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The joint replenishment problem involving multiple suppliers offering quantity discounts

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In this article, we deal with the problem of determining the economic operating policy when a number of items are to be procured from a number of suppliers offering different quantity discounts schedules. In such inventory problems, a fixed cost is incurred with each replenishment order, independent of the suppliers as well as the items involved in the order. Further, the item involves a minor fixed cost. In such a system, it includes the supplier selection problem when considering the quantity discounts as well as the general joint replenishment problem. We develop a hybrid genetic algorithm for this NP-hard decision problem and extend it to systems with resource restrictions.

Keywords: Joint replenishment problem; Quantity discounts; Multi-suppliers; Genetic algorithm

1. Introduction

In inventory systems, cost savings can be achieved by coordinating the replenishment of several items. The joint replenishment problem (JRP) deals with the problem of coordinating the replenishment of a group of items that may be jointly ordered from a single supplier. In such systems, the ordering cost has two components: a major common ordering cost S incurred whenever an order is placed and a minor ordering cost s_i incurred if item i is ordered. During the last three decades, the JRP has received considerable attention from researchers. Arkin *et al.* (1989) proved that the JRP is an NP-hard problem, and therefore, it is unlikely that it can be solved by polynomial-time algorithms. There are only a few studies which deal with exact optimisation procedures. Goyal (1974) proposed an enumeration approach to obtain an optimal solution. Van Eijs (1993) reported that the lower bound on an optimal cycle time used by Goyal (1974) does not guarantee a global optimal solution and derived another algorithm that improves Goyal's algorithm. Viswanathan (1996) proposed tighter bounds on the basic cycle time T to improve the procedures by Goyal (1974) and Van Eijs (1993). Wildeman *et al.* (1997) presented a new solution approach based on Lipschitz optimisation to obtain a solution with an arbitrarily small deviation from the optimal. Bayindir *et al.* (2006) also proposed an efficient solution algorithm based on Lipschitz optimisation for the JRP with concave production cost functions. Porras and Dekker (2006) proposed a new solution method based on the formulation of the problem given by Wildeman *et al.* (1997) to solve the JRP with minimum order quantities.

Unlike these optimisation approaches, Silver (1975, 1976) discussed the advantages and disadvantages of coordinating replenishments and presented an extremely simple non-iterative procedure to solve it. Kaspi and Rosenblatt (1991) proposed an approach based on attempting several values of the basic cycle time between the minimum and maximum values. Then, they applied the heuristic of Kaspi and Rosenblatt (1983) to each value of the basic cycle time, which is a modified version

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of the algorithm of Silver (1975). They demonstrated that their procedure (known as the RAND method) outperforms all the available heuristics. Later, Goyal and Deshmukh (1993) proposed an improvement of the lower bound used by Kaspi and Rosenblatt (1991). Khouja *et al.* (2000) presented genetic algorithms (GAs) for the JRP and compared the performance of their GAs with that of Kaspi and Rosenblatt's heuristic algorithm (1991). Moon and Cha (2006) developed both a modified RAND algorithm and a GA for the joint replenishment problem with resource restrictions.

The general JRP models assume that the unit cost is independent of the quantity ordered. However, frequently, suppliers attempt to induce their customers to place larger orders by offering quantity discounts. These quantity discounts have been considered in many production and inventory models (Silver *et al.* 1998). Pirkul and Aras (1985) considered the problem of multiple-item economic order quantities with all-units discounts offered separately for each item. Benton and Park (1996) classified the literature on lot sizing determination under several types of discount schemes and discussed some of the significant literature in this area. Benton (1991) presented an efficient heuristic algorithm for evaluating alternative discount schedules under multi-item and multi-supplier conditions, considering the resource limitations.

In order to address the joint replenishment problem, Chakravarty (1984) proposed the grouping procedure. He considered the group discounts available on the total purchase value of a group replenishment. However, as mentioned in Silver *et al.* (1998), discounts are sometimes offered not for the total volume of a replenishment composed of several different items but for each individual item included in the replenishment. Recently, Cha and Moon (2005) solved the joint replenishment problem with quantity discounts using both a simple heuristic and a modified RAND algorithm known as QD-RAND.

The aim of our research is to fill the gap in the literature on the JRP, wherein items are procured from multiple suppliers offering quantity discounts. The purpose of this article is to develop an efficient algorithm for solving this type of problem. The following section introduces the mathematical model of this problem. In section 3, both an efficient GA and a useful proposition are developed for solving this problem. We use a numerical example to test the suggested GA and present the results of a sensitivity analysis in section 4. Further, in section 5, we also demonstrate that our GA can be easily extended to the JRP with resource restriction. Finally, we summarise the conclusions of the present work.

2. Mathematical model

Similar to the joint replenishment problem under a deterministic demand condition, the following assumptions are made:

(1) The demand rate for each item is constant and deterministic.
(2) The replenishment lead time is of a known duration.
(3) Shortages are not permitted.
(4) The entire order quantity is delivered at the same time.
(5) The price of each item is dependent on the magnitude of the replenishment of each item of each supplier. The all-units discount schedule is considered.
(6) The inventory holding cost for each item of each supplier is known and constant, independent of the price of each item.
(7) Although several suppliers can be considered while purchasing each item, it can be purchased from only one supplier.

Moreover, we introduce the following notations to discuss the JRP considering the quantity discounts of multi-suppliers.

- m: number of items
- n: number of suppliers
- i: index of item, $i=1,\ldots,m$
- j: index of supplier, $j=1,\ldots,n$
- y: index of price break
- D_i: demand rate of item i
- S: major ordering cost which is incurred whenever an order is placed
- s_{ij}: minor ordering cost which is incurred if item i purchased from supplier j is included
- h_i: inventory cost of item i per unit per unit time
- p_{ijy}: price of item i ordered from supplier j in the yth price break
- q_{ijy}: quantity of item i from supplier j that triggers the yth price break
- X_{ij}: binary variable equal to 1 if item i is ordered from the jth supplier, otherwise it is 0 (decision variable)
- T: basic cycle time which is the joint replenishment time interval (decision variable)
- k_i: integer number that determines the replenishment schedule of item i (decision variable)

According to the above assumptions and definitions, the total relevant cost per unit time to be minimised is given by

$$\mathrm{TC}(X_{ij}\text{'s}, T, k_i\text{'s}) = \frac{S + \sum_{i=1}^{m}\sum_{j=1}^{n}(s_{ij}X_{ij}/k_i)}{T} + \sum_{i=1}^{m}\frac{D_i k_i T h_i}{2} + \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}(T,k_i)D_i X_{ij} \tag{1}$$

that is subject to

$$\sum_{j=1}^{n} X_{ij} = 1, \quad \text{for } i = 1, 2, \ldots, m \tag{2}$$

where C_{ij} is the unit cost function of item i when it is purchased from supplier j. This is the step function of T and k_i. For the all-units quantity discount, the unit cost C_{ij} is represented as follows:

$$C_{ij}(T, k_i) = p_{ijy}, \text{ for } q_{ijy} \leq D_i k_i T < q_{ij(y+1)}$$

where $D_i k_i T$ is the order quantity Q_i of item i. Equation (2) implies that each item i can be purchased from only one supplier.

This problem is very complex and involves many decision variables related to purchasing. These decision variables indicate both the supplier selection (X_{ij}'s) and the replenishment schedule (T, k_i's) of each item. It is evident that this problem is NP-hard since the problem of the supplier selection includes n^m alternatives. We use the GA to determine X_{ij}'s, T, and k_i's that minimise the total relevant cost per unit time.

3. Hybrid genetic algorithm

In this section, we present a new GA approach for solving the JRP considering the quantity discounts of multi-suppliers. We will introduce the main ideas of the GA and demonstrate how we apply it to our problem. Introduced by Khouja *et al.* (2000), a GA is suitable for solving the JRP, an important feature of which is that it can be formulated as a problem having one continuous decision variable (basic cycle T) and a set of integer decision variables (m integer multiples k_i of a basic cycle T).

GAs, which have been widely used to solve operations management problems during the last decade (Aytug *et al.* 2003), are stochastic search algorithms based on the mechanism of natural selection and natural genetics. Unlike conventional search techniques, GAs begin with an initial set of (random) solutions known as a population. These initial sets of solutions can be the results from another method. Each individual in the population is known as a chromosome that represents a solution to the problem at hand. The chromosomes evolve through successive iterations that are called generations. During each generation, the chromosomes are evaluated using some measures of fitness. Generally speaking, the GA is applied to solution spaces that are too large to be exhaustively searched. It is generally accepted that any GA that is to solve a problem must have six basic components (representation, initialisation, fitness function, reproduction, crossover and mutation), but it can have different characteristics, depending on the problem being studied.

We explain our overall strategies including the chromosome style in the following order.

- Representation and initialisation
- Objective and fitness function
- Reproduction, crossover and mutation

3.1 *Representation and initialisation*

The appropriate representation of a solution plays a key role in the development of a GA. Unlike the research of Khouja *et al.*'s (2000) for solving a general JRP, the supplier selection variables (X_{ij}'s), the basic cycle T, and the replenishment schedule variables (k_i's) should be determined by considering the quantity discounts of multi-suppliers. In our GA, we can search X_{ij}'s and k_i's through the operations of the GA and determine the basic cycle T through the optimality condition of T for the given X_{ij}'s and k_i's.

For example, if we purchase 10 items from three suppliers, our chromosome can be represented as follows:

As shown in figure 1, our chromosome is composed of two parts. One is for the supplier selection, and the other for the replenishment schedule of each item. The ith gene of the first part of the chromosome indicates the index of the supplier (j) from whom item i is purchased. The ith gene of the second part of the chromosome indicates k_i, which decides the replenishment schedule of item i. Therefore, we can decide the variables X_{ij}'s and k_i's by this type of chromosome. Our chromosome requires $2m$ genes to decide the supplier selection variables (X_{ij}'s) and the replenishment schedule variables (k_i's).

In our study, we use a random number between 0 and 1 to represent each gene of our chromosome as this allows for easy decoding into a feasible solution by a simple decoding process.

1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
2	1	2	3	3	3	3	1	1	1	12	6	3	1	1	3	4	6	11	37
Supplier selection										Replenishment schedule									

Figure 1. A sample chromosome structure for $m = 10$.

3.2 *Objective and fitness function*

In this subsection, we will demonstrate how our chromosome can be decoded to a feasible solution. Each individual of the population is evaluated by the following steps.

Step 1: Each chromosome is decoded to a feasible solution (X_{ij}'s, k_i's).

Step 2: The optimal basic cycle T is determined for a given (X_{ij}'s, k_i's).

Step 3: The total relevant cost TC is computed for a given (T, X_{ij}'s, k_i's).

Our decoding process for each gene of the first part of our chromosome is as follows:

$$j = 1 + \lfloor n \times \text{Gene}(i) \rfloor, \quad X_{ij} = 1 \text{ and } X_{ij'} = 0 \text{ for } j \neq j'$$

$\lfloor G \rfloor$ is the function that finds an integer number that is less than or equal to G. Similarly, each gene of the second part of our chromosome can be decoded as follows:

$$k_i = k_i^{LB} + \left\lfloor \left(k_i^{UB} - k_i^{LB} + 1\right) \times \text{Gene}(i) \right\rfloor$$

By setting an appropriate range of k_i, we can define the solution space including the optimal solution. It is evident that considerably smaller k_i's lead to a better solution space, as far as it contains the optimal solution.

For solving the general JRP without considering the quantity discounts of multi-suppliers, Khouja *et al.* (2000) used $(k_i^{LB} = 1, k_i^{UB} = \lceil T_i^{IN}/T_{\min} \rceil)$, where $T_i^{IN} = \sqrt{2(S + s_i)/(D_i h_i)}$ is the individual optimal cycle time for product i which is obtained from the EOQ model. It is clear that $k_i^{LB} = 1$ for all i. However, if we do not consider quantity discounts, we can define considerably tighter upper bounds on k_i from the following optimality condition (Goyal 1973):

$$k_i(k_i - 1) \leq \frac{2s_i}{D_i h_i T^2} \leq k_i(k_i + 1)$$

Therefore, we can determine the upper bounds on k_i from the following equation.

$$k_i^{UB}\left(k_i^{UB} - 1\right) \leq \frac{2\max\{s_{ij}\}}{D_i h_i T_{\min}^2} \leq k_i^{UB}\left(k_i^{UB} + 1\right) \tag{3}$$

where $T_{\min} = \min(\sqrt{2s_{ij}/(D_i h_i)})$

Since quantity discounts of multi-suppliers have to be considered, k_i can have a considerably larger value as shown below:

$$k_i^{UB} = \left\lceil \frac{\max\{q_{ijy}\}}{D_i T_{\min}} \right\rceil \tag{4}$$

Equation (4) implies that k_i can take the value indicating that we purchase item i at the minimum unit cost when T is $T_{\min}$. This equation may enlarge the solution space of this problem.

Van Eijs (1993) modified $T_{\min}$ for the optimal strict cyclic strategy, so we can determine the upper bounds on k_i to use the modified $T_{\min}$.

$$k_i^{UB} = \left\lceil \frac{\max\{q_{ijy}\}}{D_i T_{\min}^{\text{Modi}}} \right\rceil \tag{5}$$

where

$$T_{\min}^{\text{Modi}} = \min\left(\sqrt{\frac{s_{ij}}{(D_i h_i)}}\right) \leq \max\left[\min\left(\sqrt{\frac{s_{ij}}{(D_i h_i)}}\right), \frac{2A}{TRC^*}\right].$$

TRC* denotes the cost of the best strategy developed thus far. Though Van Eijs recommended a dynamic lower bound on T, we use $T_{\min}^{\text{Modi}}$ to determine the fixed upper bounds on k_i.

In the next section, the four upper bounds mentioned above will be compared by using a numerical example.

To find T that minimises the total relevant cost per unit time, we present the following proposition.

Proposition 1: *For a given set of X_{ij}'s and k_i's, the optimal basic cycle time T is $T^* = \operatorname{argmin}_{T_y}\{TC(T_y)\}$, where T_y includes T_0 and all T_{ijy}, satisfying the following condition*:

$$T_{ijy} = \frac{q_{ijy}}{D_i k_i} > T_0 = \sqrt{\frac{2\left(S + \sum_{i=1}^{m}\sum_{j=1}^{n}(s_{ij}X_{ij}/k_i)\right)}{\sum_{i=1}^{m} D_i k_i h_i}}$$

where T_0 is the optimal T obtained from the first-order derivative of the total cost function of the JRP, not considering quantity discounts.

Proof: For a particular given set of X_{ij}'s and k_i's, the total cost function of this problem can be easily written as

$$TC(T) = \frac{S + \sum_{i=1}^{m}\sum_{j=1}^{n}(s_{ij}X_{ij}/k_i)}{T} + \sum_{i=1}^{m}\frac{D_i k_i T h_i}{2} + \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}(T, k_i) D_i X_{ij} = TC_1(T) + \sum_{i=1}^{m} C_i(T) D_i$$

where C_i is the unit cost function of item i, which is purchased from supplier j. This is the step function of T.

For given X_{ij}'s and k_i's, it is evident that $TC_1(T)$ is a convex function and $\sum_{i=1}^{m} C_i(T)D_i$ is a decreasing step function. If T_0 is the value that minimises TC_1, $TC(T_0)$ is always less than $TC(T)$ for $T < T_0$ because $TC_1(T) > TC_1(T_0)$ and $\sum_{i=1}^{m} C_i(T)D_i > \sum_{i=1}^{m} C_i(T_0)D_i$. For $T > T_0$, $TC_1(T)$ also increases with the increase in T. However, $TC(T_0)$ is not always less than $TC(T)$ because $\sum_{i=1}^{m} C_i(T)D_i$ decreases at the price break points of $T(T_{ijy})$. Therefore, the optimal basic cycle time T that minimises TC is T_0, or one of the price break points T_{ijy} that is larger than T_0 (figure 2). □

3.3 *Reproduction, crossover and mutation*

Various evolutionary methods can be applied to obtain a suitable solution to this problem. We employ a tournament selection ($r = 2$) for selecting individuals for reproduction. Further, we produce offsprings through a uniform crossover with probability P_c. Whenever an offspring is produced, mutation is applied with the probability P_m. The operation of mutation replaces one randomly chosen gene of the chromosome with a new random number between (0, 1).

Figure 3 shows the overall structure of our GA.

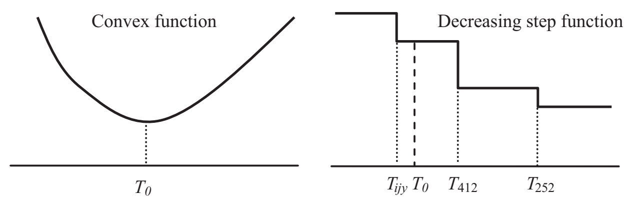

Figure 2. Graphs of $TC_1(T)$ and $\sum_{i=1}^{m} C_i(T)D_i$.

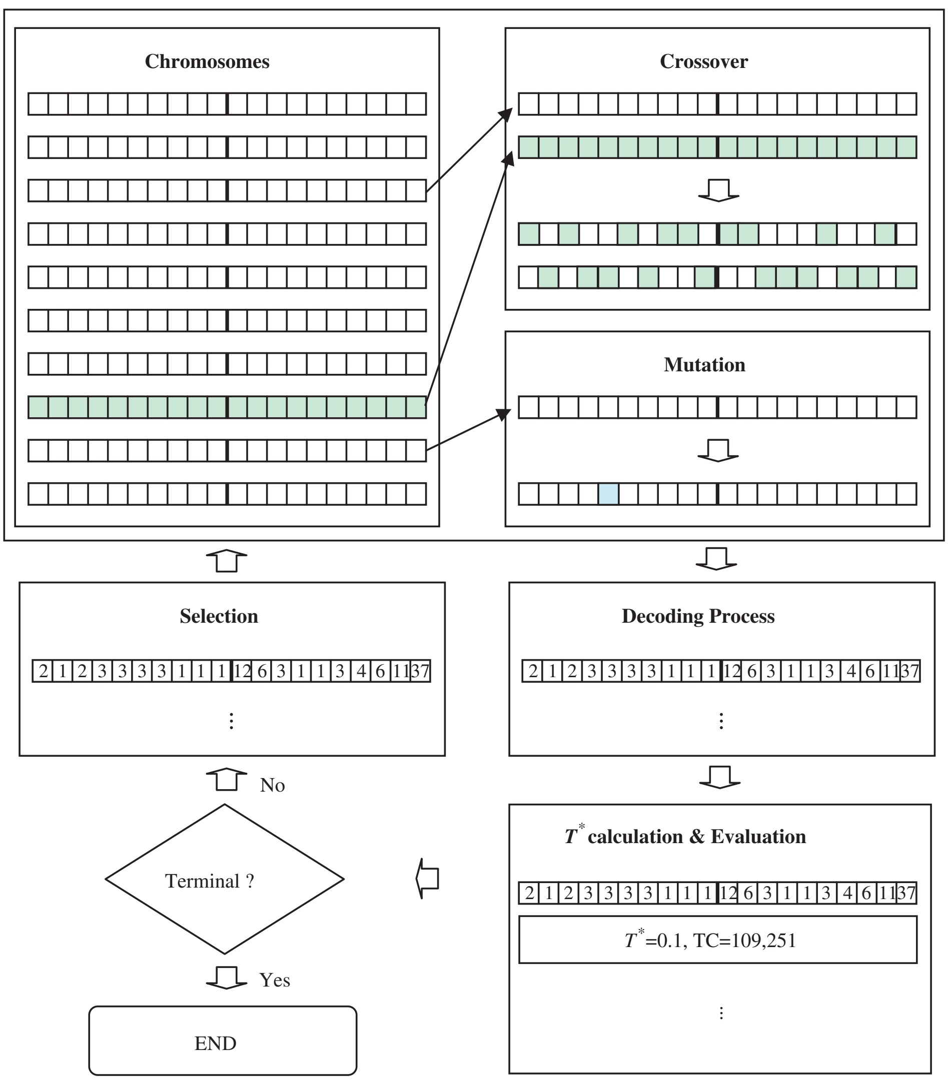

Figure 3. The structure of the proposed genetic algorithm.

4. Numerical examples

In this section, we use an example to test our GA. We consider purchasing ten items from three suppliers. The data for this example are given in tables 1 and 2. In addition, we assume that $S = \$10$.

In this example, a population size of 50 is used and the probabilities of crossover and mutation are set to 0.6 and 0.2, respectively. The termination condition is to stop if no improvement is made after 1000 generations. The GA solutions of the four upper bounds on k_i in this example are shown in table 3.

Table 3 shows that though we can create a larger solution space from equation (5), we can get the best solution from equation (4). In the case of using equation (4) for the upper bounds on k_i, both the representation and decoding solutions of the best chromosome are shown in table 4.

Table 5 shows that the supplier selection strategy is not sensitive to the major ordering cost, unlike the replenishment schedule. It appears rational that the optimal basic period T increases with the increase in the major ordering cost. This table also shows that k_i of each item is likely to converge to a smaller value for the larger major cost.

5. Resource restriction

Most real-life JRPs exist under the conditions of limited resources such as storage, transport equipment capacity, and budget. Goyal (1975) considered a JRP with one resource constraint and developed a heuristic algorithm using the Lagrangian multiplier. It is difficult to apply the heuristic algorithm to this type of problem. However, by modifying Proposition 1, we can easily extend our GA to solve the JRP with resource restriction. The following transportation restriction can be considered.

$$\sum_{i=1}^{m} D_i k_i T b_i \leq B$$

Table 1. Data for the 10-item problem.

Item	1	2	3	4	5	6	7	8	9	10
s_{i1}	5.0	19.4	9.5	8.5	2.2	8.2	10.6	4.0	20.0	16.0
s_{i2}	5.0	19.2	9.0	9.2	2.0	8.0	10.4	4.2	24.0	15.0
s_{i3}	5.2	19.4	8.4	9.2	2.4	7.8	11.2	4.2	24.0	18.0
h_i	0.50	1.94	0.95	0.85	0.22	0.82	1.06	0.40	2.00	1.60
D_i	600	900	2400	12000	18000	3000	2500	180	50	146

Table 2. Data for the discount schedules of each supplier (* No price breaks available).

j	q_{ijy}	C_{1j}	C_{2j}	C_{3j}	C_{4j}	C_{5j}	C_{6j}	C_{7j}	C_{8j}	C_{9j}	C_{10j}
1	$Q_{i1} < 150$	2.50	9.70	4.75	4.25*	1.10	4.10	5.30	2.00*	10.00*	8.00
	$150 \leq Q_{i1} < 300$	2.20	9.60	4.40	–	1.05	3.90	5.10	–	–	7.00
	$Q_{i1} \geq 300$	2.10	9.40	4.10	–	0.95	3.70	4.80	–	–	6.00
2	$Q_{i2} < 200$	2.50	9.60*	4.50	4.60	1.00*	4.00	5.20*	2.10	12.00	7.50*
	$200 \leq Q_{i2} < 400$	2.20	–	4.20	4.25	–	3.80	–	2.05	8.00	–
	$Q_{i2} \geq 400$	1.90	–	4.00	4.10	–	3.50	–	1.95	6.00	–
3	$Q_{i3} < 300$	2.60	9.70	4.20*	4.60	1.20	3.90	5.60	2.10	12.00	9.00
	$300 \leq Q_{i3} < 500$	2.30	9.50	–	4.30	1.10	3.80	5.00	1.95	9.00	6.00
	$Q_{i3} \geq 500$	2.00	9.30	–	4.00	0.90	3.40	4.60	1.90	5.00	5.00

Table 3. Comparison between the performances for various upper bounds on k_i.

Base	k_i^{UB}'s	k_i's	Supplier selection	T	TC	%
Khouja *et al.*'s	11, 6, 5, 2, 3, 4, 4, 20, 26, 16	10, 5, 3, 1, 1, 3, 3, 5, 10, 16	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	0.0667	\$109341	0.08
Equation (3)	6, 5, 3, 1, 1, 3, 3, 11, 22, 12	6, 4, 2, 1, 1, 2, 3, 4, 8, 4	1, 1, 1, 2, 3, 3, 3, 1, 1, 2	0.0833	\$109502	0.23
Equation (4)	27, 18, 7, 2, 1, 6, 7, 88, 315, 108	12, 6, 3, 1, 1, 3, 4, 6, 11, 37	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	0.0556	\$109251	–
Equation (5)	38, 25, 10, 2, 2, 8, 9, 124, 445, 153	16, 8, 4, 1, 1, 4, 5, 8, 15, 50	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	0.0417	\$109270	0.02

Table 4. Representation and decoding solutions based on Equation (4).

	Item	1	2	3	4	5	6	7	8	9	10
GA best		0.6376	0.3250	0.6159	0.6677	0.8372	0.9034	0.9894	0.3191	0.0549	0.1157
Chromosome		0.4334	0.2920	0.3758	0.2873	0.2047	0.4789	0.5461	0.0656	0.0338	0.3344
Decoding solution	j	2	1	2	3	3	3	3	1	1	1
	k_i	12	6	3	1	1	3	4	6	11	37
Optimal T	0.0556										
TC	\$109,251										

Table 5. Sensitivity analysis of the GA solution for the major ordering cost.

S	Supplier selection	Replenishment schedule	T	TC
10	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	12, 6, 3, 1, 1, 3, 4, 6, 11, 37	0.0556	\$109 251
50	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	12, 6, 3, 1, 1, 3, 4, 6, 11, 37	0.0556	\$109 971
100	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	7, 4, 2, 1, 1, 2, 2, 3, 6, 21	0.1000	\$110 389
150	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	4, 2, 1, 1, 1, 1, 2, 2, 4, 13	0.1667	\$110 761
200	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	4, 2, 1, 1, 1, 1, 2, 2, 4, 13	0.1667	\$111 061
300	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	4, 2, 1, 1, 1, 1, 1, 2, 3, 11	0.2000	\$111 565
500	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	3, 2, 1, 1, 1, 1, 1, 2, 3, 9	0.2283	\$112 547
1000	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	2, 1, 1, 1, 1, 1, 1, 1, 2, 6	0.3425	\$114 363

where b_i is the unit weight of item i and B is the maximum weight capacity of a full-truck load.

In this case, we modify Proposition 1 and propose Proposition 2 to determine an optimal T that satisfies the resource restriction

Proposition 2: *For a given set of X_{ij}'s and k_i's, if $T_1 \le T_0$, the optimal basic cycle time T is T_1. If $T_1 > T_0$, then the optimal basic cycle time T is $T^* = \text{argmin}_{T_y}\{TC(T_y)\}$, where T_y includes T_0 and all T_{ijy}, thus satisfying the following condition*:

$$\frac{B}{\sum_{i=1}^m D_i k_i b_i} = T_1 \ge T_{ijy} = \frac{q_{ijy}}{D_i k_i} > T_0$$
$$= \sqrt{\frac{2\left(S + \sum_{i=1}^m \sum_{j=1}^n (s_{ij} X_{ij}/k_i)\right)}{\sum_{i=1}^m D_i k_i h_i}}$$

T_1 denotes the largest T satisfying the resource restriction.

Proof: Unlike Proposition 1, it is evident that we need not consider T that is larger than T_1. Similar to Proposition 1, $TC_1(T)$ is a convex function and $\sum_{i=1}^m C_i(T)D_i$ is a decreasing step function. If T_0 is a value that minimises TC_1, $TC(T_1)$ is always less than $TC(T)$ for $T < T_1 < T_0$. This is because $TC_1(T) > TC_1(T_1)$ and $\sum_{i=1}^m C_i(T)D_i > \sum_{i=1}^m C_i(T_1)D_i$. For $T_0 < T < T_1$, $TC_1(T)$ also increases with the increase in T. However, $TC(T_0)$ is not always less than $TC(T)$ because $\sum_{i=1}^m C_i(T)D_i$ decreases at the price break points of $T(T_{ijy})$. Therefore, the optimal basic cycle time T that minimises TC is T_0 or one of the price break points T_{ijy} that is larger than T_0 and smaller than or equal to T_1 (figure 4). □

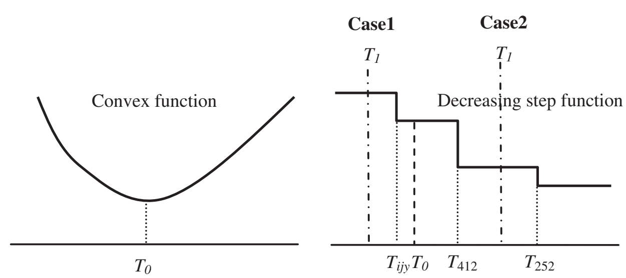

Figure 4. Graphs of $TC_1(T)$ and $\sum_{i=1}^m C_i(T)D_i$.

After adding the resource restriction to the previous example (we assume that all $b_i = 1$), we can obtain the following results from our GA.

Table 6 shows that the optimal purchasing strategy is extremely sensitive to the resource restriction. The supplier selection, the replenishment schedule and the total relevant cost change with the change in the available resources.

6. Concluding remarks

In this article, we considered a JRP involving multiple suppliers that offer quantity discounts. We developed a

Table 6. Sensitivity analysis for resource restriction

B	Supplier selection	Replenishment schedule	T_0	T_1	T^*	TC
5000	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	12, 6, 3, 1, 1, 3, 4, 6, 11, 37	0.0340	0.0659	0.0556	\$109 251
4000	2, 1, 2, 3, 3, 3, 3, 1, 1, 1	16, 8, 4, 1, 1, 4, 5, 8, 15, 50	0.0289	0.0443	0.0417	\$109 270
3000	1, 2, 1, 3, 3, 2, 1, 1, 1, 2	6, 4, 3, 1, 1, 4, 5, 5, 15, 9	0.0368	0.0417	0.0417	\$109 760
2000	2, 2, 3, 3, 3, 2, 2, 1, 1, 2	4, 4, 1, 2, 1, 5, 1, 7, 18, 10	0.0414	0.0280	0.0280	\$112 303

hybrid GA by using the optimality structure of the problem. The GA enables a feasible solution to complex problems, and allows them to be easily represented. In addition, the problem with resource restrictions can be extended using the GA. We hope that this article offers a practical approach to the daily problems faced by purchasing managers who frequently have to procure items from multiple suppliers offering quantity discounts. Furthermore, once the optimal algorithm and or heuristics procedures are developed through further research, our hybrid GA will become a valuable benchmarking algorithm.

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References

E. Arkin, D. Joneja and R. Roundy, "Computational complexity of uncapacitated multi-echelon production planning problems", *Oper. Res. Lett.*, 8, pp. 61–66, 1989.

H. Aytug, M. Khouja and F. Vergara, "Use of genetic algorithms to solve production and operations management problems: a review", *Int. J. Prod. Res.*, 41, pp. 3955–4009, 2003.

Z. Bayindir, S. Birbil and J. Frenk, "The joint replenishment problem with variable production costs", *Eur. J. Oper. Res.*, 175, pp. 622–640, 2006.

W. Benton, "Quantity discounts under conditions of multiple items, multiple suppliers and resource limitations", *Int. J. Prod. Res.*, 29, pp. 1953–1961, 1991.

W. Benton and S. Park, "A classification of literature on determining the lot size under quantity discounts", *Eur. J. Oper. Res.*, 92, pp. 219–238, 1996.

B. Cha and I. Moon, "The joint replenishment problem with quantity discounts", *OR Spectrum*, 27, pp. 569–581, 2005.

A. Chakravarty, "Joint inventory replenishments with group discounts based on invoice value", *Management Sci.*, 30, pp. 1105–1112, 1984.

S. Goyal, "Determination of economic packaging frequency for items jointly replenished", *Management Sci.*, 20, pp. 232–238, 1973.

S. Goyal, "Determination of optimum packaging frequency of items jointly replenished", *Management Sci.*, 21, pp. 436–443, 1974.

S. Goyal, "Analysis of joint replenishment inventory systems with resource restriction", *Oper. Res. Quarterly*, 26, pp. 197–203, 1975.

S. Goyal and S. Deshmukh, "A note on the economic ordering quantity for jointly replenished items", *Int. J. Prod. Res.*, 31, pp. 2959–2961, 1993.

M. Kaspi and M. Rosenblatt, "An improvement of Silver's algorithm for the joint replenishment problem", *IIE Trans.*, 15, pp. 264–269, 1983.

M. Kaspi and M. Rosenblatt, "On the economic ordering quantity for jointly replenished items", *Int. J. Prod. Res.*, 29, pp. 107–114, 1991.

M. Khouja, Z. Michalewicz and S. Satoskar, "A comparison between genetic algorithms and the RAND method for solving the joint replenishment problem", *Prod. Plan. Control*, 11, pp. 556–564, 2000.

I. Moon, and B. Cha, "The joint replenishment problem with resource restrictions", *Eur. J. Oper. Res.*, 173, pp. 190–198, 2006.

H. Pirkul and O. Aras, "Capacitated multiple item ordering problem with quantity discounts", *IIE Transactions*, 17, pp. 206–211, 1985.

E. Porras and R. Dekker, "An efficient optimal solution method for the joint replenishment problem with minimum order quantities", *Eur. J. Oper. Res.*, 174, pp. 1595–1615, 2006.

E. Silver, "Modifying the economic order quantity (EOQ) to handle coordinated replenishment of two or more items", *Prod. Invent. Manag.*, 16, pp. 26–38, 1975.

E. Silver, "A simple method of determining order quantities in jointly replenishments under deterministic demand", *Management Sci.*, 22, pp. 1351–1361, 1976.

E. Silver, D. Pyke and R. Peterson, *Inventory Management and Production Planning and Scheduling*, 3rd ed., New York: John Wiley & Sons, 1998.

M. Van Eijs, "A note on the joint replenishment problem under constant demand", *J. Oper. Res. Soc.*, 44, pp. 185–191, 1993.

S. Viswnanthan, "A new optimal algorithm for the joint replenishment problem", *J. Oper. Res. Soc.*, 44, pp. 185–191, 1996.

R. Wildeman, J. Frenk and R. Dekker, "An efficient optimal solution method for the joint replenishment problem", *Eur. J. Oper. Res.*, 99, pp. 433–444, 1997.

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