IIE Transactions (1998) **30**, 1009–1017

The effect of the stabilization period on the economic lot scheduling problem

ILKYEONG MOON,[1] JUHO HAHM[2] and CHUL LEE[3]

[1] *Department of Industrial Engineering, Pusan National University, Pusan 609-735, Korea*
E-mail: ikmoon@hyowon.pusan.ac.kr
[2] *Department of Industrial Engineering, Seoul National University, Seoul 151, Korea*
[3] *Manufacturing Technology Center, LG Electronics, Ltd., Changwon 641, Korea*

Received May 1996 and accepted November 1997

The Economic Lot Scheduling Problem (ELSP) is the problem of scheduling production of several items in a single facility, so that demands are met without stockouts or backorders, and the long run average inventory carrying and setup costs are minimized. One of the general assumptions in the ELSP is that the yield rates of a given manufacturing process are constant, or 100%, after setup. However, this assumption may not be true for certain manufacturing processes, in which the yield rates are quite low just after setup, and then increase over time. This period is called a stabilization period and yield rates gradually increase during this period until they reach the target rates, which are set empirically or strategically. The purpose of this paper is to clarify the effect of the stabilization period by applying the stabilization period concept to the ELSP, which has been widely applied to many production systems. In this paper, the problem is tackled in three stages: Firstly, we formulate a model and develop an algorithm, which provides a lower bound for a minimum cost. Secondly, we develop a heuristic procedure using the time-varying lot size approach. Finally, we solve a special case of the ELSP to find an upper bound using the common cycle approach.

1. Introduction

The Economic Lot Scheduling Problem (ELSP) is the problem of scheduling production of several items in a single facility, so that demands are met without stockouts or backorders, and the long run average inventory carrying and setup costs are minimized. This problem occurs in many production situations [1] including:

- Metal forming and plastics production lines (press lines, and plastic and metal extrusion machines), where each product requires a different die to be set up on the machine.
- Assembly lines, which produce several products and/or different product models (electric appliances, motor cars, etc.).
- Blending and mixing facilities (for paints, beverages, animal food, etc.), in which different products are poured into different containers.
- Weaving production lines (for textiles, carpets, etc.), on which the main products are manufactured in different colors, widths, and grades.

Typically, it is more economic to purchase one high-speed machine capable of producing a number of products, than to purchase many dedicated machines. This situation leads to the question of how one should schedule production on this high-speed machine. The issue is one of selecting both a sequence, in which the products will be manufactured, and a batch size for each product run. The issue of batching arises because the system usually incurs a setup cost and/or a setup time when the machine switches from one product to a different product. The cost may be due to cleaning or to scrap losses occurring when machine settings are adjusted for the next product. Setup times imply a downtime during which the machine cannot produce, which, in turn, implies a need to carry more inventory. This problem has attracted the attention of many researchers, partly because it features many frequently encountered scheduling problems, and simply because it seems to be difficult to solve.

In the ELSP, it is typically assumed that production and demand rates are known item-dependent constants, while setup times and setup costs are known item-dependent, but sequence-independent constants. In addition research on the ELSP has focused on cyclic schedules, i.e., schedules that are repeated periodically. Because of its nonlinearity and complexity, the ELSP is generally known as an NP-hard problem [2]. Many heuristic approaches have been developed for this problem. Basically, there are three approaches:

(1) *The common cycle approach*: This approach restricts all the products' cycle times to an equal length. Then it

finds the optimal common cycle time. This approach has the advantage of always finding a feasible schedule, and it consists of a very simple procedure. This procedure, however, gives solutions far from the lower bound in some situations [3–5].

(2) *The basic period approach*: This approach allows different cycle times for different products, but restricts each product's cycle time to be an integer multiple k of a time period called a basic period. All lots of each item are of the same size. Under this approach, it is NP-hard to find a feasible solution, given the number of production runs per cycle for each of the items. This approach, in general, gives better solutions than the Common Cycle approach. However, its main drawback is the difficulty of ensuring feasibility [2,6,7].

(3) *The time varying lot size approach*: This approach allows the lot sizes for a given product to vary over a cyclic schedule. It explicitly handles the difficulties caused by setup times and always gives a feasible schedule. This approach usually gives better solutions than the previous two approaches [8–11].

One phenomenon that occurs in the production system is that the yield rates are quite low just after setup, under the influence of the process parameters. Then, the yield rates increase over time while the process parameters are adjusted, until they reach the target rates, which are set either empirically or strategically. This period is called the *stabilization period* and the yield rates gradually increase during this period. We have observed this phenomenon for the operation of a manufacturing line for color monitors in which various kinds of panels and funnels are produced in a single facility. However, the effect of the stabilization period has not been seriously discussed in the published ELSP literature. It has usually been assumed that the yield rates of a given manufacturing process are constant, or 100%, after setup. This assumption may lead to faulty decision-making, which may result in significant additional operating costs. Recently, there have been several studies which consider the stabilization period. The yield rate is assumed to be an increasing function implying a learning effect. Hahm *et al.* [12] have applied the stabilization period concept to an economic production quantity model. Bourland *et al.* [13] solved a product cycling problem assuming that the yield rate is an increasing function of the production quantity. Urban [14] has assumed that the defect rate of the process is a function of the run length, and investigated both positive and negative learning effects in an economic production quantity model.

The purpose of this research is to clarify the effect of the stabilization period, and to develop an algorithm to find an efficient production schedule (production sequence and production run times) by applying the stabilization period concept to the Economic Lot Scheduling Problem (ELSP), which has been widely applied to many production systems. We call an ELSP with a stabilization period the Stabilization-ELSP.

The organization of this paper is as follows: Problem descriptions, assumptions, and notation are presented in the next section. In Section 3, we develop an algorithm which provides a lower bound for the Stabilization-ELSP. This is followed, in Section 4, by the development of a heuristic procedure for the Stabilization-ELSP, using the time-varying lot size approach. We solve a special case of the Stabilization-ELSP, using the common cycle approach, which gives an upper bound for the Stabilization-ELSP in Section 5. In Section 6, computational tests are performed to compare bounds with the heuristic solutions. Finally, concluding remarks in Section 7 close the paper.

2. Preliminaries

The following assumptions are used in the Stabilization-ELSP:

(1) Multiple items compete for the use of a single facility.

(2) Demand rates, production rates, setup costs, and inventory loss costs for all items are known constants.

(3) Backorders are not allowed.

(4) Costs (rework or scrap) are incurred for defective units.

(5) The non-defective yield rate function increases with time and is deterministic during the stabilization period.

(6) The target yield rates are set strategically by manufacturing process characteristics and production is continued until the given target rate is reached.

Note that the last assumption might be strong in the make-to-order production environment. The ELSP is more suitable for the make-in-advance production environment, in which the demand rates are quite stable. In addition, the fine-tuning process might be regarded as a setup process, and it is usually undesirable to stop production before the setup ends.

The following notation is used in the model:

i = item index $i = 1, 2, \ldots, m$;
p_i = constant production rate $i = 1, 2, \ldots, m$;
r_i = constant demand rate $i = 1, 2, \ldots, m$;
h_i = known inventory holding cost $i = 1, 2, \ldots, m$;
π_i = known loss cost $i = 1, 2, \ldots, m$;
K_i = known setup cost $i = 1, 2, \ldots, m$;
s_i = known setup time $i = 1, 2, \ldots, m$;
T = cycle length;
Q_i = non-defective production quantity $i = 1, 2, \ldots, m$;
$\gamma_i(t)$ = yield rate function $i = 1, 2, \ldots, m$;
G_i = target yield rate $i = 1, 2, \ldots, m$;

$I_{\max i}$ = maximum inventory level $\quad i = 1, 2, \ldots, m$;
$r_i/p_i = \rho_i \quad i = 1, 2, \ldots, m$;
$r_i/G_i p'_i = \rho'_i \quad i = 1, 2, \ldots, m$;
$I_i(t)$ = inventory level at time $t \quad i = 1, 2, \ldots, m$.

First, we define the following time periods related to the yield rate function $\gamma_i(t)$ (see Fig. 1).

t_{1_i} = elapsed time from the start of the production until the inventory level drops to zero;
t_{g_i} = elapsed time from the start of the production until the production rate reaches the target yield rate G_i (stabilization period);
$t_{2_i} = t_{g_i} - t_{1_i}$.

If the non-defective production rate exceeds the demand rate at the beginning of production, that is $\gamma_i(0)p_i \geq r_i$, t_{1_i} becomes zero. However, if the non-defective production rate does not exceed the demand rate at the beginning of production, the inventory level will be zero during the stabilization period. Consequently, t_{1_i} can be defined as follows:

$$t_{1_i} = \begin{cases} \gamma_i^{-1}(\rho_i), & \gamma_i(0) < \rho_i, \\ 0, & \gamma_i(0) \geq \rho_i. \end{cases}$$

In order to formulate the problem, we first need to derive the total inventory for item i during a cycle length T_i, and the total of defective units incurred during the production of item i.

Proposition 1. *The total inventory for item i during a cycle length T_i (denoted as I_{T_i}), and the total of defective units incurred during the production of item i (denoted as L_{T_i}), can be derived as follows:*

$$I_{T_i} = \frac{I_{\max i}^2}{2(1-\rho'_i)r_i} + B_i$$

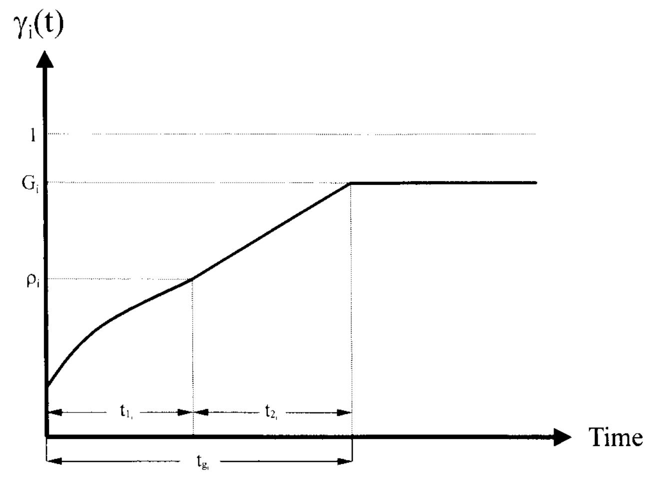

Fig. 1. Non-defective yield rate function.

$$\text{where} \quad B_i = \int_0^{t_{g_i}} I_i(t)\mathrm{d}t - \frac{1}{2}\left\{\frac{S_{1i}^2}{r_i} + \frac{S_{2i}^2}{G_i p_i - r_i}\right\}.$$

$$L_{T_i} = (G_i^{-1} - 1)Q_i + \int_0^{t_{g_i}} p_i(1 - G_i^{-1}\gamma_i(t))\mathrm{d}t.$$

Proof: The details of the proof are described in Appendix A. ∎

3. A lower bound for the stabilization-ELSP model

In order to be able to assess the performance of feasible solutions, it is desirable to have an easily calculable lower bound on the average cost. The procedure to find a lower bound can be formulated as follows: the objective function denotes the average cost (including holding, setup, and loss costs) per unit time. The capacity constraint is explicitly considered. However, the synchronization constraint, stating that no two items can be scheduled to be produced at the same time, is ignored. Consequently, the value of the program results in a lower bound on the average cost over all cyclic schedules.

Stabilization-LB

$$\mathrm{Min}_{T_1,\ldots,T_m} \sum_{i=1}^{m} \left[\frac{R_i}{T_i} + H_i T_i + U_i\right],$$

subject to

$$\sum_{i=1}^{m} \frac{S_i}{T_i} \leq \kappa,$$

$$T_i \geq \tau_i \quad i = 1, 2, \ldots, m,$$

where

$$R_i = K_i + \pi_i \int_0^{t_{g_i}} p_i\left(1 - \frac{\gamma_i(t)}{G_i}\right)\mathrm{d}t + \frac{h_i}{2r_i}\left(\frac{A_i^2}{1-\rho'_i} + 2r_i B_i\right),$$

$$H_i = \frac{1}{2}h_i(1-\rho'_i)r_i,$$

$$U_i = \pi_i r_i(G_i^{-1} - 1) - A_i h_i,$$

$$S_i = s_i + t_{g_i} - G_i^{-1}\left(\int_0^{t_{g_i}} \gamma_i(t)\mathrm{d}t\right),$$

$$\kappa = 1 - \sum_{i=1}^{m} \frac{r_i}{G_i p_i},$$

$$\tau_i = \int_0^{\max(t_{g_i}, T_{\min i})} \frac{p_i}{r_i}\gamma_i(t)\mathrm{d}t.$$

In the above program, reorder intervals (cycle times) are decision variables. The objective function and the constraint set are convex. Therefore, the optimal points of the Stabilization-LB are points which satisfy the Karush–Kuhn–Tucker (KKT) conditions. The KKT conditions are stated below. Let λ and v_i be the Lagrange multipliers corresponding to $\sum_{i=1}^{m}(S_i/T_i) \le \kappa$, $T_i \ge \tau_i$, respectively. Then the KKT conditions for program **LB** are:

$$T_i = \sqrt{\frac{R_i + \lambda S_i}{H_i - \sum_{i=1}^{m} v_i}} \quad i = 1, 2, \ldots, m. \tag{1}$$

$$\lambda \ge 0 \text{ complementary slackness (c.s.)}$$

$$\text{with } \sum_{i=1}^{m}(S_i/T_i) \le \kappa. \tag{2}$$

$$v_i \ge 0 \text{ c.s. with } T_i \ge \tau_i \quad i = 1, 2, \ldots, m. \tag{3}$$

After computing a KKT point (**T**) satisfying the above conditions, we obtain production frequencies and round them off to power-of-two integers [15]. These frequencies can then be used to obtain a production sequence, using a bin-packing heuristic [8]. The final time-varying lot sizes can be optimally selected [11]. We present an algorithm to find a KKT point satisfying (1)–(3). Note that it requires only a one-dimensional line search.

Algorithm

Step 1. (Check if $\lambda = 0$, $\sum_{i=1}^{m} v_i = 0$ gives an optimal solution)
Find T_i's from the following equations:

$$T_i = \sqrt{\frac{R_i}{H_i}} \quad i = 1, 2, \ldots, m$$

If $\sum_{i=1}^{m}(S_i/T_i) \le \kappa$ and $T_i \ge \tau_i \ \forall i$, stop. T_i's are optimal.
Else, go to *Step 2*.

Step 2. Start from an arbitrary $\lambda > 0$.

Step 3. Solve the following equation for T_i.

$$T_i = \max\left\{\sqrt{\frac{R_i + \lambda S_i}{H_i}},\ \tau_i\right\} \quad i = 1, 2, \ldots, m.$$

Step 4. If $\sum_{i=1}^{m}(S_i/T_i) < \kappa$, reduce λ and go to *Step 3*.
If $\sum_{i=1}^{m}(S_i/T_i) > \kappa$, increase λ and go to *Step 3*.
If $\sum_{i=1}^{m}(S_i/T_i) = \kappa$, stop. T_i's are optimal.

4. Stabilization-ELSP model

4.1. *Formulation*

The problem is to determine a production schedule, and to provide a complete specification of which items are to be produced, when, and in what quantities. Some items may be produced several times during a cycle, and we allow different runs of an item within a cycle to differ in size. This is known as a time-varying lot-size approach [8]. The notation is similar to that of Dobson [8], except for the stabilization period.

The problem can be viewed as one of deciding on a cycle length T, a production sequence $f^1, f^2, \ldots, f^n$ ($f^j \in 1, 2, \ldots, n$), $n \ge m$, production times (which do not include stabilization period) $t^1, t^2, \ldots, t^n$, and idle times $u^1, u^2, \ldots, u^n$, so that the production sequence is executable in the chosen cycle length. The cycle length can be repeated indefinitely, demand is met and the total cost per unit time (setup plus holding plus loss) is minimized.

We will use subscripts to refer to the ith item: $p_i, s_i, t_{g_i}, K_i, h_i$, etc. Over the cycle $n \ge m$, setups and stabilization periods will occur, producing items $f^1, f^2, \ldots, f^n$. We will use superscripts to refer to the data related to the item produced at the jth position in this sequence: $p^j, s^j, t_g{}^j, K^j, h^j$, etc. That is, $p^j = p_{f^j}, \ldots, h^j = h_{f^j}$. Consider the jth item in the production sequence: its production involves a setup time s^j, production time excluding the stabilization period t_3^j, some subsequent idle time u^j, and then some other items are produced before the production of item f^j resumes (see Fig. 2).

Let J_i be the positions in a given sequence within which item i is produced, that is, $J_i = \{j : f^j = i\}$. Let L_k be the positions in a given sequence from k (when f^k is produced), up to, but not including the position in the sequence where item f^k is produced again. The definition of L_k assumes that the sequence $f^1, f^2, \ldots, f^n$ is viewed as circular (f^1 follows f^n). We define $\mathscr{F}$ to be the set of all possible finite sequences of items. With this notation, the Stabilization-ELSP can be represented as follows:

Stabilization-ELSP

$$\inf_{f \in \mathscr{F}} \mathrm{Min}_{\mathbf{t} \ge \mathbf{0}, \mathbf{u} \ge \mathbf{0}, T \ge 0} \ \frac{1}{T}\left(\sum_{j=1}^{n} K^j + \sum_{j=1}^{n} h^j I_T^j + \sum_{j=1}^{n} \pi^j L_T^j\right),$$

subject to

$$\sum_{j \in J_i}\left(\int_0^{t_g^j} \gamma^j(t)\mathrm{d}t + G^j t_3^j\right)p_i = r_i T \quad i = 1, 2, \ldots, m. \tag{4}$$

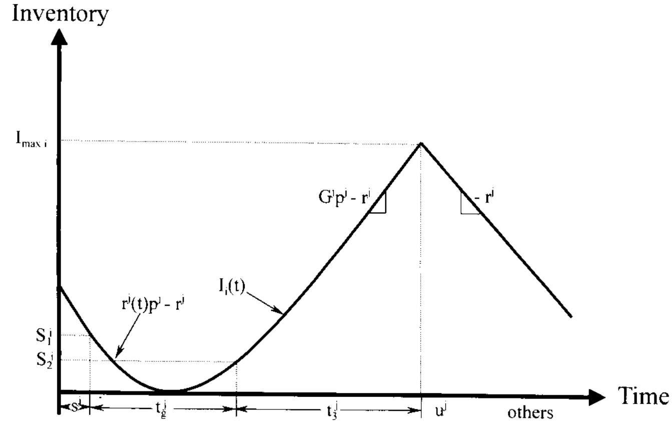

Fig. 2. Inventory for a single product.

$$\sum_{j\in L_k}(t_g^j + t_3^j + s^j + u^j) = \left(\frac{\int_0^{t_g^k}\gamma^k(t)\mathrm{d}t + G^k t^k}{r^k}\right)p^k$$

$$k = 1, 2, \ldots, n. \quad (5)$$

$$\sum_{j=1}^{n}(t_g^j + t_3^j + s^j + u^j) = T. \quad (6)$$

$$\mathbf{t} \geq \mathbf{0}, \mathbf{u} \geq \mathbf{0}, T \geq 0,$$

where

$$I_T^j = \int_0^{t_g^j} I^j(t)\mathrm{d}t + \tfrac{1}{2}[2S_2^j + t_3^j(G^j p^j - r^j)]t_3^j + \frac{[S_2^j + t_3^j(G^j p^j - r^j)]^2 - [S_1^j]^2}{2r^j},$$

$$L_T^j = \int_0^{t_g^j} p^j(1 - \gamma^j(t))\mathrm{d}t + (1 - G^j)p^j t_3^j.$$

Constraint (4) implies that we must allocate enough time for each product i to meet its demand, $r_i T$, over the cycle. The total production time consists of two parts: one is the stabilization period $(t_g^j)_{j=1}^n$, and the other is the amount of time from the end of the stabilization period to the end of production t_g^j, that is $(t_3^j)_{j=1}^n$. Constraint (5) says that we must produce enough of product i each time to last until the next time product i is produced. Constraint (6) means that a cycle consists of setup times, production times, and idle times. In the above model, the decision variables are $\mathbf{t} = (t_3^j)_{j=1}^n$, $\mathbf{u} = (u^j)_{j=1}^n$, and T.

4.2. *A heuristic algorithm*

To solve the program Stabilization-ELSP, we use the time-varying lot size approach proposed by Dobson [8]. The basic idea of the time-varying lot size approach is to decompose the problem into a combinatorial part and a continuous part. In the combinatorial part, we first determine production frequencies. Then the production frequencies are rounded off to power-of-two integers using an algorithm proposed by Roundy [15]. Finally, the items are bin packed with respect to frequencies and average loads, resulting in a production sequence. The continuous part takes the production sequence as given and computes actual production times and idle times [11]. The heuristic algorithm can be described as follows:

Algorithm

Step 1. Find the production frequencies by solving the **LB**. In Section 3, we suggested an algorithm to solve the program **LB**. Let the optimal cycle length for program **LB** be T_i^*, and the relative production frequency be x_i for item i. Then x_i is determined by the following equation:

$$x_i = \frac{\mathrm{Max}\{T_i^*\}}{T_i^*} \quad i = 1, 2, \ldots, m.$$

Step 2. Round off the frequencies to power-of-two integers. It has been shown by Roundy [15] that additional costs do not exceed 6% when we convert the real value of production frequencies to power-of-two integers. The converion of production frequencies to power-of-two integers enables the determination of the production sequences to be easily accomplished in Step 3. Let y_i be the production frequency for item i, which is a power-of-two integer. Then y_i is determined as follows:

$$\text{If } x_i \in \left[\frac{1}{\sqrt{2}}2^p, 2^p\sqrt{2}\right), \text{ then } y_i = 2^p \; p = 0, 1, \ldots.$$

Step 3. Find a production sequence using the bin-packing heuristic suggested by Dobson [8]. Given these new frequencies $(y_i)_{i=1}^m$, the bin-packing heuristic attempts to spread them out as evenly as possible. By minimizing the maximum height of the bin, the heuristic finds an efficient production sequence $(f^j)_{j=1}^n$ (see Dobson [8] for details).

Step 4. Solve for **t** and **u**, given **f**. If we assume that there are no idle times (that is, $(u^j)_{j=1}^n = 0$) for a given production sequence $(f^j)_{j=1}^n$, we can find $(t_3^j)_{j=1}^n$ using Equation (5). This approximation is good for a highly-loaded facility.

5. A special case: common cycle model

If the cycle times of all items are restricted to be of equal length (that is, each item is produced once in the common cycle), then the problem is reduced to finding an optimal common cycle time. This is clearly a simpler version of the ELSP, and is known as the Common Cycle approach [3]. We demonstrate that we can obtain a closed-form solution for this problem. This approach provides an upper bound on the average cost for the Stabilization-ELSP.

Stabilization-CC

$$\mathrm{Min}\sum_{i=1}^{m}\frac{(K_i + \pi_i L_{T_i} + h_i I_{T_i})}{T},$$

$$= \sum_{i=1}^{m}\left(K_i + \pi_i\int_0^{t_{gi}} p_i\left(1 - \frac{\gamma_i(t)}{G_i}\right)\mathrm{d}t + \frac{h_i}{2r_i}\left(\frac{A_i^2}{1-\rho_i'} + 2r_iB_i\right)\right)\Big/T$$

$$+ \sum_{i=1}^{m}\left(\frac{1}{2}h_i(1-\rho_i')r_iT + \pi_i r_i(G_i^{-1} - 1) - A_ih_i\right),$$

subject to

$$T \geq \left(\sum_{i=1}^{m} s_i + \sum_{i=1}^{m} t_{g_i} - \sum_{i=1}^{m} \frac{\int_0^{t_{g_i}} \gamma_i(t)\mathrm{d}t}{G_i}\right) \Big/ \left(1 - \sum_{i=1}^{m} \frac{r_i}{G_i p_i}\right), \quad (7)$$

$$T \geq \int_0^{\max(t_{g_i}, T_{\min i})} \frac{p_i}{r_i}\gamma_i(t)\mathrm{d}t, \quad i = 1, 2, \ldots m. \quad (8)$$

The total amount of time to produce a batch consists of the setup plus production times. We need a constraint on the total time in a cycle. Constraint (7) has been derived as follows:

$$\sum_{i=1}^{m}(s_i + t_{p_i}) \leq T,$$

$$\sum_{i=1}^{m} s_i + \sum_{i=1}^{m} t_{g_i} + \sum_{i=1}^{m} \frac{r_i T - \int_0^{t_{g_i}} \gamma_i(t)\mathrm{d}t p_i}{G_i p_i} \leq T.$$

Consequently,

$$T \geq \left(\sum_{i=1}^{m} s_i + \sum_{i=1}^{m} t_{g_i} - \sum_{i=1}^{m} \frac{\int_0^{t_{g_i}} \gamma_i(t)\mathrm{d}t}{G_i}\right) \Big/ \left(1 - \sum_{i=1}^{m} \frac{r_i}{G_i p_i}\right).$$

Constraint (8) reflects the assumption (6), that the production is continued until the given target rate is reached. t_{g_i}, $T_{\min i}$ satisfy the following equations:

$$\int_0^{T_{\min i}} p_i \gamma_i(t)\mathrm{d}t = r_i T_{\min i} \quad i = 1, 2, \ldots m,$$

$$\gamma(t_{g_i}) = G_i \quad i = 1, 2, \ldots m.$$

The solution to the Stabilization-CC satisfies the synchronization constraint, stating that no two items can be scheduled to be produced at the same time. It is therefore feasible, and consequently the optimal solution among common cycle schedules. It can be shown that the objective function, say $K(T)$, is a convex function. Consequently, if we ignore constraints (7) and (8), an unconstrained optimal solution, say T^c, is obtained by:

$$\frac{\mathrm{d}K(T)}{\mathrm{d}T} = -\sum_{i=1}^{m}\left(K_i + \pi_i \int_0^{t_{g_i}} p_i\left(1 - \frac{\gamma_i(t)}{G_i}\right)\mathrm{d}t + \frac{h_i}{2r_i}\left(\frac{A_i^2}{1-\rho_i'} + 2r_i B_i\right)\right) \Big/ T^2 + \sum_{i=1}^{m}\frac{1}{2}h_i(1-\rho_i')r_i = 0.$$

Consequently:

$$T^c = \sqrt{\frac{\left(\sum_{i=1}^{m}\left(K_i + \pi_i \int_0^{t_{g_i}} p_i\left(1 - \frac{\gamma_i(t)}{G_i}\right)\mathrm{d}t + \frac{h_i}{2r_i}\left(\frac{A_i^2}{1-\rho_i'} + 2r_i B_i\right)\right)\right)}{\sum_{i=1}^{m}\frac{1}{2}h_i(1-\rho_i')r_i}}.$$

Since we ignored constraints (7) and (8), this T^c may not be optimal. If we include constraints (7) and (8), we obtain additional conditions on the optimal cycle length as follows:

$$T \geq T^s \equiv \left(\sum_{i=1}^{m} s_i + \sum_{i=1}^{m} t_{g_i} - \sum_{i=1}^{m} \frac{\int_0^{t_{g_i}} \gamma_i(t)\mathrm{d}t}{G_i}\right) \Big/ \left(1 - \sum_{i=1}^{m} \frac{r_i}{G_i p_i}\right)$$

$$T \geq T^i \equiv \max\left\{\int_0^{t_{g_i}} \frac{p_i}{r_i}\gamma_i(t)\mathrm{d}t, \quad \int_0^{T_{\min i}} \frac{p_i}{r_i}\gamma_i(t)\mathrm{d}t, \quad i = 1, 2, \ldots m\right\}.$$

Consequently, the optimal cycle length T^* and the corresponding economic production quantities Q_i^*'s, are as follows:

$$T^* = \mathrm{Max}\{T^c, T^s, T^i\},$$
$$Q_i^* = r_i T^* \quad i = 1, 2, \ldots, m.$$

6. Computational experiments

We replicated Dobson's experiments with an inventory holding cost rate of 20%. The non-defective yield rate function was assumed to be a linear function (that is: $\gamma_i(t) = a_i t + b_i$ where, a_i, b_i are constants). We also applied the heuristic procedure to two sets of problems (100 problems in each set). The data sets were generated randomly from uniform distributions on the given intervals (See Table 1). In two sets of experiments, the loss costs were randomly selected at 10 times the holding cost. We report the mean and the maximum ratio of the average cost of the heuristic to the lower bound (See Table 2). The results are as reliable as those reported by Dobson [8].

Table 1. Distributions for randomly generated data for test problems

Parameters	*Set 1*	*Set 2*
Number of items (units)	[3, 10]	[3, 10]
Production rate (units/day)	[4000, 20 000]	[1500, 30 000]
Demand rate (units/day)	[1000, 2000]	[500, 2000]
Setup times (hours)	[1, 4]	[1, 8]
Setup cost ($)	[50, 100]	[10, 350]
Holding cost ($/day)	[0.2, 1]	[0.001, 1.4]
Loss cost ($)	[2, 10]	[0.01, 14]
Target yields rate	[0.8, 1]	[0.8, 1]
Yields rate function (a_i)	[20, 70]	[20, 70]
Yields rate function (b_i)	[0.1, 0.5]	[0.1, 0.5]

Table 2. Computational results for test problems

Model	*Set 1*		*Set 2*	
	Mean	*Maximum*	*Mean*	*Maximum*
Stabilization-CC	1.032	1.078	1.041	1.094
Stabilization-ELSP	1.014	1.040	1.016	1.060

The error in the heuristic solution, compared to the lower bound, was only a few percent on average and 4–6% maximum.

7. Concluding remarks

The Economic Lot Scheduling Problem has been studied by many researchers. It captures many important features of real and frequently encountered scheduling problems. Because of the non-linearity and combinatorial properties of the problem, most researchers have focused on the development of a heuristic algorithm to find a near-optimal solution, which is commonly compared against a lower bound. Unfortunately only a few researchers have paid attention to the basic assumptions of the ELSP which is important since they may not be appropriate when applied to many real production environments. As pointed out by Silver [16] in his review, if the quantitative models are to be more useful as aids for managerial decision-making, they must represent and formulate more realistic problems. This paper has been motivated by the need for such problem formulation. We have applied the stabilization period concept to the ELSP.

8. Acknowledgements

The authors are grateful to the useful comments suggested by Professor Robin Roundy and anonymous reviewers which improved both the content and the presentation of this paper significantly. This work was supported by the Korea Science and Engineering Foundation (KOSEF) under Grant No. KOSEF 961-1012-090-1.

References

[1] Boctor, F. (1987) The G-group heuristic for single machine lot scheduling. *International Journal of Production Research*, **25**, 363–379.

[2] Hsu, W. (1983) On the general feasibility test for scheduling lot sizes for several products on one machine. *Management Science*, **29**, 93–105.

[3] Hanssmann, F. (1962) *Operations Research in Production and Inventory Control*, John Wiley and Sons, New York.

[4] Johnson, L. and Montgomery, D. (1974) *Operations Research in Production Planning, Scheduling, and Inventory Control*, John Wiley and Sons, New York.

[5] Jones, P. and Inman, R. (1989) When is the economic lot scheduling problem easy? *IIE Transactions*, **21**, 11–20.

[6] Bomberger, E. (1966) A dynamic programming approach to a lot size scheduling problem. *Management Science*, **12**, 778–784.

[7] Elmaghraby, S. (1978) The economic lot scheduling problem (ELSP): review and extensions. *Management Science*, **24**, 587–598.

[8] Dobson, G. (1987) The economic lot scheduling problem: achieving feasibility using time-varying lot sizes. *Operations Research*, **35**, 764–771.

[9] Gallego, G. and Moon, I. (1992) The effect of externalizing setups in the economic lot scheduling problem. *Operations Research*, **40**, 614–619.

[10] Gallego, G. and Moon, I. (1995) Strategic investment to reduce setup times in the economic lot scheduling problem. *Naval Research Logistics*, **42**, 773–790.

[11] Zipkin, P. (1991) Computing optimal lot sizes in the economic lot scheduling problem. *Operations Research*, **39**, 56–63.

[12] Hahm, J., Kim, S. and Lee, G. (1994) Determination of the economic production quantity for a manufacturing process with stabilization period (*in Korean*). *Journal of the Korean Institute of Industrial Engineers*, **20**, 93–104.

[13] Bourland, K., Brown, M. and Yano, C. (1997) The product cycling problem under increasing yield rates. *IIE Transactions*, **29**, 417–422.

[14] Urban, T. (1997) Analysis of production systems when run length influences product quality. Working Paper. The University of Tulsa, Oklahoma.

[15] Roundy, R. (1989) Rounding off to powers of two in continuous relaxation of capacitated lot sizing problems. *Management Science*, **35**, 1433–1442.

[16] Silver, E. (1993) Modelling in support of continuous improvements towards achieving world class operations. *Perspectives in Operations Management*. Sarin, R. (ed.), Kluwer Academic Publishers, Boston MA, pp. 23–44.

Appendix A

Proof of Proposition 1

First, we derive the total inventory for item i during a cycle length T_i (denoted as I_{T_i}). We define the following notation (See Fig. 3):

t_{3_i} = amount of time from the end of the stabilization period to the end of production;

t_{p_i} = total production time during a cycle length T (that is, $t_{p_i} = t_{1_i} + t_{2_i} + t_{3_i}$);

t_{4_i} = amount of time from the end of production to the beginning of the next production;

S_{1_i} = inventory level at the beginning of the stabilization period;

S_{2_i} = inventory level at the end of the stabilization period.

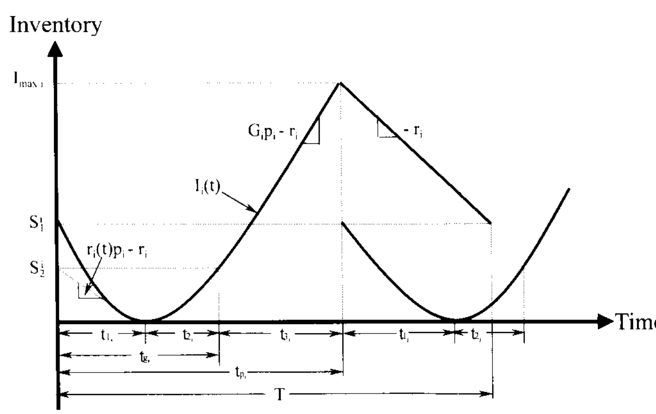

Fig. 3. Inventory level associated with the yield rate function.

Clearly, the following equations will hold using the above definitions:

$$Q_i = r_i T_i.$$

$$S_{1_i} = t_{1_i} r_i - \int_0^{t_{1_i}} p_i \gamma_i(t)\mathrm{d}t.$$

$$S_{2_i} = \int_{t_{1_i}}^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t - t_{2_i} r_i.$$

S_{1_i} is an inventory level, at which production must be started in order to prevent backorders. If production is started when the inventory level is lower than S_{1_i}, the non-defective production rate does not exceed the demand rate and backorders are incurred. S_{2_i} is an inventory level, at which the non-defective production rate reaches the target rate.

Next, we derive t_{3_i} from the definition:

$$\begin{aligned} Q_i &= \int_0^{t_{p_i}} p_i \gamma_i(t)\mathrm{d}t \\ &= \int_0^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t + \int_{t_{g_i}}^{t_{p_i}} p_i \gamma_i(t)\mathrm{d}t \\ &= \int_0^{t_{g_i}} p_i \gamma_i(t)dt + G_i p_i t_{3_i}. \end{aligned}$$

Consequently,

$$t_{3_i} = \frac{Q_i - \int_0^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t}{G_i p_i}. \tag{A1}$$

Equation (A1) means that t_{3_i} is determined by the non-defective production quantity Q_i.

The maximum inventory level $I_{\max i}$ is equivalent to S_{2_i} plus the amount of inventory accumulation during t_{3_i}. Using Equation (A1), we obtain the following Equation (A2):

$$\begin{aligned} I_{\max i} &= S_{2_i} + t_{3_i}(G_i p_i - r_i) \\ &= S_{2i} + \frac{Q_i - \int_0^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t}{G_i p_i}(G_i p_i - r_i), \\ &= (1 - \rho_i')Q_i + S_{2i} - (1 - \rho_i')\int_0^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t \\ &= (1 - \rho_i')Q_i - A_i, \end{aligned} \tag{A2}$$

$$\text{where} \quad A_i = (1 - \rho_i')\int_0^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t - S_{2_i}.$$

$I_i(t)$, inventory level at time t_i, is given as:

$$I_i(t) = S_{1_i} + \int_0^{t_i} p_i \gamma_i(t)\mathrm{d}t - r_i t_i.$$

The level of inventory accumulated during the stabilization period t_{g_i} is obtained by integrating $I_i(t)$, and the level of inventory accumulated during t_{3_i} and t_{4_i} is obtained by computing the areas of two trapezoids in Fig. 3. Therefore, the total inventory I_{T_i} can be derived as follows:

$$\begin{aligned} I_{T_i} &= \int_0^{t_{g_i}} I_i(t)\mathrm{d}t + \frac{1}{2}(I_{\max i} + S_{2_i})t_{3_i} + \frac{1}{2}(I_{\max i} + S_{1_i})t_{4i}, \\ &= \int_0^{t_{g_i}} I_i(t)\mathrm{d}t + \frac{1}{2}(I_{\max i} + S_{2_i})\frac{(I_{\max i} - S_{2_i})}{(G_i p_i - r_i)} \\ &\quad + \frac{1}{2}(I_{\max i} + S_{1_i})\frac{(I_{\max i} - S_{1_i})}{r_i}, \\ &= \int_0^{t_{g_i}} I_i(t)\mathrm{d}t + \frac{1}{2}I_{\max i}^2\left\{\frac{(G_i p_i)}{(G_i p_i - r_i)r_i}\right\} \\ &\quad - \frac{1}{2}\left\{\frac{S_{1_i}^2}{r_i} + \frac{S_{2_i}^2}{G_i p_i - r_i}\right\}, \\ &= \frac{1}{2}\frac{I_{\max i}^2}{(1 - \rho_i')r_i} + \int_0^{t_{g_i}} I_i(t)\mathrm{d}t - \frac{1}{2}\left\{\frac{S_{1i}^2}{r_i} + \frac{S_{2i}^2}{G_i p_i - r_i}\right\}, \\ &= \frac{1}{2}\frac{I_{\max i}^2}{(1 - \rho_i')r_i} + B_i, \end{aligned}$$

$$\text{where } B_i = \int_0^{t_{g_i}} I_i(t)\mathrm{d}t - \frac{1}{2}\left\{\frac{S_{1_i}^2}{r_i} + \frac{S_{2_i}^2}{G_i p_i - r_i}\right\}.$$

Now, we derive the total number of defective units incurred during the production of item i (denoted as L_{T_i}). We obtain the following equation using Equation (A1):

$$\begin{aligned} L_{T_i} &= \int_0^{t_{g_i}} p_i(1 - \gamma_i(t))\mathrm{d}t + (1 - G_i)p_i t_{3_i}, \\ &= \int_0^{t_{g_i}} p_i(1 - \gamma_i(t))\mathrm{d}t + (1 - G_i)p_i\left(\frac{Q_i - \int_0^{t_{g_i}} p_i \gamma_i(t)\mathrm{d}t}{G_i p_i}\right), \\ &= (G_i^{-1} - 1)Q_i + \left\{\int_0^{t_{g_i}} (1 - \gamma_i(t))\mathrm{d}t \right. \\ &\quad \left. - (G_i^{-1} - 1)\int_0^{t_{g_i}} \gamma_i(t)\mathrm{d}t\right\}p_i, \\ &= (G_i^{-1} - 1)Q_i + \int_0^{t_{g_i}} p_i(1 - G_i^{-1}\gamma_i(t))\mathrm{d}t. \end{aligned}$$

Biographies

Ilkyeong Moon is an Associate Professor of Industrial Engineering at Pusan National University in Korea. He received his B.S. and M.S. degrees in Industrial Engineering from Seoul National University, Korea, and a Ph.D. in Operations Research from Columbia University in 1991. His research interests include production planning and control (especially, JIT applications), modeling and analysis of manufacturing systems, and inventory control. His work has appeared in journals such as: *Computers & Industrial Engineering*, *Computers & Operations Research*, *IIE Transactions*, *International Journal of Production Economics*, *International Journal of Production Research*, *Journal of the Operational Research Society*, *Management Science*, *Naval Research Logistics*, *Omega*, *Operations Research*, and *The Engineering Economist*. He currently serves on the editorial boards of *International Journal of Industrial Engineering* and *International Journal of Operations and Quantitative Management*.

Juho Hahm is an Associate Professor of Industrial Engineering at Seoul National University in Korea. He received his B.S. and M.S. degrees in Industrial Engineering from Seoul National University, Korea, and a Ph.D. in Industrial and Operations Engineering from the University of Michigan. His research interests involve production and inventory system, logistic system analysis and design, business process reengineering, and enterprise resource planning. He has published papers in *Computers & Industrial Engineering*, *IIE Transactions*, *International Journal of Production Economics*, and *Transportation Science*.

Chul Lee is a researcher in the Manufacturing Technology Center at LG Electronics in Korea. He received his B.S. and M.S. degrees in Industrial Engineering from Pusan National University, Korea. His main research interests are production scheduling and simulation.