Modeling and optimization of energy efficient routing in wireless sensor networks

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ABSTRACT

Wireless sensor networks (WSNs) have important applications in remote environmental monitoring and target tracking. The development of WSNs in recent years has been facilitated by the availability of sensors that are smaller, less expensive, and more intelligent. The design of a WSN depends significantly on its desired applications and must take into account factors such as the environment, the design objectives of the application, the associated costs, the necessary hardware, and any applicable system constraints. In this study, we propose mathematical models for a routing protocol (network design) under particular resource restrictions within a wireless sensor network. We consider two types of constraints: the distance between the linking sensors and the energy used by the sensors. The proposed models aim to identify energy-efficient paths that minimize the energy consumption of the network from the source sensor to the base station. The computational results show that the presented models can be used efficiently and applied to other network design contexts with resource restrictions (e.g., to multi-level supply chain networks).

Keywords:
OR in telecommunications
Wireless sensor networks
Energy efficiency
Mixed integer linear program

1. Introduction

Wireless sensor networks (WSNs) have gained worldwide attention in recent years, particularly with the proliferation of Micro-Electro-Mechanical Systems (MEMS) technology, which has facilitated the development of smart sensors. These sensors are small, and require limited processing and computing resources, and are inexpensive compared to traditional sensors. These sensor nodes can sense, measure, and gather information from their environment and transmit the sensed data to the user based on local decision processes. Thousands of mini-computers equipped with sensors are deployed in a particular environment. After activation, the sensors form a self-organized network and provide data. The trend towards wireless communication is increasingly changing electronic devices in almost every sphere of life [1]. Networks of small sensor nodes, called sensor networks, facilitate the monitoring and analysis of complex phenomena over large regions and for long periods of time. Recent advances in sensor network research have made it possible to develop small and inexpensive sensor nodes that can obtain significant amounts of data about physical values. A WSN is a special type of ad hoc network composed of a large number of nodes that are equipped with different sensor devices. This network type has been supported by technological advances in low-power wireless communications and by the silicon integration of various functionalities, including sensing, communication, and processing [2]. A WSN consists of spatially distributed autonomous sensors that cooperatively monitor physical or environmental conditions. In addition to one or more sensors, each node in a sensor network is typically

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equipped with a radio transceiver or other wireless communications device, a small microcontroller, and an energy source, usually a battery. The cost of sensor nodes is similarly variable, ranging from hundreds of dollars to a few pennies, depending on the size of the sensor network and the level of complexity that is required of individual sensor nodes. The size and cost constraints on sensor nodes result in corresponding constraints on resources such as energy, memory, computational speed, and bandwidth [3]. A sensor network is normally a wireless ad hoc network, which means that each sensor supports a multi-hop (multi-level) routing algorithm (such that several nodes may forward data packets to the base station).

In general, a WSN is composed of hundreds or even thousands of small, low cost, low power sensor nodes. These nodes are deployed in an ad hoc fashion to register physical phenomena and usually communicate with each other through wireless communication channels. The design of most wireless sensor networks is based on an ad hoc (multi-hop) network technology that organizes and maintains a group of moving objects equipped with a communication device in an area in which there are no fixed base stations or access points [4]. Although ad hoc network technologies are capable of constructing sensor networks, the design and use of sensor networks to monitor stationary nodes such as construction sites, historic buildings, and bridges can be further simplified to reduce power consumption and overhead. The simplest approach to routing in wireless sensor networks is direct transmission, in which each node transmits its own data directly to the sink. If the base station is far away, the cost of sending data directly to that station will become too large, and the nodes will die quickly.

The development of WSNs was motivated by military applications such as battlefield surveillance. WSNs are now used in many industrial and civilian application areas, including industrial process monitoring and control, machine health monitoring, environment and habitat monitoring, healthcare applications, home automation, traffic control, and scientific exploration in dangerous environments [5]. Domingo [6] examined how to extend hop length while maintaining good energy efficiency and packet size optimization in body sensor networks (BSNs). They show that the optimal packet length for improving the energy efficiency depends on the type of BSN (in-body or on-body). In logistics, WSNs can use a range of sensors to detect the presence of vehicles ranging from motorcycles to train cars.

A routing protocol specifies how routers communicate with each other, disseminating information that enables them to select the appropriate route between any two nodes in a computer network. The choice of route is executed via routing algorithms. Each router has a priori knowledge of only the networks that are directly attached to it. A routing protocol shares this information first with its immediate neighbors and then throughout the network. In this way, routers gain knowledge of the topology of the network. Routing protocols designed for data transfer in traditional ad hoc networks cannot be used with sensor networks. This is because sensor nodes have limited battery power and because data transmission or reception consumes more power consumption than sensor nodes do sensing and computation operations. Therefore, it is desirable to conserve the energy of the nodes in the network while routing query responses back to the sink node.

2. Literature review

There has been considerable research conducted in the area of routing in wired networks [7,8]. Wired networks, unlike wireless sensor networks, are not limited by energy, node failure due to physical issues, or lack of a centralized controller. Therefore, it is easier to design and model wired network systems. Conversely, due to the inherent problems with multi-hop wireless sensor networks, the design of routing protocols poses many new challenges, and much work has been done in this area.

In recent years, several studies have proposed more efficient algorithms for routing protocols [9–16]. Hur et al. [9] proposed an adaptive routing algorithm for location-aware applications, using their addressing scheme in large-scale WSNs. Mizonian et al. [10] proposed a new analytical model for calculating the RRT (reliable real-time) degree in multi-hop WSNs, where the RRT degree is the percentage of real-time data that the network can reliably deliver on time from any source to its destination. Al-Karaki et al. [11] presented a cluster-based algorithm that can be used to address the routing problem under in-network aggregation without sacrificing data quality in WSNs. They focused on the joint problem of data routing with data aggregation and routes, maximizing the network lifetime via data aggregation and in-network processing techniques. Eom et al. [12] proposed an efficient and refined approximation method for quality of service (QoS) metrics of isolated cell of wireless networks. The proposed method is based on state space merging of two-dimensional Markov chains. Ahn and Park [13] proposed an improved optimal algorithm using a virtual infrastructure for the minimum connected dominating set problem in WSNs. Samadian and Noorhosseini [16] proposed a probabilistic support vector machine (SVM)-based method in WSNs. Their method provided even more improvement on the accuracy of the sensor node locations against a post processing step for PSVM.

The development of a reliable and energy-efficient protocol is important to various WSN applications. Depending on the application, a network may consist of hundreds or thousands of nodes. Each sensor node uses the protocol stack to communicate with other nodes and the sink. Hence, the protocol stack must be capable of energy efficient communication and must be able to work efficiently across multiple sensor nodes. Sohrabi et al. [17] presented a suite of algorithms used to self-organize wireless sensor networks in which there exist a large, scalable number of mostly static nodes with highly limited energy resources. Iqbal et al. [18] proposed a novel dynamic clustering algorithm for load-balanced routing based on route efficiency. The algorithm exploits the pattern and load of the traffic and the energy dissipation rate for each node on the route to calculate the node and route efficiency levels. Chang et al. [19] proposed an efficient color-theory-based energy efficient routing (CEER) algorithm for prolonging the life time of each sensor node. Their approach is unique in that it can efficiently
choose a better, more energy-aware routing path by comparing the RGB values associated with neighboring nodes. Zhua et al. [20] proposed ERAPL, which stands for ‘Energy-efficient Routing Algorithm to Prolong Lifetime’. This algorithm is able to dramatically prolong the network lifetime while efficiently expending energy. In ERAPL, a data gathering sequence (DGS) is constructed that is used to avoid mutual transmission and loop transmission across nodes. Each node proportionally transmits traffic to the links confined in the DGS. The aim is to optimize the network lifetime. Moreover, genetic algorithms are used to obtain an optimal solution for the proposed programming problem. Jia et al. [21] proposed a novel coverage control scheme based on a multi-objective genetic algorithm. The minimum number of sensors is selected in a densely deployed environment, and full coverage is preserved. To replace the binary detection sensor model used in the previous work, researchers used a more precise detection model in combination with the coverage control scheme. Alfieri et al. [22] proposed methods of exploiting sensor spatial redundancy by defining subsets of sensors that are active in different time periods, thereby allow sensors to save energy when inactive. They presented both a column generation method and a heuristic algorithm for maximizing network lifetime. Aneja et al. [23] studied a strong minimum energy topology design problem in wireless sensor networks. They provided a lower bound for the problem and novel formulations used to develop a branch-and-cut algorithm. Bari et al. [24] proposed the optimal routing schemes for maximizing network lifetime in two-tiered sensor networks. They developed integer linear program formulations, maximizing the lifetime of the relay node network while maintaining energy efficiency. They also proposed a technique for reconfiguring upper-tier architecture to avoid failed nodes. Bari et al. [25] used a genetic algorithm to efficiently schedule data gathering among relay nodes, thus significantly extending the lifetime of the relay node network. Zhang et al. [26] presented an energy-efficient adaptive sensor scheduling for a target monitoring algorithm in a local monitoring region of WSNs. They derived an optimization problem to satisfy sensor scheduling with the joint detection probability.

The minimum energy routing problem tries to optimize the performance of a single user (an end-to-end connection) by minimizing its energy consumption. The typical way to solve this problem is to use a shortest path algorithm for wired networks. However, there is a difference between the WSN design problem in this study and the existing WSN design problem. The existing WSN design problem finds the shortest path in the given graph, whereas the aim of this study is to find the shortest path between nodes and the energy. In this study, we present mathematical models for the WSN design problem that includes only distance and energy restrictions.

The remainder of this paper is organized as follows. Section 3 proposes mathematical models that consider resource restrictions. Section 4 presents numerical examples for the presented models. In Section 5, we present computational experiments, adjusting some of the parameters to reflect real world applications. Finally, we present our conclusions in Section 6.

3. Mathematical models

We develop mathematical models for WSN design under resource restrictions. More specifically, we consider the maximum distance between the linking nodes and the energy of the nodes, keeping in mind that sensors have limited available energy. A sensor sends out radio signals in order to connect to other nodes. However, a radio signal cannot reach long distances because of technical limitations. Because sensors are self-organized, they also have energy limitations. Therefore, each sensor is on standby in the sleep mode, which makes it possible to minimize energy consumption. When an event occurs, the sensor enters the wake-up mode and receives information. Then, it searches for other sensors with the aim of sending information to the base station (sink node) (Fig. 1).
WSN models have several requirements. There are several sensor nodes and a sink node (a base station). The source sensor must reach the sink node. We assume that the location of the sink does not change the entire duration of the process of collecting data from the sensor nodes. We also assume that sensor nodes are aware of their geographical coordinates in the field. Each sensor has resource constraints (with regard to energy and linking distance), and the sink node has no energy constraints. In Fig. 2, we schematize the notation used in the mathematical modeling.

3.1. WSN Model 1

WSN design modeling is similar to the shortest path problem (SPP). However, the latter type of model is used to find the shortest path between nodes when a graph is given, whereas this model is used to find the shortest path within the range of possible distances between two nodes in a graph that is not given. A sensor consumes energy at a constant rate when it connects with other sensors. Therefore, we consider the energy used in the standby mode and during information transmission. We assume that all of the sensors’ energy levels are initially the same and that the transmission energy used is in direct proportion to the distance between the sensors. In Model 1, we consider both types of resource limitations, those on distance and energy use. Model 1 aims to avoid severance in a single period, and it can be applied to battlefield landmine and home security. The objective of Model 1 is to minimize the sum of the amounts of energy used. The notation is as follows.

Indices

\( i, j, k \)  
indices of node \((i, j, k = 1, 2, \ldots, N)\)

\( N \)  
number of nodes \((i, j, k) \leq N\)

\( s \)  
source node index

\( n \)  
sink node index

Parameters

(1) Distance

\( d_{ij} \)  
distance from node \(i\) to node \(j\), for all \(i, j \leq N\)

\( ld \)  
maximum linking distance (constant)

(2) Energy

\( e_{i0} \)  
initial energy of node \(i\), for all \(i \leq N\), \((e_{i0} > 0)\)

\( cw \)  
energy consumed during sensor wake-up (constant)

\( ct \)  
rate of energy consumption during transmission (constant)

Decision variables

\( X_{ij} \)  
1, if node \(i\) and \(j\) are linked; 0, otherwise, for all \(i, j \leq N\)

\( E_{i} \)  
energy of node \(i\), for all \(i \leq N\)

WSN Model 1 is formulated as the following mixed integer linear program (MILP):

\[
\text{MIN } \sum_{i \leq N, i \neq n} e_{i0} - \sum_{i \leq N, i \neq n} E_{i},
\]

subject to

\[
\sum_{i \leq N, i \neq k} X_{ik} - \sum_{j \leq N, j \neq k} X_{kj} = 0, \quad \text{all } k \leq N, k \neq s, k \neq n, \tag{2}
\]

\[
X_{ij} \leq 1, \quad \text{all } i, j \leq N, i \neq j, \tag{3}
\]

\[
\sum_{j \leq N, j \neq s} X_{sj} = 1, \tag{4}
\]

\[
\sum_{i \leq N, i \neq n} X_{ni} = 1, \quad \text{all } j \leq N, \tag{5}
\]

\[
d_{ij} \cdot X_{ij} \leq ld, \quad \text{all } i, j \leq N, i \neq j, \tag{6}
\]

\[
E_{i} - e_{i0} + cw \cdot \sum_{j \leq N, j \neq i} X_{ij} + ct \cdot \sum_{j \leq N, j \neq i} d_{ij} \cdot X_{ij} = 0, \quad \text{all } i \leq N, i \neq n, \tag{7}
\]
Objective function (1) minimizes the sum of the amounts of energy used by the nodes. Constraint (2) is the flow conservation constraint, under which the total flow from node $i$ to node $k$ is equal to the total flow from node $k$ to node $j$ except in the case of the source node $s$ and sink node $n$. Constraint (3) eliminates sub-tours. Constraints (4) and (5) indicate that there must be flow out of source node $s$ and into sink node $n$. Constraint (6) imposes limits on the linking distance. Constraint (7) defines the remaining energy of the nodes. The remaining energy of a node is equal to the initial energy minus the wake-up energy and the transmission energy associated with the linked nodes. Constraint (8) ensures that the decision variable $X_{ij}$ takes a value of 0 or 1. Constraint (9) ensures that the decision variable $E_i$ takes a non-negative value.

### 3.2. WSN Model 2

In Model 2, we consider multi-period to prolong the lifetime of WSNs. Model 2 can be applied to multi-period situations in which constant monitoring is required such as industrial process monitoring and control, machine health monitoring, and environment and habitat monitoring. The objective of Model 2 is the same as that of Model 1 except that we consider multiple periods. For our mathematical modeling, we use and extend the notation used for Model 1.

**Indices**
- $t$: index of periods
- $T$: set of periods ($t \in T$)
- $et$: end period index

**Parameters**
- $E_{0i}$: initial energy of node $i$, for all $i \in N$, ($E_{0i} > 0$)

**Decision variables**
- $X_{ijt}$: 1, if node $i$ and $j$ are linked in period $t$; 0 otherwise, for all $i, j \in N$, $t \in T$
- $E_{it}$: energy of node $i$ in period $t$, for all $i \in N$, $t \in T$

WSN Model 2 is formulated as the following mixed integer linear program (MILP):

\[
\begin{align*}
\text{MIN} & \quad \sum_{i \in N \forall j \in n} E_{0i} - \sum_{i \in N \forall j \in n} E_{ijt}, \\
\text{subject to} & \quad \sum_{i \in N \forall j \in k} X_{ijt} - \sum_{j \in N \forall k} X_{kjt} = 0, \quad \forall k \in N, t \in T, k \neq s, k \neq n \\
& \quad X_{ijt} + X_{jkt} \leqslant 1, \quad \forall i, j \in N, t \in T, i \neq j, \\
& \quad \sum_{j \in N \forall j \neq s} X_{ijt} = 1, \quad \forall t \in T, \\
& \quad \sum_{i \in N \forall j \neq n} X_{ijt} = 1, \quad \forall j \in N, t \in T, \\
& \quad d_{ij} \cdot X_{ijt} \leqslant \text{ld}, \quad \forall i, j \in N, t \in T, \\
& \quad E_{it} - E_{i,t-1} + c_{w} \cdot \sum_{j \in N \forall s} X_{ijt} + c_{t} \cdot \sum_{j \in N \forall s} d_{ij} \cdot X_{ijt} = 0, \quad \forall i \in N, i \neq n, t \in T, \\
& \quad X_{ij} \in \{0, 1\}, \quad \forall i, j \in N, \\
& \quad E_{it} \geqslant 0 \quad \forall i \in N, t \in T.
\end{align*}
\]

Objective function (10) minimizes the sum of the total energy consumed at nodes by the end of all periods. Constraints (11) through (18) are similar to those for WSN Model 1 except that the period index $t$ is considered.

### 4. Numerical examples

We consider ten sensor nodes and one sink node in the numerical examples. The sink node is located at the external point of the sensor nodes. We use 11 selected nodes from the C201 example of the C2 type problem for the VRPTW Benchmark.
Problems (2011). The distance data \((d_{ij})\) for the pairs of nodes are shown in Table 1. We use the same values for the initial energy \((e_{io} = 10)\) and energy consumption \((c_w = 1, c_t = 0.1)\) for the sensor nodes. In addition, we assign node 11 as the sink node and make the distance limit \((ld)\) 15; we consider five periods.

The mathematical models were coded and solved using ILOG OPL Development Studio 5.5 with the ILOG CPLEX 11.0 engine [27]. ILOG OPL Development Studio utilizes primal or dual variants of the simplex method, barrier interior point method, and branch-and-bound algorithm internally. The computation time was less than five seconds using an Intel Core 2 2.66 GHz CPU with 3 GB RAM and the Microsoft Windows XP operating system. Table 2 shows the number of variables, number of constraints, and average objective function values of Models 1 and 2. We consider five periods in Model 2. If we consider more than five periods, the number of constraints and variables can be expected to increase. In Table 2, the value in parentheses is the average total distance. This value is calculated so that the two models can be compared using the same criteria. In Model 2, we ascertain that the total distance increases because there are empty energy sensors nearby. Table 3 shows the

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Distance matrix from node (i) to node (j) ((d_{ij})).</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
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<td>10</td>
<td>12</td>
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<td>11</td>
<td>19</td>
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<table>
<thead>
<tr>
<th>Table 2</th>
<th>Comparison results for Models 1 and 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
<td>Number of variables</td>
</tr>
<tr>
<td>1</td>
<td>132</td>
</tr>
<tr>
<td>2</td>
<td>656</td>
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</table>

(): Total distance.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Specific results for Models 1 and 2 based on the numerical example.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source node</td>
<td>Objective function value</td>
</tr>
<tr>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>Node 1</td>
<td>5.7 (27)</td>
</tr>
<tr>
<td>Node 2</td>
<td>1-5-2-11</td>
</tr>
<tr>
<td>Node 3</td>
<td>7.8 (38)</td>
</tr>
<tr>
<td>Node 4</td>
<td>3-7-5-2-11</td>
</tr>
<tr>
<td>Node 5</td>
<td>2.2 (12)</td>
</tr>
<tr>
<td>Node 6</td>
<td>4-11</td>
</tr>
<tr>
<td>Node 7</td>
<td>3.7 (17)</td>
</tr>
<tr>
<td>Node 8</td>
<td>5-2-11</td>
</tr>
<tr>
<td>Node 9</td>
<td>1.4 (4)</td>
</tr>
<tr>
<td>Node 10</td>
<td>6-11</td>
</tr>
<tr>
<td>Node 11</td>
<td>5.3 (23)</td>
</tr>
<tr>
<td>Node 12</td>
<td>7-5-2-11</td>
</tr>
<tr>
<td>Node 13</td>
<td>6.2 (32)</td>
</tr>
<tr>
<td>Node 14</td>
<td>8-5-2-11</td>
</tr>
<tr>
<td>Node 15</td>
<td>2.6 (36)</td>
</tr>
<tr>
<td>Node 16</td>
<td>9-10-5-2-11</td>
</tr>
<tr>
<td>Node 17</td>
<td>5.8 (28)</td>
</tr>
<tr>
<td>Node 18</td>
<td>10-5-2-11</td>
</tr>
<tr>
<td>Sum</td>
<td>47.1 (221)</td>
</tr>
</tbody>
</table>

(): Total distance.
comparison results for each node. The values in parentheses in Table 3 refer to the total distance from each source node to
the sink node. The objective function values for Model 2 are larger than those for Model 1 because this model considers mul-
tiple periods. We can calculate the average total distance for Model 2 by dividing the total distance by five which is the num-
ber of periods. Based on these results, we determine that the average total distance for Model 2 is similar to that of Model 1
except in the case of nodes 3 and 5. The reason for these differences is that the next nodes for nodes 3 and 5 are far away
because there are empty energy sensor nodes nearby.

The graphical configuration of the network for source node 3 in Model 1 is shown in Fig. 3. The optimal network for source
node 3 links nodes 7, 5, and 2 to the sink node. The objective function value of this network is 7.8 and the total distance is 38.
The energy consumptions figures for each node are as follows: node 3 consumed 2.5, node 7 consumed 1.6, node 5 consumed
2.3, and node 2 consumed 1.4. Table 4 shows the comparison between the optimal network and the feasible networks in
terms of the total distance and the total energy consumed.

5. Computational experiments

To explore the applications of our findings within WSN design, we experimented with larger networks using WSN Model
2. The distance data for the sensors were drawn from the 100 customer problems by VRPTW Benchmark Problems [28]. The
benchmark set contains six different subsets: R1, R2, RC1, RC2, C1, and C2. The customers are randomly distributed in R1
and R2, and clustered in C1 and C2. For the groups RC1 and RC2, the clustered and random distributions are mixed. Because x and
y coordinates are identical for all problems within each type (i.e., for R, C, and RC), we used C1, R1, and RC1 problem types in
the benchmark set. The sink node in all instances is located almost at the center. The test problems require different param-
eters. The number of periods (T) is 5, 10, and 15 for C1, R1, and RC1, respectively. Likewise, the distance limits (ld) are 5(10),
15, and 30. We used the limit of 10 for R1 and RC1 types because distances between nodes of less than 5 are rare for these
types. The initial energy levels (ei0) of the sensors are 10, 20, and 30, respectively. In this experiment plan, we must solve a
total of 2,700 problems. Therefore, we randomly select 10 source nodes and analyze 270 problems for a given number of
periods T. Table 5 shows the number of variables and number of constraints for the test problems.

The computation times for all problems were less than 10 s using an Intel Core 2 2.66 GHz CPU with 3 GB RAM and the
Microsoft Windows XP operating system. Table 6 shows the results for the test problems. From Table 6, we ascertain that the
objective function values differ according to the distance limit. The objective function value increases as the limit decreases.
These results indicate that under tight limits, the next node selected is closer, and several nodes will have to be passed to
reach the sink node. We also ascertain that when the number of periods increases, the objective values associated with dis-
tance limits of 15 and 30 are not very different. This means that there are many nodes within a distance of 15. Regarding

<table>
<thead>
<tr>
<th>Table 4</th>
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<tr>
<td>Comparison between the networks in terms of total distance and energy consumed by source node 3.</td>
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</table>

<table>
<thead>
<tr>
<th>Network Design</th>
<th>Total Distance</th>
<th>Total Consumed Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal network design (3-7-5-2-S)</td>
<td>38</td>
<td>7.8</td>
</tr>
<tr>
<td>Feasible network design 1 (3-1-5-2-S)</td>
<td>40</td>
<td>8.0</td>
</tr>
<tr>
<td>Feasible network design 2 (3-7-5-4-S)</td>
<td>42</td>
<td>8.2</td>
</tr>
<tr>
<td>Feasible network design 3 (3-7-5-4-2-S)</td>
<td>42</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Fig. 3. Optimal sensor network design in source node 3.
problem type, we ascertain that R1 (random) type has lower objective values than C1 (clustered) and RC1 (random and clustered) types. This means that either the source node or the sink node is included in the other cluster. Figs. 4–6 graphically illustrate the average objective function values for the various periods. Fig. 4 shows the average objective function values for five periods. Based on this figure, we determine that R1 type has the smallest values and that the RC1 type has the largest values. Fig. 5 shows the average objective function values in 10 periods. From this figure, we see that R1 type is associated with the smallest values and RC1 type with the largest values, as when there are five periods. Fig. 6 shows the average objective function values in 15 periods. This figure indicates that again, R1 type has the smallest values and RC1 type has the largest values; however, the difference is quite small. From Figs. 4–6, we ascertain that RC1 type has the largest objective function value and that the difference is larger than that in the 5- and 10-period examples. Therefore, we ascertain that as the number of periods increases, the difference between the objective function values of C1 and R1 types decreases, whereas

![Image](image-url)

**Fig. 4.** Comparison of the average objective function values for 5 periods.
the difference between R1 and RC1 values increase. Finally, as the distance limit increases, the difference between the objective function values associated with C1 and R1 types decreases, whereas the difference between R1 and RC1 values increases.

6. Conclusions

This study considered wireless sensor network design under resource restrictions. We developed mathematical models for network design by simultaneously considering multiple periods and distance and energy limits. The mathematical models were coded and solved using ILOG OPL Development Studio 5.5 with the ILOG CPLEX 11.0 engine. The computation time for each experiment was less than 10 s. The computation experiments demonstrate the usefulness of the presented models. Therefore, the proposed models can be suitably used in real situations in designing wireless sensor networks. The developed models can also be used in logistics network design under various resource restrictions. Even though the mathematical models do not require significant computation time, much more time may be needed for large-scale WSNs. Therefore,
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