Journal of the Operational Research Society (2002) 53, 620–629 0160-5682/02 $15.00

www.palgrave-journals.com/jors

Economic lot scheduling problem with imperfect production processes and setup times

I Moon*, BC Giri and K Choi

Pusan National University, Pusan, South Korea

Deteriorating production processes are common in reality. Although every production process starts in an 'in-control' state to produce items of acceptable quality, it may shift to an 'out-of-control' state, owing to ageing, at any random time and produce defective items. In the present article, we study the Economic Lot Scheduling Problem (ELSP) with imperfect production processes having significant changeovers between the products. The mathematical models are developed for the ELSP using both the common cycle approach and the time-varying lot sizes approach, taking into account the effects of imperfect quality and process restoration. Numerical examples are cited to illustrate the solution procedures and to compare the performances of the solution methodologies adopted to solve the ELSP.

Journal of the Operational Research Society (2002) **53**, 620–629. doi:10.1057/palgrave.jors.2601350

Keywords: economic lot scheduling problem; capacity constraint; imperfect production processes; common cycle approach; time-varying lot sizes approach

Introduction

The Economic Lot Scheduling Problem (ELSP) is the problem of finding the production sequence, production times and idle times of several products in a single facility (machine) on a repetitive basis so that the demands are made without stockouts or backorders and average inventory holding and setup costs are minimized. The problem is NP hard[1,2] as it requires to satisfy simultaneously the production capacity constraint and synchronization constraint (only one product can be produced at a time). Since the products must be made on the same facility, production of each product would be in lots or batches. The issue of batching arises because the system usually incurs a setup cost and/or a setup time when the machine switches from one product to another. Setup cost is the cost of changing over production equipment among families and within families, the cost due to cleaning or to scrap losses when machine settings are adjusted for the next product. Setup time is downtime during which the machine can not produce, which in turn implies a need to carry more inventory. The setup cost and setup time depend only on the item going into production.

The ELSP has been studied by researchers extensively over the past 40 years by assuming typically that the production and demand rates of each item are known to be product-dependent constants and setup cost and setup times are known to be product-dependent but sequence-independent constants. To solve the ELSP, researchers have followed so far one of the following two approaches:[3]

(i) *Analytic approaches* that achieve the optimum of a restricted version of the original problem. For example, Common Cycle (CC) approach[4] which restricts all the products' cycle times to equal length. The main advantage of this approach is that it always provides a feasible schedule. Jones and Inman[5] provided a detailed analysis of conditions under which the CC-approach provides optimal or near-optimal solutions.

(ii) *Heuristic approaches* that achieve good solutions for the original problem. These approaches have been found to be more effective in determining the optimal solution than analytic approaches. Two well-known approaches for heuristic algorithm are the Basic Period (BP) approach[6] and the time-varying lot sizes approach.[7–9] The BP approach requires every item to be produced at equally spaced intervals of time that are integer multiples of a basic time period. Under this approach, it is NP hard to find a feasible schedule, given the number of production runs per cycle for each item. Time-varying lot sizes approach allows different lot sizes for any given products during a cyclic schedule. It explicitly handles the difficulties caused by setup times and always gives a feasible schedule.

Literature review

A comprehensive review of early ELSP literature up to 1976 can be found in the well-cited paper by Elmaghraby.[3] Since

**Correspondence:* I Moon, Department of Industrial Engineering, Pusan National University, Pusan 609-735, South Korea.
E-mail: ikmoon@pusan.ac.kr

then numerous research articles have been published in order to include new approaches and extensions to this problem. Delporte and Thomas,[7] Dobson[8] and Roundy[9] contributed to the time-varying lot sizes approach which was introduced originally by Maxwell.[10] Matthew[11] proposed another approach which does not require strict regularity of cycle lengths. Zipkin[12] introduced an approach in which some items may be produced several times during a cycle and the different runs of an item can differ in size. Gallego and Roundy[13] extended the time-varying lot sizes approach to the ELSP which allows backorders. Dobson[14] extended his early work[8] by allowing the setup times to be sequence dependent. Gallego and Shaw[2] showed that the ELSP is strongly NP hard under the time-varying lot sizes approach with or without the Zero Switch Rule (ZSR) restriction, giving theoretical justification to the development of heuristics. Allen[15] modified the ELSP to allow production rates to be decision variables. He developed a graphical method to find the production rates and cycle times for a two product problem. Silver,[16] Moon *et al*,[17] Gallego,[18] Moon and Christy[19] and Khouja[20] showed that production rate reduction is more profitable for under-utilized facilities. Khouja[21] provided a similar extension for systems with high utilization. Gallego and Moon[22] examined a multiple product factory that employs a cyclic schedule to minimize holding and setup costs. When setup times are reduced, at the expense of setup costs, by externalizing internal setup operations, they showed that dramatic savings are possible for high utilized facilities. Gallego and Moon[23] developed an ELSP with the assumption that setup times can be reduced by a one-time investment. Hwang *et al*[24] and Moon[25] further showed that both setup reduction and quality improvement can be achieved through investment. Khouja *et al*[26] used genetic algorithms (GAs) for solving the ELSP which is formulated using the BP approach. Moon *et al*[27] developed a hybrid GA based on the time-varying lot sizes approach to solve the ELSP. The stabilization period (during which yield rates gradually increase until they reach the target rates) concept to the ELSP was introduced by Moon *et al*.[28] For a more recent review of the ELSP we refer interested readers to Silver *et al*.[29]

A majority of research efforts assume that the output of the production facility is of perfect quality, besides the standard assumptions of the ELSP such as constant demand and production rates, no shortages and infinite time horizon. However, there are many production processes where the production facility starts in the 'in-control' state producing items with high or perfect quality but the facility may deteriorate with time and shift at a random time to an 'out-of-control' state and begin to produce non-conforming items. Many authors[30–34] extended the classical Economic Production Quantity (EPQ) model considering the effect of imperfect production processes. However, recently Ben-Daya and Hariga[35] studied the effect of imperfect production processes on the ELSP. They developed mathematical models for the ELSP taking into account the effect of imperfect quality and process inspection during the production run so that the shift to an 'out-of-control' state can be detected and restoration can be made earlier. In developing the models, they did ignore the setup times for the products. If there is a significant time required to setup the machine then Ben-Daya and Hariga's[35] analysis is incomplete, for the frequency of setup may impose time requirements which would exceed the time available. If we assume that the demand and production rates of item i are, respectively, d_i and p_i and the common cycle length is T, then $(1-\rho)T$ is the time available for machine setups, where $\rho = \sum_i \rho_i = \sum_i (d_i/p_i)$. If the setup time for product i is assumed to be s_i, then $\sum_i s_i$ time must be devoted to change over. Therefore, the cycle length must satisfy the inequality:

$$T \geqslant \frac{\sum s_i}{1-\rho}$$

Ben-Daya and Hariga[35] applied the CC-approach to find the solution of the ELSP. Though the CC-approach, a simpler version of the ELSP, always finds a feasible schedule, it restricts the cycle times of all items to be of equal length and provides solutions far from lower bound in some situations.[4,5,36]

The purpose of this research is two fold: (i) to re-investigate the problem of Ben-Daya and Hariga[35] under the CC-approach with the restriction on the cycle length stated above; and (ii) to formulate and study the problem using the time-varying lot sizes approach proposed by Dobson.[8] The organization of the paper is as follows: assumptions and notation are presented in the next section. The Imperfect Process Model (IPM) is developed first using the CC-approach in the subsequent section where we assume that the process may shift from an 'in-control' state to the 'out-of-control' state while producing a lot. But at the end of each production run the process is restored back to the 'in-control' state. This could be due to maintenance measures which are part of the setup of the production process. So we assume that the setup cost of production includes the cost of restoring the machine to the 'good' state. The CC-approach is also applied to deal with imperfect process with inspection and restoration model (IPMWIR) where the process is inspected at regular intervals of time during the production of each product and if the system is found to be 'out-of-control', necessary actions are taken to restore it to the 'in-control' state. In the following sections, we formulate and solve both the models (IPM and IPMWIR) using the time-varying lot sizes approach. Then, numerical illustrations and a comparative study based on the outcomes of the CC-approach and the time-varying lot sizes approach are performed. The concluding remarks together with future research directions are given in the last section.

Basic assumptions and notation

The ELSP is formulated with the following basic assumptions:

(i) Multiple items compete for the use of a single facility.
(ii) Demand rates, production rates, setup costs, holding costs, process inspection and restoration costs for all items are known constants. Setup times are independent of the production sequence.
(iii) No backlogging of demand is permitted.
(iv) Production capacity is sufficient to meet the total demand.
(v) For each product, the production process starts in the 'in-control' state to produce items of acceptable quality. The process shifts to the 'out-of-control' state at any random time and starts producing a constant fraction of non-conforming items.
(vi) The time to shift (from 'in-control' state to 'out-of-control') distribution is exponential with mean θ_i for item i.

The following notation is used in developing the models:

i = item index, $i = 1, 2, \ldots, m$
p_i = constant production rates, $i = 1, 2, \ldots, m$
d_i = constant demand rates ($d_i < p_i$), $i = 1, 2, \ldots, m$
$\rho_i = d_i/p_i$, $i = 1, 2, \ldots, m$
h_i = known holding costs, $i = 1, 2, \ldots, m$
A_i = known setup costs, $i = 1, 2, \ldots, m$
s_i = known setup times, $i = 1, 2, \ldots, m$
T = cycle length
α_i = constant fraction of non-conforming items, $i = 1, 2, \ldots, m$
t_i = length of the production run for item i, $i = 1, 2, \ldots, m$
u_i = constant cost incurred by producing a defective item i, $i = 1, 2, \ldots, m$
v_i = known inspection cost while the facility is producing item i, $i = 1, 2, \ldots, m$

Imperfect Process Model (IPM) under CC-approach

In this section, we assume that once a shift occurs the process stays in the 'out-of-control' state until the setup of the next production and the items produced are of poor quality. Let t be the elapsed time for which the process remains in the 'in-control' state before a shift occurs. Then the expected number of non-conforming items produced while processing the ith product is given by

$$E(N_i) = \int_0^{t_i} \alpha_i p_i (t_i - t) \frac{1}{\theta_i} \, \mathrm{e}^{-t/\theta_i} \, \mathrm{d}t$$

Integrating and using the approximation

$$\mathrm{e}^{-t_i/\theta_i} \approx 1 - \frac{t_i}{\theta_i} + \frac{1}{2}\left(\frac{t_i}{\theta_i}\right)^2$$

by McClaurin series when $\theta_i \gg t_i \quad \forall\, i$ we obtain,

$$E(N_i) = \frac{\alpha_i p_i t_i^2}{2\theta_i} \quad \text{where} \quad t_i = \frac{d_i T_i}{p_i}.$$

Therefore, the expected quality related cost per unit time due to the production of non-conforming items is given by

$$E(\mathrm{QC}) = \sum_{i=1}^{m} \frac{u_i}{T_i} E(N_i) = \sum_{i=1}^{m} \frac{u_i \alpha_i d_i^2 T_i}{2 p_i \theta_i}.$$

The expected total cost (which is the sum of the setup costs, holding costs and quality-related costs) per unit time is given by

$$ETC = \sum_{i=1}^{m} \left[\frac{A_i}{T_i} + (H_i + Q_i) T_i\right] \tag{1}$$

where $H_i = \frac{1}{2} h_i d_i (1 - \rho_i)$, $Q_i = u_i \alpha_i d_i^2 / 2 p_i \theta_i$ and $T_i = p_i t_i / d_i$; $i = 1, 2, \ldots, m$.

In the CC-approach, we have $T_1 = T_2 = \cdots = T_m = T$ (say). So the above expression for the expected total cost can be written as

$$ETC_{1,C} = \frac{A}{T} + (H + Q) T \tag{2}$$

where

$$A = \sum_{i=1}^{m} A_i, \quad H = \sum_{i=1}^{m} H_i \quad \text{and} \quad Q = \sum_{i=1}^{m} Q_i$$

It is easy to show that $ETC_{1,C}$ is minimized by

$$T_1^* = \sqrt{\frac{A}{H + Q}} \tag{3}$$

However, before we accept T_1^* as the optimal cycle length, we must consider the time required for setups during the cycle. Since the total setup time per cycle plus the total production time per cycle must be no more than the cycle length, we have the following constraint on T:

$$\sum_{i=1}^{m} \left(s_i + \frac{d_i T}{p_i}\right) \leqslant T \tag{4}$$

or,

$$T \geqslant \frac{\sum_{i=1}^{m} s_i}{\kappa} \equiv T_{\min} \text{ (say)} \tag{5}$$

where $\kappa = 1 - \sum_{i=1}^{m} d_i/p_i =$ the long-run proportion of time available for setups.

Since $ETC_{1,C}(T)$ is convex in T, the optimal cycle length should be equal to $\max\{T_1^*, T_{\min}\}$. The expected total cost of the model then can be found by evaluating Equation (2) for this optimal cycle length. It is to be mentioned here that Ben-Daya and Hariga[35] neglected the setup times for the production runs and considered T_1^* as the optimum cycle length.

Imperfect Process Model With Inspection and Restoration (IPMWIR) under CC-approach

Let us assume that n_i inspections are carried out during the production of item i. Then the process inspection cost is equal to $\sum_{i=1}^{m} n_i v_i$ and the expected quality cost per unit time is equal to $\sum_{i=1}^{m}(T_i/n_i)Q_i$. If the restoration cost is assumed to be a linear function of its detection delay $[c(\tau) = r_0 + r_1\tau, r_0 > 0, r_1 \geqslant 0]$ and restoration time is negligible, then the expected restoration cost per unit time can be obtained as $[\sum_{i=1}^{m}(r_0 d_i/p_i\theta_i) + \sum_{i=1}^{m}(R_i T_i/n_i)]$ where

$$R_i = (r_1\theta_i - r_0)\frac{d_i^2}{2p_i^2\theta_i^2};$$

r_0, r_1 being the cost parameters for the process restoration cost function (see Ben-Daya and Hariga[35] for derivations). Thus the expected total cost of the model can be obtained as

$$ETC = \sum_{i=1}^{m}\left[\frac{A_i}{T_i} + H_i T_i + (Q_i + R_i)\frac{T_i}{n_i} + v_i\frac{n_i}{T_i} + \frac{r_0 d_i}{p_i\theta_i}\right] \quad (6)$$

Hence, the expected total cost per unit time under the common cycle approach is given by

$$ETC_{2,C} = \frac{1}{T}\left[A + \sum_{i=1}^{m} n_i v_i\right] + T\left[H + \sum_{i=1}^{m}\frac{(R_i + Q_i)}{n_i}\right] + \sum_{i=1}^{m}\frac{r_0 d_i}{p_i\theta_i} \quad (7)$$

where

$$A = \sum_{i=1}^{m} A_i, H = \sum_{i=1}^{m} H_i = \frac{1}{2}\sum_{i=1}^{m} h_i d_i(1-\rho_i) \text{ and } Q_i = \frac{u_i\alpha_i d_i^2}{2p_i\theta_i}.$$

Here the number of inspections $n_i (i = 1, 2, \ldots, m)$ are positive integers to be treated as the decision variables. Therefore, our aim is to determine the values of the variables n_i $(i = 1, 2, \ldots, m)$ and T which minimize $ETC_{2,C}$ subject to the capacity constraint

$$T \geqslant \frac{\sum_{i=1}^{m} s_i}{\kappa} \equiv T_{\min} \quad (8)$$

We first ignore the constraint (8) and assume that n_is are real rather than positive integers. Then the necessary conditions for the minimum of $ETC_{2,C}$ give

$$n_i = T\sqrt{\frac{R_i + Q_i}{v_i}}, \quad i = 1, 2, \ldots, m \quad (9)$$

and

$$T = \sqrt{\frac{A + \sum_{i=1}^{m} n_i v_i}{H + \sum_{i=1}^{m}((R_i + Q_i)/n_i)}} \quad (10)$$

We now develop the following algorithm to find the optimal values of the decision variables n_i $(i = 1,2,\ldots,m)$ and T satisfying constraint (8).

Algorithm I

Step 1. Start with T^o = any positive real number.

Step 2. Determine $n_i^o = T^o\sqrt{(R_i + Q_i)/v_i}$, $i = 1, 2, \ldots, m$.

Step 3. Find $T' = \sqrt{\dfrac{A + \sum_{i=1}^{m} n_i^o v_i}{H + \sum_{i=1}^{m}\left((R_i + Q_i)/n_i^o\right)}}$

Step 4. Evaluate $n_i' = T'\sqrt{(R_i+Q_i)/v_i}$, $i = 1, 2, \ldots, m$.

Step 5. If $|n_i' - n_i^o| < \varepsilon_{1i}$, for $i = 1, 2, \ldots, m$ go to *Step 6*. Otherwise, assign $n_i^o = n_i'$ for $i = 1, 2, \ldots, m$ and go to *Step 3*.

Step 6. Determine $n_i^* = \text{Round off}(n_i')$, $i = 1, 2, \ldots, m$, $T_2' = (T')_{n_i^*}$ and then $T_2^* = \max\{T_{\min}, T_2'\}$.

Step 7. If $T_2^* = T_2'$, then $n_i^*(i = 1, 2, \ldots, m)$ are the optimal number of inspections. Otherwise, determine n_i^* from Equation (9) by rounding off the RHS when $T = T_{\min}$.

Step 8. Finally evaluate $ETC_{2,C}^* = (ETC_{2,C})_{n_i^*, T_2^*}$.

Imperfect Process Model (IPM) under time-varying lot sizes approach

We now formulate the problem under time-varying lot sizes approach proposed by Dobson.[8] We follow the notation similar to Dobson.[8] The problem can be viewed as one of deciding on a cycle length T, a production sequence $\mathbf{f} = (f^1, f^2, \ldots, f^n)$, $n \geqslant m$, $f^j \in \{1, 2, \ldots, m\}$ which may contain repetitions, productions times $\mathbf{t} = (t^1, t^2, \ldots, t^n)$ and idle times $\mathbf{w} = (w^1, w^2, \ldots, w^n)$ so that the production sequence is executable in the chosen cycle length, the cycle length can be repeated indefinitely, demand is met and the inventory cost per unit time is minimized. We will use subscripts to refer to the ith item: p_i, d_i, h_i, A_i, s_i, α_i, θ_i, n_i etc. and superscripts to refer to the data related to the item produced at the jth position in the sequence: p^j, d^j, h^j, A^j, s^j, α^j, θ^j, n^j etc.; that is, $p^j = p_{f^j}$, $d^j = d_{f^j}$ etc. Let F be the set of all possible finite sequences of products and J_i denote the positions in a given sequence where product i is produced, that is, $J_i = j | f^j = i$. Let L_k be the positions in a given sequence from k, up to but not including the position in the sequence where product f^k is produced again. Using this notation, the ELSP for the IPM can be formulated as follows:

$$\inf_{f\in F} \text{Min}_{\mathbf{t}\geqslant \mathbf{0}, \mathbf{w}\geqslant \mathbf{0}, T>0} \times \frac{1}{T}\left[\sum_{j=1}^{n} A^j + \frac{1}{2}\sum_{j=1}^{n}\left\{h^j\left(\frac{p^j}{d^j} - 1\right) + \frac{u^j\alpha^j}{\theta^j}\right\}p^j(t^j)^2\right] \quad (11)$$

subject to

$$\sum_{j\in J_i} p_i t^j = d_i T, \quad i = 1, 2, \ldots, m \quad (12)$$

$$\sum_{j \in L_k} (t^j + s^j + w^j) = \frac{p^k t^k}{d^k}, \quad k = 1, 2, \ldots, n \tag{13}$$

$$\sum_{j=1}^{n} (t^j + s^j + w^j) = T \tag{14}$$

$\mathbf{t} \geqslant \mathbf{0}$, $\mathbf{w} \geqslant \mathbf{0}$, $T > 0$.

Constraint (12) ensures that we must allocate enough production time to each product i to meet its demand $d_i T$ over the cycle. Constraint (13) implies that we must produce enough of a product each time to last until the next time the same product is produced again. Constraint (14) means that the cycle time T must be the sum of production, setup and idle times for all items produced in the cycle.

A lower bound on expected total cost

In order to find a lower bound, we consider the objective function as the expected total cost (including setup cost, holding cost, and quality related cost) per unit time, subject to the constraint (16) given below. However, the synchronization constraint, stating that no two items can be scheduled to produce at the same time, is ignored. Consequently, the value of the following non-linear program results in a lower bound on the average total cost.

$$\text{Min}_{T_1, T_2, \ldots, T_m} \sum_{i=1}^{m} \left[\frac{A_i}{T_i} + (H_i + Q_i) T_i \right] \tag{15}$$

subject to

$$\sum_{i=1}^{m} \frac{s_i}{T_i} \leqslant \kappa \tag{16}$$

$$T_i \geqslant 0, \quad i = 1, 2, \ldots, m \tag{17}$$

where

$$H_i = \frac{1}{2} h_i d_i (1 - \rho_i), \; Q_i = \frac{u_i \alpha_i \rho_i d_i}{2\theta_i} \quad \text{and} \quad \kappa = 1 - \sum_{i=1}^{m} \rho_i.$$

Clearly the objective function and the constraint set are convex in T_is. So there exists a unique optimal point for the above lower bound model. Let λ_1 and μ_{1i} $(i = 1, 2, \ldots, m)$ be the Lagrange multipliers corresponding to the constraints (16) and (17), respectively. Then the Karush–Kuhn–Tucker (KKT) necessary conditions for the optimal point give

$$T_i = \sqrt{\frac{A_i + \lambda_1 s_i}{H_i + Q_i - \mu_{1i}}}, \quad i = 1, 2, \ldots, m. \tag{18}$$

For non-trivial T_is $(i = 1, 2, \ldots, m)$, one of the KKT conditions gives $\mu_{1i} = 0 \; \forall \, i$ and therefore, we can apply the following line search procedure on λ to find the optimal values of T_is $(i = 1, 2, \ldots, m)$.

Algorithm II

Step 1. Set $\lambda_1 = 0$ and find T_is $(i = 1, 2, \ldots, m)$ from Equation (18).
If $\sum_{i=1}^{m} (s_i/T_i) \leqslant \kappa$ then T_is are optimal. Stop. Otherwise, go to *Step 2.*

Step 2. Start with an arbitrary $\lambda_1 > 0$.

Step 3. Compute $T_i = \sqrt{(A_i + \lambda_1 s_i)/(H_i + Q_i)} \quad \forall \, i$.

Step 4. If $\sum_{i=1}^{m} (s_i/T_i) \leqslant \kappa$ then go to *Step 5.*
Otherwise, increase λ_1 and go to *Step 3*.

Step 5. If $|\sum_{i=1}^{m} (s_i/T_i) - \kappa| < \varepsilon_2$ then T_is are optimal. Stop.
Otherwise, reduce λ_1 and got to *Step 3*.

In the time-varying lot sizes approach, we can think the problem as consisting of two parts: a combinatorial part (the specification of **f**) and a continuous part (the determination of **t**, **w** and T, for given **f**). In the combinatorial part, we first determine these production frequencies utilizing the lower bound solution and then the production frequencies are rounded off to power-of-two integers which enable us to determine an efficient production sequence by bin-packing heuristic suggested by Doll and Whybark[37] and Dobson.[8] The continuous part takes the production sequence as given and computes the actual production times and idle times.[12]

Procedural steps to obtain **f**, **t** *and* **w**

1. *Find the production frequencies*
   Let the optimal cycle lengths of the lower bound model be T_i^*s. Then the relative production frequencies x_is can be determined by the relation
   $$x_i = \frac{\text{Max}_i\{T_i^*\}}{T_i^*}, \quad i = 1, 2, \ldots, m$$
2. *Round off the frequencies to power-of-two integers*
   The production frequencies x_is can be rounded off to power-of-two integers as
   $$y_i = 2^p \quad \text{if} \quad x_i \in \left[\frac{1}{\sqrt{2}} 2^p, \sqrt{2} 2^p \right], \quad p = 0, 1, \ldots$$
   Roundy[9] showed that the additional costs when the real values of the production frequencies are converted to power-of-two integers do not exceed 6%.
3. *Determine the production sequence by the bin-packing heuristic*
   Given the frequencies y_i, the bin-packing heuristic attempts to spread them out as evenly as possible.[8] For each product i, the production time duration z_i for the lots are estimated by assuming that the lots will be equally spaced. If there are b bins where $b = \max_{1 \leqslant i \leqslant m} y_i$, then y_i items of height $z_i \, \forall \, i$ are to allocate in b bins with the restriction that a product with frequency y_i must have all its lots placed in the bins equally spaced. While

assigning the items to bins, a variation of the Longest Processing Time (LPT) rule is used in which the items are ordered lexicographically by (y_i, z_i). By minimizing the maximum height of the bins, the heuristic finds an efficient production sequence **f**.

4. *Solve for* **t** *and* **w**, *given* **f**
   We will solve equations (13) by assuming that there are no idle times ($\mathbf{w}=\mathbf{0}$). This assumption fits well for a highly loaded facility. Consideration of positive idle times, which is the case of an under-utilized facility, makes the problem rather complex non-linear programming. Yet the problem can be handled through the solution of a parametric quadratic program and a few EOQ like calculations; see reference 12.

Imperfect Process Model With Inspection and Restoration (IPMWIR) under time-varying lot sizes approach

The formulation of the model, in this case, is similar to the previous section except that of the objective function which is given by

$$\inf_{f\in F}\text{Min}_{\mathbf{t}\geqslant\mathbf{0},\mathbf{w}\geqslant\mathbf{0},T>0} \times\frac{1}{T}\left[\sum_{j=1}^{n}A^j+\frac{1}{2}\sum_{j=1}^{n}\left\{h^j\left(\frac{p^j}{d^j}-1\right)+\frac{u^j\alpha^j}{n^j\theta^j}\right\}p^j(t^j)^2 +\sum_{j=1}^{n}n^jv^j+\sum_{j=1}^{n}\frac{(r_1\theta^j-r_0)(t^i)^2}{2(\theta^j)^2n^j}+\sum_{j=1}^{n}\frac{r_0t^j}{\theta^j}\right] \tag{19}$$

subject to

$$\sum_{j\in J_i}p_it^j=d_iT,\quad i=1,2,\ldots,m \tag{20}$$

$$\sum_{j\in L_k}(t^j+s^j+w^j)=\frac{p^kt^k}{d^k},\quad k=1,2,\ldots,n \tag{21}$$

$$\sum_{j=1}^{n}(t^j+s^j+w^j)=T \tag{22}$$

$\mathbf{t}\geqslant\mathbf{0}$, $\mathbf{w}\geqslant\mathbf{0}$, $T>0$.

The additional terms in the objective function (19) are due to the inspection and process restoration costs. Explanation of the other terms and constraints are as given in the previous section.

To find a lower bound for this model, we consider the objective function on the expected total cost per unit time which include setup cost, holding cost, quality related cost, inspection cost and restoration cost and we ignore the synchronization constraint as in the previous section.

Problem LB

$$\text{Min}_{T_1,T_2,\ldots,T_m}\sum_{i=1}^{m}\left[\frac{A_i+n_iv_i}{T_i}+\left(H_i+\frac{Q_i+R_i}{n_i}\right)T_i+\frac{r_0d_i}{p_i\theta_i}\right] \tag{23}$$

subject to

$$\sum_{i=1}^{m}\frac{s_i}{T_i}\leqslant\kappa \tag{24}$$

$$T_i\geqslant 0,\quad i=1,2,\ldots,m \tag{25}$$

where

$$H_i=\frac{1}{2}h_id_i(1-\rho_i),\quad Q_i=\frac{u_i\alpha_i\rho_id_i}{2\theta_i}$$

$$R_i=(r_1\theta_i-r_0)\frac{d_i^2}{2\theta_i^2p_i^2}\quad\text{and}\quad\kappa=1-\sum\nolimits_{i=1}^{m}\rho_i$$

The associated Lagrangian function L of the above constrained optimization problem can be written as

$$L=\sum_{i=1}^{m}\left[\frac{1}{T_i}(A_i+n_iv_i)+T_i\left(H_i+\frac{Q_i+R_i}{n_i}\right)+\frac{r_0d_i}{p_i\theta_i}\right] +\lambda_2\left(\sum_{i=1}^{m}\frac{s_i}{T_i}-\kappa\right)-\sum_{i=1}^{m}\mu_{2i}T_i$$

where λ_2 and μ_{2i} ($i=1, 2,\ldots, m$) are Lagrangian multiplier corresponding to the constraints (24) and (25), respectively. The number of inspections n_i ($i=1, 2,\ldots, m$) are positive integers to be treated as decision variables. If we first assume that n_i ($i=1, 2,\ldots, m$) are real variables then the KKT necessary conditions for the minimum of L give,

$$\frac{1}{T_i^2}(A_i+n_iv_i+\lambda_2s_i)-\left(H_i+\frac{Q_i+R_i}{n_i}\right)+\mu_{2i}=0 \tag{26}$$

and

$$\frac{v_i}{T_i}-T_i\frac{Q_i+R_i}{n_i^2}=0 \tag{27}$$

For non-trivial T_is, $i=1, 2,\ldots, m$, we have $\mu_{2i}=0\quad\forall i$. Thus from Equations (26) and (27), we get

$$\frac{v_in_i^2}{Q_i+R_i}=\frac{A_i+n_iv_i+\lambda_2s_i}{H_i+((Q_i+R_i)/n_i)} \tag{28}$$

which gives,

$$n_i=\sqrt{\frac{(A_i+\lambda_2s_i)(Q_i+R_i)}{v_iH_i}},\quad i=1,2,\ldots,m. \tag{29}$$

To determine the optimal values of T_i ($i=1, 2,\ldots, m$) and the optimal (positive integer) values of n_i ($i=1, 2,\ldots, m$), we may develop the following algorithm:

Algorithm III

Step 1. Set $\lambda_2 = 0$ and determine $n_i = \sqrt{((A_i + \lambda_2 s_i)(Q_i + R_i)/v_i H_i)}$ for $i = 1, 2, \ldots, m$ and then $T_i = \sqrt{(v_i n_i^2/(Q_i + R_i))}$ for $i = 1, 2, \ldots, m$.
If $\sum_{i=1}^{m}(s_i/T_i) \leqslant \kappa$ go to *Step 6*. Otherwise, go to *Step 2*.
Step 2. Start with an arbitrary $\lambda_2 > 0$.
Step 3. Compute n_i and T_i for $i = 1, 2, \ldots, m$ by using the formulae given in *Step 1*.
Step 4. If $\sum_{i=1}^{m}(s_i/T_i) \leqslant \kappa$ then go to *Step 5*. Otherwise, increase λ_2 and go to *Step 3*.
Step 5. If $|\sum_{i=1}^{m}(s_i/T_i) - \kappa| < \varepsilon_3$ then go to *Step 6*. Otherwise, reduce λ_2 and go to *Step 3*.
Step 6. Round off n_i, $i = 1, 2, \ldots, m$ and evaluate the average total cost. Stop.

Numerical illustration

Common cycle solution of IPM

Example I Let us first consider the following example (Table 1) which was chosen by Ben-Daya and Hariga.[35]

Before finding the optimal order interval by the CC-approach we verify by computing $\sum(d_i/p_i) = 0.9942$ that it is possible to meet up all the required demands. The facility must run 99.42% of the time to meet the demands and the remaining less than 0.6% of the time is available for setups, maintenance, etc. As the machine schedule is very tight, the constraint (4), in this case, is binding. However, the situation differs when the machine schedule is less tight and significant changeovers in between the products are required.

Example II Table 2 shows 3-item data, a part of which is borrowed from Silver *et al*,[29] p 446.

Table 1 Example I data

Item i	A_i	θ_i	p_i	d_i	u_i	h_i
1	100	10	1000	500	5	0.50
2	100	12	1800	400	7	0.40
3	100	15	2300	250	20	0.80
4	150	20	1200	100	20	1.00
5	200	10	2500	200	5	1.20

Table 2 Example II data (r_0 = \$10, r_1 = \$0.1)

Item i	A_i (\$)	θ_i (*y*)	α_i	p_i (*units/y*)	d_i (*units/y*)	u_i (\$)	h_i (\$)	s_i (*y*)	v_i (\$)
1	125	1.2	0.20	5000	1850	30	12.50	0.00068	3
2	100	0.5	0.25	3500	1150	200	87.50	0.00171	3
3	110	0.8	0.30	3000	800	50	21.25	0.00091	3

Here $\sum(d_i/p_i) = 0.97$. This means that the facility must run 97% of the time, with the remaining 3% available for setups, maintenance, etc. Since $\sum s_i = 0.0033$ y, the minimum cycle length $T_{\min} = \sum s_i/(1 - \sum(d_i/p_i)) = 0.0949$ y. Using Equation (3) we obtain $T_1^* = 0.0692$ y. As $T_{\min} > T_1^*$, the optimal cycle length should be taken as $T_{\min} = 0.0949$ y or 22.7760 days, assuming one year equal to 240 days. The expected annual total cost is obtained as \$10 164.86.

For this example, if we ignore the constraint (4) (which leads to Ben-Daya and Hariga's[35] model) we find $T_1^* = 0.0692$ y or 16.6080 days (infeasible) and the expected annual total cost = \$9678.33 which is obviously less than the actual expected annual total cost.

Example III Table 3 presents 5-item data, a part of which is borrowed from Johnson and Montgomery.[36]

For this data set, $\sum(d_i/p_i) = 0.94$ and $\sum s_i = 0.39$ days. Therefore, $T_{\min} = \sum s_i/(1 - \sum(d_i/p_i)) = 6.8468$ days. Equation (3) gives $T_1^* = 1.005$ days. Clearly the optimal cycle length = 6.8468 days, giving the expected total cost per day = \$2735.28. Ben-Daya and Hariga's[35] model determines 1.005 days as the optimal cycle length and \$786.06 as the expected total cost per day. Note that the huge reduction in the expected total cost is due to infeasible cycle length obtained as a result of violating the constraint (4).

Common cycle solution of IPMWIR

Using Algorithm-I, we obtain the following results for Example II data: $T_2^* = 0.0949$ y or 22.7760 days, assuming one year equal to 240 days, $n_1^* = 2$, $n_2^* = 7$, $n_3^* = 2$ and $ETC_{2,c}^* = \$8811.58$.

Ben-Daya and Hariga's[35] model gives $T_2^* = T_2' = 0.0842$ y or 20.2080 days and the expected annual total cost = \$8748.48.

Example III data results in $T_2^* = 6.8469$ days, $n_1^* = 2$, $n_2^* = 2$, $n_3^* = 2$, $n_4^* = 1$, $n_5^* = 1$ and the associated expected total cost = \$2692.25. Ben-Daya and Hariga's[35] model determines the expected total cost as \$795.63 which is far below the actual expected total cost.

Time-varying lot sizes solution of IPM

Algorithm-II when considered for Examples II and III data provides the order periods and the corresponding lower bounds as shown in Table 4.

Table 3 Example III data (r_0 = \$10, r_1 = \$0.2)

Item i	A_i (\$)	θ_i (*days*)	α_i	p_i (*units/day*)	d_i (*units/day*)	u_i (\$)	h_i (\$)	s_i (*days*)	v_i (\$)
1	75	10	0.20	1550	300	8	0.5	0.05	2
2	90	12	0.25	1890	400	5	0.4	0.08	2
3	50	15	0.30	1415	250	10	0.8	0.06	2
4	100	25	0.20	1260	300	12	1.0	0.05	2
5	80	8	0.15	1625	200	6	0.6	0.15	2

Table 4 Lower bound for the ELSP-IPM

Data	*Order periods*	*Lower bound*
Example II	$T_1 = 0.14528$ y, or 34.8720 days $T_2 = 0.07067$ y, or 16.9680 days $T_3 = 0.15460$ y, or 37.1040 days	\$9289.36
Example III	$T_1 = 5.7053$ days $T_2 = 7.0585$ days, $T_3 = 5.3725$ days $T_4 = 4.2687$ days, $T_5 = 10.7280$ days	\$2461.8

For Example II, the relative production frequencies x_i ($i = 1, 2, 3$) of the items can be obtained from lower bound model as

$$x_1 = 1.0642, \quad x_2 = 2.1876, \quad x_3 = 1.0000.$$

Rounding off these frequencies to power-of-two integers we find

$$y_1 = 1, \quad y_2 = 2, \quad y_3 = 1$$

For each product i, we now estimate the production time duration z_i for the lots by assuming that the lots will be equally spaced. That is, we compute

$$z_i = s_i + \frac{d_i T}{p_i y_i} \quad \text{where } T = \frac{\sum x_i s_i}{1 - \rho},$$

by assuming that $\mathbf{w} = \mathbf{0}$. This gives $z_1 = 0.0572$, $z_2 = 0.0254$, and $z_3 = 0.0412$. Now bin-packing with $b = \max_i\{y_i\} = 2$ bins and y_i items of height z_i for all i, we get the production sequence $\mathbf{f} = \{2\ 1\ 2\ 3\}$. Then solving the system of equations (13) with the help of numerical computational software MATHEMATICA, we find the production time sequence $\mathbf{t} = \{0.0273, 0.0533, 0.0201, 0.0384\}$ (in years) or equivalently $\mathbf{t} = \{6.5520, 12.7920, 4.8240, 9.2160\}$ (in days). This gives the required cycle length $T = 0.1441$ y, or 34.5840 days and the expected average total cost = \$9384.82.

Applying the similar procedure to Example III data we find the following results:

Production sequence $\mathbf{f} = \{4\ 2\ 1\ 3\ 5\ 4\ 2\ 1\ 3\}$
Production time sequence $\mathbf{t} = \{1.6380, 1.3200, 1.1493, 1.0212, 1.3613, 0.9953, 1.0208, 0.9914, 0.9329\}$ (in days)
Cycle length $T = 11.06$ days and associated average total cost = \$2573.29.

Time-varying lot sizes solution of IPMWIR

To find the lower bound on IPMWIR for Examples II and III data we use Algorithm-III. The computed results are given in Table 5.

The production sequence $\mathbf{f}$ and the production time sequence $\mathbf{t}$ remain unchanged compared to IPM. However, to evaluate the expected average total cost, we need to determine n^js. Assuming n^js as real, we differentiate the objective function (19) with respect to n^j and get,

$$n^j = \frac{t^j}{\theta^j}\sqrt{\frac{u^j\,\theta^j\,\alpha^j p^j + (r_1\theta^j - r_0)}{2v^j}}, \quad j = 1, 2, \ldots, n \quad (30)$$

Hence, when t^js are known the integral values of n^js can be determined from above by rounding off the RHS of Equation (30) to the nearest integer. Example II data evaluate $n^1 = 7$, $n^2 = 3$, $n^3 = 5$, $n^4 = 4$ and the expected average total cost = \$8246.65 while Example III data find $n^1 = 9$, $n^2 = 9$, $n^3 = 9$, $n^4 = 9$, $n^5 = 9$, $n^6 = 5$, $n^7 = 7$, $n^8 = 8$, $n^9 = 8$ and the expected average total cost = \$2490.15.

Table 5 Lower bound for the ELSP-IPMWIR

Data	*Order periods and number of inspections*	*Lower bound*
Example II	$T_1 = 0.1448$ y, or 34.7520 days $n_1 = 3$ $T_2 = 0.0708$ y, or 16.9920 days $n_2 = 6$ $T_3 = 0.1536$ y, or 36.8640 days $n_3 = 4$	\$8185.97
Example III	$T_1 = 5.7827$ days, $n_1 = 9$ $T_2 = 7.1298$ days, $n_2 = 11$ $T_3 = 5.3845$ days, $n_3 = 8$ $T_4 = 4.2327$ days, $n_4 = 6$ $T_5 = 10.6100$ days, $n_5 = 9$	\$2378.06

Table 6 Example IV data (r_0 = \$150, r_1 = \$10)

Item i	*A_i (\$)*	*θ_i (days)*	*α_i*	*p_i (units/day)*	*d_i (units/day)*	*u_i (\$)*	*h_i (\$)*	*s_i (days)*	*v_i (\$)*
1	3000	80	0.12	133	20	10	0.0461	4.0	4
2	1800	75	0.08	300	24	15	0.0312	2.4	4
3	3600	140	0.10	266	30	25	0.0651	4.8	4
4	1500	90	0.06	146	36	16	0.1180	2.0	4
5	6000	210	0.15	532	40	20	0.1190	4.0	4
6	30 000	112	0.05	373	50	30	0.0847	8.0	4

Table 7 Example V data (r_0 = \$10, r_1 = \$2)

Item i	*A_i (\$)*	*θ_i (days)*	*α_i*	*p_i (units/day)*	*d_i (units/day)*	*u_i (\$)*	*h_i (\$) $\times 10^{-2}$*	*s_i (days)*	*v_i (\$)*
1	15	12.5	0.04	3075	400	0.6500	0.0130	0.125	2
2	20	8.0	0.07	8000	400	0.1775	0.0355	0.125	2
3	30	6.5	0.10	9500	800	0.1275	0.0255	0.250	3
4	10	15.0	0.03	7500	750	1.0000	0.0200	0.125	2
5	110	14.0	0.15	2000	80	2.7850	0.5570	0.500	3
6	50	18.0	0.08	6015	80	0.2675	0.0535	0.250	2
7	310	10.0	0.14	2400	104	1.5000	0.3000	1.000	3
8	130	9.0	0.10	1300	340	3.2900	0.0580	0.500	6
9	200	16.0	0.05	2000	340	0.9000	0.1800	0.700	4
10	24	20.0	0.12	15 038	400	0.0400	0.0080	0.125	2

Table 8 A comparative study of the numerical results

Data	Model	*Expected average total cost ($)* Common cycle approach (x)	Time varying lot sizes approach (y)	Lower bound ($) (z)	$(x-z)/z \times 100$	$(y-z)/z \times 100$
Example II (3 items)	IPM	10 164.86	9384.82	9289.36	9.42	1.03
	IPMWIR	8811.58	8246.65	8185.97	7.64	0.74
Example III (5 items)	IPM	2735.28	2573.29	2461.82	11.11	4.53
	IPMWIR	2658.25	2490.15	2378.06	11.78	4.71
Example IV (6 items)	IPM	1310.05	1216.12	1190.79	10.02	2.13
	IPMWIR	1279.18	1192.84	1166.74	9.64	2.24
Example IV(a) (6 items)	IPM	1878.08	1766.90	1697.82	10.62	4.07
	IPMWIR	1813.06	1715.40	1643.19	10.34	4.39
Example V (10 items)	IPM	156.44	129.37	120.49	29.84	7.37
	IPMWIR	77.92	75.51	72.99	6.75	3.45

A comparative study

To make a comparison of the gains between CC solution and time-varying lot sizes solution or the gains between IPM and IPMWIR we consider the following three additional examples:

Example IV A part of the 6-item data shown in Table 6 is taken from the Eilon problem, see Haessler and Houge,[38] p 911.

Example IV(a) The data set is identical to Example IV with the exception of the production rates that are decreased by 10%.

Example V The data set given in Table 7 is constructed from the Bomberger problem, see Haessler,[39] p 339.

We choose Example IV to consider the situation where the proportion of time available for setups is high and Example V where as many as 10 items are to be produced in a single machine. In Examples IV and IV(a) the proportions of time available for changeovers in between the products are 20 and 11%, respectively. Table 8 reflects that a 10% decrease in the production rates in Example IV (which is Example IV(a)) increases the system cost more than 40%. It is also clear from Table 8 that the expected average total costs in time-varying lot sizes approach in Examples II–V are above the lower bounds by 1–7% only whereas in the CC approach these are about 7–12% except for IPM in Example V, where the expected total cost is higher than the lower bound by 30%. This numerical study clearly demonstrates the superiority of the time-varying lot sizes approach over common cycle approach in dealing with the ELSP.

Concluding remarks

This paper re-investigates the Economic Lot Scheduling Problem (ELSP) with imperfect production processes, previously studied by Ben-Daya and Hariga,[35] under the capacity constraint *viz setup time per cycle plus the total production time per cycle must be no more than the cycle length* by applying both the Common Cycle (CC) approach and the time-varying lot sizes approach. Ben-Daya and Hariga[35] assumed negligible setup times for production setups and studied the ELSP using the CC-approach. However, when setup time exists, ie, when time is required to change the machine from the production of one product to another—the question of feasibility becomes more complex. The lot sizes must be sequenced so that the intervals of production do not overlap and the sum of the setup times does not exceed the time available for setups. In this regard the present effort is significant and meaningful. In our study, we have considered quality-related costs for possible production of non-conforming items. We did not consider any rework of the defective items either on- or off-line. So future research can investigate the case where all the imperfect items are reworked instantaneously or reworked off line without affecting the utilization of the system. Another direction may be the study of effectiveness and economy of preventive maintenance on the ELSP.

Acknowledgements—The authors are very grateful to the comments by two anonymous referees and the editor. This work was supported by grant No. R02-2001-01396 from the Basic Research Program of the Korea Science & Engineering Foundation (KOSEF).

References

1 Hsu W (1983). On the general feasibility test for scheduling lot sizes for several products in one machine. *Mngt Sci* **29**: 93–105.
2 Gallego G and Shaw X (1997). Complexity of the ELSP with general cyclic schedules. *IIE Trans* **29**: 109–113.
3 Elmaghraby S (1978). The economic lot scheduling problem: reviews and extensions. *Mngt Sci* **24**: 587–98.
4 Hanssmann F (1962). *Operations Research in Production and Inventory Control.* John Wiley and Sons: New York.
5 Jones P and Inman R (1989). When is the economic lot scheduling problem easy? *IIE Trans* **21**: 11–20.
6 Bomberger E (1966). A dynamic programming approach to a lot size scheduling problem. *Mngt Sci* **12**: 778–784.

7 Delporte C and Thomas L (1978). Lot sizing and sequencing for N products on one facility. *Mngt Sci* **23**: 1070–1079.
8 Dobson G (1987). The economic lot scheduling problem: achieving feasibility using time-varying lot sizes. *Opns Res* **35**: 764–771.
9 Roundy R (1989). Rounding off to powers of two in continuous relaxation of capacitated lot sizing problems. *Mngt Sci* **35**: 1433–1442.
10 Maxwell W (1964). The scheduling of economic lot sizes. *Nav Res Log Q* **11**: 89–124.
11 Matthew JPS (1988). The optimality of the zero-switch rule for a class of economic lot scheduling problems. *J Opl Res Soc* **39**: 1155–1161.
12 Zipkin PH (1991). Computing optimal lot sizes in the economic lot scheduling problem. *Opns Res* **39**: 56–63.
13 Gallego G and Roundy R (1992). The extended economic lot scheduling problem. *Nav Res Log* **39**: 729–829.
14 Dobson G (1992). The cyclic lot scheduling problem with sequence-dependent setups. *Opns Res* **40**: 736–749.
15 Allen SJ (1990). Production rate planning for two products sharing a single process facility: a real world case study. *Prod Inv Mngt* **31**: 24–29.
16 Silver E (1990). Deliberately slowing down output in a family production context. *Int J Prod Res* **28**: 17–27.
17 Moon I, Gallego G and Simchi-Levi D (1991). Controllable production rates in a family production context. *Int J Prod Res* **29**: 2459–2470.
18 Gallego G (1993). Reduced production rates in the economic lot scheduling problem. *Int J Prod Res* **31**: 1035–1046.
19 Moon D and Christy D (1998). Determination of optimal production rates on a single facility with dependent mold lifespan. *Int J Prod Econ* **54**: 29–40.
20 Khouja M (1999). A note on deliberately slowing down output in a family production context. *Int J Prod Res* **37**: 4067–4077.
21 Khouja M (1997). The economic lot scheduling problem under volume flexibility. *Int J Prod Econ* **48**: 73–86.
22 Gallego G and Moon I (1992). The effect of externalizing setups in the economic lot scheduling problem. *Opns Res* **40**: 614–619.
23 Gallego G and Moon I (1995). Strategic investment to reduce setup times in the economic lot scheduling problem. *Nav Res Log* **42**: 773–790.
24 Hwang H, Kim D and Kim Y (1993). Multiproduct economic lot size models with investment costs for setup reduction and quality improvement. *Int J Prod Res* **31**: 691–703.
25 Moon I (1994). Multiproduct economic lot size models with investment costs for setup reduction and quality improvement: review and extensions. *Int J Prod Res* **32**: 2795–2801.
26 Khouja M, Michalewich Z and Wilmot M (1998). The use of genetic algorithms to solve the economic lot size scheduling problem. *Eur J Opl Res* **110**: 509–524.
27 Moon I, Silver E and Choi S (2002). A hybrid genetic algorithm for the economic lot scheduling problem. *Int J Prod Res* **40**: 809–824.
28 Moon I, Hahm J and Lee C (1998). The effect of the stabilization period on the economic lot scheduling problem. *IIE Trans* **30**: 1009–1017.
29 Silver E, Pyke D and Peterson R (1998). *Inventory Management and Production Planning and Scheduling* (3rd edn). John Wiley and Sons: New York.
30 Porteus EL (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Opns Res* **34**, 137–144.
31 Rosenblatt MJ and Lee HL (1986). Economic production cycles with imperfect production processes. *IIE Trans* **18**: 48–55.
32 Khouja M and Mehrez A (1994). An economic production lot size model with imperfect quality and variable production rate. *J Opl Res Soc* **45**: 1405–1417.
33 Hariga M and Ben-Daya M (1998). The economic manufacturing lot-sizing problem with imperfect manufacturing processes: bounds and optimal solutions. *Nav Res Log* **45**: 423–433.
34 Salameh MK and Jaber MY (2000). Economic production quantity model for items with imperfect quality. *Int J Prod Econ* **64**: 59–64.
35 Ben-Daya M and Hariga M (2000). Economic lot scheduling problem with imperfect production processes. *J Opl Res Soc* **51**: 875–881.
36 Johnson L and Montgomery D (1974). *Operations Research in Production Planning, Scheduling and Inventory Control.* John Wiley and Sons: New York.
37 Doll L and Whybark C (1973). An iterative procedure for the single machine, multi-product lot scheduling problem. *Mngt Sci* **20**: 50–55.
38 Haessler RW and Hogue SL (1976). A note on the single-machine multi-product lot scheduling problem. *Mngt Sci* **22**: 909–912.
39 Haessler RW (1979). An improved extended basic period procedure for solving the economic lot scheduling problem. *AIIE Trans* **11**: 336–340.

Received February 2001;
accepted December 2001 after two revisions