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A continuous review inventory model with the controllable production rate of the manufacturer

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Abstract

Optimal operating policy in most deterministic and stochastic inventory models is based on the unrealistic assumption that lead-time is a given parameter. In this article, we develop an inventory model where the replenishment lead-time is assumed to be dependent on the lot size and the production rate of the manufacturer. At the time of contract with a manufacturer, the retailer can negotiate the lead-time by considering the regular production rate of the manufacturer, who usually has the option of increasing his regular production rate up to the maximum (designed) production capacity. If the retailer intends to reduce the lead-time, he has to pay an additional cost to accomplish the increased production rate. Under the assumption that the stochastic demand during lead-time follows a Normal distribution, we study the lead-time reduction by changing the regular production rate of the manufacturer at the risk of paying additional cost. We provide a solution procedure to obtain the efficient ordering strategy of the developed model. Numerical examples are presented to illustrate the solution procedure.

Keywords: lot sizing; lead-time reduction; production rate

1. Introduction

The flexibility and usefulness of models can be improved by a structured relaxation of their inherent assumptions. Silver (1993) suggested a wide variety of possible improvements during the operations of manufacturing, including setup reduction, increasing quality, changing production rates, etc. Given parameters are assumed to be fixed or given, including setup time, setup cost, production rate, lead-time, etc. In recent years, there has been a growing literature devoted to the effects of changing the ‘‘givens’’ (setup time, lead-time, quality level, etc.) for the inventory control decisions. A comprehensive review of this literature up to 1997 can be found in Silver et al. (1998).

In most deterministic and probabilistic inventory models, lead-time is viewed as a prescribed constant or a stochastic variable, which, therefore, is not subject to control. But in many practical situations, lead-time can be reduced at an added cost; in other words, it is controllable. By shortening the lead-time, we can lower the safety stock, reduce the loss caused by stockout, improve the service level to the customer, and therefore, increase the competitiveness in business. Through the Japanese experience of using JIT production, the advantages and benefits associated with the efforts to control the lead-time can be clearly perceived.

Liao and Shyu (1991) presented a probabilistic model in which the order quantity was predetermined and lead-time was a unique decision variable. Later, Ben-Daya and Raouf (1994) extended Liao and Shyu's (1991) model by considering both lead-time and the order quantity as decision variables where shortages were neglected. Ouyang et al. (1996) allowed shortages and extended Ben-Daya and Raouf's (1994) model by adding the stockout cost. In addition, the total amount of stockout was considered a mixture of backorders and lost sales during the stockout period. Moon and Choi (1998) and Hariga and Ben-Daya (1999) improved the model of Ouyang et al. (1996) by simultaneously optimizing the order quantity, the reorder point and lead-time. Ouyang et al. (1999) incorporated ordering cost reduction into the model of Moon and Choi (1998), where the ordering cost can be reduced by capital investment.

Kim and Benton (1995) established a linear relationship between lead-time and lot size based on the observations of Karmarkar (1987). They incorporated this lead-time/lot size relation in the classical stochastic continuous review model. In a recent paper, Hariga (2000) developed a stochastic setup cost reduction model with lead-time, lot size and setup time interaction. Pan et al. (2002) considered the lead-time crashing cost as a function of both the order quantity and the reduced lead-time. Ouyang and Chang (2002) generalized the model of Moon and Choi (1998) by modifying the lead-time crashing cost function and treating the setup cost as one of the decision variables.

In this paper, the lead-time is expressed by the relation between the lot size and the production rate of the manufacturer. Under the assumption that the stochastic demand during lead-time follows a Normal distribution, our objective is to reduce the replenishment lead-time by increasing the production rate of the manufacturer subject to the additional cost to be paid by the retailer. A retailer can negotiate the lead-time by considering the regular production rate of the manufacturer, since the manufacturer can increase his production rate up to the maximum production capacity. If the retailer wants the manufacturer to increase his production rate, the retailer has to accept an additional cost. In this case, the additional cost function is easily induced by the difference between the negotiated production rate and the regular production rate.

The remainder of this paper is organized as follows. We provide a basic continuous review model in Section 2. In Section 3, the lead-time reduction model is addressed and a solution algorithm is presented. Numerical examples to illustrate the solution procedure are shown in Section 4. Finally, Section 5 presents the conclusion of this paper.

2. The basic continuous review model

The notation used in this paper is as follows:

λ	average demand per year
μ_L	expected lead-time demand
A	fixed ordering cost per order
h	cost of holding a unit per year
π	cost per unit short
S	additional cost per unit for increasing production rate
$\eta(r)$	expected number of units short per cycle
Q	order quantity (decision variable)
P	production quantity per day (decision variable)
P_0	regular production quantity per day
$P_{\max}$	maximum production quantity per day
r	reorder point
k	safety factor
L	replenishment lead-time

In the continuous review inventory model, an order of size Q is placed when the inventory level reaches the reorder point r, i.e., there is no overshoot of the reorder point. Following the normal approximation, the expected total annual inventory cost TC can be represented as follows:

$$TC(Q,r) = \frac{\lambda A}{Q} + h\left(\frac{Q}{2} + r - \mu_L\right) + \frac{\lambda\pi}{Q}\eta(r). \tag{2.1}$$

In this basic formulation, it is assumed that the lead-time is deterministic and independent of the lot size. Kim and Benton (1995) expressed their concerns over this fixed lead-time assumption. They introduced the lead-time as a function of the lot size, setup time and unit production time as follows:

$$L = (s + pQ)\delta, \tag{2.2}$$

where s is the setup time, p is the run time per unit and δ is the shop floor queuing factor. Here, the lead-time L is expressed as the sum of the setup and process time of a production lot. The shop floor queuing factor (δ) means some, maybe a much larger, portion of the lead-time is spent in queues or material handling processes ($\delta > 1$). Since the lead-time demand X follows a Normal distribution with mean μ_L and standard deviation $\sigma\sqrt{L}$, the reorder point r is the sum of the expected lead-time demand and the Safety Stock (SS). The SS is k times the standard deviation of the lead-time demand, i.e., $r = \mu_L + k\sigma\sqrt{L}$, where k is the safety factor and σ is the standard deviation of the daily demand.

The safety factor k is determined from

$$\Phi(k) = \int_k^{\infty} \phi(z)\,\mathrm{d}z,$$

where $\Phi(k)$ is the complementary cumulative distribution of a standard Normal distribution that gives the probability of stockout per cycle at the safety factor k and $\phi(z)$ is the probability density function of the standard Normal distribution.

The expected number of units short per cycle is given by

$$\eta(r) = \int_r^\infty (x-r)f(x)\,\mathrm{d}x = \sigma\sqrt{L}\Psi(k),$$

where $\Psi(k)$ is the unit linear loss integral and is given by

$$\Psi(k) = \int_k^\infty (z-k)\phi(z)\,\mathrm{d}z = \int_k^\infty z\phi(z)\,\mathrm{d}z - k\Phi(k) = \phi(k) - k\Phi(k).$$

In this case, the expected total annual inventory cost is as follows:

$$TC(Q,k) = \frac{\lambda A}{Q} + h\left(\frac{Q}{2} + \sigma k\sqrt{L(Q,s)}\right) + \sigma\frac{\lambda\pi}{Q}\Psi(k)\sqrt{L(Q,s)}. \tag{2.3}$$

Hariga (2000) extended this basic continuous review inventory model by considering the investment in reduced setup time and the practical relationship between the lead-time, lot size and setup time. According to him, the setup time can often be reduced at very low cost by a simple procedure such as the single-minute exchange of dies and without using high technology. He considered a certain amortized investment cost $iV(s)$, where i is the cost of capital and $V(s)$ is the one-time investment cost. He also assumed that the setup cost A can be represented by a scalar multiple of the setup time, i.e. $A(s) = as$, where a represents the daily direct labor cost associated with the setup operation. In this case, the expected total annual inventory cost is given by

$$TC(Q,k,s) = \frac{a\lambda s}{Q} + h\frac{Q}{2} + \sigma\left(hk + \frac{\lambda\pi}{Q}\Psi(k)\right)\sqrt{L(Q,s)} + iV(s) \tag{2.4}$$

subject to

$$L(Q,s) = (s+pQ)\delta, \quad 0 < s \leqslant s_0,$$

where s_0 is the original setup time. Hariga (2000) assumed that the one-time investment cost is a convex and strictly decreasing function of s.

3. The lead-time reduction model

3.1. Model formulation

In this model, the lead-time, the lot size and the production rate of a manufacturer are connected by the following relation:

$$L(Q,P) = \frac{Q}{P}, \quad 0 < P_0 \leqslant P \leqslant P_{\max},$$

where P_0 is the regular production rate and $P_{\max}$ is the maximum production rate of the manufacturer.

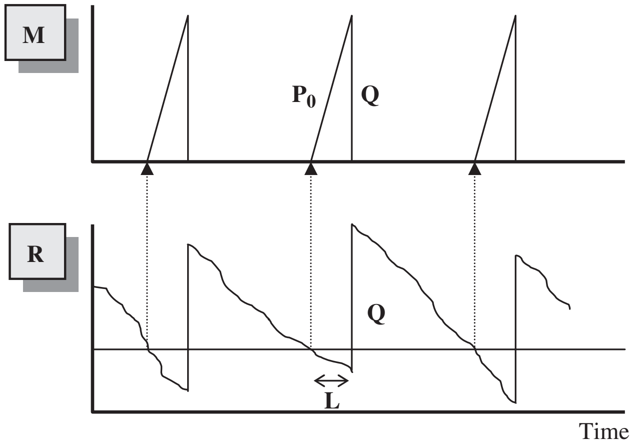

Fig. 1. Behavior of the inventory system.

Inventory is continuously reviewed and replenishments are made whenever the inventory level falls to the reorder point r. When the retailer places an order, the manufacturer starts to produce at the production rate (less than or equal to the maximum production rate) according to the contract with the retailer. The behavior of the inventory system over time is shown in Fig. 1.

Our aim is to reduce the replenishment lead-time by increasing the production rate of the manufacturer. If the retailer tells the manufacturer to increase his production rate, the retailer will be asked by the manufacturer for an additional cost in order to accomplish this by overtime, etc. In this case, the additional cost that is induced by the difference between the desired production rate and the regular production is given by

$$\frac{\lambda}{Q}(P-P_0)L(Q,P)S=\left(1-\frac{P_0}{P}\right)\lambda S.$$

Therefore, the expected total annual cost to be minimized is

$$TC(Q,k,P)=\frac{\lambda A}{Q}+h\frac{Q}{2}+\sigma\left(hk+\frac{\lambda\pi}{Q}\Psi(k)\right)\sqrt{L(Q,P)}+\left(1-\frac{P_0}{P}\right)\lambda S \tag{3.1}$$

subject to

$$L(Q,P)=\frac{Q}{P},\quad 0<P_0\leqslant P\leqslant P_{\max}.$$

Taking the partial derivatives of $TC(Q,k,P)$ with respect to Q, k and P, we obtain

$$\frac{\partial TC}{\partial Q}=\frac{h}{2}-\frac{\lambda(A+\sigma\pi\Psi(k)\sqrt{L(Q,P)})}{Q^2}+\frac{\sigma}{2P\sqrt{L(Q,P)}}\left(hk+\frac{\lambda\pi}{Q}\Psi(k)\right)=0, \tag{3.2}$$

$$\frac{\partial TC}{\partial k}=\sigma\left(h-\frac{\lambda\pi}{Q}\Phi(k)\right)\sqrt{L(Q,P)}=0, \tag{3.3}$$

$$\frac{\partial TC}{\partial P} = \frac{\sigma}{2\sqrt{L(Q,P)}}\left(-\frac{Q}{P^2}\right)\left(hk + \frac{\lambda\pi}{Q}\Psi(k)\right) + \frac{P_0\lambda S}{P^2}. \tag{3.4}$$

It is difficult to prove the convexity of $TC(Q,k,P)$. However, we can show that for fixed k and P, the expected total annual inventory cost function is not convex in Q but the optimal solution can be determined uniquely. Moreover, for fixed Q and k, $TC(Q,k,P)$ is not convex in P. But for fixed Q and P, $TC(Q,k,P)$ is convex in k. The proofs are given in the following lemmas:

Lemma 1. *For fixed k, and P, TC(Q, k, P) is not convex in Q. But the optimal solution can be determined uniquely and have either the possible minimum lot size* $Q_{\min}$ *or Q that satisfies* $\frac{\partial TC}{\partial Q} = 0$.

Proof. The second-order partial derivative of $TC(Q,k,P)$ with respect to Q for fixed k and P is given by

$$\begin{aligned}\frac{\partial^2 TC}{\partial Q^2} &= 2\lambda A Q^{-3} + \frac{3\sigma\lambda\pi\Psi(k)}{4\sqrt{P}}Q^{-5/2} - \frac{\sigma hk}{4\sqrt{P}}Q^{-3/2}\\ &= Q^{-3/2}\left(2\lambda A\, Q^{-3/2} + \frac{3\sigma\lambda\pi\Psi(k)}{4\sqrt{P}}Q^{-1} - \frac{\sigma hk}{4\sqrt{P}}\right).\end{aligned} \tag{3.5}$$

As the first and second terms within the bracket converge to zero for large Q, it is obvious that $\frac{\partial^2 TC}{\partial Q^2} < 0$ for large Q and clearly, $\left|\frac{\partial^2 TC}{\partial Q^2}\right|_{Q=\infty} = 0$. Therefore, $TC(Q,k,P)$ is not convex in Q. ■

To prove the existence of the unique solution, we consider the following.

The first-order partial derivative of $TC(Q,k,P)$ with respect to Q for fixed k and P is given by

$$\begin{aligned}\frac{\partial TC}{\partial Q} &= \frac{h}{2} + \frac{\sigma hk}{2\sqrt{P}}Q^{-1/2} - \lambda A Q^{-2} - \frac{\sigma\lambda\pi\Psi(k)}{2\sqrt{P}}Q^{-3/2}\\ &= \frac{h}{2} + Q^{-1/2}\left(\frac{\sigma hk}{2\sqrt{P}} - \lambda A\, Q^{-3/2} - \frac{\sigma\lambda\pi\Psi(k)}{2\sqrt{P}}Q^{-1}\right).\end{aligned} \tag{3.6}$$

In equation (3.6), the last two terms within the bracket converge to zero for large Q and it is certain that $\left|\frac{\partial TC}{\partial Q}\right|_{Q=\infty} = \frac{h}{2} > 0$.

If $\left|\frac{\partial TC}{\partial Q}\right|_{Q=1} = \frac{h}{2} + \frac{\sigma hk}{2\sqrt{P}} - \lambda A - \frac{\sigma\lambda\pi\Psi(k)}{2\sqrt{P}} \geqslant 0$, $\frac{\partial TC}{\partial Q} \geqslant 0$, for all Q, this indicates that TC is a strictly increasing function for Q and the optimal solution is the possible minimum lot size $Q_{\min}$. If $\left|\frac{\partial TC}{\partial Q}\right|_{Q=1} = \frac{h}{2} + \frac{\sigma hk}{2\sqrt{P}} - \lambda A - \frac{\sigma\lambda\pi\Psi(k)}{2\sqrt{P}} < 0$, it is clear that the sign of the first-order partial derivative changes from negative to positive only once for $1 \leqslant Q < \infty$. This means that $\frac{\partial TC}{\partial Q} = 0$

has a unique solution for fixed k and P. The sign of the first-order derivative indicates that TC function gradually increases after a point of inflection. Thus, for fixed k and P, there exists an optimal solution that can be determined uniquely in Q.

Lemma 2. *For fixed Q and k, $TC(Q,k,P)$ is strictly increasing or decreasing or a concave function when $P_0 \leqslant P \leqslant P_{\max}$. Therefore, the optimal production rate P minimizing $TC(Q,k,P)$ is either P_0 or $P_{\max}$.*

Proof. For fixed Q and k, equation (3.4) can be easily written as

$$\frac{\partial TC}{\partial P} = \frac{1}{P^2}(Z_1 - Z_2\sqrt{P}), \quad \text{where } Z_1 = P_0\lambda S,\ Z_2 = \frac{\sigma\sqrt{Q}}{2}\left(hk + \frac{\lambda\pi}{Q}\Psi(k)\right).$$

As Z_1, $Z_2 > 0$, the trend of the total cost function $TC(Q,k,P)$ depends on the sizes of Z_1 and Z_2.

Case 1

$Z_1 - Z_2\sqrt{P_0} \leqslant 0$: $TC(Q,k,P)$ is strictly decreasing as P increases in the interval $P_0 \leqslant P \leqslant P_{\max}$. In this case, $P_{\max}$ is the optimal production rate minimizing $TC(Q,k,P)$.

Case 2

$Z_1 - Z_2\sqrt{P_{\max}} \geqslant 0$: $TC(Q,k,P)$ is strictly increasing as P increases in the interval $P_0 \leqslant P \leqslant P_{\max}$. In this case, P_0 is the optimal production rate minimizing $TC(Q,k,P)$.

Case 3

$Z_1 - Z_2\sqrt{P_0} \geqslant 0$ and $Z_1 - Z_2\sqrt{P_{\max}} \leqslant 0$: $TC(Q,k,P)$ is a concave function in the interval $P_0 \leqslant P \leqslant P_{\max}$. In this case, the optimal production rate that minimizes $TC(Q,k,P)$ for fixed Q and k can be selected as either P_0 or $P_{\max}$ by comparing $TC(Q,k,P_0)$ and $TC(Q,k,P_{\max})$.

We know that the optimal production rate is always P_0 or $P_{\max}$. from Lemma 2.

It is easy to see that for fixed Q and P, the expected total annual inventory cost function is convex in k as the second-order partial derivative

$$\frac{\partial^2 TC}{\partial k^2} = \frac{\sigma\lambda\pi}{\sqrt{P}} Q^{-1/2}\,\phi(k) \geqslant 0 \quad \text{for all } k$$

After some manipulations, equations (3.2) and (3.3) can be rewritten as

$$Q = \sqrt{\frac{2\lambda\left(A + \sigma\pi\Psi(k)\sqrt{L(Q,P)}\right)}{h\left\{1 + \dfrac{\sigma}{P\sqrt{L(Q,P)}}\left(k + \dfrac{\Psi(k)}{\Phi(k)}\right)\right\}}}, \tag{3.7}$$

$$\Phi(k) = \frac{hQ}{\lambda\pi}. \tag{3.8}$$

3.2. Solution procedure

We now present an iterative procedure similar to the one used in most inventory textbooks (e.g. Hadley and Whitin, 1963) to solve the classical continuous review problem.

Algorithm

Step 1. Set $Q = \left\lceil \sqrt{\frac{2A\lambda}{h}} \right\rceil$. If $Q < Q_{\min}$, $Q = Q_{\min}$.

Compute k from equation (3.8) and decide P from Lemma 2. Here, $\lceil x \rceil$ means the smallest integer larger than or equal to x.

Step 2. Check the sign of $\left|\frac{\partial TC}{\partial Q}\right|_{Q=1} = \frac{h}{2} + \frac{\sigma h k}{2\sqrt{P}} - \lambda A - \frac{\sigma\lambda\pi\Psi(k)}{2\sqrt{P}}$.

If $\left|\frac{\partial TC}{\partial Q}\right|_{Q=1} \geqslant 0$, $Q' = Q_{\min}$.

Otherwise, compute Q' from equation (3.7).

If $Q' < Q_{\min}$, $Q' = Q_{\min}$.

Otherwise, set $Q' = \lceil Q' \rceil$.

Compute k' from equation (3.8), decide P' from Lemma 2.

Step 3. If $Q' = Q$, stop iteration and calculate $TC(Q, k, P)$ and $TC(Q', k', P')$.

If $TC(Q, k, P) < TC(Q', k', P')$, then the near-optimal solution is (Q, k, P).

Otherwise, the near-optimal solution is (Q', k', P').

Else if $Q' \neq Q$, set $Q \leftarrow Q'$. $k \leftarrow k'$, $P \leftarrow P'$. Go to Step 2.

The proposed iterative procedure outlined above starts with an EOQ solution. The first value of the safety factor is found by solving equation (3.8) and the production rate is selected as either P_0 or $P_{\max}$ by Lemma 2. In the next iteration, a new value for Q (called Q') is computed from equation (3.7) and Lemma 1 using the lot size, the safety factor, and the production rate of the first iteration. The modified values of k and P are determined from equation (3.8) and Lemma 2 using Q'. The iterative procedure stops when two successive values of the lot size are equal. To find the near-optimal solution, we have to compare the TCs of the last two iterations.

4. Numerical examples

To illustrate the solution procedure, we consider the example problem. The data for this example problem are as follows:

Average demand rate $\lambda = 36500$ units per year, standard deviation of demand $\sigma = 50$ units per day, fixed ordering cost $A = \$4000$ per order, inventory holding cost $h = \$500$ per unit per year, shortage cost $\pi = \$1000$ per unit short, regular production rate $P_0 = 200$ units per day, maximum production rate $P_{\max} = 300$ units per day and minimum lot size $Q_{\min} = 100$.

Tables 1 and 2 present the results of the solution procedures for each case of the additional cost $S = \$1.8$ per unit and \$1.9 per unit.

Using a complete enumeration for different integer values of Q, it is examined that our solution procedure finds the optimal solution in each case. Figures 2 and 3 show respectively the graph of the total cost function for iterations 1 and 2 in Table 2 when Q and k are fixed. Clearly, the behavior of the cost function verifies Lemma 2. The two cases indicate case 3 in Lemma 2. The graphs are drawn using Mathematica 4.0.

Note that Figs 2 and 3 indicate that the optimal production rates of iterations 1 and 2 are 300 and 200, respectively. And the two figures show that the optimal strategy for P can be reached at

Table 1
Results for $S = \$1.8$ per unit

Iteration	Q	Q'	k	P	L	SS	r	TC
1	764	699	2.309	300	2.547	184	439	\$509,713.34
2	699	692	2.343	300	2.330	179	412	\$507,786.67
3*	692*	691	2.346*	300*	2.307	178	409	\$507,764.98*
4	691	691	2.347	300	2.303	178	408	\$507,765.16

*indicates the optimal values of decision variables.

Table 2
Results for $S = \$1.9$ per unit

Iteration	Q	Q'	k	P	L	SS	r	TC
1	764	699	2.309	300	2.547	184	439	\$510,930.01
2	699	679	2.343	200	3.495	219	568	\$508,870.62
3	679	677	2.353	200	3.395	217	556	\$508,668.36
4*	677*	676	2.354*	200*	3.385	217	555	\$508,666.59*
5	676	676	2.355	200	3.380	216	554	\$508,667.01

*indicates the optimal values of decision variables.

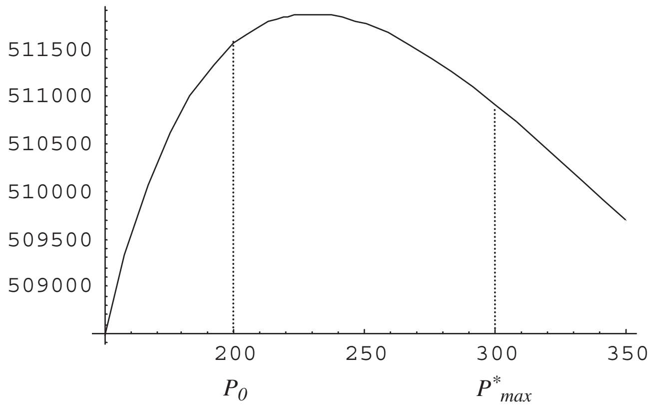

Fig. 2. Graph of TC for iteration 1 in Table 2 (optimal solution is $P_{\max}$).

the maximum production rate of the manufacturer. For iteration 4 in Table 2, the graphs of the TC and its first-order derivative with respect to Q are shown in Figs 4 and 5. These figures show the case of

$$\left|\frac{\partial TC}{\partial Q}\right|_{Q=1} = \frac{h}{2} + \frac{\sigma hk}{2\sqrt{P}} - \lambda A - \frac{\sigma\lambda\pi\Psi(k)}{2\sqrt{P}} < 0.$$

Both Tables 1 and 2 show that if we shorten the lead-time, we can lower the SS. Moreover, as expected from the total annual cost function, if the additional cost S is high, we should

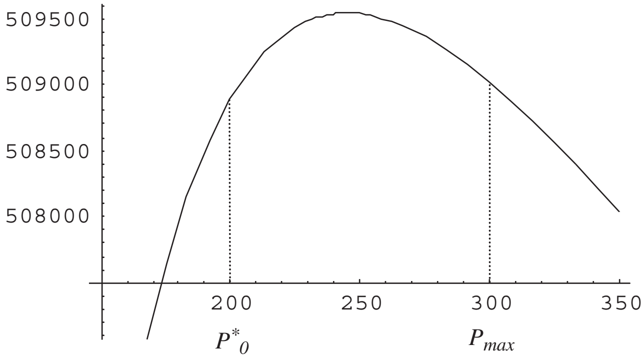

Fig. 3. Graph of TC for iteration 2 in Table 2 (optimal solution is P_0).

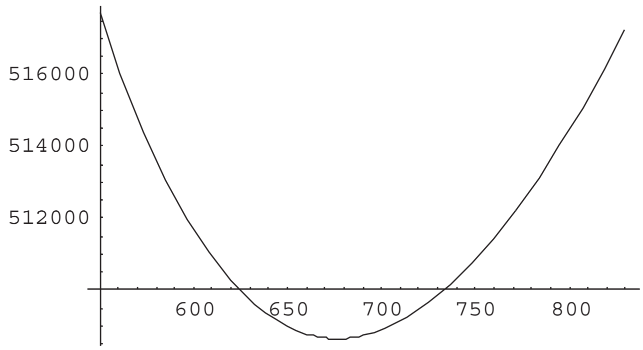

Fig. 4. Graph of TC for iteration 4 in Table 2.

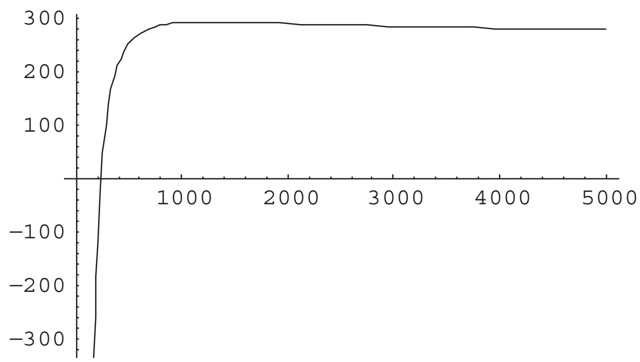

Fig. 5. Graph of $\partial TC/\partial Q$ for iteration 4 in Table 2.

not increase the production rate. Table 3 shows the percentage changes in the total cost function TC for the changes of additional cost S. From Table 3 it is clear that the percentage saving in the total cost is low, as expected, for higher additional cost is required to increase the production rate.

Table 3
Results for the different values of S

S	Q	k	P	L	SS	r	TC	Saving (%)
2.0	677	2.354	200	3.385	217	555	\$508,666.59	–
1.9	677	2.354	200	3.385	217	555	\$508,666.59	–
1.8	692	2.346	300	2.307	178	409	\$507,764.98	0.18
1.5	692	2.346	300	2.307	178	409	\$504,114.98	0.89
1.0	692	2.346	300	2.307	178	409	\$498,031.65	2.09
0.5	692	2.346	300	2.307	178	409	\$491,948.32	3.29

5. Conclusion

In this paper, we develop a stochastic lead-time reduction model with the lead-time depending on the ordering lot size and the production rate of the manufacturer. We consider the situation where the replenishment lead-time can be reduced by increasing the production rate of the manufacturer. At the time of contract with a manufacturer, the retailer can negotiate the lead-time by considering the regular production rate of the manufacturer, since the manufacturer usually has the option of increasing his regular production rate up to the maximum production capacity. If the retailer wants the manufacturer to increase his production rate, he will be asked by the manufacturer for an additional cost in order to accomplish this by overtime, etc. If the manufacturer asks for a relatively high additional cost, the retailer's decision of increasing the production rate is not profitable as evident from the numerical experiment.

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