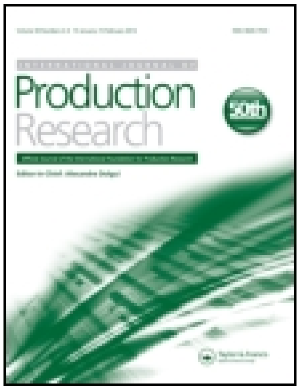

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Economic lot and supply scheduling problem: a time-varying lot sizes approach

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We study the economic lot and supply scheduling problem (ELSSP) that arises in the distribution and manufacturing industries. The ELSSP involves the simultaneous scheduling of end-item production and inbound transportation of input materials over an infinite time horizon to minimise the average costs of inventory, production set-up and transportation. We present a new methodology based on a time-varying lot sizes approach for the ELSSP. We also provide computational experiments showing that the developed algorithm outperforms the existing heuristic for improved integrated scheduling.

Keywords: economic lot and supply scheduling problem; vehicle routing; time-varying lot sizes approach

1. Introduction

The economic lot and supply scheduling problem (ELSSP) is used to find production sequences and times as well as the idle times, for several end products in a single facility. It also helps planners determine simultaneous delivery quantities, dates and routes for the collection of the input materials and for the generation of end products. Unlike the traditional economic lot and scheduling problem (ELSP), through the ELSSP, routing decisions, determined by the production schedule of end products, are imposed on input materials used to make these goods. The coordination of inbound transportation and production scheduling is the main focus of the ELSSP as firms try to secure the best trade-off between inventory and production set-up costs and transportation routing expenses. The problem arises in various manufacturing industries, especially those in just-in-time systems.

Well known as NP-hard (Hsu 1983), especially under the time-varying lot sizes approach, the ELSP was first introduced by Rogers (1958), and many good heuristic approaches have been developed for it. The following three types of approaches are typically used for solving the problem (Moon, Silver, and Choi 2002).

(1) The common cycle approach restricts all of the end items' cycle times to equal length so that this approach always finds a feasible schedule. The solution, however, serves as an upper bound, unlike other means of solving the ELSP, because in some situations, it creates solutions far from the lower bound.
(2) The basic period approach allows for different cycle times for specific end items. The cycle time for every end item is represented by an integer multiple interval of a basic period. An excellent review on this approach can be found in Elmaghraby (1978). Under this approach, it is NP-hard to find the existence of a feasible solution (Hsu 1983). This approach, in general, gives better solutions than the restricted version of the original problem. However, the feasibility of the approach is difficult to demonstrate.
(3) The time-varying lot sizes approach allows for different lot sizes of end items during a cyclic schedule. Dobson (1987) proved that any production sequence can be converted into a feasible schedule under specific conditions. This heuristic approach has been found to be more effective than either the common cycle or basic period methods.

In the literature, a variety of modifications to the ELSP have been studied extensively to include considerations of product shelf life, deteriorating production, stabilisation period, sequence dependent set-up and a family of products (refer to Silver, Pyke, and Peterson 1998 for more details). Several researchers (Silver 1995; Viswanathan and Goyal 1997; Liu, Wu, and Zhou 2008) considered a limit on shelf life. Additional research has addressed the ELSP with a deteriorating or imperfect production process. Moon, Hahm, and Lee (1998) applied the stabilisation period concept, in which yield production rates gradually increase during the starting period of production, to the ELSP. In 1992, Dobson extended previous work (Dobson 1987) to include sequence dependent set-up times. Ham, Hitomy, and Yoshida (1985) made the first contribution to the group technology ELSP (GT-ELSP), which involves finding a feasible production schedule for several families of end items

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produced on a dedicated single machine. Kuo and Inman (1990) developed a simple and practical heuristic using the common cycle approach for products in specific technology groups. Moon, Cha, and Bae (2006) reviewed and improved the heuristic of Kuo and Inman (1990) and developed a hybrid genetic algorithm for the group technology ELSP.

The literature features several multiple-stage coordination models of the ELSP. One variant of the coordination model is the economic lot and supply scheduling problem (ELSSP), which considers supply of raw or input materials. Gallego and Joneja (1994) extended the traditional ELSP to include production models that account for the ordering and holding costs of raw materials. Kuhn and Liske (2011) presented an ELSSP that simultaneously concerns a raw material supply routing and holding costs. A second similar variant of the coordination model is the economic lot and delivery scheduling problem (ELDSP), which considers the situation of inbound production schedules and outbound deliveries. The concept of the ELDSP was first presented by Hahm and Yano (1992, 1995) who developed an efficient heuristic algorithm. Jensen and Khouja (2004) improved the heuristic algorithm and suggested an optimal algorithm. Clausen and Ju (2006) suggested a more efficient algorithm that combines the previous algorithms. More recently, Osman and Demirli (2012) presented a new formulation for multi-stage supply chains and a hybrid algorithm based on a decomposition approach. In studying production sequence decisions with the ELDSP, researchers considered direct transportation costs between multiple stages.

Recently, Haksöz and Pinedo (2011) and Huang and Yao (2013) extended the single-facility ELSP to a multiple machines ELSP that deals with set-up and inventory costs. The inventory-routing problem (IRP) integrates two components of supply chain management: inventory control and vehicle routing (Campbell and Savelsbergh 2004). However, the IRP, unlike the ELSSP, does not address production issues.

As discussed earlier, ELSSP deals with all three aspects of the traditional ELSP (production and inventory decisions) with routing decisions in an integrated way. Recently, Kuhn and Liske (2011) studied the effects of input material routing issues on the ELSP. They developed a heuristic algorithm through the common cycle approach in which the sequence of a production run is not important because the cycle times for all end items are considered equal. Our main goal is to develop a time-varying lot sizes approach for solving the ELSSP to improve Kuhn and Liske's (2011) heuristic.

This paper is organised as follows. Assumptions and notation are presented in Section 2 as is a mathematical model based on the time-varying lot sizes approach to the ELSSP. In Section 3, we develop a heuristic algorithm for the ELSSP. In Section 4, computational examples illustrate the solution procedure and allow readers to compare the results with other heuristic solutions. The paper ends with concluding remarks in Section 5.

2. ELSSP model

2.1 *Assumptions and notation*

The following assumptions are used in the ELSSP:

(1) Multiple end items compete for the use of a single facility.
(2) Demand rates, production rates, set-up costs and set-up times for all end items are known constants.
(3) Backorders are not allowed.
(4) Each input material belongs to only one end item according to bill of materials.
(5) Each input material corresponds to one dedicated supplier.
(6) The travel times of the routes are neglected.
(7) Each route contains only input materials for a single end item.
(8) The delivery dates of the input materials are equal to the production start time of their associated end items.
(9) The delivery quantities of an input material required during a production cycle are not allowed to partitioned over several routes.

Note that assumptions (4)–(9) were used in Kuhn and Liske (2011). Assumptions (6) and (8), taken together, mean that travel times are neglected in manufacturing industries, especially those in just-in-time systems, but the vehicle routing cost based on travel distances is considered and is changed by the relationship between production cycle length and vehicle capacity. In addition, assumptions (7) and (9) together suggest that input materials within an end item should be collected together and divided into routes with one-vehicle capacity (i.e. one route). Thus, according to the relationship between production cycle length and vehicle capacity, the sum of vehicle routing, inventory holding and set-up costs is changed.

The following notation is used in the model:

- i index for input materials, $i = 1, 2, \ldots, n$
- j index for end items, $j = 1, 2, \ldots, m$
- p_j constant production rate of end item j (units/unit time)
- d_j constant demand rate of end item j (units/unit time)

h_j^E known inventory holding cost of end item j ($/unit/unit time)
h_i^I known inventory holding cost of input material i ($/unit/unit time)
A_j known set-up cost for end item j ($)
s_j known set-up time for end item j (unit time)
T_j production cycle length for end item j (unit time)
R_j vehicle routing cost for end item j
a_{ij} quantity of input material i needed for end item j
Q vehicle capacity
q_i delivery quantity of input material i

2.2 *ELSSP model under the common cycle approach*

In this section, we first explain and reformulate Kuhn and Liske's (2011) ELSSP model under the common cycle approach and introduce ELSSP model under the time-varying lot sizes approach in the next section.

Figure 1 shows a common cycle for three end items which consist of three input materials based on each end items' bill of materials. The total cost (which is the sum of the set-up costs, holding costs and input material routing and holding costs) per unit time is given by

$$\mathrm{TC} = \sum_{j=1}^{m}\left[\frac{A_j}{T_j} + H_j^E T_j + \frac{R_j}{T_j} + H_j^I T_j\right] \tag{1}$$

where $H_j^E = h_j^E d_j(1 - d_j/p_j)/2$ and $H_j^I = \sum_{i=1}^{n} h_i^I a_{ij} d_j (d_j/p_j)/2$; $j = 1, 2, \ldots, m$.

R_j is vehicle routing costs for end item j. According to the capacity of vehicle and cycle length T, R_j can be calculated as total vehicle routing costs by one vehicle or more than one vehicles for end item j. The delivery quantity of input material i at a supplier is $q_i(T) = \sum_{j=1}^{m}(a_{ij} d_j T)$. R_j can be calculated by VRP (vehicle routing problem) model that is the minimum routing costs of all suppliers for end item j. The delivery quantity, q_i is increased by cycle length T and the vehicle routing cost R_j is changed by delivery quantity of each input material i.

In the common cycle approach, we have $T_1 = T_2 = \ldots = T_m = T$ (say). Thus, the above expression for the total cost can be written as

$$\mathrm{TC} = \frac{(A + R)}{T} + (H^E + H^I)T, \tag{2}$$

where $A = \sum_{j=1}^{m} A_j$, $R = \sum_{j=1}^{m} R_j$, $H^E = \sum_{j=1}^{m} H_j^E$ and $H^I = \sum_{j=1}^{m} H_j^I$.

If R is constant, TC is a convex function. If R is not constant, the function TC is also a piecewise convex (Kuhn and Liske 2011). They suggest a search algorithm by the determination of the intervals as which TC is convex, and by calculation of the local minimum of TC within the interval.

If R is constant, it is easy to show that TC is minimised by

$$T^* = \sqrt{\frac{A + R}{H^E + H^I}}. \tag{3}$$

Moreover, before we accept T^* as the minimum cycle length within the interval, we must consider the time required for set-ups during the cycle. Because the total set-up time per cycle plus the total production time per cycle must be no more than the cycle length, we have the following constraint on T:

$$\sum_{j=1}^{m}\left(s_j + \frac{d_j T}{p_j}\right) \le T \tag{4}$$

$$\text{or,} \quad T \ge \frac{\sum_{j=1}^{m} s_j}{\kappa} \equiv T_{\min}\ (\text{say}) \tag{5}$$

where $\kappa = 1 - \sum_{j=1}^{m} d_j/p_j$, the long-run proportion of time available for set-ups. Because $\mathrm{TC}(T)$ is convex in T, therefore, the minimum cycle length should be equal to $\max\{T^*, T_{\min}\}$ within the interval of constant R.

According to assumption (9), the delivery quantity $q_i(T)$ has to be smaller than or equal to the vehicle capacity, Q. Due to the dependence of $q_i(T)$ and T, an upper bound $T_{\max}$ is considered as follows:

$$T_i \le \frac{Q}{\sum_{j=1}^{m} a_{ij} d_j}, \quad \forall i \tag{6}$$

$$T_{\max} = \min\{T_i\} \tag{7}$$

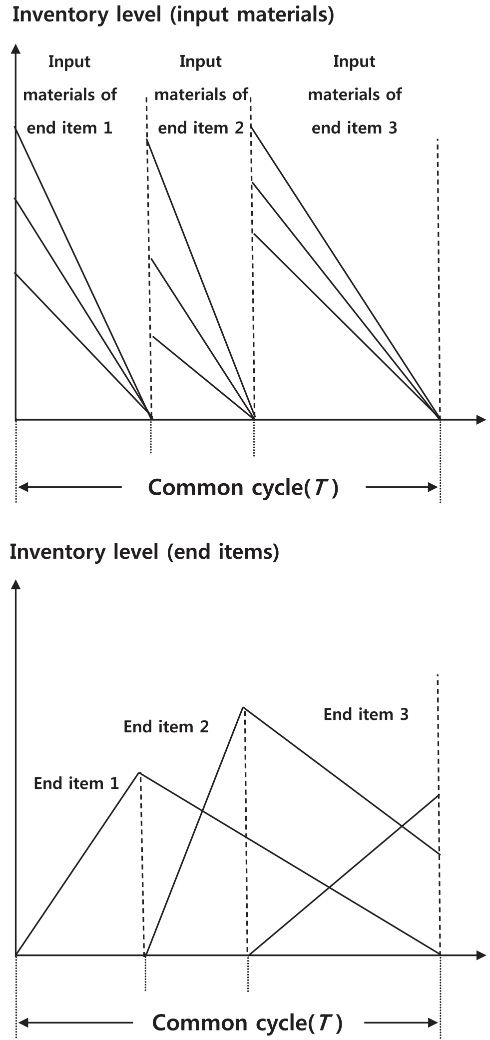

Figure 1. Production schedule for three end items in the common cycle approach.

The $T_{\max}$ is the maximum cycle length that every input materials i can be transported by one vehicle. The total cost of the ELSSP model then can be found by evaluating several intervals between $T_{\min}$ and $T_{\max}$ (see Kuhn and Liske 2011 for more details).

2.3 *ELSSP model under the time-varying lot sizes approach*

We modify the Kuhn and Liske's (2011) formulation using Dobson's (1987) original formulation of the ELSP. Here, subscripts i and j are used to indicate input materials i and end items j and superscript k indicates the end item produced at the kth position in the sequence. Let $\mathcal{F}$ be the set of all possible finite sequences of end items. Here K_j denote the positions in a given sequence where end item j is produced, that is, $K_j = \{k \mid f^k = j\}$. Let L_l be the positions in a given sequence from l, up to but not including the position in the sequence where product f^l is produced again. The complete formulation of the ELSSP is as follows:

$$\inf_{f \in \mathcal{F}} \ \text{Min}_{t \geq 0\ u \geq 0\ T > 0}$$

$$\frac{1}{T}\left(\sum_{k=1}^{K} R^k + \sum_{i=1}^{n}\sum_{k=1}^{K}\frac{1}{2}h_I^k a^{ik} p^k (t^k)^2 + \sum_{k=1}^{K}\frac{1}{2}h_E^k (p^k - d^k)\left(\frac{p^k}{d^k}\right)(t^k)^2 + \sum_{k=1}^{K} A^k\right) \tag{8}$$

subject to

$$\sum_{k \in K_j} p_j t^k = d_j T, \quad j = 1, \ldots, m \tag{9}$$

$$\sum_{k \in L_l} (t^k + s^k + u^k) = (p^l / d^l) t^l, \quad l = 1, \ldots, L \tag{10}$$

$$\sum_{k=1}^{K} (t^k + s^k + u^k) = T. \tag{11}$$

The objective function consists of vehicle routing costs, inventory holding cost of input materials and end items, and set-up costs. Constraints (9) ensure that we allocate enough time to each end item j to meet its demand $d_j T$ over the cycle. Constraints (10) mean that we must produce enough of an end item each time to last until the next time that the same end item is produced. Constraint (11) simply states that the cycle time T must be the sum of production, set-up and idle times for all the end items produced in the cycle.

The classical ELSP can be stated as follows. There is a single facility on which m distinct products are to be produced. We try to find a cycle length T, a production sequence $\mathbf{f} = (f^1, \ldots, f^K)$, where $f^j \in \{1, \ldots, m\}$, production time durations $\mathbf{t} = (t^1, \ldots, t^K)$ and idle time durations $\mathbf{u} = (u^1, \ldots, u^K)$, so that the production sequence can be completed in the chosen cycle, the cycle can be repeated over time, demand can be fully met and the total of inventory and set-up costs is minimised (Silver, Pyke, and Peterson 1998). Moreover, the ELSSP considers delivery quantities, dates and routes for the collection of the input materials from suppliers simultaneously with the ELSP.

Define

$$\kappa = 1 - \sum_{j=1}^{m} \frac{d_j}{p_j}.$$

Note that κ is the long-run proportion of time available for set-ups. For infinite horizon problems, $\kappa > 0$ is a necessary condition for the existence of a feasible schedule. Dobson (1987) showed that if $\kappa > 0$, then any production sequence can be converted into a feasible schedule by allowing time-varying production runs and a sufficiently large cycle length.

2.4 *A lower bound for the ELSSP model*

A lower bound on the minimum average cost can be easily obtained by considering each end item in isolation and calculating economic production quantities. This approach is known as the independent solution (IS) because the capacity issue caused by use of a single machine to make several end products is ignored. A tight lower bound has been implicitly suggested by Bomberger (1966) and rediscovered in several different ways by other researchers (Dobson 1987; Gallego and Moon 1992). The benefit of the nonlinear program that we propose is that the lower bound is quite tight for the total average cost for the ELSSP.

ELSSP-LB

$$\mathrm{Min}_{T_1,\ldots,T_m} \sum_{j=1}^{m} \left[\frac{A_j}{T_j} + \frac{h_j^E d_j T_j}{2} \left(1 - \frac{d_j}{p_j}\right) + \frac{R_j}{T_j} + \sum_{i=1}^{n} \frac{h_i^I a_{ij} d_j T_j}{2} \left(\frac{d_j}{p_j}\right) \right] \tag{12}$$

subject to

$$\sum_{j=1}^{m} \frac{s_j}{T_j} \le \kappa, \tag{13}$$

$$T_j \ge 0, \quad j = 1, \ldots, m. \tag{14}$$

The objective function is a lower bound on the average cost of producing m different end items on a single machine. The constraint is the proportion of time available for set-ups. The objective function and the constraint set are convex in T_j's. Therefore, the optimal points of the LB model are points which satisfy the Karush–Kuhn–Tucker (KKT) conditions as follows:

$$A_j - (H_j^E + H_j^I) T_j^2 + R_j + \lambda s_j = 0, \quad j = 1, 2, \ldots, m \tag{15}$$

$$\lambda \left[\kappa - \sum_{j=1}^{m} \frac{s_j}{T_j} \right] = 0, \tag{16}$$

$\lambda \geq 0$ complementary slackness with $\sum_{j=1}^{m}(s_j/T_j) \leq \kappa$.

The above conditions are derived by assuming that T_j's are non-trivial. Equation (15) yields

$$T_j = \sqrt{\frac{A_j + R_j + \lambda s_j}{H_j^E + H_j^I}}, \quad \forall j. \tag{17}$$

We can use the following procedure to find the optimal T_j.

Algorithm for Lower Bound

(Step 1) Check if $\lambda = 0$ gives an optimal solution.
Find T_js from the following sub Steps.

(Step 1-1) For each end item j, set $v = 2$ and calculate $T_{\text{right}}, T_{\max}, T'_{\text{right}}$ where $T_{\text{right}} = \min\{Q/\sum_{i=1}^{n} a_{ij}d_j\}$, $T_{\max} = \min_i\{Q/a_{ij}d_j\}$, $T'_{\text{right}} = T_{\text{right}}$.

(Step 1-2) Calculate q_i, R_j sequentially where $q_i = \sum_{j=1}^{m} (a_{ij}d_jT_{\text{right}})$ and find $T_j = \sqrt{(A_j + R_j)/(H_j^E + H_j^I)}$.

(Step 1-3) If $T_j > T_{\text{right}}$, set $T_{\text{left}} = T_{\text{right}}$ and update $T_{\text{right}} = \min\{vT'_{\text{right}}, T_{\max}\}$. Calculate q_i, R_j sequentially where $q_i = \sum_{j=1}^{m} (a_{ij}d_jT_{\text{right}})$ and find T_j. If T_j is larger than $T_{\max}$, T_j can be $T_{\max}$. Set $v = v + 1$ and repeat Step 1-3.
Otherwise, consider next end item j.

(Step 2) If $\sum_{j=1}^{m}(s_j/T_j) \leq \kappa$, then the T_js are an optimal solution.
Otherwise, go to Step 3.

(Step 3) Start with an arbitrary $\lambda > 0$.

(Step 4) Compute T_js from the following sub Steps.

(Step 4-1) For each end item j, set $v = 2$ and calculate $T_{\text{right}}, T_{\max}, T'_{\text{right}}$ where $T_{\text{right}} = \min\{Q/\sum_{i=1}^{n} a_{ij}d_j\}$, $T_{\max} = \min_i\{Q/a_{ij}d_j\}$, $T'_{\text{right}} = T_{\text{right}}$.

(Step 4-2) Calculate q_i, R_j sequentially where $q_i = \sum_{j=1}^{m} (a_{ij}d_jT_{\text{right}})$ and find $T_j = \sqrt{(A_j + R_j + \lambda s_j)/(H_j^E + H_j^I)}$.

(Step 4-3) If $T_j > T_{\text{right}}$, set $T_{\text{left}} = T_{\text{right}}$ and update $T_{\text{right}} = \min\{vT'_{\text{right}}, T_{\max}\}$. Calculate q_i, R_j sequentially where $q_i = \sum_{j=1}^{m} (a_{ij}d_jT_{\text{right}})$ and find T_j. If T_j is larger than $T_{\max}$, T_j can be $T_{\max}$. Set $v = v + 1$ and repeat Step 4-3.
Otherwise, consider next end item j.

(Step 5) If $\sum_{j=1}^{m}(s_j/T_j) < \kappa$, reduce λ. Go to Step 4.
If $\sum_{j=1}^{m}(s_j/T_j) > \kappa$, increase λ. Go to Step 4.
If $\sum_{j=1}^{m}(s_j/T_j) = \kappa$, stop. The T_js are optimal.

Let the optimal cycle length for each end item j in program **ELSSP-LB** be T_j^*.

3. A heuristic algorithm

To solve the ELSSP, we use the time-varying lot sizes approach proposed by Dobson (1987). We first determine production frequencies by solving the ELSSP-LB model. Then we round off the production frequencies to power-of-two integers using an algorithm proposed by Roundy (1989). Finally, the end items are packed into bins with respect to frequencies and average loads, resulting in a production sequence. The continuous part consists of actual production run, idle times and cycle length by taking the combinatorial part (the production sequence) as given (Zipkin 1991).

The heuristic algorithm can be described as follows:

Step 1. Find the relative production frequencies by solving the **ELSSP-LB** model. We suggested an one-dimensional line search algorithm to solve the **ELSSP-LB** model. Let the optimal cycle length for end item j in program **ELSSP-LB** be T_j^*. Also, let x_j represent the relative production frequency for end item j. Then, x_j is determined by the following:

$$x_j = \frac{\text{Max}_j \{T_j^*\}}{T_j^*}, \quad j = 1, 2, \ldots, m.$$

Step 2. Round the production frequencies obtained in Step 1 to power-of-two integers. Roundy (1989) proved that the additional costs does not exceed 6% when the real values of the production frequencies are converted to the powers-of-two integers. Let y_j be the production frequency which is the power-of-two integers for end item j, then y_j is determined as follows:

$$y_j = 2^p \quad \text{if} \quad x_j \in \left[\frac{1}{\sqrt{2}}2^p, \sqrt{2}2^p\right], \quad p = 0, 1, \ldots.$$

Step 3. Find an efficient production sequence **f** using the bin-packing heuristic suggested by Dobson (1987). Using the production frequencies y_j, spread end items out as evenly as possible in b bins where $b = \max_j\{y_j\}$. While assigning end items to the bins, we use a variation of the longest processing time (LPT) rule in which end items are ordered lexicographically by (y_j, v_j). v_j is the estimated production time duration of end item j using the same cycle length where $v_j = s_j + (d_j T / p_j y_j)$. By minimising the maximum height of the bins, the heuristic finds an efficient production sequence **f** (see Dobson (1987) for more details).

Step 4. Solve for production times **t** and idle times **u** for the given production sequence **f**. If we assume that there are no idle times (that is, **u** = **0**), we can easily find **t** using Equation (18) under **f** and **u** (see Dobson 1987 for more details). This approximation works very well for a highly loaded facility. Moreover, simultaneous computations for the variables **t** and **u** lead to a rather complex nonlinear programming problem. However, Zipkin (1991) showed that an optimal solution can be obtained by a parametric quadratic programme and a few EOQ-like calculations.

$$t = (I - P^{-1}L)^{-1}P^{-1}L(s + u). \tag{18}$$

We assume identical idle times preceding the lots, expect for the first position of sequence, and thus we divide the available idle times into equal segments. We can solve for idle times **u** using Equation (19). The idle time value should not be less than zero. We set T as the common cycle length and K as the number of positions in the production sequence; if position k is the first position of sequence, then u^k is 0. Otherwise,

$$u^k = \left[T - \sum_{j=1}^{m}\left(s_j + T d_j / p_j\right)\right] / (K - 1). \tag{19}$$

After calculating the feasible production time using Equation (18) with **f** and **u**, we calculate R^k using a new cycle length for the input material i. The delivery quantity of input material i at a supplier is recalculated with $T = t^k p^k / d^k$ and $q_i(T) = \sum_{j=1}^{m}(a_{ij} d_j T)$. The total average cost of the schedule is calculated using Equation (8).

Step 5. We can modify idle times based on the total cost function under the time-varying lot sizes approach. The total average cost function (8) can be restated as follows.

$$\text{TC} = \frac{1}{T}\left(\sum_{k=1}^{K} R^k + \sum_{k=1}^{K} H_k^E (t^k p^k / d^k)^2 + \sum_{k=1}^{K} H_k^I (t^k p^k / d^k)^2 + \sum_{k=1}^{K} A^k\right) \tag{20}$$

where $H_k^E = h_E^k d^k (1 - d^k / p^k)/2$ and $H_k^I = \sum_{i=1}^{n} h_I^k a^{ik} d^k (d^k / p^k)/2$.
Because we applied the common cycle length for the idle time calculation, the values for the inventory holding costs of end items and input materials have been changed to those shown in Equation (20). Thus, we try to balance between changed inventory holding costs and vehicle routing and set-up costs. To revise cycle length T, we set T to αT in Equation (20) such that

$$\text{TC} = \frac{1}{\alpha T}\left[\sum_{k=1}^{K}\left(R^k + A^k\right) + \alpha^2 \sum_{k=1}^{K}\left(H_k^E (t^k p^k / d^k)^2 + H_k^I (t^k p^k / d^k)^2\right)\right]. \tag{21}$$

We set $dTC/d\alpha$ to zero and subsequently get the α to cycle length ratio that minimises the total average cost.

$$\alpha = \sqrt{\frac{\sum_{k=1}^{K}\left(R^k + A^k\right)}{\sum_{k=1}^{K}\left[H_k^E (t^k p^k / d^k)^2 + H_k^I (t^k p^k / d^k)^2\right]}} \tag{22}$$

After finding the updated idle times **u** using αT in Equation (19), we can update the feasible production times, vehicle routing costs and the total average cost of the schedule.

4. Computational experiments

Example I Modified Kuhn and Liske (K&L)'s problem

Table 1. Data for the modified Kuhn and Liske's problem ($\kappa = 0.008$ case).

End item j	Production rate (p_j) (units/day)	Demand rate (d_j) (units/day)	Holding cost (h_j^E) (\$/unit/day)	Set-up cost (A_j) (\$)	Set-up time (s_j) (days)
1	5	1	5	1000	0.02
2	30	4	5	3500	0.05
3	3	1	5	1000	0.02
4	40	5	5	3500	0.05
5	15	3	5	1000	0.05

We modified and solved Kuhn and Liske's (2011) problem consisting of 25 input materials and 5 end items. The modified data related to the end items for Kuhn and Liske's problem are shown in Table 1. To obtain a highly loaded facility example, we changed production and demand rates as well as set-up times. We obtained $\sum_{j=1}^{m}(d_j/p_j) = 0.992$, which means that the facility must run 99.2% of the time to meet the demands; the remaining time is only for set-up or leaving the machinery idle. The data related to the input materials are considered to be the same as those in Kuhn and Liske's problem and are shown in Table 2. We only changed the vehicle capacity Q from 900 to 1500 to get a feasible range of cycle length.

We first show the detailed steps of the heuristic as follows.

Step 1. We compute a lower bound and associated cycle length for each item.

$$T_1 = 45.42, \quad T_2 = 27.06, \quad T_3 = 35.29, \quad T_4 = 23.50, \quad T_5 = 21.65,$$

with a lower bound \$2176.26.

Step 2. We compute production frequencies rounded to power-of-two integers.

$$y_1 = 1, \quad y_2 = 2, \quad y_3 = 1, \quad y_4 = 2, \quad y_5 = 2.$$

Step 3. We generate a production sequence using bin-packing heuristic. We use the following lexicographic order to assign end items to the bins.

$$(y_5, v_5) = (2, 4.59) \geq^L (y_2, v_2) = (2, 3.08) \geq^L (y_4, v_4) = (2, 2.89)$$
$$\geq^L (y_3, v_3) = (1, 15.16) \geq^L (y_1, v_1) = (1, 9.10).$$

The resulting production sequence is as follows:

$$\mathbf{f} = (5, 2, 4, 3, 5, 2, 4, 1).$$

Step 4. We compute production times $\mathbf{t}$ and idle times $\mathbf{u}$ using Equations (18) and (19), respectively, for the given sequence and total average cost using Equation (8).

$$\mathbf{u} = (0, 0.0056, 0.0056, 0.0056, 0.0056, 0.0056, 0.0056, 0.0056) \text{ and}$$
$$\mathbf{t} = (5.46, 3.40, 3.10, 15.17, 3.64, 2.67, 2.59, 9.10)$$

with total average cost \$2208.73.

Step 5. We compute α and αT using Equation (22). We update idle times $\mathbf{u}$, production times $\mathbf{t}$, and total average cost.

$$\alpha = 1.0120, \ \mathbf{u} = (0, 0.0060, 0.0060, 0.0060, 0.0060, 0.0060, 0.0060, 0.0060) \text{ and}$$
$$\mathbf{t} = (5.50, 3.42, 3.12, 15.28, 3.67, 2.69, 2.61, 9.17)$$

with total average cost \$2208.59.

We improved the Kuhn and Liske's heuristic by 1.40% and the sequential solution by 3.11%. This small differential is quite meaningful because the average cost penalty found by the Kuhn and Liske's heuristic is only a few percentage points different from the lower bound.

We also considered a sequential approach used in Kuhn and Liske (2011). We first solved the ELSP by the common cycle approach and then computed the corresponding total costs of the VRP and inventory holding cost of input materials. We

Table 2. Bill of materials and supplier positions of the Kuhn and Liske's problem.

Input material	BOM information		Supplier positions		Holding cost
i	a_{ij}	j	x_i^b	y_i^b	h_i^I
1	3	1	0	0	1
2	2	1	1500	0	1
3	1	1	300	1200	1
4	2	1	1000	1500	1
5	3	1	950	950	1
6	1	2	500	0	1
7	2	2	1200	300	1
8	3	2	0	500	1
9	2	2	1500	1500	1
10	1	2	550	550	1
11	2	3	300	300	1
12	3	3	1000	500	1
13	1	3	500	1500	1
14	2	3	1000	1000	1
15	3	3	950	550	1
16	1	4	0	500	1
17	2	4	1500	500	1
18	3	4	0	1000	1
19	2	4	1500	1000	1
20	1	4	900	750	1
21	3	5	500	500	1
22	2	5	1000	0	1
23	1	5	500	1000	1
24	2	5	1200	1200	1
25	3	5	550	950	1

Table 3. Results for modified Kuhn and Liske's problem ($\kappa = 0.008$ case).

Lower bound (LB)	Proposed heuristic (TV)	K&L's heuristic (CC)	Sequential solution (SS)
2176.26 ($T = 45.42$)	2208.59 ($T = 45.84$)	2239.85 ($T = 27.51$)	2279.38 ($T = 22.80$)

found the average total cost of the ELSP to be \$1105.971 and the cycle length to be 22.80. The average vehicle routing cost of input materials is \$912.447 and the average inventory holding cost of input materials is \$260.965. The total average cost of the sequential approach is, therefore, \$2279.383. Table 3 summarises the results of the proposed algorithm for modified Kuhn and Liske's problem.

Example II Mallya's problem.

We solved Mallya's (1992) five-item problem with Kuhn and Liske's (2011) 25 input materials data. The data for Mallya's problem are shown in Table 4. We verified by computing $\sum_{j=1}^{m}(d_j/p_j) = 0.979$. The data related to the input materials are considered to be the same as those in Kuhn and Liske's problem except changing inventory holding costs for input materials h_i^I to 0.005. We also changed the vehicle capacity Q from 900 to 300,000 to get larger production cycles time than the time required for set-ups during the cycle.

We first show the detailed steps of the heuristic as follows.

Step 1. We compute a lower bound and associated cycle length for each item.

$$T_1 = 47.46, \quad T_2 = 75.46, \quad T_3 = 49.68, \quad T_4 = 32.72, \quad T_5 = 101.61,$$

with a lower bound \$1109.13.

Table 4. Data for Mallya's problem ($\kappa = 0.021$ case).

End item j	Production rate (p_j) (units/day)	Demand rate (d_j) (units/day)	Holding cost (h_j^E) (\$/unit/day)	Set-up cost (A_j) (\$)	Set-up time (s_j) (days)
1	1800	474	0.0013265	80	0.20
2	2500	413	0.0008820	140	0.35
3	4000	528	0.0013685	60	0.15
4	3200	985	0.0009870	100	0.25
5	1500	166	0.0003780	60	0.15

Table 5. Results for Mallya's problem ($\kappa = 0.021$ case).

Lower bound (LB)	Proposed heuristic (TV)	K&L's heuristic (CC)	Sequential solution (SS)
1109.13 ($T = 101.61$)	1161.09 ($T = 104.82$)	1211.38 ($T = 52.41$)	1211.38 ($T = 52.41$)

Table 6. Distributions for randomly generated data for test problems.

Parameters	Set
Number of end items (units)	[5, 15]
Number of input materials (units)	[25]
Production rate of end item (units/unit time)	[1000, 4000]
Demand rate of end item (units/unit time)	[80, 1000]
Holding cost of end item (\$/unit/unit time)	[0.0005, 0.001]
Set-up cost for end item (\$)	[50, 150]
Set-up time for end item (unit time)	[0.1, 0.4]
Holding cost of input material (\$/unit/unit time)	[0.0001, 0.001]

Step 2. We compute production frequencies rounded to power-of-two integers.

$$y_1 = 2, \quad y_2 = 1, \quad y_3 = 2, \quad y_4 = 4, \quad y_5 = 1.$$

Step 3. We generate a production sequence using bin-packing heuristic. We use the following lexicographic order to assign end items to the bins.

$$(y_4, v_4) = (4, 8.07) \geq^L (y_1, v_1) = (2, 13.58) \geq^L (y_3, v_3) = (2, 6.86)$$

$$\geq^L (y_2, v_2) = (1, 17.14) \geq^L (y_5, v_5) = (1, 11.39).$$

The resulting production sequence is as follows:

$$\mathbf{f} = (4, 1, 4, 3, 2, 4, 1, 4, 3, 5).$$

Step 4. We compute production times $\mathbf{t}$ and idle times $\mathbf{u}$ using Equations (18) and (19), respectively, for the given sequence and total average cost using Equation (8).

$$\mathbf{u} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \text{ and}$$

$$\mathbf{t} = (6.92, 15.10, 11.10, 6.88, 17.32, 5.76, 12.50, 8.50, 6.95, 11.60)$$

with total average cost \$1161.09.

Table 7. Computational results for five end-item problems.

No.	$1-\kappa$	Total cost ($)				Improvement (%)		LB gap (%)	
		SS	CC	TV	LB	SS-TV	CC-TV	CC-LB	TV-LB
1	0.94	769.45	379.72	367.50	350.86	52.24	3.22	8.22	4.74
2	0.96	800.23	394.42	404.90	373.59	49.40	−2.66	5.57	8.38
3	0.97	617.69	518.28	513.97	445.56	16.79	0.83	16.32	15.35
4	0.97	601.49	510.17	501.89	433.83	16.56	1.62	17.60	15.69
5	0.93	793.77	435.41	431.70	405.47	45.61	0.85	7.39	6.47
6	0.92	801.74	456.91	431.10	403.13	46.23	5.65	13.34	6.94
7	0.98	460.72	423.59	416.00	381.40	9.70	1.79	11.06	9.07
8	0.97	604.36	522.96	496.03	446.49	17.92	5.15	17.13	11.09
9	0.91	680.88	342.09	325.67	308.20	52.17	4.80	11.00	5.67
10	0.99	463.80	463.80	462.35	429.30	0.31	0.31	8.04	7.70
11	0.91	770.47	475.79	435.87	395.58	43.43	8.39	20.28	10.19
12	0.97	715.53	509.24	488.77	451.40	31.69	4.02	12.81	8.28
13	0.91	965.23	414.84	402.27	366.20	58.32	3.03	13.28	9.85
14	0.91	843.78	460.03	440.76	415.91	47.76	4.19	10.61	5.97
15	0.91	849.70	493.89	469.79	423.32	44.71	4.88	16.67	10.98
16	0.99	534.03	534.03	523.31	469.47	2.01	2.01	13.75	11.47
17	0.91	772.18	412.88	385.52	362.97	50.07	6.63	13.75	6.21
18	0.90	950.73	497.85	489.29	448.24	48.54	1.72	11.07	9.16
19	0.96	755.15	476.16	469.43	426.80	37.84	1.41	11.56	9.99
20	0.96	674.24	405.00	404.75	351.30	39.97	0.06	15.28	15.21
21	0.95	755.15	414.96	410.42	370.29	45.65	1.10	12.06	10.84
22	0.92	761.66	436.12	420.31	371.80	44.82	3.62	17.30	13.05
23	0.94	803.63	478.29	463.43	412.72	42.33	3.11	15.89	12.29
24	0.98	573.84	456.71	453.88	389.68	20.90	0.62	17.20	16.48
25	0.96	832.19	601.37	552.45	460.37	33.61	8.13	30.63	20.00
26	0.95	964.27	511.09	523.43	439.68	45.72	−2.41	16.24	19.05
27	0.90	779.73	398.59	390.57	355.06	49.91	2.01	12.26	10.00
28	0.97	706.57	502.26	502.43	448.29	28.89	−0.03	12.04	12.08
29	0.99	480.98	480.98	463.58	424.12	3.62	3.62	13.41	9.30
30	0.97	713.42	465.28	458.53	404.25	35.73	1.45	15.10	13.43
31	0.91	769.40	409.09	406.23	371.49	47.20	0.70	10.12	9.35
32	0.91	679.77	445.44	394.79	344.90	41.92	11.37	29.15	14.46
33	0.91	989.01	521.83	511.90	445.97	48.24	1.90	17.01	14.78
34	0.95	835.65	441.48	441.44	393.88	47.17	0.01	12.08	12.07
35	0.92	866.40	487.95	463.70	417.77	46.48	4.97	16.80	10.99
36	0.94	742.39	420.34	410.44	364.32	44.71	2.35	15.37	12.66
37	0.94	897.95	478.28	481.62	411.33	46.36	−0.70	16.28	17.09
38	0.93	766.39	410.17	397.88	378.74	48.08	2.99	8.30	5.06
39	0.92	965.94	472.01	465.66	401.33	51.79	1.35	17.61	16.03
40	0.94	811.90	395.16	371.52	340.95	54.24	5.98	15.90	8.97
41	0.98	509.05	499.47	471.52	441.47	7.37	5.60	13.14	6.81
42	0.96	787.16	456.60	415.18	375.50	47.26	9.07	21.60	10.57
43	0.98	484.73	402.75	394.20	366.85	18.68	2.12	9.79	7.46
44	0.99	442.88	433.30	423.35	389.43	4.41	2.30	11.26	8.71
45	0.97	512.95	436.74	431.26	395.03	15.93	1.25	10.56	9.17
46	0.95	795.74	433.19	408.84	357.17	48.62	5.62	21.29	14.47
47	0.96	785.14	450.87	442.95	383.54	43.58	1.76	17.56	15.49
48	0.95	791.40	539.51	530.18	462.37	33.01	1.73	16.68	14.67
49	0.93	751.59	421.92	413.83	367.37	44.94	1.92	14.85	12.65
50	0.91	682.66	405.32	359.00	334.93	47.41	11.43	21.02	7.19
Max.						58.32	11.43	30.63	20.00
Ave.						37.00	3.06	14.66	11.07

Table 8. Computational results for test problems.

No. of end items	Improvement (%)				LB gap (%)			
	SS-TV		CC-TV		CC-LB		TV-LB	
	Max.	Ave.	Max.	Ave.	Max.	Ave.	Max.	Ave.
5	58.32	37.00	11.43	3.06	30.63	14.66	20.00	11.07
10	59.84	33.34	11.34	3.61	26.74	14.24	18.53	10.04
15	58.08	24.01	11.77	4.13	24.92	13.45	17.56	8.67

Table 9. Results of the ratio of the total average cost for test problems.

No. of end items		Mean ratio	Minimum ratio	Maximum ratio
5	CC/LB	1.1466	1.0557	1.3063
	TV/LB	1.1107	1.0474	1.2000
	CC/TV	1.0326	0.9741	1.1290
10	CC/LB	1.1424	1.0846	1.2674
	TV/LB	1.1004	1.0490	1.1853
	CC/TV	1.0384	0.9799	1.1280
15	CC/LB	1.1345	1.0710	1.2492
	TV/LB	1.0867	1.0453	1.1756
	CC/TV	1.0443	0.9875	1.1334

Step 5. We compute α and αT using Equation (22). We update idle times $\mathbf{u}$, production times $\mathbf{t}$, and total average cost.

$$\alpha = 0.7508, \ \mathbf{u} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \ \text{and}$$

$$\mathbf{t} = (6.92, 15.10, 11.10, 6.88, 17.32, 5.76, 12.50, 8.50, 6.95, 11.60)$$

$$\text{with total average cost } \$1161.09.$$

We improved the Kuhn and Liske's heuristic and the sequential solution by 4.15%. In this example, relatively small set-up and inventory holding costs of end items compared to the large routing and inventory holding costs of input materials result in the same solution of Kuhn and Liske's heuristic and the sequential approach. The imbalance between costs of end items and input materials diminishes the effect of the integrated scheduling. Table 5 summarises the results of the proposed algorithm for Mallya's problem.

In addition, we performed computational experiments to compare the performance of our proposed heuristic with that of Kuhn and Liske (2011). The data-set was generated randomly from uniform distributions on the intervals (see Table 6). Three test problems of different sizes are considered. Each test problem consists of 50 randomly generated problems. The ELSP is more meaningful and difficult to solve when κ is small. Thus, we only used problems with $\kappa \leq 0.1$.

Table 7 compares the performances of the algorithms for five end-item problems. Table 8 summarises the results of the proposed algorithm for three sets of test problems that feature different numbers of end items. In Tables 7 and 8, the headings SS-TV and CC-TV refer to the percentage improvement between the sequential solution (SS) or the common cycle (CC) approach of Kuhn and Liske (2011) and the proposed heuristic under the time-varying (TV) lot sizes approach. The improvement is calculated as follows: SS-TV = (SS-TV)/SS × 100, CC-TV = (CC-TV)/CC × 100. The headings CC-LB and TV-LB refer to the percentage gap between the common cycle (CC) approach of Kuhn and Liske (2011) or the proposed heuristic under the time-varying (TV) lot sizes approach and the lower bound (LB). The LB gap is calculated as follows: CC-LB = (CC-LB)/LB × 100, TV-LB = (TV-LB)/LB × 100. As shown in the results, the proposed heuristic outperforms the other heuristics. In Table 7, the proposed heuristic outperforms K&L's heuristic in 46 out of 50 problems. Because both algorithms find a solution for large problems (15 items) in a few seconds, we omitted computation times.

We compare the ratio of the total average cost of the CC to an LB with the ratio of the total average cost of the proposed heuristic (TV) to the same LB, as shown in Table 9 which also reports the ratio of the total average cost of the CC heuristic to that of the TV heuristic. Table 9 confirms that the proposed heuristic outperforms the CC heuristic.

5. Conclusions

Integrated scheduling is an important issue in the competitive manufacturing environment and it is interrelated with transportation problems. We have studied the economic lot and supply scheduling problem (ELSSP) by dealing with three issues: production, inventory and inbound transportation routing in an integrated way. We have assumed that each route contains input materials for a single end item. We simplified and formulated the ELSSP model under the common cycle and the time-varying lot sizes approaches, respectively. We also developed a heuristic algorithm through which we applied the existing lower bound and time-varying lot sizes approach to solve the ELSSP. Computational experiments show that the proposed heuristic algorithm outperforms an existing heuristic. Meta-heuristic algorithms might be developed which might be quite competitive to the heuristic algorithm developed in this study. Developing mathematical models and heuristic algorithms for sequence dependent set-up times/costs might be an interesting extension of this study.

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