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Theory and Methodology

The effects of inflation and time-value of money on an economic order quantity model with a random product life cycle

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Abstract

For several decades, the Economic Order Quantity (EOQ) model and its variations have received much attention from researchers. Recently, there has been an investigation into an EOQ model incorporating a random product life cycle and the concept of time-value of money. This paper extends the previous research in several areas. First, we investigate the impact of inflation on the choice of replenishment quantities. Second, the unit cost, which has been inadvertently omitted in the previous research, is included in the objective function to properly model the problem. Third, we consider the normal distribution as a product life cycle in addition to the exponential distribution. Fourth, we develop a simulation model which can be used for any probability distribution.

Keywords: Inventory; Inflation; Time-value of money

1. Introduction

The Economic Order Quantity (EOQ) model initiated by Harris [13] has been extended many ways to improve the practicality of the model. See, for example, [3,4], etc. One of the important extensions is to properly recognize the time-value of money in determining the optimal order quantity. Trippi and Lewin [21] have adopted the Discounted Cash-Flows (DCF) approach for the analysis of the basic EOQ model. Kim et al. [14] extend Trippi and Lewin's work by applying the DCF approach to various inventory systems. Chung [5] has studied the DCF approach for the analysis of the basic EOQ model in the presence of the trade credit. Recently, Haneveld and Teunter [11] apply the DCF approach to the basic EOQ model with a Poisson demand process.

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Gurnani [9] has applied the DCF approach to the finite planning horizon EOQ model in which the planning horizon is a given constant. Gurnani [10] has claimed that an infinite planning horizon does not exist in real life, and a finite horizon inventory model (accounting for the time-value of money) is theoretically superior and of greater practical utility. Chung and Kim [7] prove that Gurnani's [9] model is essentially identical to an infinite planning horizon model since the planning horizon is assumed to be a given constant. As pointed out by Gurnani [9], an infinite planning horizon occurs rarely because costs are likely to vary disproportionately and product specifications and design are prone to change, abandonment or substitution by another product due to rapid technological development. This phenomenon can be observed frequently in high-technology product markets. Chung and Kim [7] have also suggested that the assumption of an infinite planning horizon is not realistic, and called for a new model which relaxes this assumption. Moon and Yun [16] answered this request by developing a finite planning horizon EOQ model where the planning horizon is a random variable following an exponential distribution. The unit cost, which does not affect the replenishment quantity (or cycle length) in the infinite planning horizon model, was not included in their model. However, the unit cost does affect the replenishment quantity in the finite planning horizon model, as will be demonstrated later.

One of the assumptions in most derivations of the inventory model has been a negligible level of inflation. Unfortunately, many countries have recently been confronted with fluctuating inflation rates that often have been far from negligible [20]. Silver et al. [20] investigate the impact of inflation on the choice of replenishment quantities in the basic EOQ model. There have been several studies which consider various inflationary situations. Bose et al. [2] have investigated an EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation. Hariga and Ben-Daya [12] have developed time-varying lot-sizing models with linear trend in demand, taking into account the effects of inflation and time value of money.

The objectives of this study are fourfold. Firstly, we include the unit cost, which has been inadvertently omitted in [16], in the model to properly derive the optimal order quantity. Secondly, the impact of inflation on the choice of order quantities is investigated. Thirdly, we consider the normal distribution as a product life cycle in addition to the exponential distribution. Fourthly, a simulation model is developed to be used for any probability distribution.

In Section 2 we introduce the notation and mathematically formulate the problem. Analytical results for both exponential and normal distributions are developed. In Section 3, we perform some computational results to show the cost savings compared with the solution of Moon and Yun [16]. A simulation model which can be used for any probability distribution is presented in Section 4 to complement the analytical work. The paper concludes with Section 5 which includes some possible research problems.

2. Analytical modeling

The following notation will be used:

Q	the order quantity
T	the cycle length
p	the product life cycle (random variable)
$f(p)$	the probability density function of p
D	the demand rate per year
S	the ordering cost per order
c	the unit cost
h	the inventory carrying cost per unit per year ($\equiv ic$ where i is the inventory carrying charge rate)

α the discount rate representing the time-value of money
f the inflation rate

We assume that $\alpha > f$, that is the interest rate is larger than the inflation rate, which is a practical assumption. As pointed out by Seo and Kim [18], the true optimal order quantities may vary over the cycle except for the exponential distribution case which possesses a memoryless property. However, we assume that all future replenishments will be of the same size as the current replenishment to obtain a tractable analytical result. As claimed by Silver et al. [20], this assumption is not as serious as it would first appear. In particular, at the time of the next replenishment we would recompute a new value for that replenishment, reflecting any changes in cost (and other) parameters that have taken place in the interim. In addition to this, we employ discounting so that assumptions about future lot sizes should not have an appreciable effect on the current decision.

Present value of cash flows for the first cycle: The present value of cash flows for the first cycle is

$$S + cQ + h\int_0^T (Q - tD)\,\mathrm{e}^{-\alpha t}\,\mathrm{e}^{ft}\,\mathrm{d}t. \tag{1}$$

Eq. (1) can be simplified as

$$S + cDT + \frac{hD}{(\alpha - f)}\left[T + \frac{1}{(\alpha - f)}\left(\mathrm{e}^{-(\alpha-f)T} - 1\right)\right]. \tag{2}$$

Present value of cash flows up to the beginning of the last cycle: If we assume that the planning horizon p fully accommodates first k cycles, and ends during $(k+1)$th cycle, then the present value of the total ordering costs and inventory carrying costs up to the beginning of $(k+1)$th cycle is

$$(S + cDT)\sum_{i=0}^{k} \mathrm{e}^{-i\alpha T}\,\mathrm{e}^{ifT} + \frac{hD}{(\alpha - f)}\left[T + \frac{\mathrm{e}^{-(\alpha-f)T} - 1}{(\alpha - f)}\right]\sum_{i=0}^{k-1} \mathrm{e}^{-i\alpha T}\,\mathrm{e}^{ifT}. \tag{3}$$

Eq. (3) can be simplified as

$$(S + cDT)\left[\frac{1 - \mathrm{e}^{-(\alpha-f)(k+1)T}}{1 - \mathrm{e}^{-(\alpha-f)T}}\right] + \frac{hD}{(\alpha - f)}\left[T + \frac{\mathrm{e}^{-(\alpha-f)T} - 1}{(\alpha - f)}\right]\left[\frac{1 - \mathrm{e}^{-(\alpha-f)kT}}{1 - \mathrm{e}^{-(\alpha-f)T}}\right]. \tag{4}$$

Present value of inventory carrying cost during the last cycle: The present value of inventory carrying cost during the last cycle, i.e. $(k+1)$th cycle can be obtained as follows:

$$h\,\mathrm{e}^{-(\alpha-f)kT}\int_0^{p-kT} (Q - tD)\mathrm{e}^{-\alpha t}\,\mathrm{e}^{ft}\,\mathrm{d}t. \tag{5}$$

Eq. (5) can be simplified as

$$\frac{hD}{\alpha - f}\mathrm{e}^{-(\alpha-f)kT}\left[T - \frac{1}{(\alpha - f)}\right] + \frac{hD}{\alpha - f}\mathrm{e}^{-(\alpha-f)p}\left[\frac{1}{(\alpha - f)} - (k+1)T + p\right]. \tag{6}$$

Expected present value of total inventory carrying and ordering costs: Since the planning horizon p has a p.d.f. $f(p)$, the expected present value of total inventory carrying and ordering costs, say $C(T)$, is

$$
\begin{aligned}
C(T) &= \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} [(4) + (6)] f(p)\, \mathrm{d}p \\
&= \frac{S + cDT}{1 - \mathrm{e}^{-(\alpha-f)T}} \sum_{k=0}^{\infty} \left[1 - \mathrm{e}^{-(\alpha-f)(k+1)T}\right] \int_{kT}^{(k+1)T} f(p)\, \mathrm{d}p \\
&\quad + \frac{hD}{(\alpha - f)(1 - \mathrm{e}^{-(\alpha-f)T})} \left[T + \frac{\mathrm{e}^{-(\alpha-f)T} - 1}{\alpha - f}\right] \sum_{k=0}^{\infty} \left[1 - \mathrm{e}^{-(\alpha-f)kT}\right] \int_{kT}^{(k+1)T} f(p)\, \mathrm{d}p \\
&\quad + \frac{hD}{\alpha - f} \left(T - \frac{1}{\alpha - f}\right) \sum_{k=0}^{\infty} \mathrm{e}^{-(\alpha-f)kT} \int_{kT}^{(k+1)T} f(p)\, \mathrm{d}p \\
&\quad + \frac{hD}{\alpha - f} \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \left(\frac{1}{\alpha - f} - (k+1)T + p\right) \mathrm{e}^{-(\alpha-f)p} f(p)\, \mathrm{d}p. \qquad (7)
\end{aligned}
$$

2.1. Exponential distribution case

Masters [15] develops a basic EOQ model in which the product life cycle follows an exponential distribution. He shows that the traditional approach of inflating the inventory carrying charge rate is appropriate for items subject to sudden obsolescence. Even though the time-value of money is not considered in his model, we show that the approximation is quite good in the computational section later. If the planning horizon p follows an exponential distribution with parameter λ, we can obtain an exact optimal cycle length using the following propositions.

Proposition 1.

$$
\begin{aligned}
C(T) = {} & \frac{S + cDT}{1 - \mathrm{e}^{-(\alpha-f+\lambda)T}} + \frac{hD\left[\mathrm{e}^{-(\alpha-f+\lambda)T} + (\alpha - f)T - 1\right]}{(\alpha - f)^2\left[1 - \mathrm{e}^{-(\alpha-f+\lambda)T}\right]} + \frac{h\lambda D[2(\alpha - f) + \lambda]}{(\alpha - f)^2(\alpha - f + \lambda)^2} \\
& - \frac{h\lambda DT}{(\alpha - f)(\alpha - f + \lambda)[1 - e^{-(\alpha-f+\lambda)T}]}. \qquad (8)
\end{aligned}
$$

Proof. See Appendix A.

Proposition 2.

(a) *$C(T)$ has a unique local minimum on $[0, \infty)$.*

(b) *The optimal cycle length, T^*, satisfies*

$$
\frac{(\alpha - f + \lambda)^2 \mathrm{e}^{-(\alpha-f+\lambda)T^*}}{i[1 - \mathrm{e}^{-(\alpha-f+\lambda)T^*}[1 + T(\alpha - f + \lambda)]] + [1 - \mathrm{e}^{-(\alpha-f+\lambda)T^*}](\alpha - f + \lambda) - T^*(\alpha - f + \lambda)^2 \mathrm{e}^{-(\alpha-f+\lambda)T^*}} = \frac{cD}{S}. \qquad (9)
$$

Proof. (a) The first derivative of $C(T)$, say $C'(T)$, is as follows:

$$C'(T) = \frac{hD\left[1 - e^{-(\alpha-f+\lambda)T}[1 + T(\alpha - f + \lambda)]\right]}{(\alpha - f + \lambda)[1 - e^{-(\alpha-f+\lambda)T}]^2} - \frac{(S + cDT)(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T} - cD\left[1 - e^{-(\alpha-f+\lambda)T}\right](\alpha - f + \lambda)}{(\alpha - f + \lambda)[1 - e^{-(\alpha-f+\lambda)T}]^2}.$$

$C'(T)$ has the same sign as the numerator of $C'(T)$, say $f(T)$:

$$f(T) = hD\left[1 - e^{-(\alpha-f+\lambda)T}[1 + T(\alpha - f + \lambda)]\right] - (S + cDT)(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T} + cD(\alpha - f + \lambda)\left[1 - e^{-(\alpha-f+\lambda)T}\right].$$

$f(T)$ is a strictly increasing function since

$$f'(T) = hDT(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T} + (S + cDT)(\alpha - f + \lambda)^3 e^{-(\alpha-f+\lambda)T} > 0.$$

Since $f(0) = -S(\alpha - f + \lambda)^2 < 0$, and $f(\infty) = hD + cD(\alpha - f + \lambda) > 0$, there exists a unique local minimum on $[0, \infty)$.

(b) From $C(0) = C(\infty) = \infty$ and the result of (a), the optimum cycle length T^* satisfies $C'(T) = 0$. The equation $C'(T^*) = 0$ can be rewritten as Eq. (9). □

Remark 1. It is known that the unit cost c does not affect the optimal order quantity (or optimal cycle length) in the infinite planning horizon EOQ model. However, it is clear that the unit cost c affects the optimal order quantity (or optimal cycle length) em from Eq. (9). Consequently, we can obtain a better solution than that of Moon and Yun [16]. We will illustrate this using numerical examples later.

Corollary 1. (a)

$$I(T) = \frac{(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T}}{i[1 - e^{-(\alpha-f+\lambda)T}[1 + T(\alpha - f + \lambda)]] + [1 - e^{-(\alpha-f+\lambda)T}](\alpha - f + \lambda) - T(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T}}$$

is strictly decreasing in T.

(b) *The optimal cycle length increases (decreases) as ordering cost increases (decreases). It decreases (increases) as unit cost and/or demand rate increases (decreases).*

Proof. (a) Let the left-hand side of Eq. (9) as $I(T)$. It is clear that the numerator of $I(T)$ strictly decreases as T increases. The denominator of $I(T)$ is strictly increasing in T since its first derivative is $(i + \alpha - f + \lambda)(\alpha - f + \lambda)^2 T e^{-(\alpha-f+\lambda)T} > 0$ for all $T > 0$. Consequently, $I(T)$ is a strictly decreasing function of T.

(b) The optimal cycle length is an intersection of $I(T)$, which is strictly decreasing, and cD/S. The results follow directly from the behavior of the optimal cycle length as cD/S changes. □

From Proposition 2 and Corollary 1, the optimal cycle length can be easily obtained by the following algorithm which requires only a line search. Then, an optimal lot size is computed using $Q^* = DT^*$.

Algorithm

Step 0. Start from $T = \sqrt{2S/cD(i + \lambda - f)}$ (an optimal cycle length for the basic EOQ modified for inflation and with the carrying charge rate approximately adjusted for the risk of obsolescence).

$$T_{\text{low}} = 0, \; T_{\text{high}} \gg T.$$

Step 1. Compute

$$I(T) = \frac{(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T}}{i[1 - e^{-(\alpha-f+\lambda)T}[1 + T(\alpha - f + \lambda)]] + [1 - e^{-(\alpha-f+\lambda)T}](\alpha - f + \lambda) - T(\alpha - f + \lambda)^2 e^{-(\alpha-f+\lambda)T}}.$$

Step 2.
If $I(T) > cD/S$, increase T and go to Step 1 (e.g. $T_{\text{low}} = T$, $T = (T_{\text{low}} + T_{\text{high}})/2$.)
If $I(T) < cD/S$, decrease T and go to Step 1 (e.g. $T_{\text{high}} = T$, $T = (T_{\text{low}} + T_{\text{high}})/2$.)
If $I(T) = cD/S$, stop. $T^* = T$ and $Q^* = DT^*$.

Remark 2. Note that we can develop a more sophisticated search algorithm similar to that of Chung and Lin [6]. However, it only takes less than a second to find an optimal solution on an IBM Pentium II personal computer with 266 MHz clock speed. Consequently, the above algorithm based on a simple line search is sufficient enough to be used in practice.

2.2. Normal distribution case

If the planning horizon p follows a normal distribution with mean μ and variance σ^2, $C(T)$ can be represented as follows:

Proposition 3.

$$\begin{aligned} C(T) = \sum_{k=0}^{\infty} \Bigg\{ \Bigg[(S + cTD)\left(\frac{1 - e^{-(\alpha-f)(k+1)T}}{1 - e^{-(\alpha-f)T}}\right) + \frac{hD}{\alpha - f}\left(T + \frac{1}{\alpha - f}(e^{-(\alpha-f)T} - 1)\right)\left(\frac{1 - e^{-(\alpha-f)kT}}{1 - e^{-(\alpha-f)T}}\right) \\ + \frac{hDe^{-(\alpha-f)kT}}{(\alpha - f)}\left(T - \frac{1}{\alpha - f}\right)\Bigg]\left[\Phi\left(\frac{(k+1)T - \mu}{\sigma}\right) - \Phi\left(\frac{kT - \mu}{\sigma}\right)\right] \\ + \frac{hD}{\alpha - f} e^{\frac{(\alpha-f)^2\sigma^2 - 2(\alpha-f)\mu}{2}}\Bigg[\sigma\left[\phi\left(\frac{kT - \mu + (\alpha - f)\sigma^2}{\sigma}\right) - \phi\left(\frac{(k+1)T - \mu + (\alpha - f)\sigma^2}{\sigma}\right)\right] \\ + \left(\frac{1}{\alpha - f} - (k+1)T + \mu - (\alpha - f)\sigma^2\right)\Bigg[\Phi\left(\frac{(k+1)T - \mu + (\alpha - f)\sigma^2}{\sigma}\right) \\ - \Phi\left(\frac{kT - \mu + (\alpha - f)\sigma^2}{\sigma}\right)\Bigg]\Bigg]\Bigg\}, \end{aligned}$$

where ϕ and Φ denote the probability density function and the cumulative distribution function of the standard normal distribution, respectively.

Proof. First, we derive the following equations:

$$\int_{kT}^{(k+1)T} f(p)\,dp = \Phi\left(\frac{(k+1)T - \mu}{\sigma}\right) - \Phi\left(\frac{kT - \mu}{\sigma}\right),$$

$$\int_{kT}^{(k+1)T} e^{-(\alpha-f)p} f(p)\,dp = e^{[(\alpha-f)^2\sigma^2-2(\alpha-f)\mu]/2}\left[\Phi\left(\frac{(k+1)T-\mu+(\alpha-f)\sigma^2}{\sigma}\right) - \Phi\left(\frac{kT-\mu+(\alpha-f)\sigma^2}{\sigma}\right)\right],$$

$$\int_{kT}^{(k+1)T} p e^{-(\alpha-f)p} f(p)\,dp$$
$$= e^{[(\alpha-f)^2\sigma^2-2(\alpha-f)\mu]/2}\left[\sigma\left[\phi\left(\frac{kT-\mu+(\alpha-f)\sigma^2}{\sigma}\right) - \phi\left(\frac{(k+1)T-\mu+(\alpha-f)\sigma^2}{\sigma}\right)\right] + \left(\mu-(\alpha-f)\sigma^2\right)\left[\Phi\left(\frac{(k+1)T-\mu+(\alpha-f)\sigma^2}{\sigma}\right) - \Phi\left(\frac{kT-\mu+(\alpha-f)\sigma^2}{\sigma}\right)\right]\right].$$

By substituting (4) and (6) into (7), and simplifying the expression using the above equations, we can obtain $C(T)$. □

Since we cannot obtain an explicit expression for $C(T)$, we use an approximation scheme as follows: The idea is to obtain an upper bound on k for a given T so that the summation in $C(T)$ can be computed. If we use

$$\left\lfloor \frac{\mu + 3.1\sigma}{T} \right\rfloor \tag{10}$$

where $\lfloor x \rfloor$ denotes a largest integer which is less than or equal to x, then the probability that p is larger than kT is less than 0.1%. Using this upper bound on k, we can compute $C(T)$. Thus, we can find an approximate optimal solution by varying T and compare $C(T)$ values.

Remark 3. We may use another approximation scheme. The expressions in $C(T)$ become smaller and smaller for larger k beyond $(k+1)T = \mu$. A bound on k can be determined on the residual of $C(T)$. For example, we can sum up the expressions in $C(T)$ until the present value of the kth cycle becomes less than, say $\$\Delta$. The smaller the value of Δ we use, the greater the precision and the greater the computational time. In the next section, we will compare the two approximation schemes.

3. Computational results

Let the cycle length determined from the basic EOQ model modified for inflation be T^{D1} (see Chapter 5 in [20]).

$$T^{D1} = \sqrt{\frac{2S}{cD(i-f)}}. \tag{11}$$

Masters [15] shows that the following reorder interval is optimal when we ignore the time-value of money and the product life cycle is a random variable which follows an exponential distribution with parameter λ:

$$T = \sqrt{\frac{2S}{cD(i+\lambda)}}. \tag{12}$$

If we modify (12) for inflationary situation, we obtain

$$T^{D2} = \sqrt{\frac{2S}{cD(i+\lambda-f)}}. \tag{13}$$

Denote the cycle length determined from the algorithm of Moon and Yun [16] to be T^0. It means that T^0 is an optimal cycle length ignoring the effects of inflation and the unit cost. Let T^* be the optimal cycle length from our algorithm in which inflation and the unit cost have been properly considered. The cost penalties associated with using T^{D1} (obtained from the basic EOQ model ignoring the time-value of money), T^{D2} (modified from the formula in [15]), and T^0 (that ignores inflation and the unit cost) can be found from the following equations, respectively:

$$\frac{C(T^{D1})-C(T^*)}{C(T^*)} \times 100,$$

$$\frac{C(T^{D2})-C(T^*)}{C(T^*)} \times 100,$$

$$\frac{C(T^0)-C(T^*)}{C(T^*)} \times 100.$$

We solve several examples to see the cost savings of using the proper value of cycle length in the presence of inflation.

Case 1 shows the basic example. If the carrying charge rate is increased from 0.3 to 0.45 while all other parameters are fixed, then an optimum cycle length is decreased to 0.0966 yr for the exponential distribution (see Case 2 of Table 1) and 0.1127 for the normal distribution (see Case 2 of Table 2). If the

Table 1
Computational results for main examples (exponential distribution case)

		Case 1[a]	Case 2[b]	Case 3[c]	Case 4[d]	Case 5[e]	Case 6[f]
Cycle length	T^{D1}	0.2236	0.1690	0.3162	0.2582	0.1581	0.2236
	T^{D2}	0.1195	0.1085	0.1690	0.1054	0.0845	0.0913
	T^0	0.1788	0.1465	0.2507	0.1788	0.1272	0.1761
	T^*	0.1043	0.0966	0.1469	0.0905	0.0740	0.0832
Expected total present cost	$C(T^{D1})$	$18,779	$18,689	$19,692	$27,997	$36,295	$10,801
	$C(T^{D2})$	$18,296	$18,420	$18,993	$26,881	$35,623	$10,205
	$C(T^0)$	$18,523	$18,562	$19,310	$27,311	$35,946	$10,532
	$C(T^*)$	$18,281	$18,408	$18,670	$26,859	$35,603	$10,200
Cost saving	$\frac{C(T^{D1})-C(T^*)}{C(T^*)}$	2.72%	1.53%	3.81%	4.24%	1.94%	5.90%
	$\frac{C(T^{D2})-C(T^*)}{C(T^*)}$	0.08%	0.06%	0.12%	0.08%	0.06%	0.05%
	$\frac{C(T^0)-C(T^*)}{C(T^*)}$	1.32%	0.83%	1.79%	1.68%	0.96%	3.25%

[a] Case 1. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2$/yr, $f = 0.1$/yr, $\lambda = 0.5$/yr.
[b] Case 2. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.45\$/\$/\text{yr}$, $\alpha = 0.2$/yr, $f = 0.1$/yr, $\lambda = 0.5$/yr.
[c] Case 3. Computational results for $D = 1000$ units/yr, $S = \$100$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2$/yr, $f = 0.1$/yr, $\lambda = 0.5$/yr.
[d] Case 4. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$15$/unit, $i = 0.2\$/\$/\text{yr}$, $\alpha = 0.2$/yr, $f = 0.1$/yr, $\lambda = 0.5$/yr.
[e] Case 5. Computational results for $D = 2000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2$/yr, $f = 0.1$/yr, $\lambda = 0.5$/yr.
[f] Case 6. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2$/yr, $f = 0.1$/yr, $\lambda = 1$/yr.

Table 2
Computational results for main examples (normal distribution case)

		Case 1[a]	Case 2[b]	Case 3[c]	Case 4[d]	Case 5[e]
Cycle length	T^{D1}	0.2236	0.1690	0.3162	0.2582	0.1581
	T^0	0.1821	0.1480	0.2536	0.1821	0.1291
	T^*	0.1291	0.1127	0.1821	0.1127	0.0899
Expected total present cost	$C(T^{D1})$	\$35,571	\$35,690	\$36,845	\$52,778	\$69,365
	$C(T^0)$	\$35,312	\$35,542	\$36,444	\$52,059	\$69,013
	$C(T^*)$	\$35,150	\$35,447	\$36,232	\$51,743	\$68,783
Cost saving	$\frac{C(T^{D1})-C(T^*)}{C(T^*)}$	1.20%	0.69%	1.69%	2.00%	0.85%
	$\frac{C(T^0)-C(T^*)}{C(T^*)}$	0.46%	0.27%	0.59%	0.61%	0.33%

[a] Case 1. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 4$ yr, $\sigma^2 = 1$ yr^2.
[b] Case 2. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.45\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 4$ yr, $\sigma^2 = 1$ yr^2.
[c] Case 3. Computational results for $D = 1000$ units/yr, $S = \$100$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 4$ yr, $\sigma^2 = 1$ yr^2.
[d] Case 4. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$15$/unit, $i = 0.2\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 4$ yr, $\sigma^2 = 1$ yr^2.
[e] Case 5. Computational results for $D = 2000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 4$ yr, $\sigma^2 = 1$ yr^2.

ordering cost is increased from \$50 to \$100 while all other parameters are fixed, then an optimum cycle length is increased to 0.1469 yr for the exponential distribution (see Case 3 of Table 1) and 0.1821 yr for the normal distribution (see Case 3 of Table 2). If the unit cost is increased from \$10 to \$15 and the carrying charge rate is reduced to 0.2 (in order to keep h fixed) while all other parameters are fixed, then an optimum cycle length is decreased to 0.0905 yr for the exponential distribution (see Case 4 of Table 1) and 0.1127 yr for the normal distribution (see Case 4 of Table 2). If the demand rate is increased from 1000 to 2000 units/yr while all other parameters are fixed, then an optimum cycle length is decreased to 0.0740 yr for the exponential distribution (see Case 5 of Table 1) and 0.0899 yr for the normal distribution (see Case 5 of Table 2). If the mean of life cycle is reduced to 1 yr while all other parameters are fixed, then an optimum cycle length is decreased to 0.0832 yr for the exponential distribution (see Case 6 of Table 1). All the behaviors of the optimal cycle length conform to the result of Corollary 1. In addition, it has been shown that T^{D2} works extremely well for exponential distribution which justifies the work of Masters [15].

To compare the two approximation schemes in Remark 3, we solve Case 1 in Table 2 under the each approximation scheme. Under the scheme in (10), $T^* = 0.1291$ with the objective value of \$35,150. The summation has been computed up to 50 cycles. Meanwhile, if we sum up until the present value of the kth cycle becomes less than \$1, we get $T^* = 0.1281$ with the objective value of \$35,615. The summation has been computed up to 62 cycles.

We solve supplementary examples to compare the optimal cycle lengths when we use the same mean (that is, $\lambda = 1$ for the exponential distribution and $\mu = 1$ for the normal distribution) for the two distributions (see Tables 3 and 4). We conjecture that the optimal cycle length for exponential distribution is shorter than that for normal distribution. This conjecture is based on the fact that the exponential distribution is skewed to left. Among four out of five cases, the optimal cycle length for exponential distribution is shorter than that of normal distribution. If we use the same variance in addition to the same mean, we think that the optimal cycle length for exponential distribution will always be shorter than that for the normal distribution.

Table 3
Computational results for supplementary examples (exponential distribution case)

		Case 1[a]	Case 2[b]	Case 3[c]	Case 4[d]	Case 5[e]
Cycle length	T^{D1}	0.2236	0.1690	0.3162	0.2582	0.1581
	T^{D2}	0.0913	0.0861	0.1291	0.0778	0.0645
	T^0	0.1761	0.1448	0.2455	0.1761	0.1259
	T^*	0.0832	0.0792	0.1170	0.0707	0.0591
Expected total present cost	$C(T^{D1})$	\$10,801	\$10,613	\$11,560	\$16,235	\$20,566
	$C(T^{D2})$	\$10,205	\$10,260	\$10,687	\$14,945	\$19,743
	$C(T^0)$	\$10,532	\$10,478	\$11,146	\$15,532	\$20,207
	$C(T^*)$	\$10,200	\$10,256	\$10,679	\$14,940	\$19,737
Cost saving	$\frac{C(T^{D1})-C(T^*)}{C(T^*)}$	5.90%	3.48%	8.25%	8.67%	4.20%
	$\frac{C(T^{D2})-C(T^*)}{C(T^*)}$	0.05%	0.04%	0.07%	0.04%	0.03%
	$\frac{C(T^0)-C(T^*)}{C(T^*)}$	3.25%	2.16%	4.37%	3.97%	2.39%

[a] Case 1. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\lambda = 1/\text{yr}$.
[b] Case 2. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.45\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\lambda = 1/\text{yr}$.
[c] Case 3. Computational results for $D = 1000$ units/yr, $S = \$100$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\lambda = 1/\text{yr}$.
[d] Case 4. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$15$/unit, $i = 0.2\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\lambda = 1/\text{yr}$.
[e] Case 5. Computational results for $D = 2000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\lambda = 1/\text{yr}$.

Table 4
Computational results for supplementary examples (normal distribution case)

		Case 1[a]	Case 2[b]	Case 3[c]	Case 4[d]	Case 5[e]
Cycle length	T^{D1}	0.2236	0.1690	0.3162	0.2582	0.1581
	T^0	0.1799	0.1420	0.2453	0.1799	0.1285
	T^*	0.0871	0.0818	0.1227	0.0730	0.0600
Expected total present cost	$C(T^{D1})$	\$11,103	\$10,925	\$11,811	\$16,646	\$21,210
	$C(T^0)$	\$10,854	\$10,785	\$11,421	\$16,007	\$20,872
	$C(T^*)$	\$10,544	\$10,604	\$11,021	\$15,452	\$20,425
Cost saving	$\frac{C(T^{D1})-C(T^*)}{C(T^*)}$	5.30%	3.03%	7.17%	7.73%	3.85%
	$\frac{C(T^0)-C(T^*)}{C(T^*)}$	2.94%	1.71%	3.63%	3.59%	2.19%

[a] Case 1. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 1$ yr, $\sigma^2 = 0.3\ \text{yr}^2$.
[b] Case 2. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.45\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 1$ yr, $\sigma^2 = 0.3\ \text{yr}^2$.
[c] Case 3. Computational results for $D = 1000$ units/yr, $S = \$100$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 1$ yr, $\sigma^2 = 0.3\ \text{yr}^2$.
[d] Case 4. Computational results for $D = 1000$ units/yr, $S = \$50$, $c = \$15$/unit, $i = 0.2\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 1$ yr, $\sigma^2 = 0.3\ \text{yr}^2$.
[e] Case 5. Computational results for $D = 2000$ units/yr, $S = \$50$, $c = \$10$/unit, $i = 0.3\$/\$/\text{yr}$, $\alpha = 0.2/\text{yr}$, $f = 0.1/\text{yr}$, $\mu = 1$ yr, $\sigma^2 = 0.3\ \text{yr}^2$.

4. Simulation modeling

To complement the analytical modeling in Section 2, we develop a simulation model which can be used for any distribution. The simulation program has been coded using GAUSS [1]. The outline of the simulation algorithm is as follows:

Simulation algorithm

Step 1. Start from a T.
Repeat Steps 2–4 number of replications given, say 500.
Step 2. Generate a random product life cycle p which follows a specific distribution.
Step 3. Compute $k = \lfloor p/T \rfloor$. This is the number of cycles for the product life cycle using T.
Step 4. Compute $C(T)$ using Eqs. (4) and (6) with the k value computed in Step 3.
Step 5. Compute the average $C(T)$ for a given T. Go to Step 2 after increasing T by Δ, say 0.0001 yr. Repeat until T reaches a high value which is not appropriate for reorder interval, say 1 yr.

To verify whether the above simulation algorithm works well, we first run it with exponential and normal distributions for case 1 in Section 3. The detailed results are shown in Table 5. We set the number of replications to 500, and $\Delta = 0.0001$ yr. The solutions from the simulation are very close to the analytical solutions. The small deviations might be caused from slight deviations of the mean and variance of the generated random variables from the specified mean and variance. The mean and variance of the 500 random product life cycle generated for the exponential distribution are 2.0161 yr and 4.0433 yr^2, respectively. Note that the mean and the variance have been set to 2 yr and 4 yr^2, respectively.

In many cases, obsolescence may arise in a more orderly fashion, perhaps due to a more regular product life cycle or due to a form of item phase-out or planned obsolescence [15]. Masters [15] suggests that the product life cycle might better be represented by another distribution such as the lognormal. In Table 6, we summarize the simulation results for normal, lognormal, and gamma under the same mean and variance. The data used for Cases 1–3 in Table 2 have been used for this simulation. As expected, the optimal cycle lengths for lognormal and gamma are shorter than that for normal distribution. This result is due to the skewness of the lognormal and gamma distributions. Let T^F be the optimal cycle length when the true distribution is F (for example, N = Normal, L = Lognormal, G = Gamma), and $C^F(T^F)$ be the corresponding objective value. Suppose we use T^{N} (optimal cycle length for normal distribution) when the true distribution is Lognormal, the associated cost penalty is as follows:

Table 5
Comparison of simulation results with analytical results

Distribution	Analytical result		Simulation result	
	T^*	$C(T^*)$	T^*	$C(T^*)$
Exponential	0.1043	$18,281	0.1040	$18,387
Normal	0.1291	$35,150	0.1301	$34,822

Table 6
Simulation results for several different distributions

Case	Distribution	T^F	$C^F(T^F)$	$C^F(T^{\mathrm{N}})$	$\frac{C^F(T^{\mathrm{N}})-C^F(T^F)}{C^F(T^F)}$
1	Normal	0.1301	$34,822	–	–
	Lognormal	0.1257	$35,511	$35,524	0.04%
	Gamma	0.1285	$35,079	$35,136	0.16%
2	Normal	0.1127	$35,514	–	–
	Lognormal	0.1086	$35,992	$36,009	0.05%
	Gamma	0.1113	$35,482	$35,520	0.11%
3	Normal	0.1829	$35,728	–	–
	Lognormal	0.1788	$35,611	$35,651	0.11%
	Gamma	0.1799	$36,438	$36,495	0.16%

$$\frac{C^{\mathrm{L}}(T^{\mathrm{N}}) - C^{\mathrm{L}}(T^{\mathrm{L}})}{C^{\mathrm{L}}(T^{\mathrm{L}})}.$$

For all the cases we have tested, these values are very small (see Table 6). Thus, it can be an additional justification for using normal distribution as a product life cycle.

5. Concluding remarks

As pointed out by Silver [19] in his review, if the quantitative models are to be more useful as aids for managerial decision-making, they must represent and formulate more realistic problems. This paper has been motivated by the need for such problem formulation, and we have investigated the effect of inflation and time-value of money in an EOQ model with a random product life cycle. It is worthwhile to note that the optimal cycle length may be very insensitive to inflation in the presence of discounts because the breakpoint value is often the best order quantity over a wide range of parameter values [8].

Park and Son [17] have applied the DCF approach to four classical inventory models including a basic EOQ model, an Economic Production Quantity (EPQ) model, an EOQ model with shortages, and an EPQ model with shortages. In this study, we have only obtained the results for a basic EOQ model. Product life cycles have been assumed to be random variables which follow any appropriate distribution. Analytical results for an exponential distribution and a normal distribution have been presented. A simulation model which can be used for any distribution has been developed to complement the analytical result. We did not present analytical formulations for other distributions such as Gamma, Lognormal, Weibull, etc.

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Appendix A. Proof of Proposition 1

$$\begin{aligned} C(T) = {} & \frac{S + cDT}{1 - \mathrm{e}^{-(\alpha-f)T}} \sum_{k=0}^{\infty} \left[1 - \mathrm{e}^{-(\alpha-f)(k+1)T}\right] \int_{kT}^{(k+1)T} \lambda \mathrm{e}^{-\lambda p}\, \mathrm{d}p \\ & + \frac{hD}{(\alpha-f)(1 - \mathrm{e}^{-(\alpha-f)T})} \left[T + \frac{\mathrm{e}^{-(\alpha-f)T} - 1}{\alpha - f}\right] \sum_{k=0}^{\infty} \left[1 - \mathrm{e}^{-(\alpha-f)kT}\right] \int_{kT}^{(k+1)T} \lambda \mathrm{e}^{-\lambda p}\, \mathrm{d}p \\ & + \frac{hD}{\alpha - f}\left(T - \frac{1}{\alpha - f}\right) \sum_{k=0}^{\infty} \mathrm{e}^{-(\alpha-f)kT} \int_{kT}^{(k+1)T} \lambda \mathrm{e}^{-\lambda p}\, \mathrm{d}p \end{aligned}$$

$$+\frac{hD}{\alpha-f}\left(\frac{1}{\alpha-f}-T\right)\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\mathrm{e}^{-(\alpha-f)p}\lambda\mathrm{e}^{-\lambda p}\,\mathrm{d}p$$
$$+\frac{hD}{\alpha-f}\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}(p-kT)\mathrm{e}^{-(\alpha-f)p}\lambda\mathrm{e}^{-\lambda p}\,\mathrm{d}p.$$

$C(T)$ can be represented as follows after integrating by parts.

$$\begin{aligned}C(T)=&\frac{S+cDT}{1-\mathrm{e}^{-(\alpha-f)T}}\sum_{k=0}^{\infty}\left[1-\mathrm{e}^{-(\alpha-f)(k+1)T}\right]\left[\mathrm{e}^{-\lambda kT}-\mathrm{e}^{-\lambda(k+1)T}\right]\\&+\frac{hD}{(\alpha-f)(1-\mathrm{e}^{-(\alpha-f)T})}\left[T+\frac{\mathrm{e}^{-(\alpha-f)T}-1}{\alpha-f}\right]\sum_{k=0}^{\infty}\left[1-\mathrm{e}^{-(\alpha-f)kT}\right]\left[\mathrm{e}^{-\lambda kT}-\mathrm{e}^{-\lambda(k+1)T}\right]\\&+\frac{hD}{\alpha-f}\left(T-\frac{1}{\alpha-f}\right)\sum_{k=0}^{\infty}\mathrm{e}^{-(\alpha-f)kT}\left[\mathrm{e}^{-\lambda kT}-\mathrm{e}^{-\lambda(k+1)T}\right]\\&+\frac{h\lambda D}{(\alpha-f)(\alpha-f+\lambda)}\left(\frac{1}{\alpha-f}-T\right)\left(1-\mathrm{e}^{-(\alpha-f+\lambda)T}\right)\sum_{k=0}^{\infty}\mathrm{e}^{-(\alpha-f+\lambda)kT}\\&+\frac{h\lambda D}{(\alpha-f)(\alpha-f+\lambda)}\left[\frac{1-\mathrm{e}^{-(\alpha-f+\lambda)T}}{\alpha-f+\lambda}-T\mathrm{e}^{-(\alpha-f+\lambda)T}\right]\sum_{k=0}^{\infty}\mathrm{e}^{-(\alpha-f+\lambda)kT}.\end{aligned}\qquad\text{(A.1)}$$

We can simplify Eq. (A.1) using the following equations,

$$\sum_{k=0}^{\infty}\left[\mathrm{e}^{-\lambda kT}-\mathrm{e}^{-\lambda(k+1)T}\right]=1,$$

$$\sum_{k=0}^{\infty}\mathrm{e}^{-(\alpha-f+\lambda)kT}=\frac{1}{1-\mathrm{e}^{-(\alpha-f+\lambda)T}},$$

$$\sum_{k=0}^{\infty}\mathrm{e}^{-(\alpha-f)kT}\left[\mathrm{e}^{-\lambda kT}-\mathrm{e}^{-\lambda(k+1)T}\right]=\frac{1-\mathrm{e}^{-\lambda T}}{1-\mathrm{e}^{-(\alpha-f+\lambda)T}}.$$

Consequently,

$$\begin{aligned}C(T)=&\frac{S+cDT}{1-\mathrm{e}^{-(\alpha-f+\lambda)T}}+\frac{hD\left[\mathrm{e}^{-(\alpha-f+\lambda)T}+(\alpha-f)T-1\right]}{(\alpha-f)^2[1-\mathrm{e}^{-(\alpha-f+\lambda)T}]}+\frac{h\lambda D[2(\alpha-f)+\lambda]}{(\alpha-f)^2(\alpha-f+\lambda)^2}\\&-\frac{h\lambda DT}{(\alpha-f)(\alpha-f+\lambda)[1-\mathrm{e}^{-(\alpha-f+\lambda)T}]}.\qquad\square\end{aligned}$$

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