. \quad (15)$$

Therefore, $\Delta\Phi_{pi1}$ should have an upper bound (U_{pi}):

$$\Delta\Phi_{pi1} \leq \min\left(\Delta\Phi_i, \Phi_{B\_i}\left(1 - \frac{\Pi_{SC}(q^t)}{\Pi_{SC}\left(q^*\right)}\right), \Phi_{B\_i} - \frac{\Pi_{M\_i}}{\Pi_{SC}\left(q^*\right)}\right). \quad (16)$$

Because member i cannot obtain a higher profit while operating with a lower budget, we can ascertain that the lower bound on $\Delta\Phi_{pi1}$ is zero. Meanwhile, different values of Φ_{B_i} could lead to a different range of $\Delta\Phi_{pi1}$ values, influencing the flexibility of the RS contract in terms of profit allocation. Let $I^m_{\Delta\Phi_{pi1}}$ and $\Phi^m_{B\_i}$ be the ranges of $\Delta\Phi_{pi1}$ and Φ_{B_i} in scenario m, m = Ra and Ab, respectively. Property 2 addresses $I^{Ra}_{\Delta\Phi_{pi1}}$ and $I^{Ab}_{\Delta\Phi_{pi1}}$, $i = 1,2, \ldots, N$, $i \notin T$.

Property 2. *For any set of budget constraints, when* $\frac{\Pi_{M\_i}}{\Pi_{M\_SC}} > \frac{1}{N}$ *and* $\sum_{j\in S_i^u}\Phi^{Ra}_{B\_j} < \sum_{j\in S_i^u}\Phi^{Ab}_{B\_j}$*, use of* $\Phi_{B\_i} = \Pi_{M\_i}/\Pi_{M\_SC}$ *can support a larger range of* $\Delta\Phi_{pi1}$ *than use of* $\Phi_{B\_i} = \left(\frac{\Pi_{SC}(q)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}\right)/\Pi_{SC}(q)$*,* $i = 1,2, \ldots, N$*,* $i \notin T$*.*

Property 2 shows that under some conditions, use of Φ_{B_i} in Scenario Ra offers more flexible profit allocations than when it is used in Scenario Ab.

Wholesale price restricted to certain ranges have been studied by other researchers (Xiong, Chen, and Xie 2011). If the order quantity is fixed, budget constraints can be considered as the wholesale price constraints; that is, the wholesale price constraint can be considered a special case of budget constraint. Moreover, the discussion above (and the numerical examples in Section 5) shows that members can obtain higher profits by decreasing the order quantity than they can by changing the wholesale price. In the real world, managers usually pay attention to their budget constraints and must decide the wholesale price and order quantity to satisfy them. Consequently, they may typically concentrate more on satisfying budget constraints than on determining wholesale price. Therefore, we can infer that, for supply chain coordination, budget constraints form the basis for more complex and common decisions than do wholesale price constraints.

However, it is not always true that wholesale prices are limited to certain ranges because of budget constraints. There could be other reasons members experience difficulty in changing wholesale prices (Wang and Webster 2007). We show that the RSB contract, which acknowledges budget constraints, is more flexible than the RS contract in terms of profit allocation when wholesale prices are restricted. Let $I^1_{\sum_{k\in S_j}\Phi_k}$ and $I^2_{\sum_{k\in S_j}\Phi_k}$ be the ranges of $\sum_{k\in S_j}\Phi_k$ under the RS contract and the RSB contract, respectively.

Property 3. *For a range of wholesale prices,* $[\omega_j^{\min}, \omega_j^{\max}]$*, the RSB contract,* which accounts for *budget constraints, is more flexible than the RS contract,* which does not accounts for budget constraints, *in terms of profit allocation.*

Property 3 states that for the same range of wholesale prices, the RSB contract can support more profit allocation scenarios than can the RS contract. As seen from Theorems 1 and 2, the RS contract is a special case of the RSB contract considering budget constraints. That is, under certain constraints, the RSB contract offers more flexible profit allocation options, through settings of $\Delta\Phi_{pij1}$ and $\Delta\Phi_{pij2,}$ than the RS contract offers.

One implementation limitation of supply chain contracts is the administrative cost. Feasible contracts should not require large additional administrative costs that are higher than the increased profits. At first glance, the RSB contract seems to generate more additional costs than the RS contract proposed in Section 2. However, implementation of the RSB contract generates no more additional cost than the RS contract. For example, under the RS contract, the retailer must submit the amounts of sold product to the upstream members to claim the shared revenue. Other costs may result from the money transfers between the retailer and the other members. Members using an RSB contract generate similar levels of the additional administrative costs as do those using the RS contract. Real-life practice shows the economics of RS contracts in multi-stage supply chains and we infer that the RSB contract is also efficient.

Remark 2. The RSB contract is more flexible than the RS contract in terms of profit allocation, and it creates little additional administrative cost.

5. Numerical experiments

In this section, we clarify the RS and RSB contracts through numerical experiments. Six members in the supply chain are shown in Figure 2. Member 1 is the retailer, Member 2 is the manufacturer and Members 3–6 are suppliers. In this supply chain, Members 5 and 6 supply two types of material to Member 3, who produces one component. With components from Members 3 and 4, the manufacturer (Member 2) assembles the final product, and then the retailer (Member 1) sells it to customers.

Assume that the demand, X, follows a normal distribution with a mean of 100 and a standard deviation of 30 units. The retail price is \$30, and the salvage value is \$1. Other assumed problem data are depicted in Table 1.

We can obtain $R(q)$ as

$$p\left(\int_0^q xf(x)dx + \int_q^\infty qf(x)dx\right) + s\int_0^q (q-x)f(x)dx. \tag{17}$$

From Equation (2), we can ascertain that the optimal-order quantity of the supply chain under the traditional market setting is 85.15 units. The expected profits under the traditional market setting are shown in Table 2.

From Equation (1), we find that the optimal quantity for the supply chain is 124.5 units. Based on Scenario Ra, we use the percentages Φ_i^{Ra} as Φ_i, $i = 1,2,\ldots,6$. From Equation (6) in Theorem 1, we obtain all of the wholesale prices under the RS contract in Scenario Ra, namely, $(\omega_2, \omega_3, \omega_4, \omega_5, \omega_6)$ = (\$1.824, \$2.264, \$0.430, \$1.098, \$1.335). Based on Scenario Ab, we use the percentages Φ_i^{Ab} as Φ_i, $i = 1,2,\ldots,6$. With the same approach as we used with scenario Ra, we obtain all of the wholesale prices under the RS contract in Scenario Ab, namely, $(\omega_2, \omega_3, \omega_4, \omega_5, \omega_6)$ = (\$1.677, \$2.124, \$0.381, \$1.028, \$1.265). The expected profits under the RS contract are shown in Table 3 and Figure 3.

As shown in Table 3 and Figure 3, all six supply chain members obtain higher profits under the RS contract than in the traditional market setting. Hence, the members will accept the RS contract, which maximises the total profit of the supply chain. For simplicity, we only show the analysis of the profits in Scenario Ra. The analysis for Scenario Ab is similar. We can infer that the retailer requires at least \$289.3 to maintain the operation under the RS contract budget in Scenario Ra, including the production cost of \$62.2 and the purchasing cost of \$227.1. We discuss the example for an RS contract in which the retailer meets a budget less than \$289.3. As we discussed in Section 3, under the RS contract, the retailer has two choices: decreasing revenue shares (Φ_1) (Method 1) or decreasing the order quantity (q) (Method 2).

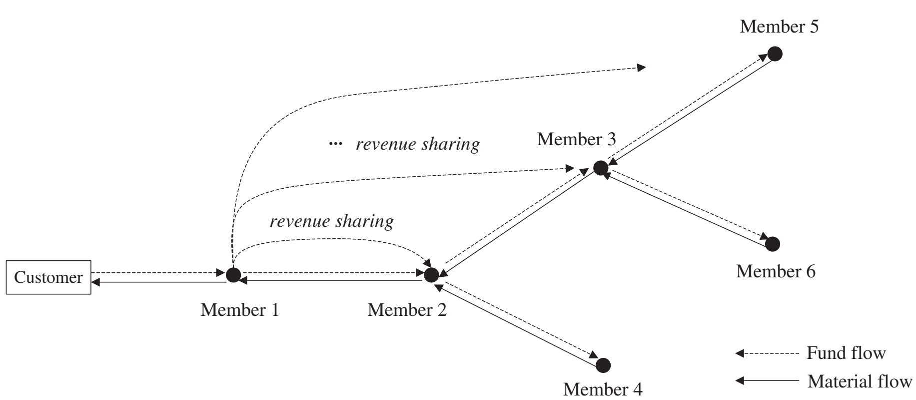

Figure 2. A supply chain with six members.

Table 1. Problem data.

Member	Cost function	Price under the traditional market setting (\$)
1	$0.5q$	
2	$0.6q$	20.5
3	$1.0q$	12.0
4	$1.2q$	3.5
5	$1.7q$	3.5
6	$2.0q$	4.0

Table 2. Expected profits ($) under the traditional market setting.

Member	1	2	3	4	5	6	Supply chain
Profit	592.9	374.7	298.0	195.9	153.3	170.3	1785.1
Percentage	33.2%	21.0%	16.7%	11.0%	8.6%	9.5%	100%
Φ_i^{Ra}	33.2%	21.0%	16.7%	11.0%	8.6%	9.5%	100%
Φ_i^{Ab}	31.1%	20.4%	16.7%	11.7%	9.6%	10.5%	100%

Table 3. Expected profits ($) under the RS contract.

Member	1	2	3	4	5	6	Supply chain
Traditional market	592.9	374.7	298.0	195.9	153.3	170.3	1785.1
Scenario Ra	681.4	430.6	342.4	225.2	176.2	195.7	2051.5
Scenario Ab	637.3	419.1	342.4	240.3	197.7	214.7	2051.5

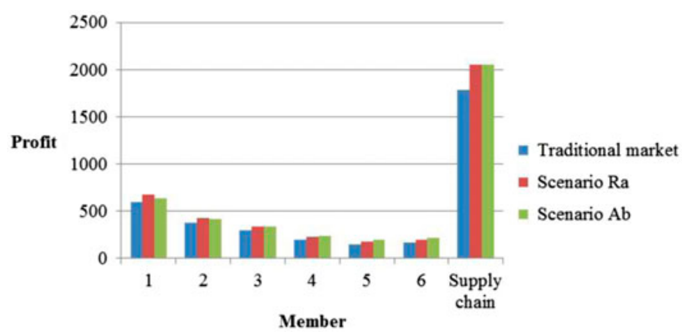

Figure 3. Comparison of profits ($).

The profits of the retailer in Scenario Ra with different budgets and under the two methods to ameliorate coordination infeasibility under the RS contract are shown in Table 4. M1 and M2 represent Methods 1 and 2, respectively.

As shown in Table 4, when the retailer chooses to decrease Φ_1, the supply chain can still obtain the maximum profit. However, the retailer can always earn a higher profit by decreasing q than by decreasing Φ_1. Consequently, the retailer would prefer to decrease the order quantity, which decreases the total profit of the supply chain. This preference means that the RS contract, which does not include consideration of budget constraints, cannot maximise the total profit of the supply chain. Therefore, in this case, the RSB contract should be used. The profits under the RSB contract are shown in Table 5.

Table 4. Expected profits ($) under different budgets and methods to ameliorate infeasibility.

	Φ_1^t(%)		ω_2($)		q		Profit of the retailer		Profit of the supply chain	
Budget ($)	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
289.3	33.2	33.2	1.824	1.824	124.5	124.5	681.36	681.38	2051.5	2051.5
250	28.7	33.2	1.509	1.824	124.5	107.5	588.78	666.24	2051.5	2005.9
200	22.9	33.2	1.103	1.824	124.5	86.0	469.79	596.79	2051.5	1796.8
150	17.2	33.2	0.704	1.824	124.5	64.5	352.85	475.92	2051.5	1432.9
100	11.5	33.2	0.305	1.824	124.5	43.0	235.92	325.26	2051.5	979.3

Table 5. Expected profits ($) under the RSB contract.

Budget ($)	$\Delta\Phi_1$	U_{p1}	$\Delta\Phi_{p11}$	Profit of the retailer M2	Profit of the retailer RS contract	Profit of the manufacturer (RS contract)	Profit of the supply chain
289.3	0	0	0	681.38	681.38	430.62	2051.5
250	4.5%	0.007	0.0035	666.24	673.81	438.19	2051.5
200	10.3%	0.041	0.0205	596.79	639.08	472.92	2051.5
150	16.0%	0.100	0.0500	475.92	578.65	533.35	2051.5
100	21.7%	0.173	0.0865	325.26	503.32	608.68	2051.5

Table 5 indicates that, under the RSB contract, $\Delta\Phi_1$ increases when the budget decreases; thus, the retailer faces a lower wholesale price and can satisfy the budget requirement. In addition, the upper bound on the penalty factor also increases. We can arbitrarily choose one positive value less than the upper bound. In this example, we use the middle value between 0 and the upper bound as the penalty factor. Table 5 shows that the retailer obtains a higher profit under the RSB contract, even in the face of budget constraints, than by decreasing the order quantity; hence, the firm will accept the RSB contract, which maximises the total profit of the supply chain. Moreover, as the manufacturer charges a lower wholesale price, the manufacturer obtains a higher profit as compensation. Because the order quantity remains the global optimal, the other members will obtain the same profits as they obtained under the RS contract. We can infer that the retailer requires at least \$270.7 (the production cost of \$62.2 and the purchasing cost of \$208.5) to maintain the firm's operation under the RS contract in Scenario Ab.

Table 6 and Figure 4 show the upper bounds of $\Delta\Phi_{p11}$ in Scenarios Ra and Ab. Note that in Table 2, Φ_1^{Ab} equals 33.2%, which is larger than 1/6, and $\sum_{j\in S_1^u}\Phi_{B\_j}^{Ra} < \sum_{j\in S_1^u}\Phi_{B\_j}^{Ab}$. We can find that for different budgets, the upper bound of $\Delta\Phi_{p11}$ in Scenario Ra, U_{p1}^{Ab}, is higher than in Scenario Ab, U_{p1}^{Ab}. This result means that in this case the RSB contract in Scenario Ra offers more flexible profit allocation than offered by Scenario Ab. This result is the same as shown for Property 2.

6. Managerial implications and concluding remarks

We showed that the supply chain profit can be maximised if all supply chain members sign a coordinating contract. Next, we proposed an RS contract in a multi-stage supply chain, which is described by a branching tree. We assign ω_i

Table 6. Upper bounds of $\Delta\Phi_{p11}$ in Scenarios Ra and Ab under different budgets.

Budget ($)	U_{p1}^{Ra} (%)	U_{p1}^{Ab} (%)
289.5	0.0	0.0
270.7	0.1	0.0
250	0.7	0.2
200	4.1	2.7
150	10.0	8.0
100	17.3	15.2

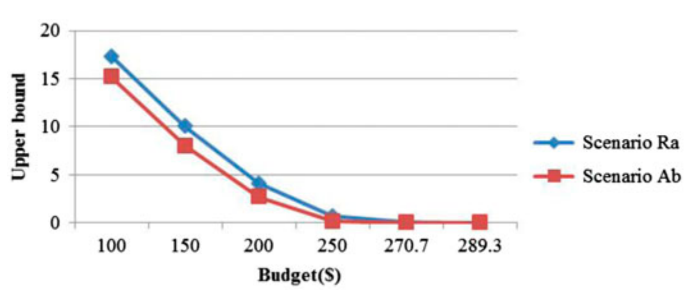

Figure 4. Comparison of the upper bounds of $\Delta\Phi_{p11.}$

and Φ_i for all i to coordinate the multi-stage supply chains in question. Mangers can allocate the supply chain profit among all of supply chain members by setting (Φ_1,Φ_2, … ,Φ_N). For any (Φ_1,Φ_2, … ,Φ_N), (ω_2, ω_3, … ,Φ_N) is also determined to coordinate the supply chain. Under this type of RS contract, all of the members obtain higher profits than they would in the traditional market setting, and the total profit of the supply chain is maximised. Under the RS contract, the total profit is divided among all of the members in the supply chain.

Our results indicate that, under the RS contract, the wholesale price charged by a member is smaller than its total costs, including the production and purchasing costs. The revenue only comes after the retailer's sale (i.e. is postponed). Due to the uncertain demand and the potential bankruptcy of the retailer, supply chain members experience more pressure to maintain budgets and cash flow. Therefore, budget constraint is more important to the cash flow safety of the companies. The analysis of supply chain coordination also shows that supply chain coordination under budget constraints is more complicated than under wholesale price constraints. We originally demonstrate that, under the RS contract, budget-constrained managers may choose to decrease order quantity to lower wholesale prices charged by upstream members. In this case, supply chain coordination cannot be achieved. In addition, when some members fail to meet their required budget, the RS contract may no longer be feasible. Managers and researchers can obtain insights into supply chain coordination under budget constraints.

Based on this concern, we modified the RS contract by considering budget constraints and defined an RSB contract, which includes a penalty factor to modify the revenue sharing mechanism. With new wholesale price settings and revenue sharing mechanisms, the member without a sufficient budget transfers a part of the revenue to the direct upstream members and faces a lower wholesale price than when under the RS contract. In this case, the member can maintain operations with a relatively low budget and still obtain a higher profit than when in traditional market conditions (without a coordinated supply chain). If a terminal member faces a budget constraint, the proposed RSB contract cannot solve the problem.

However, using the RSB contract, supply chain coordination can still be achieved when some members have budget constraints. Two methods for optimally setting Φ_i allows for varying flexibility in profit allocation. Our numerical results offer insight into ways supply chain coordination can be achieved by setting appropriate decision variables in an RSB contract. We also showed that the RSB contract is more flexible than the RS contract in terms of profit allocation when the wholesale prices are restricted to certain ranges. Therefore, we expect that managers facing budget constraints, in particular, will find our analysis of RS contracts in multi-stage supply chains insightful.

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Appendix. Proofs

Proof of Property 1. Equation (1) shows that the channel-wide optimal-order quantity of the supply chain q^* satisfies

$$F(q^*) = \frac{p - C}{p - s}.$$

For simplicity, we only prove the property when one terminal member, member t, is not part of the coordinated supply chain. The proofs of the other cases are similar. Because member t is not covered by the coordinating contract, it charges the same wholesale price per unit, ω_{M_t}, as the wholesale price in a traditional market setting. It is obvious that $\omega_{M\_t} > C_t$. Let q_1 denote the retailer's optimal-order quantity in this case. Because $N-1$ members sign the contract, q_1 satisfies

$$F(q_1) = \frac{p - C - \omega_{M\_t} + C_t}{p - s}.$$

Then, we can infer that $q_1 < q^*$ and $\Pi_{SC}(q_1) < \Pi_{SC}(q^*)$.

Proof of Theorem 1. In Case 1, i refers to a terminal member. Then, $S_i = \{i\}$.
By substituting Equation (6) into (3), the expected profit of member i is obtained as:

$$\Pi_i(\Phi_i, q) = \Phi_i R(q) + \omega_i q - C_i q = \Phi_i R(q) - \Phi_i C q = \Phi_i \Pi_{SC}(q).$$

In Case 2, member i is neither a terminal member nor the retailer. Therefore, the total payment from member i to direct upstream member(s) is

$$\sum_{j\in S_i^d}\omega_j q=\sum_{j\in S_i^u}C_j q-\sum_{j\in S_i^u}\Phi_j Cq \tag{A1}$$

By substituting Equations (6) and (A1) into (4), the profit of member i is obtained as:

$$\Pi_i(\Phi_i,q)=\Phi_i R(q)+\omega_i q-C_i q-\sum_{j\in S_i^d}\omega_j q=\Phi_i R(q)-\sum_{j\in S_i}\Phi_j Cq+\sum_{j\in S_i^u}\Phi_j Cq=\Phi_i\Pi_{SC}(q).$$

In Case 3, member i is the retailer and $i = 1$.

Noting that $\omega_1 = 0$, from Equations (4) to (6), we can obtain the expected profit of member i:

$$\Pi_1(\Phi_1,q)=\Phi_1 R(q)-C_1 q-\sum_{j\in S_1^d}\omega_j q=\Phi_1 R(q)-\left(1-\sum_{j=2}^{N}\Phi_j\right)Cq=\Phi_1\Pi_{SC}(q).$$

Therefore, the expected profit of member i is $\Pi_i(\Phi_i,q)=\Phi_i\Pi_{SC}(q)$, $i = 1,2,\dots,N$.

Proof of Theorem 2. If the budget of member i decreases from Y_i to Y_i^t, then the cost of member i is $C_i(q)+\sum_{j\in S_i^d}\omega_j q=\sum_{k\in S_i}C_k q-\left(\Delta\Phi_i+\sum_{k\in S_i^u}\Phi_{B\_k}\right)Cq=Y_i^t$. This result means that the required budget of member i equals Y_i^t. The expected revenue of member i is:

$$\left(\Phi_{B\_i}-\Delta\Phi_{pi1}+\Delta\Phi_{pi2}\frac{Cq}{R(q)}\right)R(q)+\omega_i q=\left(\Phi_{B\_i}-\Delta\Phi_{pi1}+\Delta\Phi_{pi2}\frac{Cq}{R(q)}\right)R(q)+\sum_{j\in S_i}C_j q-\sum_{j\in S_i}\Phi_{B\_j}Cq.$$

Hence, the expected profit of member i is

$$\left(\Phi_{B\_i}-\Delta\Phi_{pi1}+\Delta\Phi_{pi2}\frac{Cq}{R(q)}\right)R(q)-(\Phi_{B\_i}-\Delta\Phi_i)Cq. \tag{A2}$$

As $\Delta\Phi_{pij1}-\Delta\Phi_{pij2}=\Delta\Phi_{ij}$ and $\sum_{j\in S_i^d}\Delta\Phi_{ij}=\Delta\Phi_i$, we obtain

$$\sum_{j\in S_i^d}\Delta\Phi_{pij1}-\sum_{j\in S_i^d}\Delta\Phi_{pij2}=\Delta\Phi_i. \tag{A3}$$

Considering that $\sum_{j\in S_i^d}\Delta\Phi_{pij1}=\Delta\Phi_{pi1}$ and $\sum_{j\in S_i^d}\Delta\Phi_{pij2}=\Delta\Phi_{pi2}$, from Equation (A3) we can obtain

$$\Delta\Phi_{pi1}-\Delta\Phi_{pi2}=\Delta\Phi_i. \tag{A4}$$

Using Equations (A2) and (A4), we obtain the expected profit of member i as

$$\Pi_i\left(\Phi_{B\_i},\Delta\Phi_{pi1},q\right)=\left(\Phi_{B\_i}-\Delta\Phi_{pi1}\right)R(q)-\left(\Phi_{B\_i}-\Delta\Phi_{pi1}\right)Cq=\left(\Phi_{B\_i}-\Delta\Phi_{pi1}\right)\Pi_{SC}(q).$$

The cost of member j is $C_j q+\sum_{k\in S_j^d}\omega_k q=\sum_{k\in S_j}C_k q-\sum_{k\in S_j^u}\Phi_{B\_k}Cq$. The expected revenue of member j is

$$\left(\Phi_{B\_j}+\Delta\Phi_{pij1}-\Delta\Phi_{pij2}\frac{Cq}{R(q)}\right)R(q)+\omega_j q=\left(\Phi_{B\_j}+\Delta\Phi_{pij1}-\Delta\Phi_{pij2}\frac{Cq}{R(q)}\right)R(q)+\sum_{k\in S_j}C_k q-\left(\Delta\Phi_{ij}+\sum_{k\in S_j}\Phi_{B\_k}\right)Cq.$$

Therefore, the expected profit of member j is

$$\left(\Phi_{B\_j}+\Delta\Phi_{pij1}-\Delta\Phi_{pij2}\frac{Cq}{R(q)}\right)R(q)-\left(\Delta\Phi_{ij}+\Phi_{B\_j}\right)Cq=\left(\Phi_{B\_j}+\Delta\Phi_{pij1}\right)R(q)-\left(\Delta\Phi_{ij}+\Delta\Phi_{pij2}+\Phi_{B\_j}\right)Cq.$$

Because $\Delta\Phi_{pij1}-\Delta\Phi_{pij2}=\Delta\Phi_{ij}$, the expected profit of member j is

$$\Pi_j\left(\Phi_{B\_j},\Delta\Phi_{pij1},q\right)=\left(\Phi_{B\_j}+\Delta\Phi_{pij1}\right)R(q)-\left(\Phi_{B\_j}+\Delta\Phi_{pij1}\right)Cq=\left(\Phi_{B\_j}+\Delta\Phi_{pij1}\right)\Pi_{SC}(q)$$

From Theorem 1, we can infer that the profits of the other members remain as the chain is coordinated. Thus, the expected profits of all of the members are affine transformations of the expected profit of the supply chain. All of the members will accept q^*.

Proof of Property 2. Let $q^{t(m)}$, $\Delta\Phi_i^m$, and ωi^m be q^t, $\Delta\Phi_i$, and ω_i in scenario m under the RS contract, respectively; m = Scenario Ra, Ab.

Noting that $\Delta\Phi_{pi1}\le\min\left(\Delta\Phi_i,\Phi_{B\_i}\left(1-\frac{\Pi_{SC}(q^t)}{\Pi_{SC}(q^*)}\right),\Phi_{B\_i}-\frac{\Pi_{M\_i}}{\Pi_{SC}(q^*)}\right)$, when $\Phi_{B\_i}=\Pi_{M\_i}/\Pi_{M\_SC}$, we obtain

$$I_{\Delta\Phi_{pi1}}^{Ra}=\left[0,\min\left(\Delta\Phi_i^{Ra},\frac{\Pi_{M\_i}}{\Pi_{M\_SC}}\left(1-\frac{\Pi_{SC}\left(q^{t(Ra)}\right)}{\Pi_{SC}(q^*)}\right),\frac{\Pi_{M\_i}}{\Pi_{M\_SC}}-\frac{\Pi_{M\_i}}{\Pi_{SC}(q^*)}\right)\right].$$

When $\Phi_{B\_i} = \left(\frac{\Pi_{SC}(q)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}\right)/\Pi_{SC}(q)$, we obtain

$$I^{Ab}_{\Delta\Phi_{pi1}} = \left[0, \min\left(\Delta\Phi_i^{Ab}, \frac{\frac{\Pi_{SC}(q^*)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}}{\Pi_{SC}(q^*)}\left(1 - \frac{\Pi_{SC}\left(q^{t(Ab)}\right)}{\Pi_{SC}(q^*)}\right), \frac{\frac{\Pi_{SC}(q^*)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}}{\Pi_{SC}} - \frac{\Pi_{M\_i}}{\Pi_{SC}}\right)\right].$$

By Method 1, we obtain

$$Y_i^t = C_i q^* + \sum_{j\in S_i^u} C_j q^* - \left(\sum_{j\in S_i^u} \Phi_{B\_j}^{Ra} + \Delta\Phi_i^{Ra}\right) Cq^* = C_i q^* + \sum_{j\in S_i^u} C_j q^* - \left(\sum_{j\in S_i^u} \Phi_{B\_j}^{Ab} + \Delta\Phi_i^{Ab}\right) Cq^*,$$

Because $\sum_{j\in S_i^u} \Phi_{B\_j}^{Ra} < \sum_{j\in S_i^u} \Phi_{B\_j}^{Ab}$, we can infer that $\Delta\Phi_i^{Ra} > \Delta\Phi_i^{Ab}$.

By Method 2, based on $\sum_{j\in S_i^u} \Phi_{B\_j}^{Ra} < \sum_{j\in S_i^u} \Phi_{B\_j}^{Ab}$ and Equation (10), we obtain $\sum_{j\in S_i^d} \omega_j^{Ra} > \sum_{j\in S_i^d} \omega_j^{Ab}$ and $q^{t(Ra)} < q^{t(Ab)}$. Then, we infer that $1 - \frac{\Pi_{SC}\left(q^{t(Ra)}\right)}{\Pi_{SC}(q^*)} > 1 - \frac{\Pi_{SC}\left(q^{t(Ab)}\right)}{\Pi_{SC}(q^*)} > 0$.

Because $\Pi_{M\_i}/\Pi_{M\_SC} > 1/N$, we obtain

$$\frac{\Pi_{M\_i}}{\Pi_{M\_SC}} - \frac{\frac{\Pi_{SC}(q^*)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}}{\Pi_{SC}(q^*)} = \frac{(N\Pi_{M\_i} - \Pi_{M\_SC})\left(\Pi_{SC}(q^*) - \Pi_{M\_SC}\right)}{N\Pi_{M\_SC}\Pi_{SC}(q^*)} > 0.$$

Therefore, we infer that

$$\frac{\Pi_{M\_i}}{\Pi_{M\_SC}}\left(1 - \frac{\Pi_{SC}\left(q^{t(Ra)}\right)}{\Pi_{SC}(q^*)}\right) - \frac{\frac{\Pi_{SC}(q^*)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}}{\Pi_{SC}(q^*)}\left(1 - \frac{\Pi_{SC}\left(q^{t(Ab)}\right)}{\Pi_{SC}(q^*)}\right) > 0,$$

and

$$\frac{\Pi_{M\_i}}{\Pi_{M\_SC}} - \frac{\Pi_{M\_i}}{\Pi_{SC}(q^*)} - \frac{\frac{\Pi_{SC}(q^*)-\Pi_{M\_SC}}{N} + \Pi_{M\_i}}{\Pi_{SC}(q^*)} + \frac{\Pi_{M\_i}}{\Pi_{SC}(q^*)} = \frac{(N\Pi_{M\_i} - \Pi_{M\_SC})\left(\Pi_{SC}(q^*) - \Pi_{M\_SC}\right)}{N\Pi_{M\_SC}\Pi_{SC}(q^*)} > 0$$

Hence, considering Equation (16), we obtain $I^{Ra}_{\Delta\Phi_{pi1}} \supset I^{Ab}_{\Delta\Phi_{pi1}}$, $i = 1,2,\ldots,N, i \notin T$. □

Proof of Property 3. The function of the wholesale price charged by member j under the RS contract is $\omega_j = \sum_{k\in S_j} C_k - \sum_{k\in S_j} \Phi_{B\_k} C$. Therefore, we obtain $\omega_j^{\min} \le \sum_{k\in S_j} C_k - \sum_{k\in S_j} \Phi_{B\_k} C \le \omega_j^{\max}$ and

$$\frac{\sum_{k\in S_j} C_k - \omega_j^{\max}}{C} \le \sum_{k\in S_j} \Phi_{B\_k} \le \frac{\sum_{k\in S_j} C_k - \omega_j^{\min}}{C}.$$

This result means that the profit allocation range under the RS contract is

$$I^1_{\sum_{k\in S_j}\Phi_k} = \left[\frac{\sum_{k\in S_j} C_k - \omega_j^{\max}}{C}, \frac{\sum_{k\in S_j} C_k - \omega_j^{\min}}{C}\right], \quad j = 2, 3, \ldots, N.$$

The function of the wholesale price charged by member j under the RSB contract (which accounts for budget constraints) is $\omega_j = \sum_{k\in S_j} C_k - \left(\Delta\Phi_{ij} + \sum_{k\in S_j} \Phi_{B\_k}\right) C$.

Therefore, we obtain $\omega_j^{\min} \le \sum_{k\in S_j} C_k - \left(\Delta\Phi_{ij} + \sum_{k\in S_j} \Phi_{B\_k}\right) C \le \omega_j^{\max}$ and

$$\frac{\sum_{k\in S_j} C_k - \omega_j^{\max}}{C} \le \Delta\Phi_{ij} + \sum_{k\in S_j} \Phi_{B\_k} \le \frac{\sum_{k\in S_i} C_k - \omega_j^{\min}}{C}.$$

Because $\Delta\Phi_{pij1} - \Delta\Phi_{pij2} = \Delta\Phi_{ij}$, we infer that

$$\frac{\sum_{k\in S_j} C_k - \omega_j^{\max}}{C} + \Delta\Phi_{pij2} \le \sum_{k\in S_j} \Phi_{B\_k} + \Delta\Phi_{pij1} \le \frac{\sum_{k\in S_i} C_k - \omega_j^{\min}}{C} + \Delta\Phi_{pij2}.$$

This result means that the profit allocation range under the RSB contract is

$$I^2_{\sum_{k\in S_j}\Phi_k} = \left[\frac{\sum_{k\in S_j} C_k - \omega_j^{\max}}{C} + \Delta\Phi_{pij2}, \frac{\sum_{k\in S_i} C_k - \omega_j^{\min}}{C} + \Delta\Phi_{pij2}\right].$$

As Theorem 2 shows, under the RSB contract, the expected profits of members i and j are $(\Phi_{B\_i} - \Delta\Phi_{pi1})\Pi_{SC}(q)$ and $(\Phi_{B\_j} + \Delta\Phi_{pij1})\Pi_{SC}(q)$, respectively. Because member i has a budget constraint and $\Delta\Phi_{ij} < 0$, member j charges a lower wholesale price. Consequently, $\Delta\Phi_{pij1}$ should be greater than or equal to 0 for all $j \in S_i^d$. From $\Delta\Phi_{pij1} - \Delta\Phi_{pij2} = \Delta\Phi_{ij}$, we can infer that $\Delta\Phi_{pij2}$ is greater than or equal to $-\Delta\Phi_{ij}$. When $\Delta\Phi_{ij} = \Delta\Phi_{pij2} = 0$, $I^1_{\sum_{k\in S_j}\Phi_k} = I^2_{\sum_{k\in S_j}\Phi_k}$. Because $\Delta\Phi_{pij2}$ can also be set as any value between $-\Delta\Phi_{ij}$ and 0, some profit allocation scenarios under the RSB contracts are infeasible under RS contracts. The RSB contract offers more flexible profit allocations than does the RS contract.