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Multi-level supply chain network design with routing

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Recently, the multi-level and multi-facility industrial problem in supply chain management (SCM) has been widely investigated. One of the key issues, central to this problem in the current SCM research area is the interdependence among the location of facilities, the allocation of facilities, and the vehicle routing for the supply of raw materials and products. This paper studies the supply chain network design problem, which involves the location of facilities, allocation of facilities, and routing decisions. The proposed problem has some practical applications. For example, it is necessary for third party logistics (3PL) companies to manage the design of the network and to operate vehicle transportation. The purpose of this study is to determine the optimal location, allocation, and routing with minimum cost to the supply chain network. The study proposes two mixed integer programming models, one without routing and one with routing, and a heuristic algorithm based on LP-relaxation in order to solve the model with routing. The results show that a developed heuristic algorithm is able to find a good solution in a reasonable time.

Keywords: supply chain network design; vehicle routing; LP-relaxation; heuristic algorithm

1. Introduction

The problem with network configuration (design) is the need to specify the structure through which products flow from their source points to their demand points. This involves determining the facilities, if any, to be used; how many there should be, where they should be located, the products and customers assigned to them, the transport services used between them, the sourcing, inter-facility, and distribution to customers product flows, and the inventory levels maintained in the facilities (Ballou 2004).

In recent years, many developments in logistics were connected to the need for information in an efficient supply chain flow. The supply chain is often represented as a network known as a supply chain network (SCN) that comprises nodes that represent facilities (suppliers, manufacturers, distribution centres and customers). A multi-level SCN is a sequence of multiple SCN levels. The flow can only be transferred between two consecutive stages. The multi-level SCN problem involves the choice of facilities (manufacturers and distribution centres) to be opened and the distribution network design that must satisfy the demand with minimum cost. Today's logistics environments are characterised by globalisation. Therefore, the competitive strategy of manufacturers

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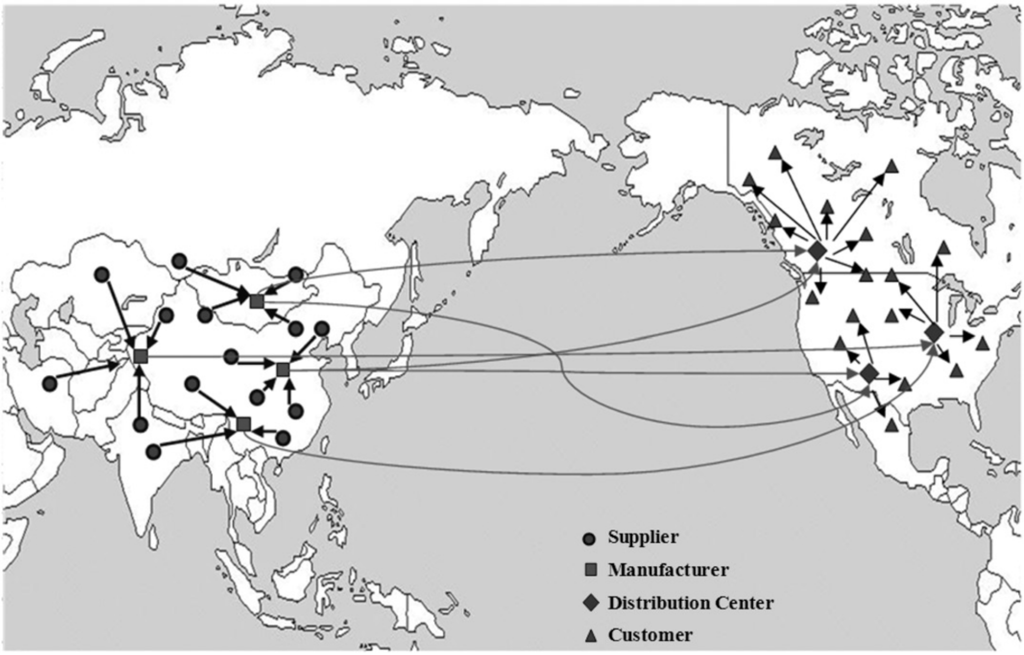

Figure 1. Circumstances of global supply chain network.

and distribution centres has a significant impact on network design decisions within the supply chain (Chopra and Meindl 2004). Manufacturers and distribution centres tend to focus on cost leadership and on finding the lower labour cost and closer location for their manufacturing and distribution facilities (Figure 1). Both academics and practitioners have recognised the interdependence between the manufacturer, the location of the distribution centre, and vehicle routing. However, no attempts have been made to incorporate routing in the location analysis of whole supply chain networks. This study considers four level supply chains that have multi-suppliers, multi-manufacturers, multi-distribution centres, and multi-customers. The solution to the supply chain network design with routing (SCNDR) problem can be defined as one that consists of the location and vehicle routing problems of manufacturers and distribution centres which have to be solved simultaneously. A much more general form of the manufacturer location model needs to be considered in the design of the entire supply chain network, from the supplier to the customer. The study considers a supply chain in which suppliers send materials to manufacturers by vehicle and manufacturers send the products to distribution centres by ship and distribution centres send products to customers by vehicle. Location and capacity allocation decisions need to be made for manufacturers and distribution centres. Multiple distribution centres may be used to satisfy customer demand and multiple manufacturers may be used to transport to distribution centres. It is also assumed that units have been appropriately adjusted so that one unit of input from a supply source produces one unit of finished product.

This study presents a mixed integer programming formulation for the SCNDR problem. This problem combines the location-allocation problem (LAP) and the multi-depot vehicle routing problem (MDVRP). It is a larger and more complex problem, which

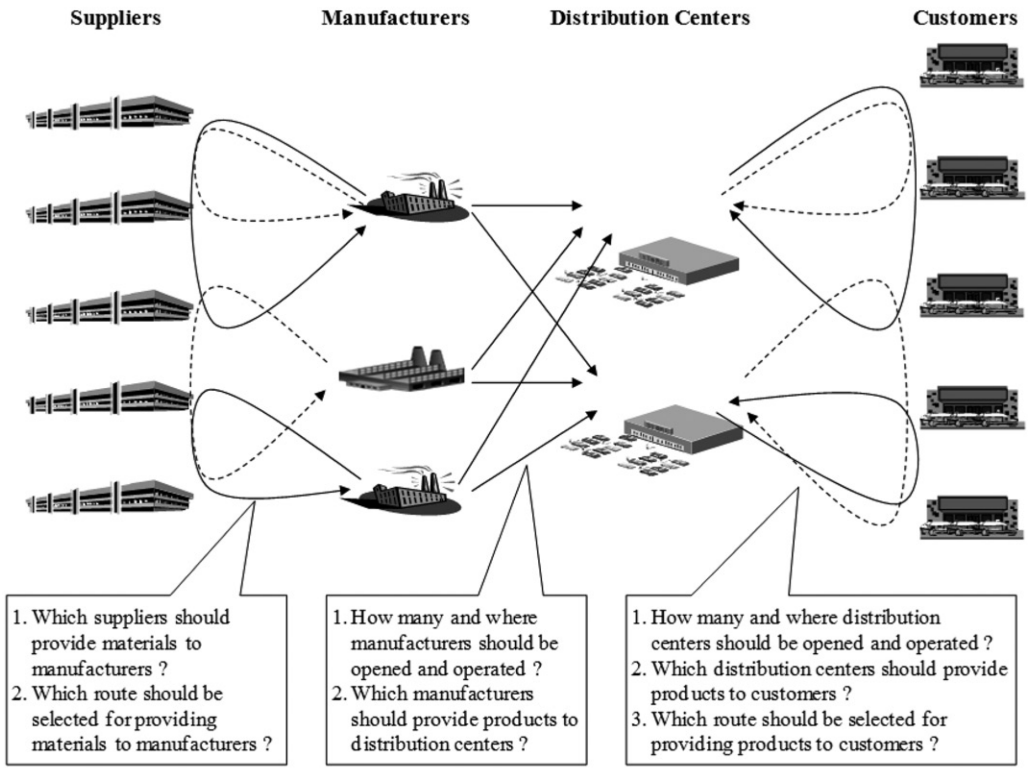

Figure 2. Demonstration of the SCNDR problem.

cannot be solved using existing mixed integer programming techniques. The SCNDR problem proposed here is an NP-hard problem combining the capacitated LAP with MDVRP. Consequently, this study presents a heuristic algorithm that offers an excellent solution quality for the SCNDR problem, in a reasonable amount of computational time. The problems of each supply chain level to be solved by this paper are as demonstrated in Figure 2.

2. Literature review

The competitive nature of today's business environment has resulted in an increase in cooperation among individual companies as members of a supply chain. Accordingly, third party logistics (3PL) companies must operate supply chains for a number of different members who wish to improve their logistics operations while decreasing the related costs. As a result of the dynamic environment in which these supply chains must operate, 3PL companies must make a sequence of inter-related decisions over time.

All companies that aim to be competitive in the market need to pay attention to their organisation as it relates to the entire supply chain. In particular, companies need to analyse the supply chain in order to improve the customer service level without a concurrent uncontrolled growth in costs. In short, companies need to increase the efficiency of their logistics operations. It is therefore of fundamental importance to optimise the flow of goods (and information) among the actors of the supply chain, (the suppliers, manufacturers, distribution centres, and customers). Ambrosino and Scutella (2005) stated that vertical as well as horizontal integrations are required for the optimisation of the flows in the supply chain, and for the optimisation of all related activities. This implies that agreements need to be reached between subjects that operate at different levels of the supply chain (vertical integration), and between actors of the same level (horizontal integration).

Distribution network design problems involve strategic decisions, which influence tactical and operational decisions (Crainic and Laporte 1997). In particular, they involve facility location, transportation and inventory decisions, which affect the cost of the distribution system and the quality of customer service. Therefore, distribution network design problems are central for each company. Abdelmaguid and Dessouky (2006) introduced a new genetic algorithm (GA) approach using a randomised version of a previously developed construction heuristic to generate the initial random population for the integrated inventory distribution problem. Ko and Lee (2007) developed a network design model for an express courier service company using a recursive optimisation and simulation procedure.

Sha and Che (2006) proposed a mathematical model using the genetic algorithm, the analytical hierarchy process, and the multi-attribute utility theory in order to solve the fundamental multi-echelon supply chain network problem that maximises the total overall utility of fulfilling demands. However, the routing problem is not considered in their model. Canel and Khumawala (1997) proposed the international facility location problem (IFLP) model by including multiple periods so that timing of location changes can be more carefully evaluated. Webb (1968) and, more recently, Salhi and Rand (1989), recognised the error introduced into location problems when the interdependence between routing and location decisions is ignored. Since the aforementioned research was published, some further research has focused on the relationships between facilities and transportation costs, stressing that the location of distribution facilities and the routing of vehicles from facilities require decisions to be made interdependently. In particular, in recent years, some location-routing problems (LRP) arising in the context of distribution network design problems have been investigated. Fields other than logistics, such as the communication network design and ship routing also have a tendency to consider problems related to routing between facilities (Lee *et al.* 2002, Gunnarsson *et al.* 2006, Wang *et al.* 2007, Lee *et al.* 2008). Over the past few decades, a great deal of research has been carried out relating to the location-routing problem (LRP). The LRP consists of two sub-problems; the facility location problem (FLP) and the vehicle routing problem (VRP). Both of these are shown to be NP-hard (Karp 1972). Therefore, the SCNDR problem also belongs to the class of NP-hard problems since the SCNDR problem consists of the capacity location-allocation problem (CLAP) and MDVRP in the multi-level supply chain. Laporte *et al.* (1988) studied how the multi-depot vehicle-routing problem and LRP can be transformed into a constrained assignment problem through graphical representation, which can solve up to 80 demand nodes. Another LRP model involves the use of the Clarke-Wright savings method in a case with stochastically processed demands (Chan *et al.* 2001). Exact solutions can only be obtained for not very large problems (from mixed integer programming or branch-and-bound algorithm with special branching rules). These can serve as lower bounds for heuristics validation. Breedam (2001) suggested building hybrid meta-heuristics in order to combine their best features. Wu *et al.* (2002) presented a method for solving the multi-depot location-routing problem (MDLRP). They relaxed some unrealistic assumptions such as homogeneous fleet type and limited number of available vehicles. They solved the problem by dividing it into two sub-problems, i.e., the location-allocation problem and the general vehicle routing problem. Each sub-problem is then solved in a sequential and iterative manner by the simulated annealing algorithm. Meta-heuristics have become prominent approaches in tackling complex, multi-objective problems (Jones *et al*. 2002). Lin and Kwok (2006) applied meta-heuristics of a tabu search and simulated annealing on real data and simulated data, so as to compare their performances under both simultaneous and sequential routing. They also proposed a new statistical procedure to compare the two algorithms on the strength of their multi-objective solutions.

The subsequent information in this paper is organised into the following sections. In Section 3, the study develops two mixed integer programming models for the supply chain network design (SCND) problem and the SCNDR problem. In Section 4, we develop a heuristic algorithm for the developed SCNDR model. Computational experiments for the developed heuristic algorithm are presented in Section 5. Finally, conclusions from this study are presented in Section 6.

3. Mathematical models

The study uses an assumed situation for formulating a mixed integer programming model. In the supply chain network, there are multi-suppliers, multi-manufacturers, multi-distribution centres (DCs), and multi-customers. We consider a single item and a capacity limitation for the supplier, the manufacturer, and the DC. The quantity of product transported to a customer must cover the demand, while a single DC supplies one customer. The objective of this study is to find a solution that will minimise the sum of the transportation (routing) cost, variable (operating) cost, and the fixed cost. The transportation (routing) cost is dependent on the distance between the two selected points. Fixed cost (e.g., for acquiring land, for infrastructure construction or for leasing existing facilities) will be incurred when these facilities are used. The variable cost of operation for manufacturers and for DCs is assumed to be linear. The study developed two mixed integer programming models (one without routing and one with routing) in order to reveal how the SCNDR model (with routing) can reduce the total cost. Given the previous problem statement, an approach used for solving the problem is to formulate the problem using a mixed integer programming model. The model used for solving the problem is explained below.

3.1 *SCND (supply chain network design) model – without routing*

The notation is given as follows:

Indices

i index of suppliers;
I set of suppliers ($i \in I$);
j index of potential manufacturers;
J set of potential manufacturers ($j \in J$);
k index of potential distribution centres;
K set of potential distribution centres ($k \in K$);
l index of customers;
L set of customers ($l \in L$).

Costs

(1) Transportation cost:

TC_{ij}^{SM} transportation cost per mile from supplier i to manufacturer j, for all $i \in I$, $j \in J$;

TC_{jk}^{MD} transportation cost per mile from manufacturer j to distribution centre k, for all $j \in J$, $k \in K$;

TC_{kl}^{DC} transportation cost per mile from distribution centre k to customer l, for all $k \in K$, $l \in L$.

(2) Fixed cost:

FC_{j}^{M} fixed cost of opening manufacturer j, for all $j \in J$;

FC_{k}^{D} fixed cost of opening distribution centre k, for all $k \in K$.

(3) Variable cost:

VC_{j}^{M} variable cost (operating cost) per unit at manufacturer j, for all $j \in J$;

VC_{k}^{D} variable cost (operating cost) per unit at distribution centre k, for all $k \in K$.

Parameters

(1) Capacity & demand:

C_{i}^{S} supply capacity at supplier i, for all $i \in I$;

C_{j}^{M} production capacity at manufacturer j, for all $j \in J$;

C_{k}^{D} capacity at distribution centre k, for all $k \in K$;

C_{l}^{C} demand of customer l, for all $l \in L$;

CV_{j}^{M} capacity of vehicle at manufacturer j, for all $j \in J$;

CV_{k}^{D} capacity of vehicle at distribution centre k, for all $k \in K$.

(2) Distance:

d_{ij}^{SM} distance from supplier i to manufacturer j (mile), for all $i \in I$, $j \in J$;

d_{jk}^{MD} distance from manufacturer j to distribution centre k (mile), for all $j \in J$, $k \in K$;

d_{kl}^{DC} distance from distribution centre k to customer l (mile), for all $k \in K$, $l \in L$;

BM a large number.

Decision variables

X_{ij}^{SM} quantity transported from supplier i to manufacturer j, for all $i \in I$, $j \in J$;

X_{jk}^{MD} quantity transported from manufacturer j to distribution centre k, for all $j \in J$, $k \in K$;

X_{kl}^{DC} quantity transported from distribution centre k to customer l, for all $k \in K$, $l \in L$;

Y_{ij}^{SM} 1, if supplier i is allocated to manufacturer j, for all $i \in I$, $j \in J$; 0, otherwise;

Y_{jk}^{MD} 1, if manufacturer j is allocated to distribution centre k, for all $j \in J$, $k \in K$; 0, otherwise;

Y_{kl}^{DC} 1, if customer l is allocated to distribution centre k, for all $k \in K$, $l \in L$; 0, otherwise;

Z_j^{M} 1, if manufacturer j is opened, for all $j \in J$; 0, otherwise;

Z_k^{D} 1, if distribution centre k is opened, for all $k \in K$; 0, otherwise.

The SCND model is formulated as the following mixed integer programming model:

$$\begin{aligned} \min \sum_{i \in I} \sum_{j \in J} 2TC_{ij}^{\mathrm{SM}} d_{ij}^{\mathrm{SM}} Y_{ij}^{\mathrm{SM}} + \sum_{j \in J} \sum_{k \in K} TC_{jk}^{\mathrm{MD}} d_{jk}^{\mathrm{MD}} Y_{jk}^{\mathrm{MD}} + \sum_{k \in K} \sum_{l \in L} 2TC_{kl}^{\mathrm{DC}} d_{kl}^{\mathrm{DC}} Y_{kl}^{\mathrm{DC}} \\ + \sum_{j \in J} VC_j^{\mathrm{M}} \left(\sum_{i \in I} C_i^{\mathrm{S}} Y_{ij}^{\mathrm{SM}} \right) + \sum_{k \in K} VC_k^{\mathrm{D}} \left(\sum_{l \in L} C_l^{\mathrm{C}} Y_{kl}^{\mathrm{DC}} \right) \\ + \sum_{j \in J} FC_j^{\mathrm{M}} Z_j^{\mathrm{M}} + \sum_{k \in K} FC_k^{\mathrm{D}} Z_k^{\mathrm{D}} \end{aligned} \tag{1}$$

subject to:

$$\sum_{j \in J} X_{ij}^{\mathrm{SM}} \leq C_i^S, \quad \text{all } i \in I \tag{2}$$

$$\sum_{k \in K} X_{jk}^{\mathrm{MD}} \leq C_j^{\mathrm{M}} Z_j^{\mathrm{M}}, \quad \text{all } j \in J \tag{3}$$

$$\sum_{l \in L} X_{kl}^{\mathrm{DC}} \leq C_k^{\mathrm{D}} Z_k^{\mathrm{D}}, \quad \text{all } k \in K \tag{4}$$

$$\sum_{k \in K} X_{kl}^{\mathrm{DC}} \geq C_l^{\mathrm{C}}, \quad \text{all } l \in L \tag{5}$$

$$\sum_{i \in I} X_{ij}^{\mathrm{SM}} - \sum_{k \in K} X_{jk}^{\mathrm{MD}} \geq 0, \quad \text{all } j \in J \tag{6}$$

$$\sum_{j \in J} X_{jk}^{\mathrm{MD}} - \sum_{l \in L} X_{kl}^{\mathrm{DC}} \geq 0, \quad \text{all } k \in K \tag{7}$$

$$\sum_{k \in K} Y_{kl}^{DC} = 1, \quad \text{all } l \in L \tag{8}$$

$$\sum_{i \in I} X_{ij}^{\mathrm{SM}} \leq CV_j^M, \quad \text{all } j \in J \tag{9}$$

$$\sum_{l \in L} X_{kl}^{\mathrm{DC}} \leq CV_k^{\mathrm{D}}, \quad \text{all } k \in K \tag{10}$$

$$X_{ij}^{\mathrm{SM}} \leq BM\, Y_{ij}^{\mathrm{SM}}, \quad \text{all } i \in I,\ j \in J \tag{11}$$

$$X_{jk}^{\mathrm{MD}} \le BMY_{jk}^{\mathrm{MD}}, \quad \text{all } j \in J,\ k \in K \tag{12}$$

$$X_{kl}^{\mathrm{DC}} \le BMY_{kl}^{\mathrm{DC}}, \quad \text{all } k \in K,\ l \in L \tag{13}$$

$$Y_{ij}^{\mathrm{SM}}, Y_{jk}^{\mathrm{MD}}, Y_{kl}^{\mathrm{DC}}, Z_{j}^{\mathrm{M}}, Z_{k}^{\mathrm{D}} \in \{0,1\}, \quad \text{all } i \in I,\ j \in J,\ k \in K,\ l \in L. \tag{14}$$

The objective function, Equation (1), minimises the sum of the costs required to transfer the raw materials/products from the source sites (suppliers and DCs) to the destination sites (manufacturers and customers), the fixed costs for opening and the variable cost for operating facilities. The transportation costs involve the cost of transporting the raw materials from the supplier to the manufacturer, the final product from the manufacturer to the distribution centre, and from the DC to the customer. We consider the cost of the round trip in the transportation cost to compare it with the SCNDR model in which the routing cost is considered. Constraint (2) specifies that the total amount of product transported from a supplier cannot exceed the supplier's capacity. Constraint (3) specifies that the amount produced by the manufacturer cannot exceed its capacity. Constraint (4) specifies that the amount transported through a distribution centre cannot exceed its capacity. Constraint (5) specifies that the amount transported to a customer must cover the demand. Constraints (6) and (7) are flow conservation constrains between suppliers and manufacturers. They state that the amount transported out of a manufacturer cannot exceed the quantity of raw material received from the suppliers. Between manufacturers and distribution centres, they specify that the amount transported from a distribution centre cannot exceed the quantity received from the manufacturers. Constraint (8) ensures that each customer is supplied by a single distribution centre. Constraints (9) and (10) are the vehicle capacity of the manufacturer and the DC. Constraints (11), (12), and (13) are linking constraints for Y and X (if Y is 1, X has some value, otherwise X is 0). Constraint (14) specifies whether or not there is transportation between points of each level and that each manufacturer or distribution centre is either open or closed.

3.2 *SCNDR (supply chain network design with routing) model – with routing*

The notation is given as follows:

Indices

i	index of suppliers;
I	set of suppliers ($i \in I$);
j	index of potential manufacturers;
J	set of potential manufacturers ($j \in J$);
k	index of potential distribution centres;
K	set of potential distribution centres ($k \in K$);
l	index of customers;
L	set of customers ($l \in L$);
g, h	index of suppliers and manufacturers;
G, H	set of suppliers and manufacturers ($G, H = I \cup J$);
s, t	index of distribution centres and customers;
S, T	set of distribution centres and customers ($S, T = K \cup L$);

v^{M} index of vehicles belonging to manufacturers;
V^{M} set of vehicles belonging to manufacturers ($v^{\mathrm{M}} \in V^{\mathrm{M}}$);
v^{D} index of vehicles belonging to distribution centres;
V^{D} set of vehicles belonging to distribution centres ($v^{\mathrm{D}} \in V^{\mathrm{D}}$).

Costs

(1) Transportation cost:

TC_{gh}^{SM} routing cost per mile from point g to point h, for all $g \in G$, $h \in H$;
TC_{jk}^{MD} transportation cost per mile from manufacturer j to distribution centre k, for all $j \in J$, $k \in K$;
TC_{st}^{DC} routing cost per mile from point s to point t, for all $s \in T$, $t \in T$

(2) Fixed cost:

FC_j^{M} fixed cost of opening manufacturer j, for all $j \in J$;
FC_k^{D} fixed cost of opening distribution centre k, for all $k \in K$.

(3) Variable cost

VC_j^{M} variable cost (operating cost) per unit at manufacturer j, for all $j \in J$;
VC_k^{D} variable cost (operating cost) per unit at distribution centre k, for all $k \in K$.

Parameters

(1) Capacity & demand:

C_i^{S} supply capacity at supplier i, for all $i \in I$;
C_j^{M} production capacity at manufacturer j, for all $j \in J$;
C_k^{D} capacity at distribution centre k, for all $k \in K$;
C_l^{C} demand of customer l, for all $l \in L$;
$CV_{v^{\mathrm{M}}}^{\mathrm{M}}$ capacity of vehicle v^{M} at manufacturer, for all $v^{\mathrm{M}} \in V^{\mathrm{M}}$;
$CV_{v^{\mathrm{D}}}^{\mathrm{D}}$ capacity of vehicle v^{D} at distribution centre, for all $v^{\mathrm{D}} \in V^{\mathrm{D}}$.

(2) Distance:

d_{gh}^{SM} distance from point g to point h, for all $g \in G$, $h \in H$;
d_{jk}^{MD} distance from manufacturer j to distribution centre k, for all $j \in J$, $k \in K$;
d_{st}^{DC} distance from point s to point t, for all $s \in T$, $t \in T$.

(3) Temporary:

$U_{iv^{\mathrm{M}}}$, $U_{gv^{\mathrm{M}}}$ auxiliary variables for sub-tour elimination constraints in vehicle v^{M} for suppliers and manufacturers;
$W_{sv^{\mathrm{D}}}$, $W_{lv^{\mathrm{D}}}$ auxiliary variables for sub-tour elimination constraints in vehicle v^{D} for distribution centres and customers;
BM a large number.

Decision variables

$X^{\mathrm{SM}}_{ghv^m}$ 1, if point g immediately precedes point h on vehicle v^{M} for all $g \in G$, $h \in H$; 0, otherwise;

X^{MD}_{jk} 1, if manufacturer j transport to distribution centre k, for all $j \in J$, $k \in K$; 0, otherwise;

$X^{\mathrm{DC}}_{stv^D}$ 1, if point s immediately precedes point t on vehicle v^D, for all $s \in T$, $t \in T$; 0, otherwise;

Y^{SM}_{ij} 1, if supplier i is allocated to manufacturer j, for all $i \in I$, $j \in J$; 0, otherwise;

Y^{MD}_{jk} transportation quantity from manufacturer j to distribution centre k, for all $j \in J$, $k \in K$;

Y^{DC}_{kl} 1, if customer l is allocated to distribution centre k, for all $k \in K$, $l \in L$; 0, otherwise;

Z^{M}_{j} 1, if manufacturer j is opened, for all $j \in J$; 0, otherwise;

Z^{D}_{k} 1, if distribution centre k is opened, for all $k \in K$; 0, otherwise.

The SCNDR model is formulated as the following mixed integer programming model:

$$\min \sum_{v^{\mathrm{M}} \in V^{\mathrm{M}}} \sum_{g \in G} \sum_{h \in H} TC^{\mathrm{SM}}_{gh} d^{\mathrm{SM}}_{gh} X^{\mathrm{SM}}_{ghv^M} + \sum_{j \in J} \sum_{k \in K} TC^{\mathrm{MD}}_{jk} d^{\mathrm{MD}}_{jk} X^{\mathrm{MD}}_{jk} + \sum_{v^{\mathrm{D}} \in V^{\mathrm{D}}} \sum_{s \in S} \sum_{t \in T} TC^{\mathrm{DC}}_{st} d^{\mathrm{DC}}_{st} X^{\mathrm{DC}}_{stv^D}$$
$$+ \sum_{j \in J} VC^{\mathrm{M}}_{j} \left(\sum_{i \in I} C^{\mathrm{S}}_{i} Y^{\mathrm{SM}}_{ij} \right) + \sum_{k \in K} VC^{\mathrm{D}}_{k} \left(\sum_{l \in L} C^{\mathrm{C}}_{l} Y^{\mathrm{DC}}_{kl} \right)$$
$$+ \sum_{j \in J} FC^{\mathrm{M}}_{j} Z^{\mathrm{M}}_{j} + \sum_{k \in K} FC^{\mathrm{D}}_{k} Z^{\mathrm{D}}_{k} \tag{15}$$

subject to:

$$\sum_{v^{\mathrm{M}} \in V^{\mathrm{M}}} \sum_{h \in H} X^{\mathrm{SM}}_{ihv^{\mathrm{M}}} = 1, \quad \text{all } i \in I \tag{16}$$

$$\sum_{i \in I} C^{\mathrm{S}}_{i} \sum_{h \in H} X^{\mathrm{SM}}_{ihv^{\mathrm{M}}} \le CV^{\mathrm{M}}_{v^{\mathrm{M}}}, \quad \text{all } v^{\mathrm{M}} \in V^{\mathrm{M}} \tag{17}$$

$$\sum_{v^{\mathrm{M}} \in V^{\mathrm{M}}} \sum_{g \in G} \sum_{h \in H} X^{\mathrm{SM}}_{ghv^{\mathrm{M}}} \ge 1 \tag{18}$$

$$\sum_{g \in G} X^{\mathrm{SM}}_{hgv^{\mathrm{M}}} - \sum_{g \in G} X^{\mathrm{SM}}_{ghv^{\mathrm{M}}} = 0, \quad \text{all } v^{\mathrm{M}} \in V^{\mathrm{M}}, h \in H \tag{19}$$

$$\sum_{j \in J} \sum_{i \in I} X^{\mathrm{SM}}_{ijv^{\mathrm{M}}} \le 1, \quad \text{all } v^{\mathrm{M}} \in V^{\mathrm{M}} \tag{20}$$

$$\sum_{i \in I} C^{\mathrm{S}}_{i} Y^{\mathrm{SM}}_{ij} - C^{\mathrm{M}}_{j} Z^{\mathrm{M}}_{j} \le 0, \quad \text{all } j \in J \tag{21}$$

$$\sum_{h \in H} X^{\mathrm{SM}}_{ihv^{\mathrm{M}}} + \sum_{h \in H} X^{\mathrm{SM}}_{jhv^{\mathrm{M}}} - Y^{\mathrm{SM}}_{ij} \le 1, \quad \text{all } i \in I,\ j \in J,\ v^{\mathrm{M}} \in V^{\mathrm{M}} \tag{22}$$

$$U_{iv^{\mathrm{M}}} - U_{gv^{\mathrm{M}}} + (|I| + |J|)X^{\mathrm{SM}}_{igv^{\mathrm{M}}} \leq |I| + |J| - 1, \quad \text{all } i \in I,\ v^{\mathrm{M}} \in V^{\mathrm{M}},\ g \in G \tag{23}$$

$$\sum_{k \in K} Y^{\mathrm{MD}}_{jk} - \sum_{i \in I} C^{\mathrm{S}}_i Y^{\mathrm{SM}}_{ij} \leq 0, \quad \text{all } j \in J \tag{24}$$

$$\sum_{j \in J} Y^{\mathrm{MD}}_{jk} - \sum_{l \in L} C^{\mathrm{C}}_l Y^{\mathrm{DC}}_{kl} \geq 0, \quad \text{all } k \in K \tag{25}$$

$$Y^{\mathrm{MD}}_{jk} - BM\, X^{\mathrm{MD}}_{jk} \leq 0, \quad \text{all } j \in J,\ k \in K \tag{26}$$

$$\sum_{v^{\mathrm{D}} \in V^{\mathrm{D}}} \sum_{s \in S} X^{\mathrm{DC}}_{slv^{M}} = 1, \quad \text{all } l \in L \tag{27}$$

$$\sum_{l \in L} C^{\mathrm{C}}_l \sum_{s \in S} X^{\mathrm{DC}}_{slv^{\mathrm{D}}} \leq CV^{\mathrm{D}}_{v^{\mathrm{D}}}, \quad \text{all } v^{\mathrm{D}} \in V^{\mathrm{D}} \tag{28}$$

$$\sum_{v^{\mathrm{D}} \in V^{\mathrm{D}}} \sum_{s \in S} \sum_{t \in T} X^{\mathrm{DC}}_{stv^{\mathrm{D}}} \geq 1 \tag{29}$$

$$\sum_{t \in T} X^{\mathrm{DC}}_{stv^{\mathrm{D}}} - \sum_{t \in T} X^{\mathrm{DC}}_{tsv^{\mathrm{D}}} = 0, \quad \text{all } v^{\mathrm{D}} \in V^{\mathrm{D}},\ s \in S \tag{30}$$

$$\sum_{k \in K} \sum_{l \in L} X^{\mathrm{DC}}_{klv^{\mathrm{D}}} \leq 1, \quad \text{all } v^{\mathrm{D}} \in V^{\mathrm{D}} \tag{31}$$

$$\sum_{l \in L} C^{\mathrm{C}}_l Y^{\mathrm{DC}}_{kl} - C^{\mathrm{D}}_k Z^{\mathrm{D}}_k \leq 0, \quad \text{all } k \in K \tag{32}$$

$$\sum_{t \in T} X^{\mathrm{DC}}_{ktv^{\mathrm{D}}} + \sum_{t \in T} X^{\mathrm{DC}}_{tlv^{\mathrm{D}}} - Y^{\mathrm{DC}}_{kl} \leq 1, \quad \text{all } k \in K,\ l \in L,\ v^{\mathrm{D}} \in V^{\mathrm{D}} \tag{33}$$

$$W_{sv^{\mathrm{D}}} - W_{lv^{\mathrm{D}}} + (|K| + |L|)X^{\mathrm{DC}}_{slv^{\mathrm{D}}} \leq |K| + |L| - 1, \quad \text{all } l \in L,\ v^{\mathrm{D}} \in V^{\mathrm{D}},\ s \in S \tag{34}$$

$$\begin{aligned} X^{\mathrm{SM}}_{ghv^{\mathrm{M}}}, X^{\mathrm{MD}}_{jk}, X^{\mathrm{DC}}_{stv^{\mathrm{D}}}, Y^{\mathrm{SM}}_{ij}, Y^{\mathrm{DC}}_{kl}, Z^{\mathrm{M}}_j, Z^{\mathrm{D}}_k \in \{0, 1\}, \quad & \text{all } g \in G,\ h \in H,\ s \in S, \\ & t \in T,\ i \in I,\ j \in J, \\ & k \in K,\ l \in L, \\ & v^{\mathrm{M}} \in V^{\mathrm{M}},\ v^{\mathrm{D}} \in V^{\mathrm{D}} \end{aligned} \tag{35}$$

$$\begin{aligned} U_{iv^{\mathrm{M}}}, U_{gv^{\mathrm{M}}}, W_{sv^{\mathrm{D}}}, W_{lv^{\mathrm{D}}} \geq 0, \quad & \text{all } g \in G,\ s \in S,\ i \in I, \\ & l \in L,\ v^{\mathrm{M}} \in V^{\mathrm{M}},\ v^{\mathrm{D}} \in V^{\mathrm{D}}. \end{aligned} \tag{36}$$

The objective function, Equation (15), of the SCNDR model is similar to that of the SCND model. However, in the SCNDR model, we consider the routing cost instead of the round trip cost. The objective function minimises the sum of the costs required to route the raw materials/products from the source sites (suppliers and DCs) to the destination sites (manufacturers and customers), the fixed costs for opening and the variable cost for operating facilities. The transportation costs involve transporting the raw materials from the supplier to the manufacturer (routing), the final product from the manufacturer to the distribution centre, and from the DC to the customer (routing). Constraints (16)–(23) guarantee the location/allocation and routing between suppliers and manufacturers. Constraint (16) requires that each supplier be placed on a single route. This implies that the requirement of any single supplier is less than the capacity of the vehicle. Constraint (17) ensures the capacity of the manufacturer's vehicle. Constraint (18) ensures that every delivery route should be connected to a manufacturer. Constraint (19) requires that the vehicle should leave every point that is entered by the vehicle. Constraint (20) specifies that a route cannot be operated from multiple manufacturers. Constraint (21) limits the flow through a manufacturer so that it does not exceed the manufacturer's capacity. Constraint (22) ensures links between the allocation and routing: so that a supplier can be allocated to a manufacturer only if there is a route from that manufacturer going via that supplier. Constraint (23) is a sub-tour elimination constraint in which $|I|$ denotes the number of elements in set I. Constraints (24)–(26) guarantee the transportation between manufacturers and DCs. Constraint (24) ensures that the flow out of a manufacturer should be less than or equal to the flow into a manufacturer. Constraint (25) specifies that the flow out of a DC should be less than or equal to the flow into a DC. Constraint (26) specifies that if the manufacturer is allocated to the DC, the product can be transported from the manufacturer to the DC. Constraints (27)–(34) guarantee the location/allocation and routing between customers and DCs. Constraint (27) requires that each customer be placed on a single route. This implies that the requirement of any single customer is less than the capacity of the vehicle. Constraint (28) ensures the capacity of the DC vehicle. Constraint (29) ensures that every delivery route should be connected to a DC. Constraint (30) requires that the vehicle should leave every point that it has entered. Constraint (31) specifies that a route cannot be operated from multiple DCs. Constraint (32) limits the flow through a DC to the capacity of the DC. Constraint (33) ensures a link between the allocation and the routing so that a customer can be allocated to a DC only if there is a route from that DC going via that customer. Constraint (34) guarantees sub-tour elimination. Constraint (35) is a binary variable (that specifies whether or not there is transportation between points of each level and that each manufacturer or DC is either open or closed). Constraint (36) ensures that the auxiliary variables take positive values.

4. Heuristic algorithm based on LP-relaxation

The study first uses a small problem as a case study. The problem involves four suppliers, two potential manufacturers, two potential DCs, and four customers. The mixed integer programming model includes 1024 variables and 149 constraints. An increase in the number of suppliers (or customers) would cause an exponential increase in the variables and constraints. As mentioned in Section 1, even for small problems, the model for the SCNDR problem cannot be solved using the existing optimisation techniques. Therefore, the study proposes an efficient heuristic algorithm based on LP-relaxation in order to solve this problem. An increase in the number of decision variables, especially binary variables, causes an increase in the time it takes to solve an MIP problem. If the number of binary variables is reduced, the problem can be solved in less time. Hence, in advance, the study determines the binary variables for manufacturers and DCs in the MIP model and subsequently solves the whole model. Figure 3 shows the flowchart of the proposed heuristic algorithm based on LP-relaxation. The detailed procedure for this algorithm is as follows:

Step 1: Determine the lower bounds of required numbers for manufacturers and DCs, initialise T (number of iterations) and *best_ob*:

$$LM = \left\lceil \sum_{i \in I} C_i^{\mathrm{S}} / \min\left\{ C_j^{\mathrm{M}} \right\} \right\rceil,$$

$$LD = \left\lceil \sum_{l \in L} C_l^{\mathrm{C}} / \min\left\{ C_k^{\mathrm{D}} \right\} \right\rceil.$$

Step 2: Determine which manufacturers and DCs are opened, by creating binary values (0 or 1) satisfying the following conditions for manufacturers and DCs:

$$\sum_{j \in J} Z_j^{\mathrm{M}} = LM,$$

$$\sum_{k \in K} Z_k^{\mathrm{D}} = LD.$$

Step 3: If the conditions of Step 2 are satisfied, go to step 4. Otherwise, update the *tabu list* and go to Step 2.

Step 4: Solve the suppliers-manufacturers routing problem and DCs-customers routing problem with Z_j^{M}, Z_k^{D} determined in Step 2. Solve the transportation problem between manufacturers and DCs with routing results.

Step 5: Set *temp_ob* = calculated objective value. If *temp_ob* is less than *best_ob*, set *best_ob* = *temp_ob*. Otherwise, go to Step 6.

Step 6: If T is equal to T_{MAX}, the objective value is *best_ob*. Otherwise, set $T = T + 1$, update *tabu list*, and go to Step 2.

In Step 3 and Step 6, the tabu list does not find the binary value that it found previously. It is a similar function to the tabu list of the tabu search. In Steps 5 and 6, *best_ob* is defined as the best objective value and an initial *best_ob* value is set to a large number such as BM. The maximum number of iterations (T_{MAX}) is defined as follows: the number of combinations of manufacturers and DCs is $2^{|J|} \cdot 2^{|K|}$ since their values are binary. However, at least one manufacturer and one DC should be opened. Therefore, the number of combinations becomes $(2^{|J|}-1) \cdot (2^{|K|}-1)$. However, there are also lower bounds of the required numbers for the manufacturer and DC, and we denote LM and LD, respectively. Therefore, the total number of combinations is ${}_{|J|}C_{LM} \cdot {}_{|K|}C_{LD}$. Consequently, the proposed heuristic algorithm based on LP-relaxation can reduce the search area of the combinations for the binary variables to as much as $(2^{|J|}-1) \cdot (2^{|K}-1) - ({}_{|J|}C_{LM} \cdot {}_{|K|}C_{LD})$. For example, if $|J| = |K| = 10$ and $LM = LD = 5$, then 983,025 ((1024−1) · (1024–1) − ${}_{10}C_5 \cdot {}_{10}C_5 = 1,046,529 - 63,504 = 983{,}025$) iterations can be reduced.

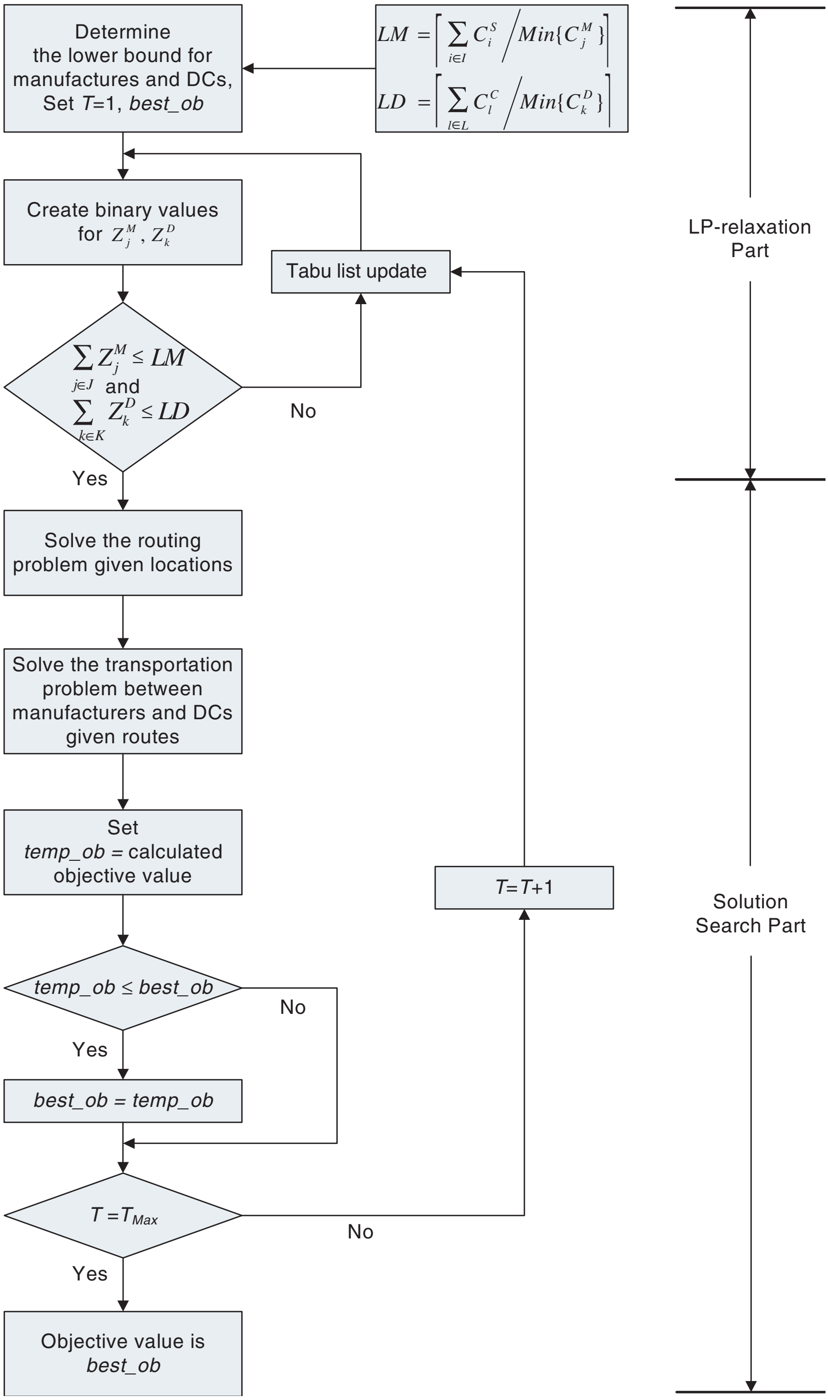

Figure 3. Flowchart of solution procedure for the SCNDR.

5. Computational experiments

The MIP models were conducted on a 3.0 GHz PC with 1 GB RAM on the Microsoft Windows XP operating system. They were solved using Xpress-MP release 2007A. An unacceptable amount of computational time is required when we use this model to find an optimal solution. We compared the SCND model (MIP) and the SCNDR model (MIP and heuristic algorithm) in order to test their validity for small problems. The heuristic algorithm was developed using Xpress-MP release 2007A and Microsoft Visual Basic 6.0. Table 1 shows the composition of the supply chain network. Table 2 shows the objective value, computational time, number of constraints, number of variables, and percentage difference (or savings) between the optimal values of the SCND model and the SCNDR model. Using these results, the study demonstrated that the SCNDR model could reduce the total cost. Moreover, the SCNDR model is more realistic. In Table 2, the computational time for finding an optimal solution increases exponentially with an increase in the number of suppliers, manufacturers, distribution centres, and customers.

Figures 4 and 5 graphically illustrate the objective value and computational time of the MIP models and the heuristic algorithm for the SCND model and SCNDR model. Due to the characteristics of the NP-hard problem, the computational time, constraints, and variables increase exponentially with the number of suppliers, manufacturers, distribution centres, and customers as shown in Figure 5. Figure 6 shows the number of constraints and variables for the SCND MIP model and the SCNDR MIP model.

In the application of the real problem, we experimented with the instances of a realistic size problem (30 suppliers, 10 potential manufacturers, 10 potential DCs, and 30 customers). The supply quantity and customer demand are generated from a uniform distribution on the ranges [5, 30]. We used the modified data from the Solomon instances (Solomon 1987) for the suppliers, manufacturers, DCs, and customer locations. The Solomon instances are divided into six groups, denoted R1, R2, C1, C2, RC1 and RC2. For all of the instances within a group, the customer locations are the same. In R1 and R2, the customer locations are randomly generated from a uniform distribution, and in C1 and C2, they are clustered. In RC1 and RC2, the customer locations are a combination of randomly generated and clustered points. The study created one instance that corresponded to each group of supplier and customer locations, C1-30, C2-30, R1-30, and RC1-30, because the Solomon instances yield only four sets of distinct customer locations. For 10 candidate manufacturers and DC locations, the study randomly generated the candidate locations from a uniform distribution. In order to calculate the distance matrix, the study calculated the Euclidean distance between all node pairs and rounded the value to the nearest integer. Table 3 shows the results of the SCNDR heuristic algorithm. In the

Table 1. Data sets for comparing MIPs and heuristic algorithm.

Problem instance	Number of suppliers	Number of manufacturers	Number of DCs	Number of customers
1	3	2	2	3
2	4	2	2	4
3	5	3	3	5
4	6	3	3	6
5	8	3	3	8

Table 2. Comparison results of MIPs and heuristic algorithm.

Problem instance	SCND MIP				SCNDR MIP					Heuristic algorithm	
	Objective value	Computational time (seconds)	Number of constraints	Number of variables	Objective value	Computational time (seconds)	Number of constraints	Number of variables	Savings (%)	Objective value	Computational time (seconds)
1	1538	1.2	32	82	1488	7.7	107	640	3.36	1488	1.5
2	1610	2.5	38	100	1532	80.2	149	1024	5.09	1532	4.6
3	1772	5.8	64	200	1618	1512.7	334	3057	9.52	1618	151.1
4	1928	15.4	72	228	1754	3188.3	439	4068	9.92	1754	256.4
5	3178	74.1	91	308	2890	15,907.3	668	6702	9.97	2890	406.3

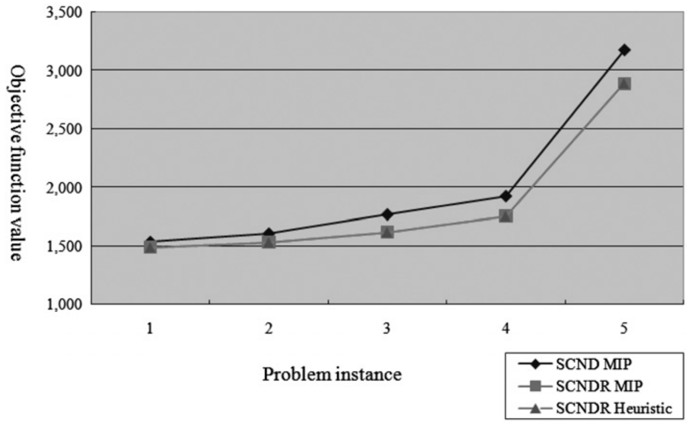

Figure 4. Comparison of the objective values among the SCND MIP, SCNDR MIP, and heuristic.

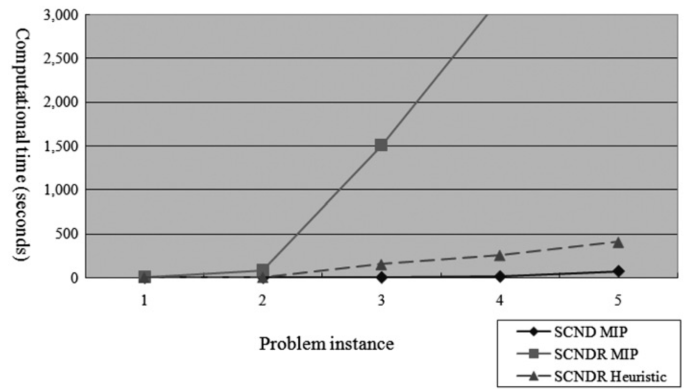

Figure 5. Comparison of the computational times among the SCND MIP, SCNDR MIP, and heuristic.

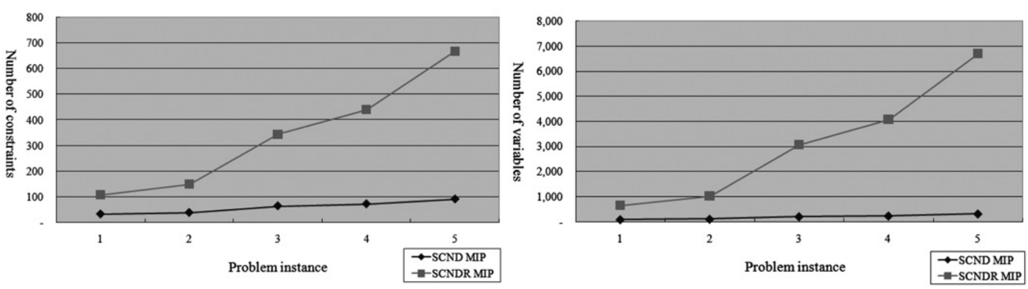

Figure 6. Comparison of the number of constraints and variables of the MIP models.

results, the study experimented with two kinds of T_{MAX} (=200 and 400) for the proposed heuristic algorithm.

In terms of T_{MAX}, the result of the C2-30 clustered instance is partly under the influence of T_{MAX} and takes a longer amount of time. On the other hand, the RC1-30

Table 3. Comparison result of SCNDR heuristic algorithm.

	$T_{MAX}=200$				$T_{MAX}=400$				
Problem instance	Number of manufacturers	Number of DCs	Objective value*	Computational time (seconds)*	Number of manufacturers	Number of DCs	Objective value*	Computational time (seconds)*	Remark
C1-30	3	4	7,725	535.7	4	2	7,190	1,385.1	7.44%
C2-30	2	3	6,436	737.4	2	3	6,406	1,567.3	0.47%
R1-30	3	2	6,760	452.4	3	2	6,640	843.5	1.78%
RC1-30	3	5	9,270	951.4	3	4	8,404	1,844.6	9.34%

Note: *Average of 10 evaluations.

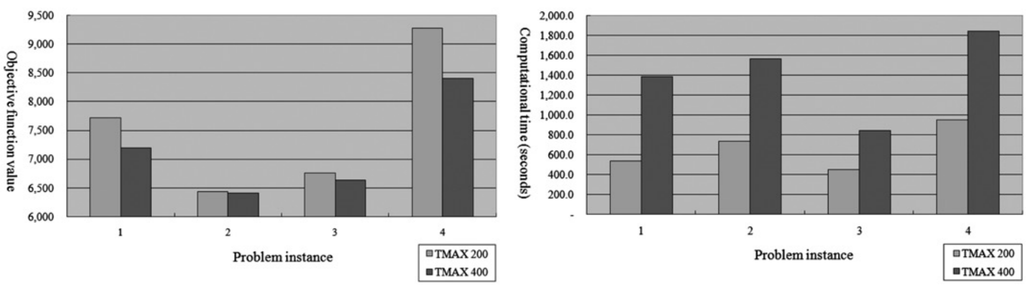

Figure 7. The differences according to T_{MAX} for the heuristic algorithm.

instance (combination of randomly generated and clustered points) is to a large extent under the influence of T_{MAX}, but takes a longer amount of time. Therefore, in the C1-30 instance type it is acceptable to use the T_{MAX} (400), but in the C2 and R1 instance types it is better to use the T_{MAX} (200) (Figure 7).

In addition, we investigate the efficiency of the proposed heuristic method by comparing it with the existing heuristic method. Since this problem consists of the location problem, the allocation problem, and the routing problem in the several layers simultaneously, it differs from that defined by the past studies and their heuristic methods are not directly applicable for this problem. To overcome this limitation, we divide this problem into three sub-problems. The first sub-problem is an LRP between DCs and customers. We first solve the DC location and allocation problem without routing. Then we solve the routing problem among one DC and its customers using Wu *et al.*'s (2002) algorithm. As a result of solving this sub-problem, we obtain the required quantities of product in each DC. The second sub-problem is a simple transportation problem between manufacturers and DCs. It can be solved by a linear program to satisfy the requirement of each DC. As a result of solving this sub-problem, we obtain the product quantity of each manufacturer and the relevant transportation scheme between manufacturers and DCs. The third sub-problem is an LRP between suppliers and manufacturers. We first solve the manufacturer location and allocation problem without routing. Then we solve the routing problem among one manufacturer and its suppliers. We call this algorithm the modified Wu *et al.*'s algorithm. Table 4 shows the computational results of both the SCNDR heuristic algorithm and the modified Wu *et al.*'s algorithm. As expected, the SCNDR

Table 4. Comparison result of the SCNDR heuristic algorithm and the modified Wu *et al.*'s (2002) algorithm.

	SCNDR heuristic algorithm ($T_{\mathrm{MAX}} = 200$)			Modified Wu *et al.*'s algorithm			
Problem instance	Number of manufacturers	Number of DCs	Objective value*	Number of manufacturers	Number of DCs	Objective value*	Remark
C1-30	3	4	7,725	3	4	7,952 (3084 + 760 + 4108)**	2.94%
C2-30	2	3	6,436	3	3	6,806 (2568 + 790 + 3448)**	5.75%
R1-30	3	2	6,760	3	3	7,100 (3654 + 850 + 2596)**	5.03%
RC1-30	3	5	9,270	4	5	9,750 (3886 + 1120 + 4744)**	5.18%

Notes: *Average of 10 evaluations;
**Supplier-manufacturer cost/manufacturer-DC cost/DC-customer cost.

heuristic algorithm outperforms the modified Wu *et al.*'s algorithm for all problem instances since the SCNDR heuristic algorithm solves the location-allocation-routing problem in the four layer supply chain network simultaneously.

6. Conclusions

This paper considered both the supply chain network design model and the supply chain network design with routing model. We formulated two mixed integer programs for these models. In order to solve these models, we proposed a heuristic algorithm based on the relaxed binary variables technique. In order to evaluate the superiority of the proposed algorithm, an experiment was performed. This experiment compared the results between the SCND MIP, the SCNDR MIP, and the SCNDR heuristic algorithm. The results showed that the computational time taken to obtain an optimal solution using Xpress-MP increases exponentially as the problem size grows, while the proposed algorithm reduced the influence of the problem size on the computational time. The computational results demonstrate the efficiency of the developed heuristic algorithm. In terms of real time application, the size of the problem might be larger than that of the test data. In such environments, it is not feasible to find an optimal solution in a reasonable amount of time. Hence, the proposed algorithm can be suitably used in the real situation of designing a global supply chain network in third party logistics companies. Developing some meta-heuristics for the SCNDR model and comparing them with our heuristic algorithm might be an interesting research problem.

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