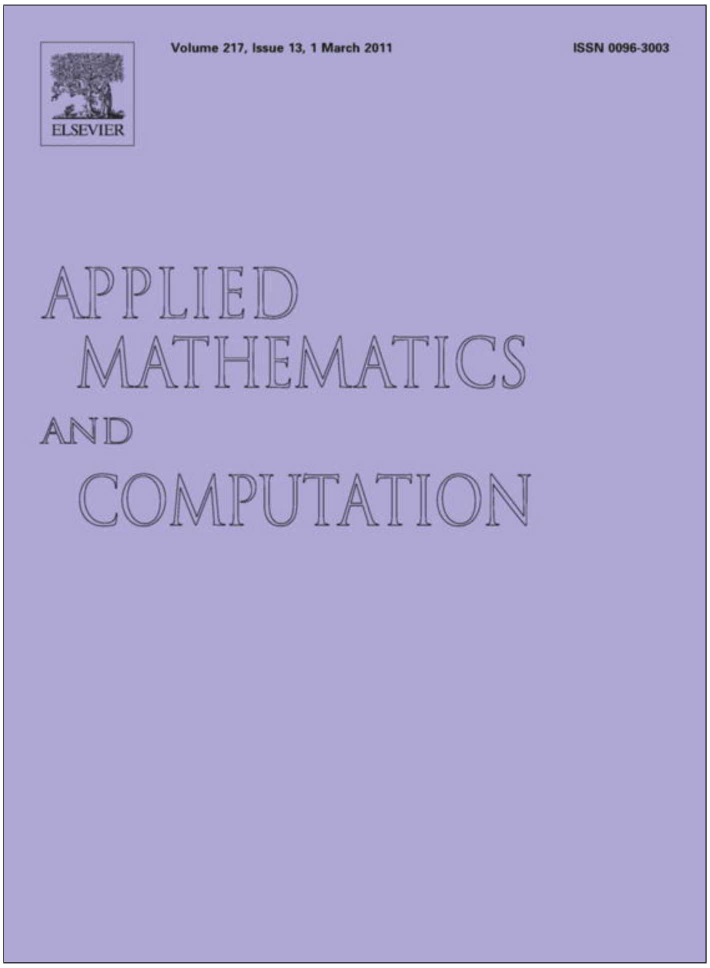
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An EPQ model with inflation in an imperfect production system

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ABSTRACT

In this paper, a production inventory model is considered for stochastic demand with the effect of inflation. Generally, every manufacturing system wants to produce perfect quality items. However, due to real-life problems (labor problems, machine breakdown, etc.), a certain percentage of products are of imperfect quality. The imperfect items are reworked at a cost. The lifetime of a defective item follows a Weibull distribution. Due to the production of imperfect quality items, a product shortage occurs. The profit function is derived by using both a general distribution of demand and the uniform rectangular distribution of demand. Computational experiments along with graphical illustrations are presented to discuss the optimality of the probability functions.

1. Introduction

In the modern world, computer controlled machines are used to increase the productivity and quality of products. Such a manufacturing system can be difficult to control owing to its complicated working system. A system breakdown sometimes occurs resulting in the production of defective items. Given these facts, several researchers and scientists from different sectors have considered models with the flexible manufacturing system (FMS). FMS offers the prospect of eliminating many of the weaknesses of the different approaches but possibly at the cost of many jobs. It consists of small or medium sized automated production lines. The ultimate aim of FMS design is to develop a manufacturing system that is extremely flexible in terms of product and volume mix, and provides high quality and low cost outputs.

In fact, productivity is the measure of inventory turnover ratio. A higher turnover ratio increases the productivity of items. For higher production, machinery systems have to pass through a long run process. During the process, machinery systems are shifted from the *in-control* to the *out-of-control* system where the manufacturing system produces defective/ imperfect quality items. These items are reworked at a cost to restore the original quality and the brand image of the company.

Generally, the classical EPQ (*economic production quantity*) models consider the production of perfect quality items. However, in reality, this is quite different due to the different types of problems. Researchers and scientists have made numerous attempts in the direction of extending the EPQ model with different types of deterministic demand. Some of them considered the EPQ model with stochastic demand. Given all these factors, we consider the expansion of the EPQ model with stochastic demand as well as the production of defective items that follow a Weibull distribution in the presence of a product shortage under the effect of inflation. This type of model has not been considered yet.

The basic *economic order quantity* (EOQ) model was developed by Harris [1]. The square-root formula by Harris is based on constant demand where shortage is not allowed. In most classical EOQ models, demand is considered to be of deterministic type. In reality, however, most demands in the market are of stochastic type. Moran [2] established a model on the storage

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system with a stochastic demand pattern. After that Wagner [3] derived the classical EOQ model with a stochastic demand pattern. Miller [4] extended the continuous time stochastic storage process model with random linear input and output. Researchers like Faddy [5], Harrison and Resnick [6], Nahmias [7] and Meyer et al. [8] extended the EOQ model with different types of stochastic demand.

Scraf [9] developed the optimization model of (s,S) policies in a dynamic inventory problem for a finite time horizon. Iglehart [10] extended the same model with an infinite time horizon. Veinott and Wagner [11] developed the (s,S) inventory model with a new computing algorithm. Among others, Archibald and Silver [12], Silver [13], Federgruen and Zipkin [14] and Zheng and Federgruen [15] extended (s,S) inventory policies with a more efficient algorithm in computational procedures. Ke et al. [16] developed optimization models and a GA-based algorithm for stochastic time–cost trade-off problem.

In this direction, several researchers like Zhou [17], Khouja and Mehrej [18] and Zhou [19] extended the EPQ model considering stochastic demand. Chen et al. [20] found Bayesian single and double variable sampling plans for a Weibull distribution with censoring. Dutta et al. [21] developed continuous review inventory model in mixed fuzzy and stochastic environment. Chiu et al. [22] presented the optimal run-time for the EPQ model with scrap, rework, and stochastic breakdowns, which was again extended by Chiu et al. [23]. Sohn et al. [24] developed an excellent model on random effects Weibull regression model for an occupational Lifetime, and it was extended by Wienke and Kuss [25]. Arizono et al. [26] developed another model with Weibull distribution. Seliaman and Ahmad [27] extended optimizing inventory decisions in a multi-stage supply chain under stochastic demands. Skouri et al. [28] discussed inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. In this direction, Xu [29] presented the optimal policy for a dynamic, non-stationary and stochastic inventory problem with capacity commitment. Recently, Perea et al. [30] developed modeling cooperation on a class of distribution problems and also Liao et al. [31] derived an excellent EPQ model for imperfect processes with imperfect repair and maintenance. Sana [32] extended an EOQ model over an infinite time horizon for perishable items with price dependent demand and partial backlogging. The deterioration rate was taken to be time proportional. Based on the partial backlogging and lost sale cases, the model developed the criterion for the optimal solution for the replenishment schedule.

The above-mentioned models did not take into account the production of defective items. In real life situations, when a machine undergoes repair for a very long time, the manufacturing system may produce defective items. The defective items should be restored to their original quality by reworking them at a cost. Depending on this policy, some researchers like Salameh and Jaber [33], Cardenas-Barron [34] and Goyal and Cardenas-Barron [35] discussed an EPQ model for imperfect quality items. Goyal et al. [36] discussed an EPQ model for imperfect quality items for a deterministic model. Sana et al. [37] investigated the EPQ model for deteriorating items with trended demand and shortages. Mandal and Roy [38] developed a Multi-item imperfect production lot size model with hybrid number cost parameters.

The effect of inflation and time-value of money cannot be ignored in global economics. Buzacott [39] first derived an EOQ model by considering the inflationary effect on costs. Bierman and Thomas [40] then extended Buzacott's [39] model under inflation with discount rates. Misra [41] then extended the EOQ model with different inflation rates for various associated costs. Later, Yang et al. [42] established various inventory models with time varying demand patterns under inflation. Some researchers like in Sarker et al. [43], Moon and Lee [44], Yang [45], Moon et al. [46] and Jolai et al. [47] – derived different types of models under inflation and time-discounting. Dey et al. [48] extended this type of model by considering a two-storage system and dynamic demand under inflation. Chern et al. [49] developed inventory lot size models for deteriorating items with fluctuating demand under inflation. Recently, Sarkar et al. [50] discussed a finite replenishment model with increasing demand under inflation. Sarkar et al. [51] studied the production inventory model with variable demand under the effect of inflation. Sarkar et al. [52] developed an inventory model with different types of stochastic demand pattern.

To the author's knowledge, such a type of model for stochastic demand with the production of defective items has not yet been considered. Therefore, our model has a new managerial insight that helps a manufacturing system/ industry gain maximum profit.

2. The mathematical model

The following assumptions and notation are considered to develop the model.

Assumptions

1. The production-inventory system in an imperfect production system produces a single item type in which some products are defective in nature and can be reworked at a cost.
2. The time horizon of the production system is finite.
3. The demand is stochastic and uniform over the time horizon.
4. The production rate of the inventory system is considered to be constant.
5. Shortages are permitted and fully backlogged.
6. The effect of inflation and time value of money is considered.
7. The lead time is zero.

Notation

Q	production lot size-considered as a decision variable
P	production rate per unit time
Q_1	on-hand inventory at time t without shortage
Q_2	on-hand inventory at time t with shortage
x	uniform demand over $[0,T]$
T	length of production inventory cycle
$f(x)$	probability density function of demand x
C_0	rework cost/defective item
C_h	holding cost/unit/unit time
C_s	shortage cost/unit/unit time
C_p	profit/unit item

We consider a production-inventory system that produces a certain percentage of defective items. The production of items started with a fixed production rate P with production lot Q and continues up to time $t_1 = \frac{Q}{P}$. During $[0,t_1]$, the inventory piles up after adjusting the uniform demand x and after reaching time t_1 it decreases gradually until the zero level at time T. The lifetime of defective item follows a Weibull distribution $\phi(t) = \alpha t^{\beta}$, $\beta > -1$ where α and β are the two parameters and t is the time to failure. Hence, the total number of defective items is:

$$= P\int_0^{t_1} \phi(t) e^{\left[-\int_0^t \phi(\tau)d\tau\right]} dt = P\left\{1 - e^{\left[-\frac{\alpha t_1^{\beta+1}}{(\beta+1)}\right]}\right\}.$$

Therefore, the expected reworking cost with the effect of inflation is

$$Z_1 = \int_0^T C_0 P\left\{1 - e^{\left[-\frac{\alpha t_1^{\beta+1}}{(\beta+1)}\right]}\right\} e^{-\sigma t}dt = C_0 P\left\{1 - e^{\left[-\frac{\alpha (Q/P)^{\beta+1}}{(\beta+1)}\right]}\right\}\left(\frac{1-e^{-\sigma T}}{\sigma}\right) = C_0 P \xi(Q/P) \text{ where } \xi(Q/P) = 1 - e^{\left[-\frac{\alpha (Q/P)^{\beta+1}}{(\beta+1)}\right]}.$$

Now, there are two cases:

Case (1): System without shortage: The mathematical state of on-hand inventory is described by the following differential equations

$$\frac{dQ_1(t)}{dt} = P - \frac{x}{T}; \quad 0 \leqslant t \leqslant t_1, \quad \text{with } Q_1(0) = 0, \quad 0 \leqslant t \leqslant t_1 \tag{1}$$

and

$$\frac{dQ_1(t)}{dt} = -\frac{x}{T} \text{ with } Q_1(T) \geqslant 0; \quad t_1 \leqslant t \leqslant T. \tag{2}$$

The solution of the system can be found using the initial conditions.

$$Q_1(t) = \left\{\begin{array}{ll} (P - \frac{x}{T})t, & 0 \leqslant t \leqslant t_1, \\ Pt_1 - \frac{x}{T}t, & t_1 \leqslant t \leqslant T. \end{array}\right\}$$

Since there is no shortage, the lot size at time $t = T$ is always greater than or equal to zero, which implies demand $x \leqslant Pt_1 = Q$.

In real life situation, inflation is a rise in the general level of prices of goods and services in an economy over a period of time. When the price level rises due to inflation, each unit of currency buys small amount of items; consequently, annual inflation is also an erosion in the holding cost – a loss of real value in the internal medium of trading. Since the value of an inventory item is no longer constant, holding costs become a function of the inflation used to determine the value of ending inventory. Hence, the expected holding cost for the inventory system with the effect of inflation is

$$\begin{aligned} &= C_h \int_0^Q \left\{\int_0^{t_1} Q_1(t)e^{-\sigma t}dt + \int_{t_1}^T Q_1(t)e^{-\sigma t}dt\right\} f(x)dx = C_h \int_0^Q \left\{\int_0^{t_1}\left(P - \frac{x}{T}\right)te^{-\sigma t}dt + \int_{t_1}^T \left(Pt_1 - \frac{x}{T}t\right)e^{-\sigma t}dt\right\} f(x)dx \\ &= C_h \int_0^Q \left\{\frac{P(1-e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma T}}{\sigma} - \frac{x}{T}\left(\frac{1-e^{-\sigma T}}{\sigma^2} - \frac{Te^{-\sigma T}}{\sigma}\right)\right\} f(x)dx. \end{aligned}$$

Therefore, the expected profit with the effect of inflation is

$$\begin{aligned} &= C_p \int_0^Q \left(\int_0^T Qe^{-\sigma t}dt\right) f(x)dx - C_h \int_0^Q \left\{\frac{P(1-e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma T}}{\sigma} - \frac{x}{T}\left(\frac{1-e^{-\sigma T}}{\sigma^2} - \frac{Te^{-\sigma T}}{\sigma}\right)\right\} f(x)dx \\ &= C_p \int_0^Q \left[\frac{Q(1-e^{-\sigma T})}{\sigma}\right] f(x)dx - C_h \int_0^Q \left\{\frac{P(1-e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma T}}{\sigma} - \frac{x}{T}\left(\frac{1-e^{-\sigma T}}{\sigma^2} - \frac{Te^{-\sigma T}}{\sigma}\right)\right\} f(x)dx. \end{aligned}$$

Case (2): System with shortage: The mathematical state of on-hand inventory, $Q_2(t)$, is described by the following differential equations.

$$\frac{dQ_2(t)}{dt} = P - \frac{x}{T}; \quad \text{with } Q_2(0) = 0, \quad 0 \leqslant t \leqslant t_1, \tag{3}$$

$$\frac{dQ_2(t)}{dt} = -\frac{x}{T}; \quad \text{with } Q_2(t_2) = 0, \quad t_1 \leqslant t \leqslant t_2, \tag{4}$$

and

$$\frac{dQ_2(t)}{dt} = -\frac{x}{T}; \quad \text{with } Q_2(T) < 0, \quad t_2 \leqslant t \leqslant T. \tag{5}$$

Using the boundary conditions for $Q_2(t)$, the solution of the system

$$Q_2(t) = \begin{Bmatrix} (P - \frac{x}{T})t, & 0 \leqslant t \leqslant t_1, \\ Pt_1 - \frac{x}{T}t, & t_1 \leqslant t \leqslant t_2, \\ -\frac{x}{T}(t - t_2), & t_2 \leqslant t \leqslant T. \end{Bmatrix}$$

and $Q_2(t_2) = 0$ which implies $t_2 = \frac{Pt_1T}{x}$. Since shortage occurs, so $Q_2(T) < 0$ implies $x > Pt_1 = Q$.

High or unpredictable inflation rates are regarded as injurious to an overall economic sector. They include disorganizations in the real market, and make it crucial for the different marketing sector to budget or plan for long-term. Inflation can operate as a drag on productivity as the business sectors are forced to move resources away from products and services in order to focus on profit and losses from currency inflation. Hence, the expected holding cost with the effect of inflation is

$$\begin{aligned} &= C_h \int_Q^\infty \left\{ \int_0^{t_1} Q_2(t)e^{-\sigma t}dt + \int_{t_1}^{t_2} Q_2(t)e^{-\sigma t}dt \right\} f(x)dx = C_h \int_Q^\infty \left\{ \int_0^{t_1} \left(P - \frac{x}{T}\right)te^{-\sigma t}dt + \int_{t_1}^{t_2} \left(Pt_1 - \frac{x}{T}t\right)e^{-\sigma t}dt \right\} f(x)dx \\ &= C_h \int_Q^\infty \left\{ \frac{P(1 - e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma t_2}}{\sigma} - \frac{x}{T}\left(\frac{1 - e^{-\sigma t_2}}{\sigma^2} - \frac{t_2 e^{-\sigma t_2}}{\sigma}\right) \right\} f(x)dx. \end{aligned}$$

The inflation also affects the shortage cost. Due to inflation, money value will decrease. Therefore, people want to buy more, i.e., due to excess demand shortage may occur. Hence, the expected shortage cost with the effect of inflation is

$$\begin{aligned} \lambda_3 &= C_s \int_Q^\infty \left\{ \int_{t_2}^T -Q_2(t)e^{-\sigma t}dt \right\} f(x)dx = C_s \int_Q^\infty \left\{ \int_{t_2}^T \frac{x}{T}(t - t_2)e^{-\sigma t}dt \right\} f(x)dx \\ &= C_s \int_Q^\infty \left\{ \frac{x}{T}\left(\frac{(t_2 - T)e^{-\sigma T}}{\sigma} + \frac{e^{-\sigma t_2} - e^{-\sigma T}}{\sigma^2}\right) \right\} f(x)dx. \end{aligned}$$

The expected profit, in the presence of shortages, becomes

$$\begin{aligned} &= C_p \int_Q^\infty \left[\int_0^T Qe^{-\sigma t}dt \right] f(x)dx - C_h \int_Q^\infty \left\{ \frac{P(1 - e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma t_2}}{\sigma} - \frac{x}{T}\left(\frac{1 - e^{-\sigma t_2}}{\sigma^2} - \frac{t_2 e^{-\sigma t_2}}{\sigma}\right) \right\} f(x)dx \\ &\quad - C_s \int_Q^\infty \left\{ \frac{x}{T}\left(\frac{(t_2 - T)e^{-\sigma T}}{\sigma} + \frac{e^{-\sigma t_2} - e^{-\sigma T}}{\sigma^2}\right) \right\} f(x)dx = C_p \frac{Q(1 - e^{-\sigma T})}{\sigma} \int_Q^\infty f(x)dx \\ &\quad - C_h \int_Q^\infty \left\{ \frac{P(1 - e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma t_2}}{\sigma} - \frac{x}{T}\left(\frac{1 - e^{-\sigma t_2}}{\sigma^2} - \frac{t_2 e^{-\sigma t_2}}{\sigma}\right) \right\} f(x)dx - C_s \int_Q^\infty \left\{ \frac{x}{T}\left(\frac{(t_2 - T)e^{-\sigma T}}{\sigma} + \frac{e^{-\sigma t_2} - e^{-\sigma T}}{\sigma^2}\right) \right\} f(x)dx. \end{aligned}$$

Therefore, the expected profit for the whole system in the presence of inflation is

$$\begin{aligned} \chi(Q) &= \text{Profit from Case 1} + \text{Profit from Case 2} - \text{Rework cost of the defective items} \\ &= C_p\left[\frac{Q(1 - e^{-\sigma T})}{\sigma}\right] - C_h \int_0^Q \left\{ \frac{P(1 - e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma T}}{\sigma} - \frac{x}{T}\left(\frac{1 - e^{-\sigma T}}{\sigma^2} - \frac{Te^{-\sigma T}}{\sigma}\right) \right\} f(x)dx \\ &\quad - C_h \int_Q^\infty \left\{ \frac{P(1 - e^{-\sigma t_1})}{\sigma^2} - \frac{Qe^{-\sigma t_2}}{\sigma} - \frac{x}{T}\left(\frac{1 - e^{-\sigma t_2}}{\sigma^2} - \frac{t_2 e^{-\sigma t_2}}{\sigma}\right) \right\} f(x)dx \\ &\quad - C_s \int_Q^\infty \left\{ \frac{x}{T}\left(\frac{(t_2 - T)e^{-\sigma T}}{\sigma} + \frac{e^{-\sigma t_2} - e^{-\sigma T}}{\sigma^2}\right) \right\} f(x)dx - \frac{C_0 P\xi(\frac{Q}{P})(1 - e^{-\sigma T})}{\sigma} \\ &= (C_pQ - C_0P\xi(Q/P))\left[\frac{(1 - e^{-\sigma T})}{\sigma}\right] - \frac{C_hP(1 - e^{-\sigma t_1})}{\sigma^2} + C_h\left[\int_0^Q \left\{ \frac{Qe^{-\sigma T}}{\sigma} + \frac{x}{T}\left(\frac{1 - e^{-\sigma T}}{\sigma^2} - \frac{Te^{-\sigma T}}{\sigma}\right) \right\} f(x)dx \right. \\ &\quad \left. + \int_Q^\infty \left\{ \frac{Qe^{-\sigma t_2}}{\sigma} + \frac{x}{T}\left(\frac{1 - e^{-\sigma t_2}}{\sigma^2} - \frac{t_2 e^{-\sigma t_2}}{\sigma}\right) \right\} f(x)dx \right] - C_s \int_Q^\infty \left\{ \frac{x}{T}\left(\frac{(t_2 - T)e^{-\sigma T}}{\sigma} + \frac{e^{-\sigma t_2} - e^{-\sigma T}}{\sigma^2}\right) \right\} f(x)dx. \end{aligned}$$

To maximize the profit function, we need the following lemma.

Lemma 1. *If* $Q^*\epsilon(0,\infty)$ *satisfies the conditions* $(C_p - C_0P\xi'(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] + \frac{C_h e^{-\sigma Q/P}}{\sigma} + C_h\left[\int_0^Q \left\{\frac{e^{-\sigma T}}{\sigma}\right\} f(x)dx + \int_Q^\infty \left\{e^{-\sigma TQ/x}\left(\frac{2}{\sigma} - \frac{TQ\sigma}{x} - \frac{1}{x\sigma} + \frac{\sigma T^2}{x^2}\right)\right\} f(x)dx\right] = C_s \int_Q^\infty \left(\frac{e^{-\sigma T}}{\sigma} - \frac{Te^{-\frac{\sigma TQ}{x}}}{x\sigma}\right) f(x)dx$ *and* $C_0P\xi''(Q/P))\left[\frac{(e^{-\sigma T}-1)}{\sigma}\right] - \frac{C_h e^{\frac{-\sigma Q}{P}}}{P} + C_h\left[\int_Q^\infty \left\{e^{-\sigma TQ/x}\left(\frac{T^2\sigma^2 Q}{x^2} - \frac{2T}{x} - \frac{T^3\sigma^2}{x^3} - \frac{\sigma T}{x} + \frac{T}{x^2}\right)\right\} f(x)dx\right] < C_s \int_Q^\infty \frac{T^2 e^{-\sigma TQ/x}}{x^2} f(x)dx$*, then* $\chi(Q^*)$ *is maximum.*

Proof. We have the profit function $\chi(Q)$ as follows

$$\chi(Q) = (C_pQ - C_0P\xi(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] - \frac{C_hP(1-e^{-\sigma t_1})}{\sigma^2} + C_h\left[\int_0^Q \left\{\frac{Qe^{-\sigma T}}{\sigma} + \frac{x}{T}\left(\frac{1-e^{-\sigma T}}{\sigma^2} - \frac{Te^{-\sigma T}}{\sigma}\right)\right\} f(x)dx\right.$$
$$\left. + \int_Q^\infty \left\{\frac{Qe^{-\sigma t_2}}{\sigma} + \frac{x}{T}\left(\frac{1-e^{-\sigma t_2}}{\sigma^2} - \frac{t_2 e^{-\sigma t_2}}{\sigma}\right)\right\} f(x)dx\right] - C_s\int_Q^\infty \left\{\frac{x}{T}\left(\frac{(t_2-T)e^{-\sigma T}}{\sigma} + \frac{e^{-\sigma t_2} - e^{-\sigma T}}{\sigma^2}\right)\right\} f(x)dx.$$

For maximization of the profit function, $\chi'(Q) = 0$ and $\chi''(Q) < 0$ must be satisfied. Hence,

$$\chi'(Q) = (C_p - C_0P\xi'(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] + \frac{C_h e^{-\sigma Q/P}}{\sigma}$$
$$+ C_h\left[\int_0^Q \left\{\frac{e^{-\sigma T}}{\sigma}\right\} f(x)dx + \int_Q^\infty \left\{e^{-\sigma TQ/x}\left(\frac{2}{\sigma} - \frac{TQ\sigma}{x} - \frac{1}{x\sigma} + \frac{\sigma T^2}{x^2}\right)\right\} f(x)dx\right] - C_s\int_Q^\infty \left(\frac{e^{-\sigma T}}{\sigma} - \frac{Te^{-\sigma TQ/x}}{x\sigma}\right) f(x)dx$$

and

$$\chi''(Q) = C_0P\xi''(Q/P))\left[\frac{(e^{-\sigma T}-1)}{\sigma}\right] - \frac{C_h e^{-\sigma Q/P}}{P} + C_h\left[\int_Q^\infty \left\{e^{-\sigma TQ/x}\left(\frac{T^2\sigma^2 Q}{x^2} - \frac{2T}{x} - \frac{T^3\sigma^2}{x^3} - \frac{\sigma T}{x} + \frac{T}{x^2}\right)\right\} f(x)dx\right]$$
$$- C_s\int_Q^\infty \frac{T^2 e^{-\sigma TQ/x}}{x^2} f(x)dx.$$

For $\chi'(Q) = 0$, we have

$$(C_p - C_0P\xi'(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] + \frac{C_h e^{-\sigma Q/P}}{\sigma} + C_h\left[\int_0^Q \left\{\frac{e^{-\sigma T}}{\sigma}\right\} f(x)dx + \int_Q^\infty \left\{e^{-\sigma TQ/x}\left(\frac{2}{\sigma} - \frac{TQ\sigma}{x} - \frac{1}{x\sigma} + \frac{\sigma T^2}{x^2}\right)\right\} f(x)dx\right]$$
$$= C_s\int_Q^\infty \left(\frac{e^{-\sigma T}}{\sigma} - \frac{Te^{-\sigma TQ/x}}{x\sigma}\right) f(x)dx$$

and from $\chi''(Q) < 0$, we have

$$C_0P\xi''(Q/P))\left[\frac{(e^{-\sigma T}-1)}{\sigma}\right] - \frac{C_h e^{-\sigma Q/P}}{P} + C_h\left[\int_Q^\infty \left\{e^{-\sigma TQ/x}\left(\frac{T^2\sigma^2 Q}{x^2} - \frac{2T}{x} - \frac{T^3\sigma^2}{x^3} - \frac{\sigma T}{x} + \frac{T}{x^2}\right)\right\} f(x)dx\right]$$
$$< C_s\int_Q^\infty \frac{T^2 e^{-\sigma TQ/x}}{x^2} f(x)dx$$

Hence, we prove the lemma. □

3. Rectangular distribution

The density function for demand x is as follows:

$$f(x) = \begin{Bmatrix} \frac{1}{b-a}, & a \leqslant x \leqslant b, \\ 0, & \text{otherwise} \end{Bmatrix}$$

If the density function follows a rectangular distribution, the associated profit function is given by

$$\begin{aligned}\chi(Q) &= (C_pQ - C_0P\xi(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] - \frac{C_hP(1-e^{-\sigma t_1})}{\sigma^2}\\
&\quad + \frac{C_h}{b-a}\left[\frac{1}{2\sigma}\left\{(Q-a)^2e^{-\sigma T} + \frac{Q^2-a^2}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(\frac{e^{-\sigma QT/b}}{b^2} - \frac{e^{-\sigma T}}{Q^2}\right) + \frac{b^2-Q^2}{2T\sigma^2}\right\}\right]\\
&\quad + \frac{C_s-C_h}{b-a}\left[\frac{e^{-\sigma TQ/b}}{bT^2\sigma^3Q} - \frac{e^{-\sigma TQ/b}}{T^3\sigma^4Q^2} - \frac{e^{-\sigma T}}{T^2\sigma^3Q^2} + \frac{e^{-\sigma T}}{T^3\sigma^4Q^2}\right] - \frac{C_s}{b-a}\left[\frac{(b^2-Q^2)e^{-\sigma T}}{2T\sigma^2} - \frac{e^{-\sigma T}(b-Q)^2}{2\sigma}\right].\end{aligned}$$

Now, for the maximum value of the profit function, the conditions $\chi'(Q) = 0$ and $\chi''(Q) < 0$ must be satisfied.

Now

$$\begin{aligned}\chi'(Q) &= (C_p - C_0P\xi'(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] + \frac{C_he^{-\sigma Q/P}}{\sigma}\\
&\quad + \frac{C_h}{b-a}\left[\frac{1}{\sigma}\left\{(Q-a)e^{-\sigma T} + \frac{Q}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(-T\sigma\frac{e^{-\sigma QT/b}}{b^3} + \frac{2e^{-\sigma T}}{Q^3}\right) - \frac{Q}{T\sigma^2}\right\}\right]\\
&\quad + \frac{C_s-C_h}{b-a}\left[-\frac{\left(1+\frac{TQ\sigma}{b}\right)e^{-\sigma TQ/b}}{bT^2\sigma^3Q^2} + \frac{\left(2+\frac{TQ\sigma}{b}\right)e^{-\sigma TQ/b}}{T^3\sigma^4Q^3} + \frac{2e^{-\sigma T}}{T^2\sigma^3Q^3} - \frac{2e^{-\sigma T}}{T^3\sigma^4Q^3}\right] - \frac{C_se^{-\sigma T}}{(b-a)\sigma}\left[b - Q - \frac{Q}{T\sigma}\right]\end{aligned}$$

and

$$\begin{aligned}\chi''(Q) &= C_0P\xi''(Q/P)\left[\frac{(e^{-\sigma T}-1)}{\sigma}\right] - \frac{C_he^{-\sigma Q/P}}{P}\\
&\quad + \frac{C_h}{b-a}\left[\frac{1}{\sigma}\left\{e^{-\sigma T} + \frac{1}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(T^2\sigma^2\frac{e^{-\sigma QT/b}}{b^4} - \frac{6e^{-\sigma T}}{Q^4}\right) - \frac{1}{T\sigma^2}\right\}\right]\\
&\quad + \frac{C_s-C_h}{b-a}\left[\frac{\left(2+\frac{2TQ\sigma}{b}+\frac{TQ^2\sigma}{b}\right)e^{-\sigma TQ/b}}{bT^2\sigma^3Q^3} - \frac{\left(2+\frac{2TQ\sigma}{b}+\frac{T^2Q\sigma^2}{b^2}\right)e^{-\sigma TQ/b}}{T^3\sigma^4Q^4} - \frac{6e^{-\sigma T}}{T^2\sigma^3Q^4} + \frac{6e^{-\sigma T}}{T^3\sigma^4Q^4}\right] + \frac{C_se^{-\sigma T}}{(b-a)\sigma}\left[1 + \frac{1}{T\sigma}\right].\end{aligned}$$

For finding the maximum value of $\chi(Q)$, $\chi'(Q) = 0$ i.e., we have

$$\begin{aligned}&(C_p - C_0P\xi'(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] + \frac{C_he^{-\sigma Q/P}}{\sigma}\\
&\quad + \frac{C_h}{b-a}\left[\frac{1}{\sigma}\left\{(Q-a)e^{-\sigma T} + \frac{Q}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(-T\sigma\frac{e^{-\sigma QT/b}}{b^3} + \frac{2e^{-\sigma T}}{Q^3}\right) - \frac{Q}{T\sigma^2}\right\}\right]\\
&\quad + \frac{C_s-C_h}{b-a}\left[-\frac{\left(1+\frac{TQ\sigma}{b}\right)e^{-\sigma TQ/b}}{bT^2\sigma^3Q^2} + \frac{\left(2+\frac{TQ\sigma}{b}\right)e^{-\sigma TQ/b}}{T^3\sigma^4Q^3} + \frac{2e^{-\sigma T}}{T^2\sigma^3Q^3} - \frac{2e^{-\sigma T}}{T^3\sigma^4Q^3}\right]\\
&\quad = \frac{C_se^{-\sigma T}}{(b-a)\sigma}\left[b - Q - \frac{Q}{T\sigma}\right]\end{aligned}$$

and $\chi''(Q) < 0$, we have

$$\begin{aligned}&C_0P\xi''(Q/P)\left[\frac{(e^{-\sigma T}-1)}{\sigma}\right] + \frac{C_h}{b-a}\left[\frac{1}{\sigma}\left\{e^{-\sigma T} + \frac{1}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(T^2\sigma^2\frac{e^{-\sigma QT/b}}{b^4} - \frac{6e^{-\sigma T}}{Q^4}\right) - \frac{1}{T\sigma^2}\right\}\right]\\
&\quad + \frac{C_s-C_h}{b-a}\left[\frac{\left(2+\frac{2TQ\sigma}{b}+\frac{TQ^2\sigma}{b}\right)e^{-\sigma TQ/b}}{bT^2\sigma^3Q^3} - \frac{\left(2+\frac{2TQ\sigma}{b}+\frac{T^2Q\sigma^2}{b^2}\right)e^{-\sigma TQ/b}}{T^3\sigma^4Q^4} - \frac{6e^{-\sigma T}}{T^2\sigma^3Q^4} + \frac{6e^{-\sigma T}}{T^3\sigma^4Q^4}\right] + \frac{C_se^{-\sigma T}}{(b-a)\sigma}\left[1 + \frac{1}{T\sigma}\right]\\
&\quad < \frac{C_he^{-\sigma Q/P}}{P}\end{aligned}$$

Therefore, we provide a lemma as follows:

Lemma 2. *$\chi(Q)$ attains its maximum value at $Q = Q^*\epsilon(0,\infty)$ if* $(C_p - C_0P\xi'(Q/P))\left[\frac{(1-e^{-\sigma T})}{\sigma}\right] + \frac{C_he^{-\sigma Q/P}}{\sigma} + \frac{C_h}{b-a}\left[\frac{1}{\sigma}\left\{(Q-a)e^{-\sigma T} + \frac{Q}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(-T\sigma\frac{e^{-\sigma QT/b}}{b^3} + \frac{2e^{-\sigma T}}{Q^3}\right) - \frac{Q}{T\sigma^2}\right\}\right] + \frac{C_s-C_h}{b-a}\left[-\frac{\left(1+\frac{TQ\sigma}{b}\right)e^{-\sigma TQ/b}}{bT^2\sigma^3Q^2} + \frac{\left(2+\frac{TQ\sigma}{b}\right)e^{-\sigma TQ/b}}{T^3\sigma^4Q^3} + \frac{2e^{-\sigma T}}{T^2\sigma^3Q^3} - \frac{2e^{-\sigma T}}{T^3\sigma^4Q^3}\right] = \frac{C_se^{-\sigma T}}{(b-a)\sigma}\left[b - Q - \frac{Q}{T\sigma}\right]$ *and* $C_0P\xi''(Q/P)\left[\frac{(e^{-\sigma T}-1)}{\sigma}\right] + \frac{C_h}{b-a}\left[\frac{1}{\sigma}\left\{e^{-\sigma T} + \frac{1}{T\sigma}(1-e^{-\sigma T})\right\} + \left\{\frac{1}{T\sigma}\left(1-\frac{1}{\sigma}\right)\left(T^2\sigma^2\frac{e^{-\sigma QT/b}}{b^4} - \frac{6e^{-\sigma T}}{Q^4}\right) - \frac{1}{T\sigma^2}\right\}\right] + \frac{C_s-C_h}{b-a}\left[\frac{\left(2+\frac{2TQ\sigma}{b}+\frac{TQ^2\sigma}{b}\right)e^{-\sigma TQ/b}}{bT^2\sigma^3Q^3} - \frac{\left(2+\frac{2TQ\sigma}{b}+\frac{T^2Q\sigma^2}{b^2}\right)e^{-\sigma TQ/b}}{T^3\sigma^4Q^4} - \frac{6e^{-\sigma T}}{T^2\sigma^3Q^4} + \frac{6e^{-\sigma T}}{T^3\sigma^4Q^4}\right] + \frac{C_se^{-\sigma T}}{(b-a)\sigma}\left[1 + \frac{1}{T\sigma}\right] < \frac{C_he^{-\sigma Q/P}}{P}$ *are satisfied.*

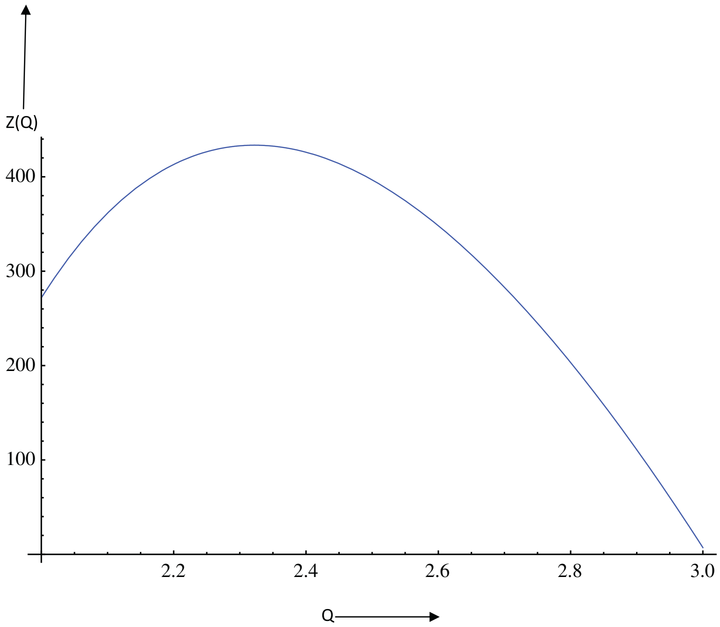

Fig. 1. Graphical Representation of production-inventory system for rectangular distribution of demand.

4. Numerical example

We first solve a numerical example with the help of Mathematica 7 software from where we get the optimal value.

Example 1. We consider the following parameter values in appropriate units: $a = 2$ units, $b = 3$ units, $\alpha = 0.2$, $\beta = 0.7$, $\sigma = 0.03$, $P = 30$ units, $C_h = \$4$, $C_p = \$3.5$, $C_0 = \$1.5$, $C_s = \$3$, $T = 2$ weeks. Then the optimal solution is $Z^* = \$433.47$, $Q^* = 2.32$ units, $t_1^* = 0.077$ weeks. Although the closed type formula for the concavity of the profit function $\chi(Q)$ is not obtained, the graphical representation of the Example (see Fig. 1) shows the *global maximum* value at $Q = 2.32$ units.

The sensitivity analysis of the key parameters and the features of the analysis have been discussed in Table 1.

Table 1
Sensitivity analysis of the numerical example.

Parameters change (in %)		Q^* (%)	Z^* (%)
α	−50	+0.0008	+0.0150
	−25	+0.0004	+0.0070
	+25	−0.0004	−0.0070
	+50	−0.0008	−0.0150
C_h	−50	7.66	−142.44
	−25	5.86	−75.52
	+25	+2.97	+65.08
	+50	+3.91	+131.94
C_p	−50	−0.05	−1.81
	−25	−0.02	−0.90
	+25	+0.02	+0.91
	+50	+0.05	+1.82
C_0	−50	+0.0008	+0.0150
	−25	+0.0004	+0.0076
	+25	−0.0004	−0.0070
	+50	−0.0008	−0.0150
C_s	−50	+4.66	85.19
	−25	+3.39	41.40
	+25	−3.03	18.36
	+50	−13.87	10.79

5. Conclusion

In the present model, the classical EPQ model is extended with stochastic demand under the effect of inflation. Due to the different types of problems during production run-time (labor problems, machinery breakdown, etc.), manufacturing systems produce a certain percentage of defective items. These items are reworked at a cost. After expending the reworking cost, the original product quality is restored. The model is described by considering a general distribution function $f(x)$ and is extended further using a particular type of distribution, namely the rectangular distribution. In the present situation, inflation is a very important factor for all sectors. Therefore, we also take inflation into account. In our model, no closed type formulas for convergence are obtained because of the complicated objective functions. However, a particular numerical illustration in Fig. 1 shows the *concave nature* of the objective functions. To the author's knowledge, such a stochastic EPQ model has not yet been discussed in the inventory literature. This model is applicable in an industry where the production rate is fixed throughout the production-run, shortages are permitted and fully backlogged and inflation is present. A possible future research direction is the study of a multi-item EPQ stochastic model for a variable production rate.

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