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Manufacturing setup cost reduction and quality improvement for the distribution free continuous-review inventory model with a service level constraint

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ABSTRACT

Over several decades, the continuous-review inventory model has been widely studied based on various assumptions and restrictions such as those related with quality improvement, service level constraint, and setup cost reduction. We extend Moon and Choi's [1] model by assuming setup cost reduction and quality improvement. A distribution free approach is employed such that only mean and standard deviation need to be known. The total system cost is minimized with respect to decision variables against the worst possible distribution scenario. The benefit of using quality improvement and setup cost reduction in this model is shown. Numerical examples show that this model offers significant improvements over existing models. Finally, sensitivity analysis of the key parameters is also presented.

1. Introduction

According to the literature, most of the inventory terms are considered as constants. Basic inventory models are based on assumptions of perfect quality products, but in reality, can the quality of products be perfectly guaranteed? No. Due to long-term use of machinery systems, equipment breakdowns, and labor problems, the production system cannot always generate perfect items, and as a result, the quality of products must be improved for benefit the brand image of the industry. In addition, according to the literature on inventory models, the setup cost is typically considered as fixed. However, setup costs can be reduced by use of an initial investment. To incorporate improvement efforts into the model, Silver [2] proposed that researchers engage in "changing the givens" by regarding constant terms as decision variables. Since Silver [2] made the suggestion, many researchers developed new research models by changing parameters into decision variables, including those for setup times and costs reduction, lead time reduction, and quality improvement, among others.

To apply probabilistic inventory models in practice, inventory managers must know the distribution function of the lead time demand. However, it is quite difficult and time consuming to determine the exact distribution. Thus, it is very important for industries and researchers to create a procedure by which the total system cost can be calculated without data on the lead time demand distribution. This problem was first successfully addressed by Scarf [3], who developed a min–max solution to the newsvendor problem in which only the mean and standard deviation of the lead time demand distribution were assumed to be known. The solution was beautifully expressed but lengthy and quite difficult to understand. Gallego and Moon [4] made Scarf's [3] ordering rule very easy to comprehend. Subsequently, they discussed different ways to apply distribution free procedures for some inventory models (Moon and Gallego [5]).

Moon and Choi [1] developed a model on the distribution free continuous review inventory system with a service level constraint. Many researchers extended this model with different approaches. Moon and Choi [6] discussed the distribution free newsboy problem with customer balking. Moon and Choi [7] developed the distribution free procedures for make-to-order (MTO) and make-in-advance (MIA) models. They also developed a two-echelon stochastic model of composite policies in a single period. Moon and Yun [8] addressed a distribution free job control problem to determine an optimal release time in which the flow time is a random variable with a known probability distribution; they acknowledged the trade-off between the penalty cost for late delivery and the holding cost for early completion.

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Ouyang and Wu [9] discussed a mixture inventory model involving variable lead times with a service level constraint. They developed an algorithm to obtain the optimal order quantity and lead time when the probability distribution of lead time demand is normal or the distribution is free. Ernst and Powell [10] extended the inventory model by including manufacturers' incentives to improve retail service levels. The results of their model indicated that optimal incentive levels for manufacturers, and the resulting shares of the manufacturers' and retailers' increased profits, are particularly sensitive to the underlying variability of demand and to the relative variability of any additional demand introduced by high service levels. Janssen et al. [11] came up with an excellent idea regarding the (Q, R, s) inventory model when the demand is modeled as a compound Bernoulli process. They presented an approximation method to compute the reorder point s in (Q, R, s) inventory model with a service level restriction.

Chu et al. [12] extended the distribution free model to an improved inventory model with a service level constraint and variable lead time. Lee et al. [13] developed a computational algorithm for an inventory model with a service level constraint as well as a lead time demand with a mixture of distributions. In this model, they considered the negative exponential backorder rate which is dependent on lead time length as affected by shortage amounts. Gallego et al. [14] discussed a model on inventory management under highly uncertain demand. Jha and Shanker [15] developed a two-echelon supply chain inventory model with controllable lead time and a service level constraint. A well-organized procedure reveals the bounds on the number of shipments. They developed an improved algorithm to obtain the optimal solution for the model. By looking at buyer and vendor savings, they also compared a joint optimization model to one in which players minimized their own costs independently.

Applying distribution free continuous-review inventory model with a service level constraint of Moon and Choi [1], Tajbakhsh [16] derived an iterative procedure to obtain an optimal solution and some closed-form expressions. Janssens and Ramaekers [17] developed a linear programming formulation for an inventory management decision problem with a service level constraint. Brito and Almeida [18] discussed a model applied to a multi-attribute utility newsvendor with partial backlogging. Jha and Shanker [19] extended Brito and Almeida's [18] model to a single-vendor multi-buyer integrated production-inventory model with controllable lead time and a service level constraint. To reduce imperfect production and setup cost, some initial investment helps in nearly every industry. A large initial investment used to fund updated machineries and other moderation to improve the system lowers each independent setup cost and helps reduce the number of imperfect items produced. Chuang et al. [20] described a periodic review inventory model with controllable setup costs and lead times. Ma and Qiu [21] developed a continuous review inventory model with a controllable lead time and setup cost reduction in the presence of a service level constraint. Sarkar and Majumder [22] discussed an integrated inventory model with vendor's setup cost reduction.

They used a logarithmic investment function but did not consider any service level constraint. Sarkar and Moon [23] derived an inventory model with setup cost reduction, quality improvement, and variable backorder rate, but they did not consider any service level constraints. They used the same logarithmic investment function for setup cost reduction and quality improvement as Ouyang et al. [24] and extended it by assuming a variable backorder rate. Nye et al. [25] used an optimal investment to reduce the setup in manufacturing systems with work-in-process inventories. Chen [26] developed a model to rework imperfect items and to offer an inspection strategy that prevents maintenance error. Pal et al. [27] discussed an inventory model for stochastic demand in an imperfect production system. Pasandideh et al. [28] optimized a bi-objective multi-product inventory model with defective items and limited orders with two separate algorithms. Recently, Sarkar et al. [29] developed a manufacturing model for derivation of a random defective rate to control the production of imperfect products. See Table 1 for a comparison of existing models and our model based on the service level constraint.

In this study, we sought to obtain the optimal cost with respect to optimal order quantity, improved quality factors, and reduced setup costs under the effect of a service level constraint by applying the distribution free approach. In our proposed model, we extend the model of Moon and Choi [1] with the intention of reducing setup costs and improving product quality. According to Porteus [30], a logarithmic expression can be used to reduce the setup cost and also improve the quality of the product. Hence, we use the logarithmic expression for both setup cost reduction and quality improvement procedures. Inclusion of the service level constraint makes the model more realistic. The rest of the paper is organized as follows: the mathematical model is shown in Section 2. In Section 3, numerical examples illustrate the model and the sensitivity analysis is shown. Finally, conclusions are given in Section 4.

2. Mathematical model

The following notation is used to develop the model.

Decision variables

A	setup cost per setup (\$/setup)
Q	order quantity (units)
r	reorder point
ϕ	probability of the production process which may go to *out-of-control* state

Parameters

A_0	initial setup cost per setup (\$/setup)
D	average demand per year (units/year)
ϕ_0	initial probability of the production process which may go to *out-of-control* state
μ	mean of the lead time demand
σ	standard deviation of the lead time demand
h	holding cost per unit per year (\$/unit/year)
α	annual fractional cost of capital investment
X	lead time demand which has a probability distribution function F
$E(x)$	expected value of x
x^+	max $\{x, 0\}$
$E(X-r)^+$	expected shortage per replenishment cycle
m	cost of replacing a defective unit (\$/defective unit)
B	percentage decrease in setup cost per dollar increase in the investment to reduce the setup cost
b	percentage decrease in *out-of-control* probability per dollar increase in the investment to reduce the *out-of-control* probability
k	safety factor
$S_A(A)$	investment for setup cost reduction
$S_\phi(\phi)$	investment for quality improvement
β	fraction of customer's demand that is satisfied regularly

The following assumptions are used to develop the model:

1. We consider a continuous-review inventory model with a stochastic lead time. The mean and standard deviation of the lead time demand distribution are known. Generally, it is very difficult to collect the information regarding the lead time demand distribution. If we know the distribution, then the expected

Table 1
Contribution of different authors.

Author(s)	Distribution free approach	Service level constraint	Optimal order quantity	Setup cost reduction	Quality improvement
Moon and Choi [1]	√	√	√		
Moon and Gallego [5]	√		√		
Ouyang and Wu [9]	√	√	√		
Ernst and Powell [10]		√	√		
Janssen et al. [11]		√	√		
Chu et al. [12]	√	√	√		
Lee et al. [13]	√	√	√		
Gallego et al. [14]		√	√		
Tajbakhsh [16]	√	√	√		
Janssens and Ramaekers [17]		√			
Brito and Almeida [18]		√			
Jha and Shanker [19]		√			
Ma and Qiu [21]	√	√	√	√	
Porteus [30]			√	√	√
This study	√	√	√	√	√

shortage amount can be easily calculated by using the known distribution. However, if we do not know the distribution of the lead time demand, then we must use statistical procedures, and the distribution free approach is the best method because we can use the lemma of Gallego and Moon [4]. Therefore, we use the distribution free approach to develop the model.

2. To make a product more perfect, the relationship between lot size and product quality is considered. The production process is assumed to be at an *in-control* state at the beginning of production. During the production process, the process may shift to an *out-of-control* state, and then it starts to produce defective units and continues until the entire lot is produced (see, for instance, Porteus [30]).
3. Additional investment is a good strategy to reduce imperfect production during the out-of-control state. For instance, to reduce an out-of-control probability from 0.00002 to 0.000018, an investment of \$200 may be needed, and again another \$200 can be used to reduce it to 0.000016, and so on. Therefore, the best way to reduce the imperfect production is by using some initial investment. We assume a capital investment $S_\phi(\phi)$ (refer to Porteus [30]) to improve the process quality and reduce *out-of-control* probability as:

$$S_\phi(\phi) = b\ln\left(\frac{\phi_0}{\phi}\right), \quad \text{for } 0 < \phi \le \phi_0$$

where $b = 1/\zeta$, and ζ = the percentage decrease in A per dollar increase in $S_\phi(\phi)$. From the investment function if $S_\phi(\phi_0) = 0$, then there is no investment for quality improvement. If there is at least some investment, then the value of ϕ will be reduced for every stage, which indicates the improvement of product quality. The benefit of using the logarithmic function is that it is convex within the range defined for the investment function.
4. The basic inventory model is generally based on the assumption of fixed setup cost. By using investment, we can reduce the setup cost of the model. The initial investment may be high, but total cost will be reduced in each stage by using the initial investment function. We assume a logarithmic investment function for this purpose (refer to Porteus [30]).
5. The service level constraint is used to make the model more realistic.

We extend the model of Moon and Choi [1] with the concept of setup cost reduction and quality improvement. We use the cost function of Moon and Choi [1] as

$$C(Q,r) = \frac{AD}{Q} + h\left(\frac{Q}{2} + r - \mu\right) \tag{1}$$

Most continuous-review inventory models are based on a fixed setup cost. However, an initial investment can be used to reduce the setup cost of the whole system. We use a logarithmic expression to reduce setup cost (see, for instance, Porteus [30]). The cost expression is as follows:

$$S_A(A) = B\ln\left(\frac{A_0}{A}\right) \quad \text{for } 0 < A \le A_0 \tag{2}$$

where $B = 1/\tau$, τ is the percentage decrease in A per dollar increase in $S_A(A)$.

During long production processes, machinery systems may produce low quality products, which may result in revenue loss and an impugned industry reputation. Therefore, to maintain the brand image of the industry, firms may choose to make some initial investments that improve the quality of products. Although this strategy for quality improvement may lead to increases in total system costs, setup cost reduction counter balances the added expense such that the total cost of the system is maintained. The investment for quality improvement is as follows:

$$S_\phi(\phi) = b\ln\left(\frac{\phi_0}{\phi}\right) \quad \text{for } 0 < \phi \le \phi_0 \tag{3}$$

where $b = 1/\zeta$ and ζ is the percentage decrease in ϕ per dollar increase in $S_\phi(\phi)$. Hence, total investment for quality improvement and setup cost reduction can be written as follows:

$$S(\phi, A) = S_\phi(\phi) + S_A(A) = Y - b\ln\phi - B\ln A$$
$$\text{where } Y = b\ln\phi_0 + B\ln A_0,\ 0 < A \le A_0, \text{ and } 0 < \phi \le \phi_0 \tag{4}$$

After a long production process, the system can get *out-of-control* state with a probability ϕ (generally, ϕ is very small and close to zero), and once it is in *out-of-control*, it produces imperfect items continuously until the entire lot is produced. Due to this relationship between lot size and *out-of-control* product quality, the expected number of defective items during a production run cycle is approximated by $DQ\phi/2$ (see Appendix A), and the cost of replacing a defective item is m. Therefore, the expected annual defective cost is: $mDQ\phi/2$ (see Appendix B).

After the absorption of the imperfect items, the expected annual total cost can be expressed as

$$\Gamma(Q, r, \phi) = C(Q, r) + \frac{mDQ\phi}{2} \tag{5}$$

Therefore, the cost function of our model becomes

$$EAC(Q, r, \phi, A) = \frac{AD}{Q} + h\left(\frac{Q}{2} + r - \mu\right) + \alpha(Y - b\ln\phi - B\ln A) + \frac{mDQ\phi}{2} \quad \text{for } 0 < A \le A_0 \text{ as well as } 0 < \phi \le \phi_0 \tag{6}$$

We use this model to obtain the minimum expected cost that is subject to a specified fill rate. This service measure is considered the fraction of customer's demand that is satisfied regularly and is denoted by β.

$$\beta = \frac{\text{Expected demand satisfied per replenishment cycle}}{\text{Expected demand per replenishment cycle}}$$

Therefore, we write

$$\beta = 1 - \frac{E(X-r)^+}{Q}$$

which moderates as

$$E(X-r)^+ = (1-\beta)Q$$

Suppose $\omega = r - \mu$, then the cost function can be represented as follows:

$$EAC(Q, \omega, \phi, A) = \frac{AD}{Q} + h\left(\frac{Q}{2} + \omega\right) + \alpha(Y - b\ln\phi - B\ln A) + \frac{mDQ\phi}{2} \quad \text{for } 0 < A \le A_0 \text{ as well as } 0 < \phi \le \phi_0 \tag{7}$$

Generally, the lead time distribution function is unknown in practice. Hence, we are unable to obtain the value of the expected shortages per replenishment cycle $E(X-r)$. Therefore, we apply the min-max distribution free approach as suggested by Scarf [3] and further simplified by Gallego and Moon [4]. Using this method, we obtain the least favorable distribution function in F for each (Q, A) and then minimize the annual total cost with respect to the decision variables. Therefore, the problem is reduced to:

$$\text{Min Max}_{F\in\mathbf{F}} EAC(Q, \omega, \phi, A) \tag{8}$$

To obtain the least favorable distribution in $\mathbf{F}$, we can use the following lemma in Gallego and Moon [4].

$$E(X-r)^+ \le \frac{1}{2}\sigma\sqrt{L}(\sqrt{1+k^2} - k) \text{ for any } F \in \mathbf{F} \tag{9}$$

Moreover, this upper bound is tight. By considering ω_β as the safety stock with respect to β, we get

$$\frac{\sqrt{\sigma^2 + \omega_\beta^2} - \omega_\beta}{2} = (1-\beta)Q$$

which gives

$$\omega_\beta = \frac{\sigma^2}{4(1-\beta)Q} - (1-\beta)Q$$

Using this and taking Q as Q_β, we obtain

$$EAC(Q_\beta, \phi, A) = \frac{AD}{Q_\beta} + \left(\beta - \frac{1}{2}\right)hQ_\beta + \frac{h\sigma^2}{4(1-\beta)Q_\beta} + \alpha(Y - b\ln\phi - B\ln A) + \frac{mDQ_\beta\phi}{2} \quad \text{for } 0 < A \le A_0 \text{ as well as } 0 < \phi \le \phi_0 \tag{10}$$

To find the necessary conditions for the minimum cost of $EAC(Q_\beta, \phi, A)$, we have

$$\frac{\partial EAC}{\partial Q_\beta} = \frac{-AD}{(Q_\beta)^2} + \left(\beta - \frac{1}{2}\right)h - \frac{h\sigma^2}{4(1-\beta)(Q_\beta)^2} + \frac{mD\phi}{2}$$

$$\frac{\partial EAC}{\partial \phi} = -\frac{\alpha b}{\phi} + \frac{mDQ}{2}$$

$$\frac{\partial EAC}{\partial A} = -\frac{\alpha B}{A} + \frac{D}{Q}$$

Therefore, we obtain the optimal values of our decision variables as

$$Q_\beta = \sqrt{\frac{4(1-\beta)AD + h\sigma^2}{2(1-\beta)\{(2\beta-1)h + mD\phi\}}} \tag{11}$$

$$\omega_\beta = \frac{\sigma^2}{4(1-\beta)Q_\beta} - (1-\beta)Q_\beta \tag{12}$$

$$A = \frac{\alpha B Q_\beta}{D} \tag{13}$$

$$\phi = \frac{2\alpha b}{mDQ_\beta} \tag{14}$$

We then test the sufficient condition for minimization of the cost function with respect to the decision variables. From the sufficient conditions, we obtain the following values:

$$\frac{\partial^2 EAC}{\partial Q_\beta^2} = \frac{2AD}{Q_\beta^3} + \frac{h\sigma^2}{2(1-\beta)(Q_\beta)^3}$$

$$\frac{\partial^2 EAC}{\partial A^2} = \frac{\alpha B}{A^2}$$

$$\frac{\partial^2 EAC}{\partial \phi^2} = \frac{\alpha b}{\phi^2}$$

$$\frac{\partial^2 EAC}{\partial Q_\beta \partial A} = \frac{\partial^2 EAC}{\partial A \partial Q_\beta} = -\frac{D}{Q_\beta^2}$$

$$\frac{\partial^2 EAC}{\partial Q_\beta \partial \phi} = \frac{\partial^2 EAC}{\partial Q_\beta \partial \phi} = \frac{mD}{2}$$

$$\frac{\partial^2 EAC}{\partial \phi \partial A} = \frac{\partial^2 EAC}{\partial \phi \partial A} = 0.$$

To find the global minimum EAC, we use the following lemma.

Lemma 1. *If the Hessian matrix for EAC is always positive definite at the optimal value (Q_β^*, ϕ^*, A^*), then EAC contains the global minimum at the optimal solution (Q_β^*, ϕ^*, A^*).*

Proof. To show the Hessian matrix for EAC is always positive definite, we must prove that all minors are positive definite.

$$H_{ii} = \begin{pmatrix} \frac{\partial^2 EAC}{\partial Q_\beta^{*2}} & \frac{\partial^2 EAC}{\partial Q_\beta^* \partial A^*} & \frac{\partial^2 EAC}{\partial Q_\beta^* \partial \phi^*} \\ \frac{\partial^2 EAC}{\partial A^* \partial Q_\beta^*} & \frac{\partial^2 EAC}{\partial A^{*2}} & \frac{\partial^2 EAC}{\partial A^* \partial \phi^*} \\ \frac{\partial^2 EAC}{\partial \phi^* \partial Q_\beta^*} & \frac{\partial^2 EAC}{\partial \phi^* \partial A^*} & \frac{\partial^2 EAC}{\partial \phi^{*2}} \end{pmatrix}$$

and the corresponding principal minors at the optimal value are

$$\det H_{11} = \det\left(\frac{\partial^2 EAC}{\partial Q_\beta^{*2}}\right) = \frac{2A^*D}{(Q_\beta^*)^3} + \frac{h\sigma^2}{2(1-\beta)(Q_\beta^*)^3} > 0$$

Table 2
The comparison between reduced setup cost and fixed setup cost.

Distribution free model with a service level constraint, quality improvement, and setup cost reduction		Distribution free model with a service level constraint, quality improvement, and fixed setup cost	
(Q, R, ϕ, A)	EAC_1	(Q, R, ϕ)	EAC_2
(74,14,0.0000241579,71)	$2371	115,14,0.00001541)	$2855

$$\det H_{22} = \det\begin{pmatrix} \dfrac{\partial^2 EAC}{\partial Q_\beta^{*2}} & \dfrac{\partial^2 EAC}{\partial Q_\beta^* \partial A^*} \\ \dfrac{\partial^2 EAC}{\partial A^* \partial Q_\beta^*} & \dfrac{\partial^2 EAC}{\partial A^{*2}} \end{pmatrix}$$

$$= \left[\frac{2A^*D}{(Q_\beta^*)^3} + \frac{h\sigma^2}{2(1-\beta)(Q_\beta^*)^3}\right]\left[\frac{\alpha B}{(A^*)^2}\right] - \left[-\frac{D}{(Q_\beta^*)^2}\right]^2$$

$$= \left[\frac{2A^*D}{(Q_\beta^*)^3} + \frac{h\sigma^2}{2(1-\beta)(Q_\beta^*)^3}\right]\left[\frac{\alpha B}{(A^*)^2}\right] - \frac{1}{(Q_\beta^*)^2}\left[\frac{\alpha B}{A^*}\right]^2$$

$$= \left[\frac{\alpha B}{(A^*)^2(Q_\beta^*)^3}\right]\left[2A^*D + \frac{h\sigma^2}{2(1-\beta)} - \alpha BQ_\beta^*\right]$$

$$= \left[\frac{\alpha B}{(A^*)^2(Q_\beta^*)^3}\right]\left[2A^*D + \frac{h\sigma^2}{2(1-\beta)} - A^*D\right]$$

$$= \left[\frac{\alpha B}{(A^*)^2(Q_\beta^*)^3}\right]\left[A^*D + \frac{h\sigma^2}{2(1-\beta)}\right] > 0$$

$$\det H_{33} = \det\begin{pmatrix} \dfrac{\partial^2 EAC}{\partial Q_\beta^{*2}} & \dfrac{\partial^2 EAC}{\partial Q_\beta^* \partial A^*} & \dfrac{\partial^2 EAC}{\partial Q_\beta^* \partial \phi^*} \\ \dfrac{\partial^2 EAC}{\partial A^* \partial Q_\beta^*} & \dfrac{\partial^2 EAC}{\partial A^{*2}} & \dfrac{\partial^2 EAC}{\partial A^* \partial \phi^*} \\ \dfrac{\partial^2 EAC}{\partial \phi^* \partial Q_\beta^*} & \dfrac{\partial^2 EAC}{\partial \phi^* \partial A^*} & \dfrac{\partial^2 EAC}{\partial \phi^{*2}} \end{pmatrix}$$

$$= \left[\frac{\alpha^2 Bb}{(A^*)^2(\phi^*)^2(Q_\beta^*)^3}\right]\left[A^*D + \frac{h\sigma^2}{2(1-\beta)}\right] - \frac{m^2D^2\alpha B}{4(A^*)^2}$$

$$= \left[\frac{\alpha^2 Bb}{(A^*)^2(\phi^*)^2(Q_\beta^*)^3}\right]\left[A^*D + \frac{h\sigma^2}{2(1-\beta)}\right] - \left[\frac{\alpha^3 Bb^2}{(A^*)^2(\phi^*)^2(Q_\beta^*)^2}\right]$$

$$= \left[\frac{\alpha^2 Bb}{(A^*)^2(\phi^*)^2(Q_\beta^*)^3}\right]\left[A^*D + \frac{h\sigma^2}{2(1-\beta)} - \alpha bQ_\beta^*\right]$$

$$= \left[\frac{\alpha^2 Bb}{(A^*)^2(\phi^*)^2(Q_\beta^*)^3}\right]\left[A^*D + \frac{h\sigma^2}{2(1-\beta)} - \frac{A^*Db}{B}\right]$$

$$= \left[\frac{\alpha^2 Bb}{(A^*)^2(\phi^*)^2(Q_\beta^*)^3}\right]\left[A^*D\left(1-\frac{b}{B}\right) + \frac{h\sigma^2}{2(1-\beta)}\right] > 0$$

Because all of the principal minors are positive definite, *EAC* contains the global minimum at the optimal solution (Q^*, ϕ^*, A^*). The cost function is a non-linear function, and continuous quality improvement and setup cost reduction are needed to obtain the optimal solution. Therefore, we use the following algorithm to obtain an optimal solution of the problem.

Solution algorithm

Step 1 Consider $A = A_0$ and $\phi = \phi_0$.
Step 2 Input h, D, σ, μ, m, α, b, and B.
Step 3 From Eq. (11), the value of Q_β is calculated.
Step 4 Utilizing the value of Q_β, the values of ω_β, A, and ϕ are calculated from Eqs. (12)–(14).
Step 5 Repeat *Step* 3–4 until no change occurs in the values of Q_β, ω_β, A, and ϕ. Denote the solution by $\tilde{Q}_\beta$, $\tilde{\omega}_\beta$, $\tilde{A}$, and $\tilde{\phi}$.
Step 6 Compare the values of $\tilde{\phi}$ with ϕ_0 and $\tilde{A}$ with A_0.
Step 6A If $\tilde{\phi} \geq \phi_0$ and $\tilde{A} \geq A_0$, then goto Step 7.
Step 6B If $\tilde{\phi} < \phi_0$ and $\tilde{A} \geq A_0$, then no setup cost reduction is needed which implies $A^* = A_0$. Utilizing the value A_0, we consider the same procedure from Step 1–5. If the new $\tilde{\phi}$ is less than ϕ_0, then the optimal solution is $(Q^*, \omega_\beta^*, \phi^*, A_0)$ and goto Step 8. Otherwise, goto Step 7.
Step 6C If $\tilde{\phi} \geq \phi_0$ and $\tilde{A} < A_0$, then no quality improvement is needed which implies $\phi^* = \phi_0$. Utilizing the value ϕ_0, we consider the same procedure from Step 1-5. If the new $\tilde{A}$ is less than A_0, then the optimal solution is $(Q^*, \omega_\beta^*, \phi_0, A^*)$ and goto Step 8. Otherwise, goto Step 7.
Step 6D If $\tilde{\phi} < \phi_0$ and $\tilde{A} < A_0$, then the optimal solution is $(Q^*, \omega_\beta^*, \phi^*, A^*)$ and goto Step 8.
Step 7 Replace A by A_0 and ϕ by ϕ_0 and follow the procedure from Step 1–5 and the optimal solution is $(Q^*, \omega_\beta^*, \phi_0, A_0)$.
Step 8 After getting the optimal solution from any of the above steps, the minimum expected annual cost can be calculated from Eq. (10).

3. Numerical examples

The following parametric values (from Lee et al. [13]) were used in the numerical examples: $D = 600$ units/year, $A_0 = \$200$/setup, $h = \$20$/unit/year, $\sigma = 7$ units/week, $\mu = 11$ units/week, $\phi_0 = 0.0002$, $m = \$75$/defective unit, $\alpha = 0.1$/dollar/year, $b = 400$, $B = 5800$.

Example 1. In this example, we compare the results between two models: one with reduced setup cost and another with fixed setup cost. The comparison is summarized in Table 2.

Example 2. In this example, we compare the results between two models: one with improved quality and another with fixed quality. The comparison is summarized in Table 3.

Table 3
The comparison between the improved quality and fixed quality.

Distribution free model with a service level constraint, quality improvement, setup cost reduction		Distribution free model with a service level constraint, setup cost reduction, and fixed quality	
(Q, R, ϕ, A)	EAC_1	(Q, R, A)	EAC_2
(74,18,0.00002415,71)	$2371	(57,21,55)	$2406

Table 4
The comparison of a model showing improved quality and reduced setup cost to a model showing fixed quality and fixed setup cost.

Distribution free model with a service level constraint, quality improvement, and setup cost reduction		Distribution free model with a service level constraint	
(Q, R, ϕ, A)	EAC_1	(Q, R)	EAC_2
(74,18,0.00002415,71)	$2371	(97,15)	$3079

Table 5
The comparison of this model with other models.

	Sarkar and Moon [23]	Moon et al. [31]	This model
Order quantity	118.87	125.03	74.00
Reorder point	76.86	32.927	14.00
Reduced setup cost	114.91	–	71.00
Reduced *out-of-control* probability	0.00001496	–	0.00002415
Total cost	3500.73	2400.61	2371.00

"–" indicates that the parameter is not included in the model.

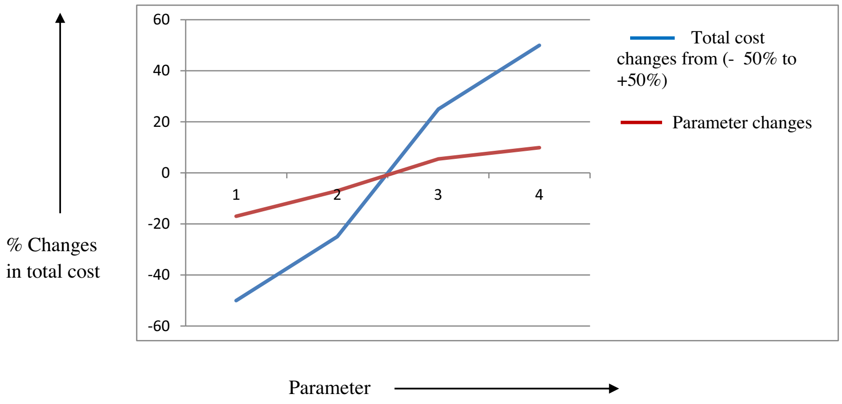

Fig. 1. Changes of parameter A_0 versus % change in total cost.

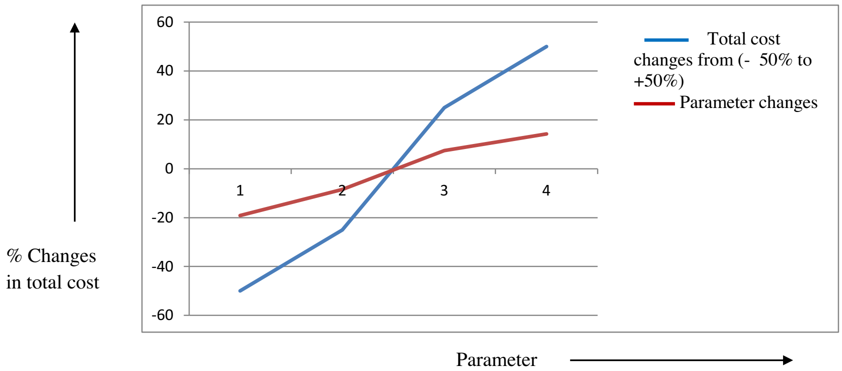

Fig. 2. Changes of parameter h versus % change in total cost.

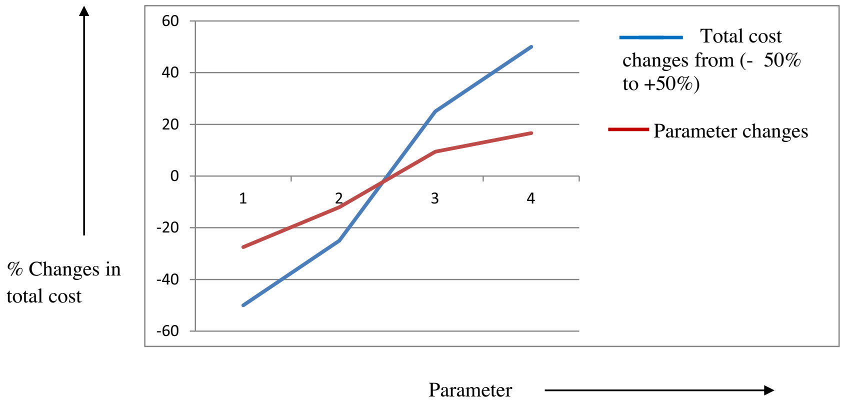

Fig. 3. Changes of parameter α versus % change in total cost.

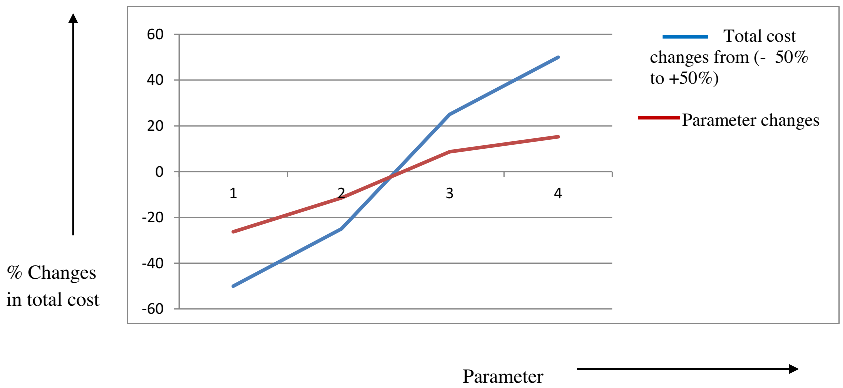

Fig. 4. Changes of parameter B versus % change in total cost.

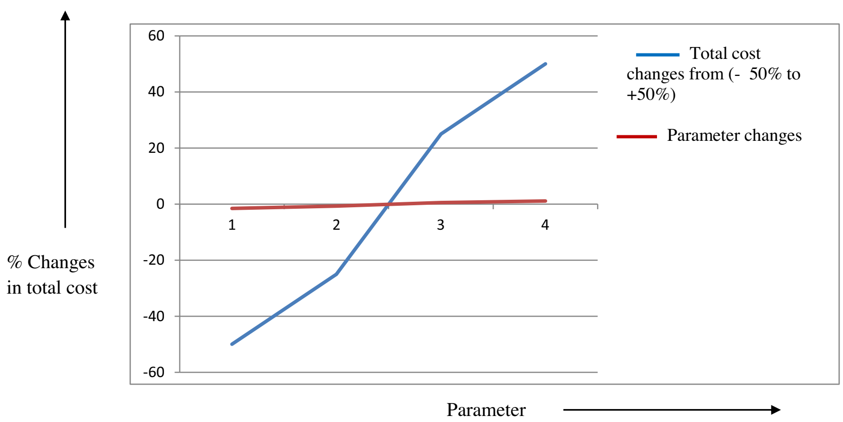

Fig. 5. Changes of parameter b versus % change in total cost.

Example 3. In this example, we compare the results between two models: one with reduced setup cost and improved quality and another with only a service level constraint. The comparison is given in Table 4.

Example 4. In this example, we compare the results between our model and models of other authors: The comparison is given in Table 5.

3.1. Sensitivity analysis

The sensitivity analysis has been done for each key parameter by changing −50%, −25%, +25%, and +50% one at a time and keeping the remaining parameters unchanged. The percentage change in the optimal cost indicates that some parameters such as α, A_0, h, and B are more sensitive than b. Parameter b shows the least sensitivity. From this analysis, we see that increasing values of all key parameters result in increased optimal costs of the whole system. The effect of system parameter changes on the optimal total cost is

Table 6
Sensitivity analysis of key parameters.

Parameters	Changes (in %)	*EAC*
A_0	−50%	−16.95
	−25%	−7.03
	+25%	+5.45
	+50%	+9.91
h	−50%	−19.10
	−25%	−8.46
	+25%	+7.39
	+50%	+14.20
α	−50%	−27.50
	−25%	−12.07
	+25%	+9.40
	+50%	+16.60
B	−50%	−26.32
	−25%	−11.46
	+25%	+8.74
	+50%	+15.23
b	−50%	−1.55
	−25%	−0.70
	+25%	+0.58
	+50%	+1.08

summarized in Table 6, and the changes are also shown graphically in Figs. 1–5.

4. Conclusions

We extended the model of Moon and Choi [1] with the consideration of setup cost reduction and quality improvement. To make more perfect products and to reduce the setup cost, two separate logarithmic expressions were used. By using the initial investment, the product quality was improved as shown in the numerical experiments. The service level constraint was used to obtain the optimal result, and a solution algorithm was developed to obtain an improved result. We constructed a lemma to show the global optimum value of the decision variables. We provided numerical results that showed that our model offers an improvement over existing models because it produces savings in the total expected cost. Furthermore, our model encourages industry managers to think about reductions in setup cost and imperfect production. By using this strategy, they can reduce both of them. The manager of an industry may utilize the proposed algorithm to decide whether to invest to improve the product quality or to reduce the setup cost. This model can be extended to the case of fuzzy demand and budget constraints. Another extension of this model is to consider inflation and the time value of money. Moreover, this model can be extended to account for discrete lead times along with the variable backorder rate.

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Appendix A.

From Porteus [30], the expected number of defective items in a lot size Q is

$$Q - \frac{\tilde{\phi}(1-\tilde{\phi}^Q)}{\phi}$$

As $\tilde{\phi} = 1-\phi \cong 1$, we use the Taylor series expansion of $\tilde{\phi}^Q$ until 2nd order as ϕ is very small and we obtain

$$\tilde{\phi}^Q = e^{(\ln\tilde{\phi})Q} \cong 1 + (\ln\tilde{\phi})Q + \frac{[(\ln\tilde{\phi})Q]^2}{2}$$

Therefore, the number of defective items are

$$\begin{aligned}
&= Q - \frac{\tilde{\phi}(1-\tilde{\phi}^Q)}{\phi}\\
&= Q - \frac{1 - 1 - (\ln\tilde{\phi})Q - \dfrac{(\ln\tilde{\phi})^2Q^2}{2}}{\phi}\\
&= Q - \frac{\dfrac{\phi}{\tilde{\phi}}Q - \dfrac{\phi^2}{2\tilde{\phi}^2}Q^2}{\phi}\\
&= Q - \frac{\phi Q - \dfrac{\phi^2Q^2}{2}}{\phi}\\
&= \frac{\phi Q^2}{2}
\end{aligned}$$

Appendix B.

The expected annual defective cost is

$$\begin{aligned}
&= mD - \frac{mD\tilde{\phi}(1-\tilde{\phi}^Q)}{\phi Q} = mD - \frac{mD\left(1 - 1 - (\ln\tilde{\phi})Q - \dfrac{(\ln\tilde{\phi})^2Q^2}{2}\right)}{\phi Q}\\
&\left[\text{since } \tilde{\phi} = 1-\phi \cong 1 \text{ and } \tilde{\phi}^Q = e^{(\ln\tilde{\phi})Q} \cong 1 + (\ln\tilde{\phi})Q + \frac{[(\ln\tilde{\phi})Q]^2}{2}\right]\\
&= mD - \frac{mD\left(\dfrac{\phi}{\tilde{\phi}}Q - \dfrac{\phi^2}{2\tilde{\phi}^2}Q^2\right)}{\phi Q} = mD - \frac{mD\left(\phi Q - \dfrac{\phi^2Q^2}{2}\right)}{\phi Q}\\
&= mD - mD\left(1 - \frac{\phi Q}{2}\right) = \frac{mD\phi Q}{2}
\end{aligned}$$

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