Pergamon

Computers ind. Engng Vol. 32, No. 1, pp. 109–113, 1997

S0360-8352(96)00200-8
0360-8352/97 $17.00 + 0.00

THE DISTRIBUTION FREE JOB CONTROL PROBLEM

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(Received 1 May 1996)

Abstract—The job control problem is to determine an optimal release time where the flow time is a random variable with a known probability distribution. There is a trade-off between the penalty cost for late delivery and the holding cost for early finish. C. Liao [*Computers & Industrial Engineering*, **22**, 163–169 (1992)] formulated this problem and provided a practical decision procedure. This paper addresses the job control problem where only the mean and the variance of the flow time are known. The motivation of this paper comes from the fact that the distributional information on the flow time is usually limited for some job shops. We provide a simple line search algorithm to find a job release time which can be used in the absence of the distributional information. Copyright © 1997 Elsevier Science Ltd

1. INTRODUCTION

Recently Liao [1] addressed the problem of accepting or rejecting arriving jobs to a job shop in which the flow times are stochastic. We call the problem the *job control problem*. The job control problem is to determine an optimal release time where the flow time is a random variable with a known probability distribution. There is a trade-off between the penalty cost for late delivery and the holding cost for early finish. Liao formulated this problem and provided a practical decision procedure. The motivation of this paper comes from the fact that the distributional information on the flow time is usually limited for some job shops. Sometimes all that is available is an educated guess of the mean and of the variance. There is a tendency to use the normal distribution under these conditions. However, the normal distribution does not provide the best protection against the occurrence of other distributions with the same mean and the same variance.

Scarf [2] addressed the newsboy problem where only the mean μ and the variance σ^2 of the demand are known without any further assummptions about the form of the distribution of the demand. Taking a conservative approach, he modeled the problem as that of finding the order quantity that maximizes the expected profit against the worst possible distribution of the demand with mean μ and variance σ^2. We call this approach the *distribution free procedure*. Gallego and Moon [3] presented a compact proof of the optimality of Scarf's ordering rule for the newsboy problem where only the mean and the variance of the demand are known. They extended the analysis to the recourse case, where there is a second purchasing opportunity; to the fixed ordering cost case, where a fixed cost is charged for placing an order; to the case of random yields; and to the multi-item case, where multiple items compete for a scarce resource. Moon and Gallego [4] applied the idea to several inventory models including continuous review model and periodic review model. Other works related to the distribution free approach include Gallego [5], Moon and Choi [6,7].

We think that the distribution free procedure is of practical value because it is optimal under limited information. Moreover, the procedure provides us with an intuitive explanation of when it is profitable (in expectation) to release earlier (respectively, later) than the due date minus the expected flow time. The goal of this paper is to apply the distribution free procedure to the job control problem studied by Liao [1].

2. PROBLEM FORMULATION

We consider the case that both the tardiness penalty cost and the carrying cost are linear functions on the duration of the time. We use a similar notation for the arriving job as in Liao [1]:

h: carrying cost for the finished job per unit time
p: tardiness penalty cost per unit time

R: reward of accepting the job
$\bar{d}$: due date
$\underset{\sim}{d}$: earliest delivery date
t: arrival time
μ: mean of the flow time
σ: standard deviation of the flow time
r: release time (decision variable)

Let X denote the random flow time. We make no assumption on the distribution F of X other than saying that it belongs to the class $\mathscr{F}$ of cumulative distribution functions with mean μ and variance σ^2. In what follows we let $y^+ = \max\{y, 0\}$.

A fixed amount of reward will be earned if a job is accepted, whereas a tardiness penalty cost for late delivery and a carrying cost for early finish will be incurred at the same time. If a job is finished after the allowed earliest delivery date, but before the due date, there will be no carrying cost since it can be delivered to the customer immediately upon finish. Customers usually allow deliveries in a range of times instead of a fixed time since in most practical situations they do know exactly when the deliveries will be needed when placing the job order. This permissible range of delivery also represents a compromise between customers and the shop organization. If customers want a fixed due date, the analysis in Section 3 can be applied. For further justifications of these assumptions, refer to Liao [1] and Baker and Scudder [8].

The expected profit can be written as

$$\pi^F(r) = R - h\int_0^{\underset{\sim}{d}-r} (\underset{\sim}{d} - r - x)\,\mathrm{d}G(x) - p\int_{\bar{d}-r}^{\infty} (x - \bar{d} + r)\,\mathrm{d}G(x)$$

$$= R - hE(\underset{\sim}{d} - r - X)^+ - p[\mu - \bar{d} + r + E(\bar{d} - r - X)^+]$$

Evidently, maximizing $\pi^F(r)$ is equivalent to minimizing

$$Z^F(r) = hE(\underset{\sim}{d} - r - X)^+ + p[\mu - \bar{d} + r + E(\bar{d} - r - X)^+] \tag{1}$$

so we concentrate on the latter problem.

Since the distribution F of X is unknown we want to minimize (1) against the worst possible distribution in $\mathscr{F}$. To this end, we need the following lemma.

Lemma 1.

$$E(\underset{\sim}{d} - r - X)^+ \leqslant \frac{[\underset{\sim}{d} - r - \mu)^2 + \sigma^2]^{1/2} + (\underset{\sim}{d} - r - \mu)}{2} \tag{2}$$

Proof: Notice that

$$(d - r - X)^+ = \frac{|\underset{\sim}{d} - r - X| + (\underset{\sim}{d} - r - X)}{2} \tag{3}$$

We can get the following inequality using Cauchy-Schwarz inequality:

$$E|\underset{\sim}{d} - r - X| \leqslant [E(\underset{\sim}{d} - r - X)^2]^{1/2} = \{E[(\underset{\sim}{d} - r - \mu) + (\mu - X)]^2\}^{1/2} = [(\underset{\sim}{d} - r - \mu)^2 + \sigma^2]^{1/2} \tag{4}$$

By taking expectation on (3) and using (4), we can get the upper bound as in (2). □

Using Lemma 1, we can obtain an upper bound on $Z^F(r)$ for all distributions with mean μ and variance σ^2:

$$Z^F(r) \leqslant Z^W(r) = h\left[\frac{[\underset{\sim}{d} - r - \mu)^2 + \sigma^2]^{1/2} + (\underset{\sim}{d} - r - \mu)}{2}\right] - p(\bar{d} - r - \mu)$$

$$+p\left[\frac{[(\bar{d} - r - \mu)^2 + \sigma^2]^{1/2} + (\bar{d} - r - \mu)}{2}\right] \tag{5}$$

When F is unknown, we want to use a release time which minimizes $Z^W(r)$. This approach is called as a min–max distribution-free rule [2–7].

The second derivative of $Z^W(r)$ is

$$\frac{d^2Z^W(r)}{dr^2} = \frac{\sigma^2}{2}\{h[\underline{d} - r - \mu)^2 + \sigma^2]^{-3/2} + p[\overline{d} - r - \mu)^2 + \sigma^2]^{-3/2}\} \geqslant 0$$

Consequently, $Z^W(r)$ is a convex function. Upon setting the first derivative of $Z^W(r)$ to zero we obtain the following equation:

$$h[(\underline{d} - r - \mu)^2 + \sigma^2]^{-1/2}(\underline{d} - r - \mu) + p[(\overline{d} - r - \mu)^2 + \sigma^2]^{-1/2}(\overline{d} - r - \mu) = p - h \quad (6)$$

Let the left hand side of (6) be $g(r)$, then we can show that $g(r)$ is decreasing in r. Let $\hat{r}^W$ be a solution of (6) which can be obtained easily using a line search. Since $Z^W(r)$ is a convex function, $\hat{r}^W$ is an unconstrained optimal solution. However, a release time must be at least as large as the job arrival time, t. Consequently, if $\hat{r}^W < t$, i.e. $g(t) < p - h$, then an optimal release time must be equal to the job arrival time, t. In summary, we can obtain a unique optimal distribution-free release time, say r^W, as follows.

Algorithm

(Step 1) If $g(t) < p - h$, then $r^W = t$. Stop. The job arrival time is an optimal solution. Otherwise, Go to Step 2.

(Step 2) Start from an arbitrary r, say $\hat{r}^W = (\underline{d} + \overline{d})/2 - \mu$. Let LB $= t$, and UB $= \underline{d}$.

(Step 3) Compute

$$g(\hat{r}^W) = h[(\underline{d} - r - \mu)^2 + \sigma^2]^{-1/2}(\underline{d} - r - \mu) + p[(\overline{d} - r - \mu)^2 + \sigma^2]^{-1/2}(\overline{d} - r - \mu).$$

(Step 4) If $g(\hat{r}^W) > p - h$, then LB$\leftarrow\hat{r}^W$. $\hat{r}^W\leftarrow$(LB + UB)/2. Go to Step 3.
If $g(\hat{r}^W) < p - h$, then UB$\leftarrow\hat{r}^W$. $\hat{r}^W\leftarrow$(LB + UB)/2. Go to Step 3.
If $g(\hat{r}^W) = p - h$, then $\hat{r}^W$ is an optimal release time. Stop.

Remark 1. If $h = p$, then $\hat{r}^W = (\underline{d} + \overline{d})/2 - \mu$. If $h > p$ (respectively, $h < p$), then $\hat{r}^W > (\underline{d} + \overline{d})/2 - \mu$ (respectively, $\hat{r}^W < (\underline{d} + \overline{d})/2 - \mu$).

Remark 2. If the distribution F is known, we can find an optimal release time r^F from the following equation and comparison with t [1]:

$$hF(\underline{d} - r) = p[1 - F(\overline{d} - r)]$$ □

If we use the release time r^W instead of r^F, the expected additional cost is equal to

$$Z^F(r^W) - Z^F(r^F).$$

This is the largest amount that we would be willing to pay for the knowledge of F. This quantity can be regarded as the *Expected Value of Additional Information* (EVAI).

Example 1. This problem is taken from Liao [1]. The mean and standard deviation of the flow time are 10 and 2, respectively. In addition, $h = \$10$, $p = \$20$, $t = 6$, $\underline{d} = 20$, and $\overline{d} = 24$. We compare the performance of r^W with r^N where $N \in \mathcal{F}$ represents the normal distribution. The results are (normal in parenthesis) $r^W = 11.438$ (11.546) and a worst case expected cost of \$12.01 (\$4.64). We show the computational steps to derive r^W in detail in order to enhance understanding of the shop floor managers.

(iteration 1)

(Step 1) Since $g(t) = 28.35 > p - h$, go to Step 2.
(Step 2) $\hat{r}^W = (\underline{d} + \overline{d})/2 - \mu = 12$ LB = 6, UB = 20.
(Step 3) $g(\hat{r}^W) = 7.1$.
(Step 4) Since $g(\hat{r}^W) < p - h$, UB = 12, LB = 6, $\hat{r}^W = 9$.

(iteration 2)
(Step 3) $g(\hat{r}^W) = 23.0$.
(Step 4) Since $g(\hat{r}^W) > p - h$, UB = 12, LB = 9, $\hat{r}^W = 10.5$

(iteration 3)
(Step 3) $g(\hat{r}^W) = 14.9$.
(Step 4) Since $g(\hat{r}^W) > p - h$, UB = 12, LB = 10.5, $\hat{r}^W = 11.25$.

(iteration 4)
(Step 3) $g(\hat{r}^W) = 10.9$.
(Step 4) Since $g(\hat{r}^W) > p - h$, UB = 12, LB = 11.25, $\hat{r}^W = 11.625$.

(iteration 5)
(Step 3) $g(\hat{r}^W) = 9.0$.
(Step 4) Since $g(\hat{r}^W) < p - h$, UB = 11.625, LB = 11.25, $\hat{r}^W = 11.438$.

(iteration 6)
(Step 3) $g(\hat{r}^W) = 10$.
(Step 4) Since $g(\hat{r}^W) = p - h$, stop.
$r^W = 11.438$ is an optimal distribution free release time.

The EVAI is

$$Z^N(r^W) - Z^N(r^N) = \$4.66 - \$4.64 = \$0.02$$ □

3. EARLINESS AND TARDINESS PENALTIES

In a Just-In-Time (JIT) production, which espouses the notion that earliness, as well as tardiness, should be discouraged. In a JIT scheduling environment, jobs that complete early must be held in finished goods inventory until their due date , while jobs that complete after their due dates may cause a customer to shut down operations. Therefore, an ideal schedule is one in which all jobs finish exactly on their assigned due dates. See Baker and Scudder [8] for further justifications on using earliness and tardiness penalties in scheduling problems.

Let $d = \underline{d} = \bar{d}$ be the due date, and h and p be the earliness penalty and the tardiness penalty, respectively. Then,

$$Z^F(r) = (h + p)E(d - r - X)^+ + p(\mu - d + r) \tag{7}$$

Using Lemma 1 again, we can find a closed-form optimal distribution-free release time which is $\max(t, \hat{r}^W)$ where

$$\hat{r}^W = d - \mu - \frac{\sigma}{2}\left(\sqrt{\frac{p}{h}} - \sqrt{\frac{h}{p}}\right). \tag{8}$$

Note that (8) allows more time (respectively, less time) than the mean flow time if and only if the ratio $p/h > 1$ (respectively, $p/h < 1$). Substituting (8) into (5) with $d = \underline{d} = \bar{d}$ we obtain, for all $F \in \mathscr{F}$, the following upper bound on the optimal expected cost

$$0 \leqslant Z^F(r^F) \leqslant \sigma\sqrt{hp}$$

Remark 3. Noting that 0 is the minimum cost when the flow time is deterministic, we can regard $\sigma\sqrt{hp}$ as the maximal fractional cost of randomness. Note also that if we were to allow μ time units, then the expected cost would be at most $\sigma(h + p)/2$. So if the fraction $\sigma(h + p)/2$ is small then no great loss is incurred by simply allowing μ units i.e., by allowing as if the problem were deterministic. □

Example 2. The data are as in Example 1 except $d = \underline{d} = \bar{d} = 24$. Again, we compare the performance of r^W with r^N where $N \in \mathscr{F}$ represents the normal distribution. The results are (normal in parenthesis) $r^W = 13.293$ (13.139) and a worst case expected cost of \$28.28 (\$21.82). The EVAI is

$$Z^N(r^W) - Z^N(r^N) = \$21.88 - \$21.82 = \$0.06$$

If we use the release time as if the problem were deterministic, i.e. $r^D = 14$ where D denotes deterministic case, the expected cost for normal distribution $Z^N(r^D) = \$23.94$. □

4. CONCLUDING REMARKS

We derived the optimal distribution-free release time for the job control problem where only the mean and the variance of the flow time are known. We expect the distribution-free release time is robust which can be conjectured from numerical examples. Shop floor managers are encouraged to use the distribution-free release time in the absence of the distributional information on the flow time. Further theoretical and empirical investigations on the robustness of the distribution-free release time might be an interesting research problem.

Acknowledgement—This work was supported by an Academic Research Grant from Pusan National University.

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