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Note on an economic lot scheduling problem under budgetary and capacity constraints

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Abstract

Setup reduction program has become an important manufacturing strategy in today's competitive business environment. Banerjee et al. (Int. J. Prod. Econom. 45 (1996) 321) studied the impact of capital investment in setup reduction on an economic lot scheduling problem with a limited budget. They followed the common cycle approach to formulate the problem and suggested a heuristic procedure for numerical computation. The solution they obtained for the model without setup reduction is infeasible perhaps due to mathematical and/or computational errors. In this note, we point out the mathematical errors involved in their paper and provide efficient solution algorithms to both setup reduction and without setup reduction models.

Keywords: Economic lot scheduling problem; Setup reduction; Budgetary and capacity constraints

1. Introduction

It has been recognized by the academics and practitioners that setup reduction is one of the key factors in the Japanese just-in-time (JIT) manufacturing philosophy. In fact, Japanese have been emphasizing process improvement for a long time by setup reduction mainly in the case of a single machine because short setup/change over times permit smaller lot sizes and that result in reduced work-in-progress (WIP), improved quality, lower waste, reduced storage space, and lower investment in inventory. In setup reduction program, the reduction of setup cost/time largely depends on capital investment. The impact of capital investment in setup cost reduction was investigated first by Porteus (1985). Since then considerable amount of research work has been carried out to study the effects of setup reduction on the EOQ/EPQ model under a variety of real-world situations. Most of the work devoted to set up reduction considers the single product situation. However, Spence and Porteus (1987), Freeland et al. (1990), Kim et al. (1995), Banerjee et al. (1996), Leschke and Weiss (1997) and Diaby (2000) extended their investigation to multi-product situation. Chandrashekar and Callarman (1998) examined the effects of setup reduction and processing time reduction in the case of multi-product multi-machine job shop. Gallego and Moon (1992) showed that in the case of single-machine multi-product situation, machine setup times can be reduced at the expense of setup costs by externalizing setup operations.

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Hwang et al. (1993) studied the multi-product economic lot size model in which setup reduction and quality improvement are assumed to be achieved by one-time initial investment. Moon (1994) reviewed and extended Hwang et al.'s (1993) work by analyzing for the general investment functions. Gallego and Moon (1995) showed that for the economic lot scheduling problem (ELSP), setup times and costs can be reduced by an initial investment that is amortized over time. For most recent review on the ELSP, refer to Moon et al. (2002) who applied the genetic algorithm to the ELSP.

Banerjee et al. (1996) studied the impact of setup reduction on production batch sizes and the schedule in batch manufacturing system, producing multiple products where the total investment in setup reduction is constrained by the availability of capital resources. They formulated the ELSP using Hanssmann's (1962) common cycle approach in which the sequence of production run is not important, and the sequence as well as the cycle length do not change from cycle to cycle. All products are produced exactly once in each cycle. Banerjee et al. (1996) also provided a solution procedure (algorithm) for the ELSP. However, the solution they obtained for the model without setup reduction is infeasible perhaps due to mathematical and/or computational errors. Our aim in this note is to point out and correct the mathematical errors involved in their paper and provide efficient solution algorithms to both setup reduction and without setup reduction model.

2. Assumptions and notation

The following assumptions are made in accordance with the classical definition of the ELSP:

(i) All parameters are known and deterministic.
(ii) Shortages are not permitted, that is, the production of each product is sufficient to meet the total demand over the cycle.
(iii) Setup and production times are continuous.
(iv) At time zero, there is just enough inventory on hand of each product to satisfy demand until the first scheduled production of the product.

To avoid any possible confusion, we use the notation of Banerjee et al. (1996) and reformulate the ELSP under the common cycle approach.

m	number of items to be produced
i	item index, $i = 1, 2, \ldots, m$
D_i	annual demand rate of product i
P_i	annual production rate of product i
H_i	annual inventory holding cost per unit of product i
K_i	amortized annual investment in setup reduction for product i
K	total amortized investment for setup reduction for all the products
N	number of common cycles for all products per year
L_i	lower limit of setup cost of product i (at the maximum reduction level)
U_i	upper limit of setup cost of product i (without any setup reduction)
a_i	setup reduction parameter of product i
S_i	current setup cost (without setup reduction) of product i
S_i^t	current setup time (without setup reduction) of product i, in hours
$S_i(K_i)$	setup cost of product i as a function of investment K_i in setup reduction
$S_i^t(K_i)$	setup time of product i expressed in hours as a function of investment K_i
b	proportionality constant between setup cost and setup time, i.e. $S_i(K_i) = bS_i^t(K_i)$
$\hat{k}$	vector of investment K_i for the products, i.e. $\hat{k} = (K_1, K_2, \ldots, K_m)$
T	total time available for setup and production of all the products over a year, in hours.
TRC	total relevant cost per year

Let us first point out the errors in Banerjee et al.'s (1996) paper which may mislead the readers:

(i) Eqs. (6) and (8) on p. 324 are incorrect. The term $\mu_1 K_i$ in each equation should be μ_1.
(ii) Eq. (14) on p. 325 should be corrected as

$$N^* = \sqrt{\sum_{i=1}^{m} D_i H_i (1 - D_i/P_i)/2 \sum_{i=1}^{m} S_i(K_i)}.$$

(iii) Though N, the number of common cycles for all products, is an integer (positive), the partial derivative of $\mathrm{TRC}(N, \hat{k})$ with respect to N is taken to obtain N^*.
(iv) The solution of the model without setup reduction reported on p. 325 is infeasible.
(v) The reference of Kim, Hayya and Hong (Gallego and Moon, 1992) mentioned in the text on p. 326 does not carry the solution of the problem using a heuristic technique based on the basic period approach.

In the following section, we formulate the ELSP under the common cycle approach when no investment is made for setup reduction and provide a simple algorithm to solve the ELSP.

3. The model without setup reduction

(P1) Minimize $\mathrm{TRC}^0(N)$

$$= N \sum_{i=1}^{m} S_i + \frac{1}{2N} \sum_{i=1}^{m} H_i D_i \left(1 - \frac{D_i}{P_i}\right) \tag{1}$$

subject to

$$T\left(1 - \sum_{i=1}^{m} \frac{D_i}{P_i}\right) - N \sum_{i=1}^{m} S_i^t \geqslant 0. \tag{2}$$

The constrained optimization problem (P1) can be solved by Lagrange method. The associated Lagrangean function L_1 can be written as

$$L_1(N, \lambda) = N \sum_{i=1}^{m} S_i + \frac{1}{2N} \sum_{i=1}^{m} H_i D_i \left(1 - \frac{D_i}{P_i}\right) - \lambda \left[T\left(1 - \sum_{i=1}^{m} \frac{D_i}{P_i}\right) - N \sum_{i=1}^{m} S_i^t\right]. \tag{3}$$

Assuming N as real, not just a positive integer, it is easy to see from above that $L_1(N, \lambda)$ is convex in N, for any given λ. Then the Karush–Kuhn–Tucker (KKT) necessary conditions for the optimal value of N give

$$N = \sqrt{\frac{\frac{1}{2} \sum_{i=1}^{m} H_i D_i (1 - D_i/P_i)}{\sum_{i=1}^{m} S_i + \lambda \sum_{i=1}^{m} S_i^t}}, \tag{4}$$

where $\lambda \geqslant 0$ is the complementary slackness (c.s.) with

$$T\left(1 - \sum_{i=1}^{m} \frac{D_i}{P_i}\right) - N \sum_{i=1}^{m} S_i^t \geqslant 0. \tag{5}$$

We now develop the following algorithm to find the optimal value of N (integer) satisfying the capacity constraint (5).

Algorithm.

Step 1: (Check if $\lambda = 0$ gives the optimal solution.) Find N^0 by rounding off the RHS of (4) to the nearest integer.

If $T(1 - \sum_{i=1}^{m} \frac{D_i}{P_i}) - N^0 \sum_{i=1}^{m} S_i^t \geqslant 0$ then go to *step 4*. Otherwise, go to *step 2*.

Step 2: (Search for the optimal value of λ.) Increase λ and find N^0 (nearest integer) from (4).

Step 3: If $T(1 - \sum_{i=1}^{m} D_i/P_i) \geqslant N^0 \sum_{i=1}^{m} S_i^t$ then go to *step 4*. Otherwise, go to *step 2*.

Step 4: $N = N^0$ is optimal. Compute $\mathrm{TRC}^0(N)$ from (1) and stop.

4. The model with setup reduction

We now formulate the ELSP when the amortized annual investments in setup reduction are made for all the products and that the associated setup reduction function is convex and twice differentiable.

(P2) Minimize $\mathrm{TRC}(N, \hat{k})$

$$= N \sum_{i=1}^{m} S_i(K_i) + \frac{1}{2N} \sum_{i=1}^{m} H_i D_i \times \left(1 - \frac{D_i}{P_i}\right) + \sum_{i=1}^{m} K_i \tag{6}$$

subject to

$$\sum_{i=1}^{m} K_i - K \leqslant 0, \tag{7}$$

$$\frac{N}{b} \sum_{i=1}^{m} S_i(K_i) - T\left(1 - \sum_{i=1}^{m} \frac{D_i}{P_i}\right) \leqslant 0. \tag{8}$$

Clearly, for a particular N, the objective function (6) and the constraint functions given in (7) and (8) are convex in K_i's, $i = 1, 2, \ldots, m$, as long as the setup reduction function is convex. Therefore, if we can find the KKT solution of the above problem then it will give a global minimum. The Lagrangean function L_2 for problem (P2) can be written as

$$L_2(N, \hat{k}, \mu_1, \mu_2)$$
$$= N \sum_{i=1}^{m} S_i(K_i) + \frac{1}{2N} \sum_{i=1}^{m} H_i D_i \left(1 - \frac{D_i}{P_i}\right)$$
$$+ \sum_{i=1}^{m} K_i - \mu_1 \left(K - \sum_{i=1}^{m} K_i\right)$$
$$- \mu_2 \left\{ T\left(1 - \sum_{i=1}^{m} \frac{D_i}{P_i}\right) - \frac{N}{b} \sum_{i=1}^{m} S_i(K_i) \right\}.$$

For any given N, the KKT necessary conditions for the minimum of L_2 require

$$\frac{\partial L_2}{\partial K_i} \equiv N\left(1 + \frac{\mu_2}{b}\right)\frac{\partial S_i(K_i)}{\partial K_i} + 1 + \mu_1 = 0, \quad i = 1, 2, \ldots, m, \tag{9}$$

where

$\mu_1 \geqslant 0$ c.s. with $K - \sum_{i=1}^{m} K_i \geqslant 0$

and

$\mu_2 \geqslant 0$ c.s. with

$$T(1 - \sum_{i=1}^{m} D_i/P_i) - (N/b) \sum_{i=1}^{m} S_i(K_i) \geqslant 0.$$

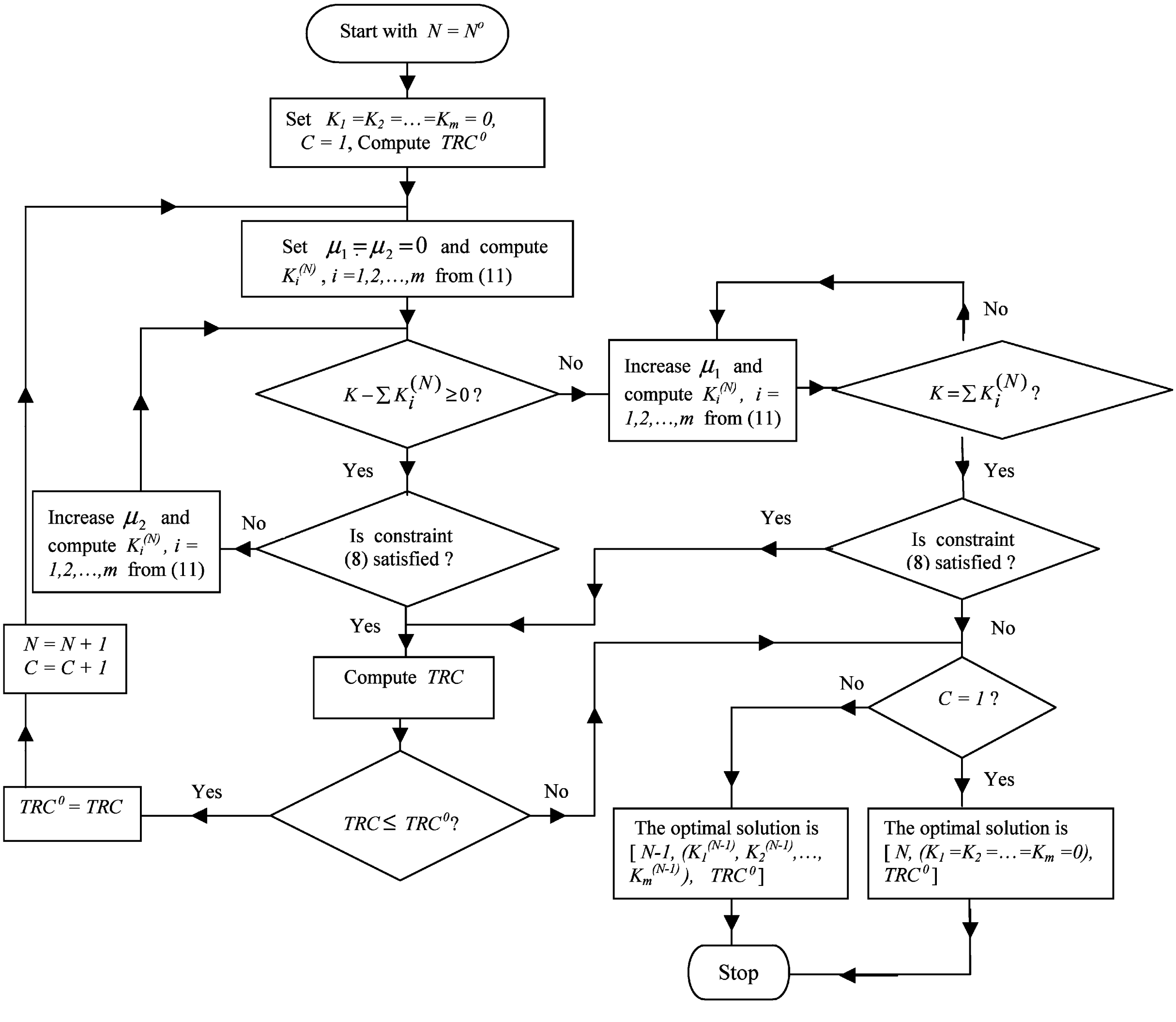

Fig. 1. Flow diagram of the solution algorithm in setup reduction model.

For the setup reduction function

$$S_i(K_i) = L_i + (U_i - L_i)\exp(-a_i K_i), \tag{10}$$

Eq. (9) gives

$$K_i = \frac{1}{a_i}\log_e\left[\frac{Na_i(1+\mu_2/b)(U_i - L_i)}{1+\mu_1}\right],$$
$$i = 1, 2, \ldots, m. \tag{11}$$

Applying line search technique on N, the optimal values of K_i's and the associated total relevant cost can be obtained. Fig. 1 shows a flow diagram of the solution algorithm. One can start the searching process with $N = 1$. However, to save CPU access time in computer, the starting value of N can be chosen as N^0, the number of common cycles of the model without setup reduction.

5. Numerical examples

We consider the same numerical example that was adopted by Banerjee et al. (1996). The data are given in Table 1.

Lower limit of the setup time: $L_i = 0.167$ hours for all i, setup reduction parameter: $a_i = 0.0005$ for all i, proportionality constant: $b = \$100$/hours of setup time, annual capacity available: 3840 hours (0.4384 year), total funds for setup reduction: $K = \$20,000$.

Solution of the model without setup reduction: The algorithm given in Section 3 yields the following optimal solution: The number of common cycles $(N) = 16$, and annual total relevant costs $(\text{TRC}^0) = \$303,483.64$. Banerjee et al. (1996) obtained $N = 31$, and $\text{TRC}^0 = \$247,471$. Though Banerjee et al.'s (1996) heuristic procedure earns lower cost, the solution is infeasible as $N = 31$ does not satisfy the capacity constraint (2).

Solution of the model with setup reduction: To find the optimal solution of the setup reduction model we follow the algorithm whose flow diagram is outlined in Fig. 1. It takes just a few seconds to find the solution using Pentium PC. The computational results are as follows: $N^* = 82$, $K_1 = \$2718$, $K_2 = \$3558$, $K_3 = \$4602$, $K_4 = \$4148$, $K_5 = \$4973$ and TRC = \$113,942.43.

Banerjee et al. (1996) obtained a feasible solution in which $N^* = 101$, $K_1 = \$3869$, $K_2 = \$3955$, $K_3 = \$4062$, $K_4 = \$4015$, $K_5 = \$4100$ and the total relevant cost, including the amortized investment in setup reduction is \$118,908. This shows that the cost savings by our solution algorithm is about 4.4%.

6. Conclusion

In recent years, setup reduction program is gaining increasing importance in various manufacturing industries, because it not only reduces inventory lot sizes, storage space and total relevant inventory costs but also improves products' quality. Banerjee et al. (1996) were the first to investigate the setup reduction in a single-machine multi-item manufacturing system under limited machining resource and investment budget. They formulated the ELSP using the common cycle approach and provided a solution algorithm. Unfortunately Banerjee et al.'s (1996) paper contained some incorrect mathematical expressions/equations and their heuristic procedure for without setup reduction model obtained infeasible solutions. This note points out and corrects those errors and provides efficient solution algorithms for both setup reduction and without setup

Table 1
Data for the example

Product i	D_i (units/year)	P_i (units/year)	H_i (\$/unit/year)	Current setup time S_i^t (h)	Current setup cost U_i (\$)
1	18,050	153,120	66	4	400
2	34,026	153,120	84	6	600
3	35,980	153,120	87.84	10	1000
4	13,404	153,120	60	8	800
5	24,576	152,120	60	12	1200

reduction models. Computational results show that the proposed algorithm earns significant cost savings in the setup reduction model.

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