Scheduling Economic Lot Sizes in Deteriorating Production Systems

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Abstract: The paper considers the economic lot scheduling problem (ELSP) where production facility is assumed to deteriorate, owing to aging, with an increasing failure rate. The time to shift from an "in-control" state to an "out-of-control" state is assumed to be normally distributed. The system is scheduled to be inspected at the end of each production lot. If the process is found to be in an "out-of-control" state, then corrective maintenance is performed to restore it to an "in-control" state before the start of the next production run. Otherwise, preventive maintenance is carried out to enhance system reliability. The ELSP is formulated under the capacity constraint taking into account the quality related cost due to possible production of non-conforming items, process inspection, and maintenance costs. In order to find a feasible production schedule, both the common cycle and time-varying lot sizes approaches are utilized. © 2003 Wiley Periodicals, Inc. Naval Research Logistics 50: 650–661, 2003.

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1. INTRODUCTION

The Economic Lot Scheduling Problem (ELSP) is the problem of finding a feasible schedule of several products on a single facility so that the demands are met without stockouts, and the long-run average inventory holding and setup costs are minimized (Silver, Pyke, and Peterson [32]). The problem occurs in many production situations (Boctor [3]) such as bottling, metal forming, and plastic production lines (press lines, plastic, and metal extrusion machines), weaving production lines (for textiles, carpets), paper production, molding and stamping operations, etc. Hsu [17] proved that even a very restricted version of the original problem is NP-hard. Over the past 40 years, the problem and many of its variants have been studied by a large number of researchers. A comprehensive review on the ELSP until the late seventies can

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be found in Elmaghraby [6]. The recent developments on the ELSP are well cited in Silver, Pyke, and Peterson [32].

In the literature, research in the ELSP has focused on cycle schedules, i.e., schedules that are repeated periodically regardless of the sequence of production. If the cycle times for all products are restricted to be equal then the approach is known as Common Cycle (CC) approach (Haessler and Hogue [14] and Hanssmann [15]). The guarantee of feasibility is the main advantage of this approach. Jones and Inman [19] and Gallego [8] provided a detailed analysis of conditions under which this approach provides optimal or near-optimal solutions.

The relaxation of the equally spaced production runs restriction was initiated by Maxwell [24] and Delporte and Thomas [4]. Dobson [5] developed a formulation of the problem that allows lot sizes and therefore cycle times to vary over time by explicitly taking into account the setup times. Zipkin [33] developed an algorithm to find the production run times and machine idle times for each product taking the production sequence of Dobson [5] as given. Gallego and Roundy [12] extended the time-varying lot sizes approach to the ELSP which allows backorders. Gallego and Shaw [13] showed that the ELSP is strongly NP-hard under the time-varying lot sizes approach, giving theoretical justification of the development of the heuristics.

Allen [1] modified the ELSP to allow production rates to be decision variables. He developed a graphical method for finding the production rates and cycle times for a two-product problem. Silver [31], Moon, Gallego, and Simchi-Levi [27], Gallego [9], and Khouja [21] showed that production rate reduction is more profitable for under-utilized facilities. Khouja [20] provided a similar extension for systems with high utilization. Gallego and Moon [10] examined a multiple product factory that employs a cyclic schedule to minimize holding and setup costs. When setup times are reduced, at the expense of setup costs, by externalizing internal setup operations, they showed that dramatic savings are possible for highly utilized facilities. Gallego and Moon [11] developed an ELSP with the assumption that setup times can be reduced by a one time investment. Hwang, Kim, and Kim [18] and Moon [26] developed an ELSP in which both setup reduction and quality improvement can be achieved through investment. Khouja, Michalewich, and Wilmot [22] and Moon, Silver, and Choi [29] used genetic algorithms (GAs) for solving the ELSP. Moon, Hahm, and Lee [28] applied the stabilization period (during which yield rates gradually increase until they reach the target rates) concept to the ELSP.

All the above research efforts assume that the production process is perfect, besides the standard assumption of the ELSP such as constant demand rates, constant production rates, no shortages, and infinite time horizon. However, owing to aging, many production processes deteriorate from "in-control" state to "out-of-control" state and produce defective items. Recently, Ben-Daya and Hariga [2] studied the effects of imperfect production processes on the ELSP. They assumed negligible setup times for the products and developed a mathematical model under the common cycle approach taking into account the effect of imperfect quality and process restoration. If significant time is required to set up the machine, then their analysis is incomplete because the frequency of setup may impose time requirements which would exceed the time available. In developing the model they assumed that the time to shift follows an exponential distribution. Although there has been empirical support for such a distribution, there are circumstances in which it is not adequate (Lawless [23]). For example, owing to aging, the hazard function of the process shift distribution will be more appropriately described by an increasing failure rate (IFR) distribution. Usually when the process shift distribution follows an IFR, preventive maintenance policy is also used to enhance system reliability.

In the present article, we consider the ELSP with imperfect production processes, where the process shift distribution is normal and positive setup time is required for machine setup. The normal distribution has an increasing hazard function that begins to increase rapidly near but before the point of median life. There are many practical situations where the failure time of components or parts (most of the mechanical components that are subjected to repeat cycle loads) can approximately be described by a normal distribution (Elsayed [7]). Examples include electric filament devices (e.g., incandescent light bulbs and toaster heating elements) and strength of wire bonds in integrated circuits (component strength is often used as an easy-to-obtain surrogate measure or indicator of eventual reliability) (Meeker and Escobar [25]). Though the normal distribution has a theoretical range extending to negative numbers, it has been proved to be a useful distribution for certain life data when mean is positive, sufficiently large and the coefficient of variation is small (Meeker and Escobar [25]). As significant changeover time is always required in some state-of-the-art production processes, we formulate the ELSP under capacity constraint, taking into account the quality related cost, process inspection and maintenance costs. In order to find a feasible production schedule, we apply both the common cycle and time-varying lot sizes approaches to the ELSP.

The paper is organized as follows. Assumptions and notation are presented in the next section. The ELSP is formulated in Section 3. A mathematical model is developed for the ELSP, in Section 4, using the common cycle approach. Section 5 deals with the time-varying lot sizes approach to the ELSP. In Section 6, a lower bound model is developed to compare the common cycle and time-varying lot-size solutions with respect to the lower bound. Finally, the concluding remarks are given in Section 7.

2. ASSUMPTIONS AND NOTATION

The assumptions of the ELSP are:

(i) Several items to produce on a single machine and only one item can be produced at a time.
(ii) Product demand rates, production rates are deterministic and constant.
(iii) Production capacity is sufficient to meet the demand.
(iv) Product setup costs and setup times are independent of production sequence. The setup time and setup cost depend only on the item going into production.
(v) Setup costs, holding costs, process inspection, and preventive maintenance costs are known constants.
(vi) For each product, the production process starts in an "in-control" state to produce items of acceptable quality. The process shifts to an "out-of-control" state at any random time and starts producing a constant fraction of non-conforming items.
(vii) The time to shift (from an "in-control" state) to an "out-of-control" state is normally distributed.
(viii) The system is inspected at the end of each production lot and maintenance is done in order to ensure that the process is perfect at the beginning of each production cycle.

We use the following notation:

m	number of items
i	index for the items, $i = 1, 2, \ldots, m$
p_i	constant production rates (units/unit time), $i = 1, 2, \ldots, m$
d_i	constant demand rates (units/unit time), $(d_i < p_i)$, $i = 1, 2, \ldots, m$
h_i	holding costs (\$/unit/unit time), $i = 1, 2, \ldots, m$
A_i	known setup costs (\$/setup), $i = 1, 2, \ldots, m$

s_i known setup times (unit time), $i = 1, 2, \ldots, m$
T_i production cycle times (unit time)
α_i constant fractions of nonconforming/defective items produced in an "out-of-control" state, $i = 1, 2, \ldots, m$
t_i length of production run (unit time) of the ith item, $i = 1, 2, \ldots, m$
u_i cost of producing a defective item (\$), $i = 1, 2, \ldots, m$
v_i known inspection costs (\$/inspection), $i = 1, 2, \ldots, m$
c_p constant preventive maintenance cost (\$)
$f(\cdot)$ probability density function of the normal distribution
$\phi(\cdot)$ probability density function of the standard normal distribution
$\Phi(\cdot)$ cumulative distribution function corresponding to ϕ
μ_i mean times to shift (unit time), $i = 1, 2, \ldots, m$
σ_i standard deviations of the time to shift (unit time), $i = 1, 2, \ldots, m$

3. MODEL FORMULATION

We assume that at the beginning of each production cycle the process is in an "in-control" state, producing items of acceptable quality. After a period of production, the process may shift to an "out-of-control" state and starts producing α $(0 \le \alpha \le 1)$ proportion of defective items until the end of the current production lot where the process is monitored through inspection. The inspection of the state of the process after each production lot is typical in many production processes. In fact, when the production is in progress, it may be impossible or expensive to interrupt the process or even it may not be possible to detect the failure point (Hariga and Ben-Daya [16]). However, at the end of each production run, if the machine is found to be in an "out-of-control" state, a repair (corrective maintenance) is performed in order to restore the process to an "in-control" state. The restoration cost $c(\tau)$ is assumed to be a function of the detection delay (the elapse time between the machine failure and the time when it is actually detected by inspection). We define $c(\tau) = r_0 + r_1\tau$, $r_0 > 0$, $r_1 \ge 0$, and τ is the detection delay time. On the other hand, if the system is found to be in an "in-control" state, then only preventive maintenance is carried out. After each preventive or corrective maintenance, the system becomes as good as new. We also assume that the amount of time needed for inspection and restoration is negligible (Ben-Daya and Hariga [2]). Inspection and maintenance times can be small compared to production cycle time. Under the problem scenario described above, preventive maintenance can be performed as a part of the setup procedure and the repair or corrective maintenance of failed units may consist of replacing the units or modules by new ones. The repair of the failed units then can be done "off-line" without affecting the production process.

The expected total cost of the model consists of the following cost components:

- Setup and inventory holding costs
- Quality related cost due to possible production of nonconforming items
- Inspection and maintenance costs

We assume that while processing the ith product, the time to failure is normal with mean μ_i and variance σ_i^2. Thus the p.d.f. of the failure distribution is

$$f_i(t) = N(t;\ \mu_i;\ \sigma_i^2) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-[(t-\mu_i)^2/2\sigma_i^2]}, \qquad -\infty < t < \infty,$$

where

$$-\infty < \mu_i < \infty \quad \text{and} \quad \sigma_i > 0.$$

Let t be the elapsed time for which the process remains in an "in-control" state before a shift occurs and N_i denotes the number of defectives of the ith product. Then

$$\begin{aligned} N_i &= 0 \qquad \text{if } t > t_i \\ &= \alpha_i p_i (t_i - t) \qquad \text{if } t < t_i. \end{aligned}$$

The expected number of defective items produced while processing the ith product is given by

$$\begin{aligned} E(N_i) &= \int_{-\infty}^{t_i} \alpha_i p_i (t_i - t)\, f_i(t)\, dt \\ &= \alpha_i p_i\, \sigma_i [z_i\, \Phi(z_i) + \phi(z_i)], \qquad \text{where} \quad z_i = \frac{t_i - \mu_i}{\sigma_i} \end{aligned} \tag{1}$$

since

$$\begin{aligned} \int_{-\infty}^{t_i} t f_i(t)\, dt &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(t_i - \mu_i/\sigma_i)} (\mu_i + z\sigma_i) e^{-(z^2/2)}\, dz \\ &= \mu_i\, \Phi\left(\frac{t_i - \mu_i}{\sigma_i}\right) - \sigma_i\, \phi\left(\frac{t_i - \mu_i}{\sigma_i}\right). \end{aligned}$$

Therefore, the expected quality related cost per unit time due to the possible production of nonconforming items is

$$\begin{aligned} E(QC) &= \sum_{i=1}^{m} \frac{u_i}{T_i} E(N_i) \\ &= \sum_{i=1}^{m} \frac{u_i \alpha_i p_i \sigma_i}{T_i} [z_i\, \Phi(z_i) + \phi(z_i)]. \end{aligned} \tag{2}$$

The inspection cost per unit time is $\sum_{i=1}^{m} \frac{v_i}{T_i}$ and the expected maintenance cost per unit time is

$$\begin{aligned} E(MC) &= \sum_{i=1}^{m} \frac{1}{T_i} \left[\int_{-\infty}^{t_i} \{r_0 + r_1(t_i - t)\} f_i(t)\, dt + c_p \int_{t_i}^{\infty} f_i(t)\, dt \right] \\ &= \sum_{i=1}^{m} \frac{1}{T_i} [(r_0 + r_1 \sigma_i z_i - c_p)\, \Phi(z_i) + r_1 \sigma_i\, \phi(z_i) + c_p], \end{aligned} \tag{3}$$

r_0 and r_1 being the cost parameters for the process restoration cost function. We assume that $c_p < r_0$. This assumption is reasonable because even when the detection delay is negligible, the corrective maintenance cost should exceed the preventive maintenance cost. Otherwise, the preventive maintenance policy becomes meaningless. In general, the cost to repair the system after failure is greater than the cost of maintaining the system before its failure.
The expected total cost (which is the sum of the setup costs, holding costs, quality related costs, inspections and maintenance costs) per unit time is

$$ETC = \sum_{i=1}^{m} \left[\frac{A_i + v_i}{T_i} + H_i T_i + \frac{u_i \alpha_i p_i \sigma_i}{T_i} \{z_i \, \Phi(z_i) + \phi(z_i)\} + \frac{1}{T_i} \{(r_0 + r_1 \sigma_i z_i - c_p) \, \Phi(z_i) + r_1 \sigma_i \, \phi(z_i) + c_p\} \right], \quad (4)$$

$$\text{where} \qquad \rho_i = d_i/p_i, \qquad H_i = \tfrac{1}{2} h_i d_i (1 - \rho_i).$$

Since $p_i t_i = d_i T_i$, the above expression for ETC can be rewritten as

$$ETC = \sum_{i=1}^{m} \left[\frac{A_i + v_i + c_p}{T_i} + H_i T_i + \frac{u_i \alpha_i p_i \sigma_i}{T_i} \{\xi_i \, \Phi(\xi_i) + \phi(\xi_i)\} + \frac{1}{T_i} \{(r_0 + r_1 \sigma_i \xi_i - c_p) \, \Phi(\xi_i) + r_1 \sigma_i \, \phi(\xi_i)\} \right], \quad (5)$$

$$\text{where} \qquad \xi_i = \frac{\rho_i T_i - \mu_i}{\sigma_i}.$$

4. ELSP—COMMON CYCLE (CC) APPROACH

In the CC-approach, we have $T_1 = T_2 = \cdots = T_m = T$ (say). Therefore, the expected total cost per unit time is

$$ETC_0(T) = \frac{A + v + mc_p}{T} + HT + \frac{1}{T} \sum_{i=1}^{m} u_i \alpha_i p_i \, \sigma_i \{\eta_i \, \Phi(\eta_i) + \phi(\eta_i)\} + \frac{1}{T} \sum_{i=1}^{m} \{(r_0 + r_1 \sigma_i \eta_i - c_p) \, \Phi(\eta_i) + r_1 \sigma_i \, \phi(\eta_i)\}, \quad (6)$$

where $A = \sum_{i=1}^{m} A_i$, $v = \sum_{i=1}^{m} v_i$, $H = \sum_{i=1}^{m} H_i$, and $\eta_i = \frac{\rho_i T - \mu_i}{\sigma_i}$, $i = 1, 2, \ldots, m$. The batch size for item i is $d_i T$. So the total time required to produce a batch of product i is $\left(s_i + \frac{d_i T}{p_i}\right)$, i.e., the setup plus processing times. Therefore, we have the following constraint on T:

$$\sum_{i=1}^{m}\left(s_i+\frac{d_iT}{p_i}\right)\leq T$$

$$\text{or}\qquad T\geq\frac{\sum_{i=1}^{m}s_i}{\kappa}\equiv T_{min}\qquad\text{(say)}\tag{7}$$

where $\kappa = 1 - \sum_{i=1}^{m} (d_i/p_i) =$ the proportion of time available for setups.

Our objective is to find the optimal value of T which minimize $ETC_0(T)$ subject to the capacity constraint (7). The expected total cost function (6) is convex provided that the second-order derivative

$$\begin{aligned}\frac{d^2(ETC_0)}{dT^2}&=\frac{2(A+v+mc_p)}{T^3}+\frac{2}{T^3}\left[\sum_{i=1}^{m}u_i\alpha_ip_i\,\sigma_i\{\eta_i\,\Phi(\eta_i)+\phi(\eta_i)\}\right.\\&\quad\left.+\sum_{i=1}^{m}\{(r_0+r_1\sigma_i\eta_i-c_p)\,\Phi(\eta_i)+r_1\sigma_i\,\phi(\eta_i)\}\right]\\&\quad-\frac{2}{T^2}\sum_{i=1}^{m}\left\{(r_1+u_i\alpha_ip_i)\rho_i\,\Phi(\eta_i)+\frac{(r_0-c_p)\rho_i}{\sigma_i}\,\phi(\eta_i)\right\}\\&\quad+\frac{1}{T}\sum_{i=1}^{m}\left\{(r_1+u_i\alpha_ip_i)\frac{\rho_i^2}{\sigma_i}\,\phi(\eta_i)-(r_0-c_p)\frac{\eta_i\rho_i^2}{\sigma_i^2}\,\phi(\eta_i)\right\}\end{aligned}\tag{8}$$

is strictly positive. Because of complexity, it is difficult to show that the RHS of (8) is strictly positive for all T and consequently the existence of the global optimal solution cannot be guaranteed. We now look for a local optimal solution. Let us first ignore the capacity constraint (7), then the necessary condition for $ETC_0(T)$ to be minimum gives

$$\begin{aligned}G(T)\equiv T^2\frac{d(ETC_0)}{dT}&=-(A+v+mc_p)+HT^2\\&\quad-\left[\sum_{i=1}^{m}\beta_i\,\sigma_i\{\eta_i\,\Phi(\eta_i)+\phi(\eta_i)\}+\sum_{i=1}^{m}(r_0-c_p)\,\Phi(\eta_i)\right]\\&\quad+T\sum_{i=1}^{m}\left\{\beta_i\rho_i\,\Phi(\eta_i)+\frac{(r_0-c_p)\rho_i}{\sigma_i}\,\phi(\eta_i)\right\}=0,\end{aligned}\tag{9}$$

$$\text{where}\qquad\beta_i=r_1+u_i\alpha_ip_i.$$

It can be easily seen from the above that $G(0) < 0$ as both $\Phi(\eta_i)$ and $\phi(\eta_i)$ are greater than zero at $T = 0$. Also $\lim_{T\to+\infty} G(T) = +\infty$ since $\Phi(\eta_i) \to 1$ and $\phi(\eta_i) \to 0$ as $T \to +\infty$. Hence there exists a nonnegative solution $T = T^*$ (say) of Eq. (9). This solution will be unique provided $G(T)$ is a strictly increasing function of T, i.e.,

$$G'(T) = 2HT + T \sum_{i=1}^{m} \{\beta_i\sigma_i - (r_0 - c_p)\eta_i\} \frac{\rho_i^2}{\sigma_i^2} \phi(\eta_i) > 0. \tag{10}$$

Note that the condition (10) is always satisfied when $T < min_i(\mu_i/\rho_i)$ since this implies $\eta_i|_T < 0$. In fact, T^* is a local minimizer as the second-order derivative of $ETC_0(T)$ at $T = T^*$ can be shown to be strictly positive. If T^* satisfies the capacity constraint (7), then it is a local optimal solution. Clearly, if $ETC_0(T)$ is convex, then its minimum is either inside the feasible region, i.e., is at T^*, or at the boundary, i.e., is at T_{min}.

5. ELSP—TIME VARYING LOT SIZES APPROACH

We now formulate the ELSP using the time-varying lot sizes approach proposed by Dobson [5]. The problem can be viewed as one of deciding on a cycle length T, a production sequence $\mathbf{f} = (f^1, f^2, \ldots, f^n)$ $[f^j \in \{1, 2, \ldots, m\}]$ and sequence may contain repetitions, productions times $\mathbf{t} = (t^1, t^2, \ldots, t^n)$ and idle times $\mathbf{w} = (w^1, w^2, \ldots, w^n)$ so that the production sequence is executable in the chosen cycle length, the cycle length can be repeated indefinitely, demand is met, and the inventory cost per unit time is minimized. n $(\geq m)$ setups will be made over the cycle to prepare the items $f^1, f^2, \ldots, f^n$. We will use subscripts to refer to the data related to the ith item (e.g., p_i, d_i, h_i, A_i, s_i, α_i, μ_i, σ_i, u_i, v_i, w_i, etc.) and superscripts to refer to the data related to the item produced at the jth position in the sequence (e.g., p^j, d^j, h^j, A^j, s^j, α^j, μ^j, σ^j, u^j, v^j, w^j, etc., where $p^j = p_{f^j}$, $d^j = d_{f^j}$, and so on). Let F be the set of all possible finite sequences of products and J_i denote the positions in a given sequence where product i is produced, that is, $J_i = j|f^j = i$. Let L_k be the positions in a given sequence from k, up to but not including the position in the sequence where product f^k is produced again. With these definitions and additional notation, the ELSP can be formulated using Eq. (5) as

$$\begin{aligned} &inf_{f\in F}\, Min_{\mathbf{t}\geq \mathbf{0},\mathbf{w}\geq \mathbf{0},T>0} \frac{1}{T}\sum_{j=1}^{n}\Bigg[(A^j + v^j + c_p) + \frac{1}{2}h^j\left(\frac{p^j}{d^j} - 1\right)p^j(t^j)^2 \\ &+ u^j\alpha^j p^j\sigma^j\left\{\left(\frac{t^j - \mu^j}{\sigma^j}\right)\Phi\left(\frac{t^j - \mu^j}{\sigma^j}\right) + \phi\left(\frac{t^j - \mu^j}{\sigma^j}\right)\right\} \\ &+ \{r_0 + r_1(t^j - \mu^j) - c_p\}\,\Phi\left(\frac{t^j - \mu^j}{\sigma^j}\right) + r_1\sigma^j\,\phi\left(\frac{t^j - \mu^j}{\sigma^j}\right) \end{aligned} \tag{11}$$

subject to

$$\sum_{j\in J_i} p_i t^j = d_i T, \qquad i = 1, 2, \ldots, m, \tag{12}$$

$$\sum_{j\in L_k} (t^j + s^j + w^j) = \frac{p^k t^k}{d^k}, \qquad k = 1, 2, \ldots, n, \tag{13}$$

$$\sum_{j=1}^{n} (t^j + s^j + w^j) = T, \tag{14}$$

$$\mathbf{t} \geq \mathbf{0}, \quad \mathbf{w} \geq \mathbf{0}, \quad T > 0.$$

Constraint (12) indicates that sufficient time should be allocated to product i to meet its demand $d_i T$ over the cycle. Constraint (13) implies that sufficient production of each product is needed each time to meet its demand until the next time the same product is produced again. Constraint (14) means that the cycle time T must be the sum of production, setup, and idle times for all items produced in the cycle.

The basic idea of solving the ELSP using the time-varying lot sizes approach proposed by Dobson [5] is to decompose the problem into two parts: a combinatorial part (the specification of $\mathbf{f}$) and a continuous part (the determination of $\mathbf{t}$, $\mathbf{w}$, and T). In the combinatorial part, the production frequencies are determined and are rounded off to power-of-two integers by using Roundy's algorithm [30]. The items are then bin packed with respect to the frequencies and average loads, resulting in a production sequence. The continuous part takes the production sequence as given and computes the actual production times and idle times [33]. The procedure that finds a feasible production schedule can be described in the following steps:

Step 1. Let the T_i^*s be the optimal cycle lengths of the lower bound model. Then the relative production frequencies x_i's can be determined by the relation

$$x_i = \frac{Max_i\{T_i^*\}}{T_i^*}, \qquad i = 1, 2, \ldots, m.$$

Step 2. Round off the relative production frequencies (x_i) to power-of-two integers (y_i) by the following rule:

$$y_i = 2^p \qquad \text{if} \quad x_i \in \left[\frac{1}{\sqrt{2}}\, 2^p, \sqrt{2}\; 2^p\right], \qquad p = 0, 1, \ldots .$$

Roundy [30] showed that the additional cost due to the conversion of the real values of the production frequencies to power-of-two integers do not exceed 6%.

Step 3. Determine the production sequence by bin-packing heuristic. Given the frequencies y_i, the bin-packing heuristic attempts to spread them out as evenly as possible [8]. For each product i, the processing time duration z_i for the lots are estimated by assuming that the lots will be equally spaced. If there be b bins where $b = max_{1\leq i\leq m}\, y_i$, then y_i items of height z_i $\forall$ i are to put in b bins with the restriction that a product with frequency y_i must have all its lots placed in the bins equally spaced. While assigning the items to bins, a variation of the Longest Processing Time (LPT) rule is used in which the items are ordered lexicographically by (y_i, z_i). By minimizing the maximum height of the bins, the heuristic finds an efficient production sequence $\mathbf{f}$.

Step 4. Solve for $\mathbf{t}$ and $\mathbf{w}$, given $\mathbf{f}$.
We solve equations (13) for $\mathbf{t}$, assuming that there is no idle time ($\mathbf{w} = \mathbf{0}$). This assumption fits well for a highly loaded facility. For positive idle times, it is required to solve a complex nonlinear programming problem which can be handled through the solution of a parametric quadratic program and a few EOQ like calculations (see Zipkin [33]).

We now develop a lower bound model for the ELSP. As no method of solving an optimal production schedule exists, the purpose of developing a lower bound model is to compare the nonoptimal feasible solutions (common cycle and time varying lot sizes solutions) with the lower bound.

6. A LOWER BOUND MODEL

We consider the objective function as the expected total cost (including setup cost, holding cost, and quality related cost, maintenance costs) per unit time, subject to the constraints (16) and (17) given below. However, the synchronization constraint, stating that no two items can be scheduled to produce at the same time, is ignored. Consequently, the value of the following nonlinear program results in a lower bound on the expected average total cost.

$$Min_{T_1,T_2,\ldots,T_m} \sum_{i=1}^{m} \Bigg[\frac{A_i + v_i + c_p}{T_i} + H_i T_i + \frac{u_i \alpha_i p_i \sigma_i}{T_i} \left\{ \left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) \Phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) + \phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) \right\} + \frac{1}{T_i} \left\{ \left(r_0 + r_1 \rho_i T_i - r_1 \mu_i - c_p \right) \Phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) + r_1 \sigma_i \, \phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) \right\} \tag{15}$$

subject to

$$\sum_{i=1}^{m} \frac{s_i}{T_i} \le \kappa, \tag{16}$$

$$T_i \ge 0, \qquad i = 1, 2, \ldots, m, \tag{17}$$

$$\text{where} \qquad H_i = \tfrac{1}{2} h_i d_i (1 - \rho_i) \qquad \text{and} \qquad \kappa = 1 - \sum_{i=1}^{m} \rho_i.$$

Let λ and ν_i $(i = 1, 2, \ldots, m)$ be the Lagrange multipliers corresponding to the constraints (16) and (17), respectively. Then the Karush–Kuhn–Tucker (KKT) necessary conditions for the optimal points give

$$A_i + v_i + c_p + (r_1 + u_i \alpha_i p_i) \left\{ -\mu_i \, \Phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) + \sigma_i \, \phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) \right\} + \frac{r_0 - c_p}{\sigma_i} \left\{ \sigma_i \, \Phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) - \phi\left(\frac{\rho_i T_i - \mu_i}{\sigma_i} \right) \right\} + \lambda s_i = (H_i - \nu_i) T_i^2, \qquad i = 1, 2, \ldots, m \tag{18}$$

$$\lambda \ge 0 \qquad \text{and} \qquad \nu_i \ge 0 \qquad \text{for} \qquad i = 1, 2, \ldots, m.$$

For nontrivial T_i's $(i = 1, 2, \ldots, m)$, we have $\nu_i = 0 \ \forall \ i$ (from one of the KKT conditions). Therefore, we can use the following algorithm based on line search technique to find the lower bound of the expected total cost per unit time.

Algorithm

Step 1. Set $\lambda = 0$ and find T_i's ($i = 1, 2, \ldots, m$) by solving Eqs. (18).
If $\sum_{i=1}^{m} \frac{s_i}{T_i} \leq \kappa$, then go to Step 6. Otherwise, go to Step 2.

Step 2. Start with an arbitrary $\lambda > 0$.

Step 3. Solve Eqs. (18) to find T_i's ($i = 1, 2, \ldots, m$).

Step 4. If $\sum_{i=1}^{m} \frac{s_i}{T_i} \leq \kappa$, go to Step 5.
Otherwise, increase λ and go to Step 3.

Step 5. If $\left|\sum_{i=1}^{m} \frac{s_i}{T_i} - \kappa\right| < \epsilon$, then go to Step 6.
Otherwise, reduce λ and go to Step 3.

Step 6. T_i's are optimal. Find the lower bound of the average total cost from (15). Stop.

7. CONCLUSION

In this article, we have studied the economic lot scheduling problem (ELSP) where the production facility is assumed to deteriorate (from an "in-control" state to an "out-of-control" state) according to a normal distribution. The assumption is motivated by the fact that the hazard function for a normal distribution is monotonically increasing so that owing to aging, the machines suffer increasing failure rate (IFR). There are many practical situations where the failure time of the machine parts or components can be described approximately by a normal distribution. As only process inspection cannot enhance system reliability, we have considered preventive maintenance operation after each inspection if the system is found to be in an "in-control" state. We have formulated the ELSP under the capacity constraint, taking into account the quality related cost due to possible production of nonconforming items, inspection, and maintenance costs. Implication of capacity constraint to the ELSP is meaningful as significant changeover times in between the products are essential in many production processes. We have applied the existing common cycle and time-varying lot sizes approaches to solve the ELSP, as both the techniques provide a feasible production schedule to the ELSP. We have also carried out numerical experiments for the ELSP. In each experiment, the time varying lot sizes solution emerges superior to the common cycle solution in comparison to the lower bound. Due to space restriction, the results are not given here.

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