



A network flow model for the optimal allocation of both foldable and standard containers



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ABSTRACT

This paper considers a multi-port and multi-period container planning problem of shipping companies that use both standard and foldable containers. A company must decide which number of empty containers of each type to purchase and reposition at each port within a defined period to minimize the total purchasing, repositioning, and storage costs.

We develop a model to optimally allocate both foldable and standard containers. We show that a single commodity minimum cost network flow algorithm solves the problem by proving that a two commodity flow problem can be modeled as a single commodity flow problem.

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1. Introduction

In this paper, we deal with a container planning problem of shipping companies. A shipping company hopes to satisfy the demand for empty containers at ports over a time. We assume that the demand at each port in each period is known and that a number of empty containers are supplied from consignees after the freight is unloaded. We also assume that the supply of containers available at each port in each period is known. There usually is an imbalance in the demand and the supply of containers at each port in each period. At deficit ports, a shipping company may use empty containers stored in inventory and/or purchase new ones to meet the demand. Shipping companies also reposition empty containers from surplus ports to deficit ports. There is a time lag between the departure and the arrival of empty containers. In the container planning process, a shipping company must decide how many empty containers to purchase and reposition at each port to minimize the total costs for purchasing, repositioning, and storage. This problem is typical of shipping companies that own both container ships and empty containers. Moreover, this kind of shipping companies own depots in many ports around the world, and they use their own vessels unless the schedule is unworkable at which time they charter or pay other shipping companies to reposition containers.

Repositioning seems to be a smart strategy to balance the flow of containers, but it also incurs cost. Recently, shipping companies

have considered the use of foldable containers to reduce the repositioning cost [5,6]. The foldable container reduces transportation costs by saving on storage space but involves folding/unfolding costs. Therefore, to generate savings, a shipping company must carefully decide the type and number of containers to use. Naturally, use of both standard and foldable containers complicates the container planning problem which we address in this paper.

A large number of research articles have been devoted to a variety of container planning problems with differences in, among other characteristics, the structure of distribution systems, assumptions on the uncertainty of demand and supply, and company objectives [2–6,8,7,9–17]. In Table 1, we compare our model to others. Two models are based on the network concept, but do not apply a min-cost component [2,11]. Moreover, they solve different problems based on different assumptions than addressed by our model.

Most of the past studies were concerned with an unfolded standard container. Two notable exceptions are the studies of Shintani et al. [13] and Moon et al. [10], both of which developed integer programming models to analyze the cost savings of foldable containers. Shintani et al. [12] considered a model for repositioning in the hinterland and Moon et al. [9] illustrated a model for repositioning between seaports. In this paper, we present a model similar to that of Moon et al. [10] and show that a seemingly two commodity (i.e. standard and foldable containers) flow problem can be modeled as a single commodity network flow problem and that therefore integral solutions can be found in polynomial time. The remainder of the paper is organized as follows: Section 2 describes the problem, and in Section 3, we

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Table 1
Summary of relevant studies.

Paper	Container type	Repositioning	Methodology
Cheang and Lim [2]	Standard	No	Decision support system
Crainic et al. [3]	Standard	Yes	MIP
Dong and Song [4]	Standard	Yes	GA and simulation
Konings [5]	Foldable	No	Economic analysis
Konings and Thijs [6]	Foldable	No	Economic analysis
Meng and Wang [8]	Standard	Yes	MIP
Li et al. [7]	Standard	Yes	Heuristic
Moon et al. [9]	Standard	Yes	MIP and hybrid GA
Moon et al. [10]	Standard and foldable	Yes	MIP and heuristic
Shen and Khoong [11]	Standard	Yes	Decision support system
Shintani et al. [12]	Standard	Yes	Hybrid GA
Shintani et al. [13]	Standard and foldable	Yes	MIP
Song and Carter [14]	Standard	Yes	Mathematical program
Song and Dong [15]	Standard	No	Simulation
Song and Dong [16]	Standard	Yes	Heuristic
Song and Zhang [17]	Standard	Yes	Dynamic program
This study	Standard and foldable	Yes	Network flow algorithm

develop the network flow model and prove its validity. Some concluding remarks are presented in Section 4.

2. Problem definition and model formulation

In our container planning problem, a shipping company must satisfy the demand for empty containers at a set of ports over a time horizon. The demand at each port in each period is assumed to be known. Either a standard or a foldable container can be used, but when a foldable container is provided in the folded state, it must undergo an unfolding operation. To satisfy the demand, the shipping company can use containers that are available via different ways: containers stocked in inventory as well as those transported to a port fully loaded and become available after the freight is unloaded in port are called “supplied containers”. We assume that the supply of empty containers to each port in each period is known. Another group, called “repositioned containers”, are empty and transported to a port for repositioning. A supplied foldable container is delivered in the unfolded state while a repositioned foldable container is in the folded state. The available containers that exceed the demand can be stocked or repositioned.

Our model is based on the assumption that the supply of empty containers sent to each port in each period is given as a parameter. In reality, however, the number of containers used to satisfy demand influences the handling of supplies in later periods. We made this assumption because of the difficulty in estimating the time needed for devanning (i.e., the process at the destination port in which containers are delivered to customers and unpacked and then returned to the port).

Despite the weakness caused by the set-container presumption, our model still has practical use, with the primary purpose being to analyze the potential cost savings of using foldable containers in ocean transportation. Through our model, one can compare the costs of various scenarios with different combinations of demands, supplies, and cost elements and can find the optimal portion of foldable containers according to different demand patterns.

For such experiments, an algorithm that quickly generates an optimal solution is invaluable. Even when our model is used for an operational purpose, the weakness of it is mitigated by the decisions made in real world situations. Specifically, supply parameters of early periods are relatively certain because the types of containers to be supplied are determined before the starting point of the planning horizon. Moreover, only the solution values of the decision variables for early periods are typically put into operation. This is, in the planning horizon, a shipping company does not determine all operational decisions for every period based on the initial solution of the model but rather implements the

model in each period after updating the parameters, including supply.

We also assume linear costs. Container transportation costs usually depend on the embarkment port and differ slightly among shipping companies. Due to the small scale of most customers, discounts are not typically applied. Because shipping companies own enough containers and only purchase replacements, they do not order many new ones. Therefore, quantity discounts for purchasing are rare, and we cannot assume quantity discounts in either the unit purchase or transportation cost.

The container planning problem involves decisions on the number of containers to purchase and reposition to minimize the sum of the costs for purchasing, repositioning, holding inventory, and unfolding/folding operations. We use the following notations to describe the parameters:

- P : set of ports, $P = \{1, 2, \dots, n_p\}$
- T : set of periods, $T = \{1, 2, \dots, n_t\}$
- W_{it}^S : number of standard containers being supplied at port i in period t
- W_{it}^F : number of foldable containers being supplied at port i in period t
- D_{it} : demand for empty containers at port i in period t
- H_{it}^S : unit storage cost of a standard container at port i in a period
- H_{it}^F : unit storage cost of a foldable container at port i in a period
- A_{it}^S : unit purchasing price of a standard container at port i
- A_{it}^F : unit purchasing price of a foldable container at port i
- C_{ij}^S : unit repositioning cost of a standard container from port i to port j
- C_{ij}^F : unit repositioning cost of a foldable container from port i to port j
- L_{it}^F : unit folding cost of a foldable container at port i
- L_{it}^U : unit unfolding cost of a foldable container at port i .

In our container planning problem, we determine the values of the following decision variables:

- x_{it}^S : number of standard containers to be used to satisfy the demand at port i in period t
- x_{it}^F : number of foldable containers to be used to satisfy the demand at port i in period t
- f_{ijt}^S : number of standard containers to be transported (for repositioning) from port i to port j in period t
- f_{ijt}^F : number of foldable containers to be transported (for repositioning) from port i to port j in period t
- y_{it}^S : number of standard containers to be purchased at port i in period t
- y_{it}^F : number of foldable containers to be purchased at port i in period t

I_{it}^S : inventory level of standard containers at port i in period t
 I_{it}^F : inventory level of foldable containers at port i in period t .

To clearly explain the problem, we describe the objective and the constraints of it using mathematical formulas constructed by the above parameters and decision variables. Our objective in addressing this problem is to minimize the total cost, which consists of purchasing, repositioning, storage, unfolding, and folding expenses. To describe the unfolding and folding costs, we need to define the number of foldable containers to be unfolded and folded at port i in period t . If we use x_{it}^F foldable containers to satisfy a portion of demand, then the supplied foldable containers are used first because the folding and unfolding operation generates cost. Therefore, if $x_{it}^F \geq W_{it}^F$, then we unfold $x_{it}^F - W_{it}^F$ containers to satisfy a portion of demand. Or, if $x_{it}^F \leq W_{it}^F$, we fold $W_{it}^F - x_{it}^F$ containers to keep in inventory or to reposition. Therefore, we present the total cost function as follows:

$$\begin{aligned} \text{Total cost} = & \sum_{i \in P} \sum_{j \in P - \{i\}} \sum_{t \in T} (C_{ij}^S f_{ijt}^S + C_{ij}^F f_{ijt}^F) \\ & + \sum_{i \in P} \sum_{t \in T} (H_i^S I_{it}^S + H_i^F I_{it}^F) \\ & + \sum_{i \in P} \sum_{t \in T} (A_i^S y_{it}^S + A_i^F y_{it}^F) \\ & + \sum_{i \in P} \sum_{t \in T} \{L_i^U [x_{it}^F - W_{it}^F]^+ + L_i^F [W_{it}^F - x_{it}^F]^+\} \end{aligned}$$

where $[x]^+ = \text{Max}[x, 0]$.

Constraints of our problem can be expressed as Eqs. (1) through (3). We must satisfy all of the demands:

$$x_{it}^S + x_{it}^F = D_{it}, \quad \forall i \in P, t \in T. \tag{1}$$

In addition, x_{it}^S (x_{it}^F) cannot exceed the number of available standard (foldable) empty containers at port $i \in P$ in period $t \in T$, which is determined by the sum of the ending inventory I_{it-1}^S (I_{it-1}^F) from period $t - 1$, the supplied empty containers, the repositioned empty containers, and the purchased containers. The available standard (foldable) empty containers in excess of x_{it}^S (x_{it}^F) can be transported to other ports for repositioning and the remainder makes up the ending inventory. These constraints are depicted in the following two equations:

$$\begin{aligned} I_{it-1}^S + W_{it}^S + \sum_{j \in P - \{i\}, \tau_{ji} < t} f_{jit-\tau_{ji}}^S + y_{it}^S \\ = x_{it}^S + \sum_{j \in P - \{i\}, \tau_{ij} + t \leq n_t} f_{ijt}^S + I_{it}^S, \quad \forall i \in P, t \in T \end{aligned} \tag{2}$$

$$\begin{aligned} I_{it-1}^F + W_{it}^F + \sum_{j \in P - \{i\}, \tau_{ji} < t} f_{jit}^F + y_{it}^F \\ = x_{it}^F + \sum_{j \in P - \{i\}, \tau_{ij} + t \leq n_t} f_{ijt}^F + I_{it}^F, \quad \forall i \in P, t \in T \end{aligned} \tag{3}$$

where τ_{ij} is the transportation time (in terms of time periods) from port i to port j .

Our problem can be defined as the problem of finding the nonnegative integer variables that minimize the total cost function while satisfying (1), (2), and (3). We refer to our problem as the *container planning problem* (CPP).

The CPP is similar to the problem addressed by Moon et al. [10], which featured an additional capacity constraint as follows:

$$f_{ijt}^S + f_{ijt}^F / N \leq K_{ijt}, \quad \forall i, j \in P, t \in T. \tag{4}$$

In Eq. (4), K_{ijt} is the capacity limit of repositioning empty containers from port i to port j during period t , and N is the number

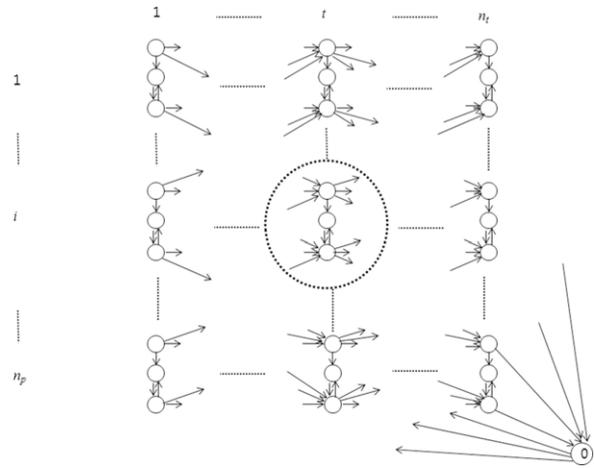


Fig. 1. The high-level structure of our network.

of folded containers that occupy the same space as a single standard container. The model by Moon et al. [10] is an integer programming problem, which requires the application of an integer programming algorithm to obtain an exact solution. However, we show in the next section that instead of integer programming algorithms, we can use an efficient minimum cost network flow algorithm to solve the CPP. Moreover, a good algorithm for the CPP is very useful to solve the problem of Moon et al. [10]. A popular solution approach for solving an integer programming problem is a branch and bound algorithm, and the success of which depends on an efficient method to obtain a lower bound. Lagrangian relaxation is a good method for obtaining a lower bound, and when we apply it to the model by Moon et al. by dualizing Eq. (4), the resulting Lagrangian sub-problem becomes the CPP.

3. Network flow model

In this section, we show that the CPP can be solved using a minimum cost network flow algorithm. For this purpose, we construct a directed network as shown in Fig. 1. We derive an optimal solution of the CPP from a minimum cost flow of the network. Our network contains $3n_p \cdot n_t + 1$ nodes. Among them, $3n_p \cdot n_t$ nodes can be categorized into $n_p \cdot n_t$ groups of three nodes. Each group corresponds to each port in each period. Three nodes, labeled as $i-t-S$, $i-t$, and $i-t-F$, constitute a group corresponding to port i in period t and are used to describe Eqs. (1), (2), and (3), respectively. The other node, labeled node 0, is added to balance the total supply and demand of a network.

In our network, arcs correspond to the decision variables of the CPP. The arcs from node 0 to nodes $i-t-S$ and $i-t-F$ correspond to y_{it}^S and y_{it}^F for each $i \in P$ and $t \in T$. The amount of the flow from node 0 to node $i-t-S$ ($i-t-F$) represents the number of purchased standard (foldable) containers at port i in period t . The arcs from node $i-(t-1)-S$ to $i-t-S$ and from node $i-(t-1)-F$ to $i-t-F$ for each $i \in P$ and $t \in T$ represent inventory. The amount of flow from node $i-(t-1)-S$ to $i-t-S$ represents the number of standard containers carried in inventory at port i at the end of period $t - 1$. The arcs from $i-n_t-S$ and $i-n_t-F$ to node 0 correspond to inventories at port i at the end of the final period n_t . The arcs between nodes associated with different ports represent repositioning. The amount of flow from node $i-t-S$ ($i-t-F$) to $j-(t+\tau_{ij})-S$ ($j-(t+\tau_{ij})-F$) represents the number of the repositioned empty standard (foldable) containers from port i in period t to port j in period $t + \tau_{ij}$ under the assumption that it takes τ_{ij} periods to transport containers from port i to port j .

The arcs from node $i-t-S$ to node $i-t$ and the arcs between $i-t$ and $i-t-F$ for each $i \in P$ and $t \in T$ play a critical role in relating the network with the CPP. The arc from node $i-t-S$ to node $i-t$

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