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The multi-item newsvendor problem with a budget constraint and fixed ordering costs

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This paper deals with a multi-item newsvendor problem subject to a budget constraint on the total value of the replenishment quantities. Fixed costs for non-zero replenishments have been explicitly considered. Dynamic programming procedures are presented for two situations: (i) where the end item demand distributions are assumed known (illustrated for the case of normally distributed demand) and (ii) a distribution free approach where only the first two moments of the distributions are assumed known. In addition, simple and efficient heuristic algorithms have been developed. Computational experiments show that the performance of the heuristics are excellent based on a set of test problems.

Keywords: inventory; newsvendor problem; budget; distribution free approach

Introduction

The single period stochastic inventory problem, the so-called newsvendor problem, has been widely studied in the literature. In its simplest form there is a single item subject to probabilistic demand with a known, usually continuous, distribution in the period of interest. Units must be acquired (at a constant marginal cost) prior to the demand being realised. There is a constant revenue per unit sold. Any units leftover have an associated unit salvage value. There can also be a unit penalty (above and beyond the lost profit) associated with each unit short. The objective is to select the replenishment quantity so as to maximise the expected profit. Most introductory textbooks in operations research, operations management, or inventory management present this problem and its solution.[1–3] One of the earliest expositions was by Karr and Geisler.[4]

A wide variety of extensions to the basic newsvendor problem have been studied. These include (with associated illustrative references): initial usable inventory,[1] a fixed cost associated with the replenishment,[3] quantity discounts,[5,6] quantity received not necessarily being equal to the quantity requested in the replenishment,[7,8] demand dependent on price level,[9–11] the repair kit problem where sets of items are needed to satisfy specific demands,[12,13] the possibility of recourse (a second replenishment or mark-down of merchandise after at least part of the demand is realised),[14–18] adjusting the forecasting procedure to reflect truncated demand during stockouts,[19–23] and the use of a different objective such as maximisation of the probability that at least a certain profit level is achieved.[24,25] An excellent review on the newsvendor problem has been recently provided by Khouza.[27]

Another important version, which is examined in this paper, is where there are multiple items to be acquired subject to a restriction on the total budget to be used for the acquisitions. Previous exposes on this topic[3,28–30] have ignored any fixed cost of a replenishment. In this paper we explicitly take account of this potentially important factor.

In practice, the distributional information about the demand is often limited. Sometimes all that is available are estimates of the mean and variance. There is a tendency to use the normal distribution under these conditions. However, the normal distribution does not offer the best protection against the occurrences of other distributions with the same mean and variance. Scarf[31] addressed a newsvendor problem where only the mean μ and the variance σ^2 of the demand are known without any further assumptions about the form of the distribution of the demand. Taking a conservative approach, he modeled the problem as that of finding the order quantity that maximises the expected profit against the worst possible distribution of the demand with mean μ and variance σ^2. The approach is called the minimax distribution free approach. Recently, there have been many related papers. Gallego and Moon[16] presented a very compact proof of the optimality of Scarf's ordering rules for the newsvendor problem and extended the analysis to several cases including a fixed ordering cost, multiple products, random yield, and the possibility of recourse. Moon and Gallego[32] applied the approach to

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several inventory models including both continuous and periodic review models. Moon and Choi[33] extended the model of Gallego and Moon[15] to the case that allows customers to balk when inventory level is low. Shore[34] derived explicit approximate solutions to the standard newsboy problem, to some (Q, r) models, and to a periodic review model in which the first three or four moments of the demand are known. Moon and Choi[35] applied the approach to the two- echelon stochastic production/inventory models in which assemble-to-order (ATO), assemble-to-make (ATM), and composite policies can be adopted. Hariga[36] extended the work of Moon and Choi[35] to the multi echelon case. Gallego *et al*[37] considered stochastic finite-horizon inventory models with discrete demand distributions that are incompletely specified by selected moments, percentiles, or a combination of moments and percentiles. Hariga and Ben-Daya[38] and Moon and Choi[39] independently solved the continuous review inventory problem in which lead time can be reduced by investment. Their models generalised the distribution free model of Ouyang and Wu[40] by simultaneously optimising both the order quantity and reorder point.

In the following section we present the notation, assumptions, and basic mathematical model for the case where we have a known distribution of demand for each item. A dynamic program is then developed to find an optimal solution. The associated computational effort required increases substantially with the number of items involved and with the available budget. Perhaps more important, dynamic programming is a difficult concept to explain to practitioners. Therefore, we subsequently present two heuristic algorithms that overcome both of these drawbacks. This is followed by computational experiments to test the heuristics. Then we deal with the distribution free situation where all we assume are known are the means and variances of the demand distributions. Computational tests to show the robustness of the distribution free solution are also presented. Concluding remarks are provided in the last section.

Problem formulation and optimal solution using dynamic programming

The notation to be used is as follows:

i = index for items ($i = 1, 2, \dots, m$)
A_i = fixed ordering cost associated with a replenishment of item i, in pounds
g_i = salvage value of item i, in pounds/unit
v_i = purchase cost of item i, in pounds/unit
B_i = penalty for not satisfying demand of item i, in pounds/unit
I_i = initial inventory of item i, in units
D_i = demand for end item i during the period, in units
$f_i(D_i)$ = probability density function of demand for item i
$F_i(D_i)$ = cumulative probability distribution of demand for item i
μ_i = expected value of demand for item i
σ_i = standard deviation of demand for end item i
W = budget available for allocation among the stocks of the m items, in pounds
S_i = order-up-to level for item i (decision variable)

Suppose that there are I_i units of item i at the beginning of the period. In addition, there is a significant fixed ordering cost A_i if a replenishment is made for item i. Initially, we assume that the budget is of sufficient size to permit each item to be ordered at its optimal, independent level. Let S_i be the order-up-to level of item i. Then, $S_i = I_i + Q_i$ where Q_i is the purchase quantity of item i. The expected cost for item i can be written as

$$C_i^F(S_i) = G_i^F(S_i) + A_i \mathbf{1}_{\{S_i > I_i\}}$$

where

$$\begin{aligned} G_i^F(S_i) &= v_i(S_i - I_i) - g_i \int_0^{S_i} (S_i - D_i) f_i(D_i) dD_i \\ &\quad + B_i \int_{S_i}^{\infty} (D_i - S_i) f_i(D_i) dD_i \\ &= (v_i - g_i)S_i - v_i I_i + g_i \mu_i + (B_i - g_i) E[D_i - S_i]^+ \end{aligned}$$

and $\mathbf{1}$ denotes an indicator function, and the superscript F denotes a general distribution function of demand. Then, the problem can be rewritten as

$$\min_{S_i \geqslant I_i} [A_i \mathbf{1}_{\{S_i > I_i\}} + G_i^F(S_i)]$$

For the normal distribution,

$$E[D_i - S_i]^+ = \sigma_i \phi\left(\frac{S_i - \mu_i}{\sigma_i}\right) + (\mu_i - S_i)\left[1 - \Phi\left(\frac{S_i - \mu_i}{\sigma_i}\right)\right]$$

Let S_i^* denote the unconstrained minimiser of $G_i^F(S_i)$, then

$$F_i(S_i^*) = \frac{B_i - v_i}{B_i - g_i}$$

By employing an argument similar to that used for the classical newsvendor model with a fixed ordering cost,[1] a replenishment should be made if $I_i < S_i^*$ and $G_i^F(I_i) > A_i + G_i^F(S_i^*)$. Since $G_i^F(S_i)$ is strictly convex and is not bounded from above, there exists a unique $s_i^* < S_i^*$ satisfying

$$G_i^F(s_i^*) = A_i + G_i^F(S_i^*)$$

Therefore, the ordering rule becomes as follows: Order up to S_i^* units if $I_i < s_i^*$, and do not order otherwise.

Now we assume that there is a restricted budget W. We develop a dynamic program to find an optimal solution. We first need to check the following steps.

Step 1 Solve a single period model with a fixed ordering cost for each item separately. Let $\mathscr{P}$ be the set of items for which it is profitable to order. That is,

$$\mathscr{P} = \{i \text{ such that } I_i < s_i^*\}$$

Step 2 If

$$\sum_{i \in \mathscr{P}} v_i(S_i^* - I_i) \leqslant W,$$

(s_i^*, S_i^*)s are optimal and we stop here. Otherwise, we go to Step 3.

Step 3 Note that we only need to consider items which belong to $\mathscr{P}$. Without loss of generality, renumber the items in $\mathscr{P}$ as 1 to m. The dynamic programming formulation consists of the following three elements.

(i) Stage i is represented by item i, $i = 1, 2, \ldots, m$.
(ii) State x_i at stage i is the amount of budget left to be assigned to items i, $i+1, \ldots, m$.
(iii) Alternative Q_i at stage i is the number of units of item i purchased. The value of Q_i may be as small as 0 and as large as $\lfloor x_i/v_i \rfloor$ where $\lfloor y \rfloor$ represents the smallest integer which is less than or equal to y (Note that in order to avoid a proliferation of possible states we are restricting the order quantities to integer values, despite the possibility of a continuous demand distribution).

Let $f_i(x_i)$ = optimal value of stages i, $i+1, \ldots, m$, given the state x_i. Let $Q_i^*(x_i)$ be the optimal order quantity at stage i given state x_i. Then the backward recursive equations are as follows:

$$f_m(x_m) = \min_{0 \leqslant Q_m \leqslant \lfloor x_m/v_m \rfloor} \{A_m \mathbf{1}_{\{Q_m > 0\}} + G_m^F(I_m + Q_m)\} \qquad x_m = 0, \ldots, W$$

$$f_i(x_i) = \min_{0 \leqslant Q_i \leqslant \lfloor x_i/v_i \rfloor} \{A_i \mathbf{1}_{\{Q_i > 0\}} + G_i^F(I_i + Q_i) + f_{i+1}(x_i - v_i Q_i)\} \qquad x_i = 0, \ldots, W, \quad i = 2, \ldots, m-1$$

$$f_1(W) = \min_{0 \leqslant Q_1 \leqslant \lfloor W/v_1 \rfloor} \{A_1 \mathbf{1}_{\{Q_1 > 0\}} + G_1^F(I_1 + Q_1) + f_2(W - v_1 Q_1)\}$$

Example 1 We assume independent normal distributions and that the budget is £10,000. The detailed data for this example are given in Table 1.

First we compute independent (s, S) values:

$$(s_1, S_1) = (34, 86), \quad (s_2, S_2) = (70, 89),$$
$$(s_3, S_3) = (23, 108), \quad (s_4, S_4) = (198, 230)$$

Table 1 Data for the example (W = £10,000)

Item	v_i	A_i	g_i	B_i	I_i	x_i	σ_i
Item 1	£35	£500	£15	£50	30	90	25
Item 2	£20	£100	£10	£40	10	80	20
Item 3	£28	£300	£15	£32	30	120	17
Item 4	£40	£200	£10	£70	20	230	60

From the above information, we compute $\mathscr{P}$ and the unconstrained optimal solution with the following results:

$$\mathscr{P} = \{1, 2, 4\}, \qquad Q_1 = S_1 - I_1 = 56,$$
$$Q_2 = S_2 - I_2 = 79,$$
$$Q_3 = 0, \qquad Q_4 = S_4 - I_4 = 210$$
$$\text{Total cost} = £17{,}577.93$$

Since

$\sum_{i \in \mathscr{P}} v_i Q_i = £11{,}940 > £10{,}000$, this solution is not feasible.

Let $(Q_1^N, \ldots, Q_m^N)$ be the optimal solution for the normal distribution. If we apply the DP, we obtain the following results:

$$(Q_1^N, Q_2^N, Q_3^N, Q_4^N) = (0, 79, 0, 210)$$

with a total cost of £17,636.77.

The restricted budget has increased the overall cost by £58.84 (or approximately 0.3%).

Two heuristic approaches

We develop here two heuristic algorithms since, as mentioned earlier, it would take too much time to solve the dynamic program if m or W become quite large. Moreover, a heuristic solution is typically much easier for a practitioner to understand than is dynamic programming. First we develop a heuristic approach based on a marginal (or greedy) allocation algorithm.[41] At each step the algorithm reduces the budget with the least increase of the objective value per unit reduction of the budget used. This is continued until a feasible solution is obtained. Any budget remaining (due to an undershoot) is filled in a reverse greedy fashion. We call this approach the marginal allocation heuristic. It is summarised as follows:

Marginal allocation heuristic

Step 1 Solve a single period model with a fixed ordering cost for each item separately. Let $\mathscr{P}$ be the set of items for which it is profitable to order. That is,

$$\mathscr{P} = \{i \text{ such that } I_i < s_i^*\}$$

Step 2 If

$$\sum_{i \in \mathscr{P}} v_i(S_i^* - I_i) \leqslant W,$$

(s_i^*, S_i^*)s are optimal and we stop here. Otherwise, we go to Step 3.

Step 3 From now on we only need to consider items which belong to $\mathscr{P}$. Because of the fixed cost per renplenishment we consider two types of greedy options. The first involves a unit reduction in the replenishment quantity of an item, whereas the second considers elimination of the entire replenishment quantity (thus saving the fixed replenishment cost). For the first option choose the item $i1$ with the least increase of cost per budget pound saved:

$$\min_i \left\{ \frac{C_i^F(S_i^* - 1) - C_i^F(S_i^*)}{v_i} \right\} \quad (1)$$

For the second option choose the item $i2$ with the least increase of cost per budget pound saved by reducing Q_i to 0:

$$\min_i \left\{ \frac{C_i^F(I_i) - C_i^F(S_i^*)}{v_i(S_i^* - I_i)} \right\} \quad (2)$$

If (1) $\leqslant$ (2), reduce S_{i1}^* to $S_{i1}^* - 1$, that is Q_{i1}^* to $Q_{i1}^* - 1$. If (1) > (2), reduce S_{i2}^* to I_{i2}, that is Q_{i2}^* to 0. Repeat Step 3 until we satisfy

$$\sum_{i \in \mathscr{P}} v_i(S_i^* - I_i) \leqslant W$$

Step 4 If $\sum_{i \in \mathscr{P}}(S_i^* - I_i) < W$, increase S_i^* with the largest decrease of cost per pound increase in the budget used:

$$\max_i \left\{ \frac{C_i^F(S_i^*) - C_i^F(S_i^* + 1)}{v_i} \right\} \quad (3)$$

Repeat this step as long as we can satisfy the budget constraint.

Now we develop another heuristic called the two stages heuristic. The heuristic attempts to proportionally assign the budget to the items which belong to $\mathscr{P}$. Note that we can easily obtain a counter example which shows that it is preferable to not order certain items which belong to $\mathscr{P}$. This implies that the problem has a combinatorial characteristic, and makes us strongly conjecture that the problem is NP-hard.[42]

Two stages heuristic

Stage 1 Solve a single period model with a fixed ordering cost for each item separately. Let $\mathscr{P}$ be the set of items for which it is profitable to purchase something. That is,

$$\mathscr{P} = \{i \text{ such that } I_i < s_i^*\}$$

Let S_i^* be the order-up-to level for item i for the unconstrained problem. If

$$\sum_{i \in \mathscr{P}} v_i(S_i^* - I_i) \leqslant W,$$

$(s_i^*, S_i^*)s$ are optimal and we stop here. Otherwise, we go to Stage 2.

Stage 2 Without loss of generality, we renumber the items in $\mathscr{P}$ as 1 to m, then solve the following problem.

$$\min_{S_1,\ldots,S_m} \sum_{i=1}^{m} \left[v_i(S_i - I_i) - g_i \int_0^{S_i} (S_i - D_i) f_i(D_i) dD_i + B_i \int_{S_i}^{\infty} (D_i - S_i) f_i(D_i) dD_i + A_i \right]$$

subject to

$$\sum_{i=1}^{m} v_i(S_i - I_i) \leqslant W \quad (4)$$

Let λ be the Lagrangian multiplier associated with the budget constraint. By computing $\partial L / \partial S_i$ after forming the Lagrangian function, $L(S_1, \ldots, S_m, \lambda)$, we obtain the following equations:

$$F_i(S_i) = \frac{B_i - (\lambda + 1)v_i}{B_i - g_i} \quad \forall i$$

The stage 2 is to find the smallest nonnegative λ such that $S_i(\lambda)$s satisfy (4). A simple line search algorithm can be used to find the optimal value of λ.

Example 2 We solve the same example as earlier, but this time using the two heuristic algorithms.

Marginal allocation heuristic

Note that Step 1 and Step 2 are exactly the same as for the DP. Let $(Q_1^M, \ldots, Q_m^M)$ be the marginal allocation heuristic solution. Then we find

$$(Q_1^M, Q_2^M, Q_3^M, Q_4^M) = (36, 70, 0, 183)$$

with a total cost of £17,837.19

Two stages heuristic

Let $(Q_1^T, \ldots, Q_m^T)$ be the two stages heuristic solution. If we solve Stage 2 using a line search algorithm, we obtain $\lambda = 0.2572$ with the following solution.

$$(Q_1^T, Q_2^T, Q_3^T, Q_4^T) = (36, 70, 0, 183)$$

with a total cost of £17,837.19

Note that the λ value implies that increasing the budget beyond £10,000 will approximately save £0.257 per pound increase in budget when this heuristic is used.

Computational results

In order to investigate the performances of the two heuristics developed in the previous section, 25 test problem instances were generated randomly from uniform distributions on the given intervals as shown below:

$m \sim DU(5, 10)$
where DU denotes a discrete uniform distribution

$W \sim U(0.5, 0.8) \times \sum_{i=1}^{m} v_i(S_i^U - I_i)$ where S_i^U denotes an unconstrained order-up-to level for item i

$$v \sim U(30, 50), \quad g \sim U(0.2, 0.5) \times v,$$
$$B \sim U(1.5, 2.0) \times v, \quad A \sim U(50, 300)$$
$$\mu \sim U(50, 150), \quad \sigma \sim U(0.1, 0.3) \times \mu,$$
$$I \sim U(0.1, 0.5) \times \mu$$

We did not do tests with more than 10 items because of the work space limit of the GAUSS[43] software which has been used to find the optimal solution. The computational results are summarised in Table 2. Here Heu1, Heu2, and OPT represent the objective value of the marginal allocation heuristic, the objective value of the two stages heuristic, and the optimal objective value using DP. The performances of both heuristics are extraordinarily good, particularly when one recognises that in practice it is difficult to estimate the values of the parameters, such as the parameters of the demand distribution for each item,[21] needed to obtain the optimal solution.

Distribution free model

We now consider the distribution free approach, that is we make no assumption on the distribution F of D_j other than saying that it belongs to the class $\mathscr{F}$ of cumulative distribution functions with mean μ_j and variance σ_j^2. Since the distribution F of D is unknown we want to minimise the total expected cost against the worst possible distribution in $\mathscr{F}$. The distribution free approach for this model involves finding the most unfavorable distribution in F for each $(S_1, \ldots, S_m)$. Our problem is to solve:

$$\min_{S_1,\ldots,S_m} \max_{F \in \mathscr{F}} \sum_{i=1}^{m} C_i^F(S_i)$$

To this end, we need to use the following proposition as in Gallego and Moon.[16]

Table 2 Results of computational tests

	Minimum ratio	*Maximum ratio*	*Mean ratio*
Heu1/OPT	1.0000	1.0130	1.0049
Heu2/OPT	1.0000	1.0203	1.0045

Proposition 1 *For any $F \in \mathscr{F}$*

$$E[D_i - S_i]^+ \leqslant \tfrac{1}{2}\{\sqrt{\sigma_i^2 + (S_i - \mu_i)^2} - (S_i - \mu_i)\}$$

Moreover, the upper bound is tight. In other words, we can always find a distribution in which the above bound is satisfied with equality for every S_i.

Using the above proposition, our cost function becomes

$$\begin{aligned}\sum_{i=1}^{m} C_i^W(S_i) &= \sum_{i=1}^{m}\{(v_i - g_i)S_i - v_iI_i + g_i\mu_i \\ &\quad + (B_i - g_i)E[D_i - S_i]^+ + A_i\mathbf{1}_{\{S_i > I_i\}}\} \\ &= \sum_{i=1}^{m}\Bigg\{(v_i - g_i)S_i - v_iI_i + g_i\mu_i \\ &\quad + \frac{B_i - g_i}{2}\left[\sqrt{\sigma_i^2 + (S_i - \mu_i)^2} - (S_i - \mu_i)\right] \\ &\quad + A_i\mathbf{1}_{\{S_i > i_i\}}\Bigg\} \\ &= \sum_{i=1}^{m}\{G_i^W(S_i) + A_i\mathbf{1}_{\{S_i > I_i\}}\}\end{aligned}$$

where W denotes a worst case distribution function of demand.

Let $h_i(x_i)$ = optimal values of stages $i, i+1, \ldots, m$, given the state x_i. Let $Q_i^*(x_i)$ be the optimal purchase quantity at stage i given state x_i. Then the backward recursive equations are as follows:

$$h_m(x_m) = \min_{0 \leqslant Q_m \leqslant \lfloor x_m/v_m \rfloor}\{A_m\mathbf{1}_{\{Q_m > 0\}} + G_m^W(I_m + Q_m)\} \qquad x_m = 0, \ldots, W$$

$$h_i(x_i) = \min_{0 \leqslant Q_i \leqslant \lfloor x_i/v_i \rfloor}\{A_i\mathbf{1}_{\{Q_i > 0\}} + G_i^W(I_i + Q_i) + h_{i+1}(x_i - v_iQ_i)\} \qquad x_i = 0, \ldots, W, \quad i = 2, \ldots, m-1$$

$$h_1(N) = \min_{0 \leqslant R_1 \leqslant \lfloor W/v_1 \rfloor}\{A_1\mathbf{1}_{\{Q_1 > 0\}} + G_1^W(I_1 + Q_1) + h_2(W - v_1Q_1)\}$$

Let optimal solutions for the normal distribution and the worst distribution be $(S_1^N, \ldots, S_m^N)$ and $(S_1^W, \ldots, S_m^W)$, respectively. We are interested in the Expected Value of Additional Information (EVAI). That is,

$$\sum_{i=1}^{m} C_i^N(S_i^W) - \sum_{i=1}^{m} C_i^N(S_i^N)$$

This is the largest amount that we would be willing to pay for the knowledge of the specific (normal) distribution in the absence of knowledge of the specific distribution. If this quantity is small, the use of the distribution free solution can be justified.

Example 3 We solve the same example as before to compare the distribution free solution with the solution for normal distribution.

$$(Q_1^N, Q_2^N, Q_3^N, Q_4^N) = (0, 79, 0, 210)$$
with a total cost of £17,636.77
$$(Q_1^W, Q_2^W, Q_3^W, Q_4^W) = (0, 77, 0, 210)$$
with a total cost of £18,163.87
(under the worst case distribution of demand)

We compute the Expected Value of Additional Information (EVAI) as follows.

$$\sum_{i=1}^{m} C_i^N(Q_i^W) - \sum_{i=1}^{m} C_i^N(Q_i^N) = £17{,}637.45 - £17{,}636.77 = £0.68$$

$$\text{(cf.)} \quad \frac{\sum_{i=1}^m C_i^N(Q_i^W)}{\sum_{i=1}^m C_i^N(Q_i^N)} = \frac{£17{,}637.45}{£17{,}636.77} = 1.00004$$

From this example, we can conjecture that the distribution free approach is very robust. We will confirm this using the computational experiment described below.

Remark 1 We notice that the distribution free solution does not use up its budget. This is quite interesting and makes the problem much more difficult to solve. For confirmation, if we substitute the normal solution (which uses £40 more budget than the distribution free solution) into the distribution free objective function, we obtain £18,165.00 (which is larger than £18,163.87 which is the optimal objective value under the distribution free approach).

Remark 2 The two heuristics developed for the general distribution can be easily modified to be used for the distribution free case. We omit the details for brevity.

In order to further investigate the robustness of the distribution free approach, we compute the following ratio for the 25 randomly generated problems of the previous section.

$$\frac{\sum_{i=1}^m C_i^N(Q_i^W)}{\sum_{i=1}^m C_i^N(Q_i^N)}$$

We report the minimum, maximum and mean values of this ratio in Table 3. The ratios are very close to 1 which indicates that use of the distribution free solution in the absence of the specific form of the distribution function produces negligible penalties when the true distributions are normal.

Table 3 Results of computational tests

Minimum ratio	*Maximum ratio*	*Mean ratio*
1.0000	1.0053	1.0005

Conclusions

In this paper we have considered a single period newsvendor problem with multiple items subject to a budget constraint on the total value of the replenishment quantities and recognising fixed costs for non-zero replenishments. A dynamic programming formulation was presented which could be used for small scale problem instances. However, for more realistic sized problems and for easier understanding by practitioners a heuristic approach is essential. Two such approaches were developed and shown to perform excellently on a set of test problems. A distribution free analysis of the problem was also presented.

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