A Simple Heuristic on a Multi-product Inventory System with Budgetary Constraint

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Abstract

The present article provides a simple heuristic rule for replenishing products in a multi-item inventory system with budgetary constraint. Numerical experiments show that it provides better cost solution than the traditional Lagrange Multiplier method and almost equal cost solution to Page and Paul's Equal Order Interval method. Moreover, the proposed heuristic rule is very easy to implement in real inventory systems as compared with the existing optimal solution methods.

Keywords: Heuristic; Multi-item inventory; Budget constraint

INTRODUCTION

In practice, most inventory systems accommodate more than one type of product. Such inventor
systems can be studied by treating each type of product independently. But, the problem becomes complicated when some constraints (like budgetary, space or availability of items) are imposed on the system. These constraints may have some impacts on the optimal order quantities and as well as on the total. variable cost of the inventory system. Multi-product inventory systems when certain restrictions are active have so far been studied by many researchers[1-7]. In the present article, we refer to the work of Page and Paul[4].

Page and Paul[4] analyzed a multi-product inventory system with a single restriction (capital budget or warehouse space constraint). We consider here the restriction of the maximum capital invested in stock at any time. If we ignore the restriction, then we have to find Qi (i =1,2,...,n; n being the number of products of the inventory system) such that the total variable inventory cost is minimized. And in that case, each Q; takes the Economic Order Quantity (EOQ) value given by

$$Q=\sqrt{\frac{2C_iY_i}{B_i}}\quad i=1,2,\ldots\ldots\ldots\ldots n \qquad (1)$$

where for the ith product C_i=the order cost per on
Y_i = the demand per unit
B_i = the stockholding cost per unit per unit time.

In general, for a set of n products, there may arise a situation when all n products have to be replenished at the same time. If the restriction of the maximum capital investment (W) at any time in inventory is active, then (1) is still valid if Q; (i = 1,2,...n) satisfy the following constraint:

$$\sum_{i=1}^{n} V_iQ_i \le W, \qquad (2)$$

where V_i is the value of one unit of product i. Otherwise, Q_i (i = 1,2,.. n) need modification to meet
the restriction. In order to satisfy the restriction (2), one can use the traditional Lagrange multiplier technique to obtain the modified Q_i such that

$$Q_{Li}=\sqrt{\frac{2C_iY_i}{B_i+2□V_i}}\quad i=1,2,\ldots\ldots\ldots\ldots n \qquad (3)$$

□ being the Lagrange multiplier associated with the capital investment constraint.

The problem thus seems to be solved but not necessarily optimally as is assumed by Hadley and Whitin[2]. Page and Paul[4] showed that Lagrange multiplier method can produce a utilization of only 50% of the capital investment (see page 817, Page and Paul[4] for reasons). They suggested the

Fixed Cycle or Equal Order Interval Method for adjusting the order intervals of the products to ,avoid the possible situation of replenishing all n products at the same time. The common order interval T for all products, they evaluated, as

$$T=\sqrt{\left(2\sum_{i=1}^{n} Ci \Big/ \sum_{i=1}^{n} B_iY_i\right)} \qquad (4)$$

They also designed the staggering of the replenishments of the products as the following:

The time interval from the replenishment of product n to

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the replenishment of product l is T_1, and the time interval from the replenishment of the product j-1. to the replenishment of product j is Tj, j = 2,3,...,n.
If M be the maximum capital investment required at the time of replenishment o1' the nth product in inventory, then following the procedure laid by Page and Paula one can get the expression for M as

$$M = \sum_{j=1}^{n} \frac{1}{2} TV_j Y_j + \frac{1}{2} \sum_{j=1}^{n} TV_j Y_j, \qquad (5)$$

where

$$T_j = (TV_j Y_j)/(\sum_{j=1}^{n} V_j Y_j), \quad j = 1,2,..........n.$$

If $M \leq W$ then T is optimal, according to Page and Paul[4]. Otherwise, its value has to be modified by evaluating the R.H.S of eqn.(5) to give M = W.

2. A SIMPLE HEURISTIC RULE

We now propose a very simple heuristic rule for staggering of the replenishments of the products under Equal Order Interval Method and provide a simple formula for obtaining the upper limit of the maximum investment in inventory. We assume that each product is replenished at equal time interval during the common order interval T, i .e. for n products, we order at a time interval of T/n. And the upper limit of the maximum investment in inventory can be obtained from the following:

Arrange the items in descending order of magnitude of the product of demand of each item and its unit value (i.e., max.$[V_i Y_i]$ is for i = I.). If INV_j denote the capital investments required at times (j-1.)T/n , j =1,2,......,n then INV_j can be expressed as

$$INV_j = \frac{T}{n} \sum_{k=1}^{n} r_{jk} V_k Y_k, \quad j = 1,2,..........n.$$

where

$$r_{jk} = n+k - j \text{ if } k \leq j, \qquad (6)$$
$$= k - j \text{ if } k > j.$$

Thus $MI_{UL} = \max (INV_j)$ gives the upper limit of the maximum investment in a cycle of duration

j

T. To have an exact utilization of the invested budget, the estimated value of T should be obtained from $W = M_{UL}$.

3. NUMERICAL EXAMPLES

For numerical illustration of the proposed heuristic rule, we first consider the same example assumed by Page and Paul4 and rearrange the items according to our heuristic. When Equal Order Interval Method is applied to the inventory system of three products, we obtain the common order interval T = 0.1. 1.5. Page and Paula's method gives the optimal solution as $T^* = 0.1139$, $Q_1^* = 227$, * $Q_2 = 56$, $Q_3^* = 113$ and the total variable cost = \$3914. Our heuristic gives the upper limit of the maximum inventory investment ($M_{UL.}$,) as \$130,000T [obtained from eqn. (6)]. The estimated value of T then can be found from $W = M_{UL.}$ as 0.108 which gives the order quantities $Q_1 = 216$, $Q_1 = 54Q_3$; = 108 and the total variable cost for the three products per year = \$3919. The computational results are summarized in Table 1. to make a comparison among the Lagrange method, Page and Paul4's method and our heuristic method.

Table 1. Comparison of the optimal results in three different methods

Method of solution	T	Q_1	Q_2	Q_3	Total cost
Lagrange Multiplier	-	145	44	114	\$4063
Page & Paul[4]	0.114	227	56	113	\$3914
Our Heuristic	0.108	216	54	108	\$3919

It is clear from the above table that our heuristic rule provides better cost performance than Lagrange Multiplier method and leads closer to the optimal cost solution of Page and Poul[4]'s method.

We consider another numerical example as given in Table 2.:

Table 2. Data for Johnson and Montgomery[3]'s example (products are arranged according to our model)

Product i	1	2	3
Demand rate (units per year) Y_i	2000	1000	1000
Order cost C_i (dollars per unit per year)	50	50	50
Stockholding cost B_i (dollars per unit per year)	16	10	4
Value V_i (dollars per unit)	80	50	20
EOQ_i (from eqn. (1))	112	100	158

The maximum allowable investment W=\$15,000.

In this example, if the Economic Order Quantities are used, ignoring the budgetary constraint, the maximum investment in inventory would be \$17,120 which is greater than W. Therefore, we should try to find the optimal solution by Lagrange Multiplier technique which when applied gives

$Q_{L1} = 88$, $Q_{L2} = 139$, $Q_{L3} = 98$ and Total cost = \$ 3451, where the Lagrange multiplier □ = 0.03.

The computational results of Page and Paul[4]'s method and our heuristic method are presented in Table 3 which indicates that the total inventory cost obtained by our heuristic is almost equal to that of Page and Paul[4].

Table 3. Computational results

Methodof solution	T	Q_1	Q_2	Q_3	Total cost
LagrangianMultiplier	-	98	88	139	$3451
Page & Paul[4]	0.081	162	81	81	$3715
Our Heuristic	0.079	158	79	79	$3716

Thus the above two numerical experiments ensure that our heuristic rule performs well. Moreover, the proposed rule is easy to implement in practice compare to Page and Paul[4]'s optimal solution method.

4. CONCLUSION

The problem of determining the optimal order quantities and the optimal reorder points in a multi-product inventory system with certain restriction has been studied by many researchers. Attempts have been made to find the optimal solution of the problem; but the methods provided are not so easy to implement in reality. The present article provides a heuristic rule which is not only very simple to implement in practice but also gives better results than traditional Lagrange Multiplier method and almost equal cost solution to the existing Page and Paul[4]'s Equal Order Interval method.

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