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Distribution free procedures for make-to-order (MTO), make-in-advance (MIA), and composite policies

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Abstract

The purpose of this paper is to study the make-to-order (MTO), make-in-advance (MIA), and composite policies in a single period two echelon stochastic model with more realistic assumptions. We relax the assumption that the cumulative distribution function of demand is completely known and merely assume that its first two moments are known.

Keywords: Inventory; Distribution free procedure; Make-to-order; Make-in-advance, Composite policy

1. Introduction

Faced with stochastic demand, managers are often confronted with decisions on whether to hold inventories in the form of raw materials, subassembles or finished products. Two extreme policies may be excercised: holding finished products or holding raw materials (or components). If the demand's realization turns out to be smaller than the available finished products, then some processing cost have unnecessarily been incurred. But if the demand's realization turns out to be larger than the available finished products, then some customers might balk and their demand will be lost.

Johnson and Montgomery [1] analyzed a two-echelon single period stochastic model consisting of a single facility that converts a purchased material into a finished product to capture the above tradeoff. The model is based on a four echelon model by Bryan et al. [2] and another multi-stage model in Hanssman [3]. Gerchak and Zhang [4] investigated the dependence of the optimal stocks on the initial inventories. Recently, Eynan and Rosenblatt [5] have pointed out that Bryan et al. [2], Johnson and Montgomery [3], and Gerchak and Zhang [4] implicitly assumed that there is no difference (costwise) of when the product is assembled. As they pointed out, the cost of producing in advance (MIA) is usually less expensive than in a rush job when customers are waiting for the conversion of materials into finished products (due to expediting costs). Furthermore, under make-to-order (MTO) one may lose the advantages of economy of scale (working in batches) and learning effects that tend to increase the cost under MTO. Finally, one may consider MIA as a case where finished units are purchased from a supplier at a lower cost per unit than if they are made to order within the plant [5]. Our model is based on that of Eynan and Rosenblatt [5], and the distribution free approach, which will be explained below, is applied to their model.

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In practice, the distributional information about the demand is often limited. Sometimes all that is available is an educated guess of the mean and of the variance. There is a tendency to use the normal distribution under these conditions. However, the normal distribution does not offer the best shield against the occurrences of other distributions with the same mean and same variance. Scarf [6] addressed a newsboy problem where only the mean μ and the variance σ^2 of the demand are known without any further assumptions about the form of the distribution of the demand. Taking a conservative approach, he modeled the problem as that of finding the order quantity that maximizes the expected profit against the worst possible distribution of the demand with the mean μ and the variance σ^2. He showed that the worst distribution of the demand has positive mass at two points and used this result to obtain a closed-form expression for the optimal-order quantity. The approach is called the minmax distribution free approach.

Recently, there have been several papers related to the distribution free approach. Gallego [7] applied the distribution free approach to a continuous review inventory model. Moon and Choi [8] solved the distribution free continuous review inventory model with a service level constraint. Moon and Gallego [9] applied the approach to several inventory models including a periodic review model. Gallego and Moon [10] presented very compact proof of the optimality of Scarf's ordering rules for the newsboy problem and extended the analysis to several cases including fixed ordering cost case, multi-product case, random yield case, and recourse case. Moon and Choi [11] extended the model of Gallego and Moon [10] to the case that allows customers balk when inventory level is low. Shore [12] derived explicit approximate solutions to the standard newsboy problem, to some (Q, r) models, and to a periodic review model in which the first three or four moments of the demand are known.

The basic model of this paper has been initially studied by Eynan and Rosenblatt [5]. In this paper we apply the distribution free approach to the basic model, and develop a procedure that provides optimal inventory levels against the worst distribution. In Section 2, we briefly review the basic model and then apply the distribution free approach to the basic model. We also compare the profit differences between MIA, MTO, and composite model and derive an equation which shows the relationship between inventory levels from the three policies. In Section 3, we extend the analysis to the case that there is a budget limit on the total inventory cost. Computational experiments are provided in Section 4.

2. Basic model and distribution free approach

The data used in this paper are as follows:

c_1 cost to purchase one unit of the finished product
c_2 cost to purchase one unit of raw material (or component)
p unit selling price of finished product
v_1 unit salvage value of finished product
v_2 unit salvage value of raw material
m cost to process one unit of raw material into one unit of finished product
μ expected demand
σ standard deviation of the demand
S_1^F on-hand inventory of the finished product at the beginning of the period when demand follows a distribution function F (decision variable)
S_2^F on-hand inventory of raw material at the beginning of the period when demand follows a distribution function F (decision variable)

We consider a single period model with stochastic demand under a two echelon production system. Demand can be satisfied from two kinds of inventories: First one is the inventory of finished products (MIA) which have been purchased at the beginning of the period (or processed from raw materials before the beginning of the period). Second one is the inventory of raw materials (MTO) which will be processed into the finished products once demand is realized. For the composite model, we use both kinds of inventories to satisfy demands. Our problem is to decide the appropriate inventory levels S_1 and S_2 to maximize the expected profit. Unlike in the Johnson and Montgomery [1] or Gerchak and Zhang [4] model, no sales are assumed to be lost (i.e. all customers will wait for conversion). However, the model in this paper can easily be extended to the case in which a fraction of customers will wait for conversion. Also, unlike in Gerchak and Zhang [4], there are no initial inventories of materials and/or finished products.

Products can be purchased or processed in advance with a cost per unit of c_1. If the unit is processed upon the receipt of an actual order, then the cost per unit is $c_2 + m$ which is larger than c_1 due to deviation from the original schedule and causing some form of system nervousness (see Carlson et al. [13]). For the problem to be meaningful, we assume that $c_2 < c_1$, $c_1 < c_2 + m < p$, $v_1 < c_1$, $v_2 < c_2$, and $c_2 - v_2 < c_1 - v_1$. Note that if $c_2 + m < c_1$, there is no advantage to purchase finished products and we should always adopt the MTO policy. Similarly, if $c_1 - v_1 < c_2 - v_2$, there is no advantage to stock raw materials and we should always adopt the MIA policy.

Let D denote the random demand with probability density function $f(D)$ and distribution function F. In what follows, we let $x^+ = \max\{x, 0\}$. Only when S_1 is exhausted, we will start to process raw materials into the finished products. The expected profit can be written as

$$\begin{aligned}\pi^F(S_1,S_2) = {} & p\int_0^{S_1+S_2} Df(D)\,\mathrm{d}D + p(S_1+S_2)\int_{S_1+S_2}^{\infty} f(D)\,\mathrm{d}D + v_1\int_0^{S_1}(S_1-D)f(D)\,\mathrm{d}D \\ & -m\int_{S_1}^{S_1+S_2}(D-S_1)f(D)\,\mathrm{d}D \\ & -mS_2\int_{S_1+S_2}^{\infty} f(D)\,\mathrm{d}D - c_1S_1 - c_2S_2 \\ & +v_2S_2\int_0^{S_1} f(D)\,\mathrm{d}D \\ & +v_2\int_{S_1}^{S_1+S_2}(S_1+S_2-D)f(D)\,\mathrm{d}D. \qquad (1)\end{aligned}$$

Noting that

$$\int_{S_1}^{\infty}(D-S_1)f(D)\,\mathrm{d}D = E[D-S_1]^+,$$

$$\int_{S_1+S_2}^{\infty}(D-S_1-S_2)f(D)\,\mathrm{d}D = E[D-S_1-S_2]^+,$$

we can write the expected profit as

$$\begin{aligned}\pi^F(S_1,S_2) = {} & (p-v_1)\mu + (v_2+m-p) \\ & \times E[D-S_1-S_2]^+ - (m+v_2-v_1) \\ & \times E[D-S_1]^+ - (c_1-v_1)S_1 - (c_2-v_2)S_2.\end{aligned}$$

Note that if $S_1 = 0$ or $S_2 = 0$, the above equation reduces to the MTO model and MIA model, respectively. Evidently, maximizing $\pi^F(S_1,S_2)$ is equivalent to minimizing the expected cost $C^F(S_1,S_2)$.

$$\begin{aligned}C^F(S_1,S_2) = {} & (p-m-v_2)E[D-S_1-S_2]^+ \\ & +(m+v_2-v_1)E[D-S_1]^+ \\ & +(c_1-v_1)S_1 + (c_2-v_2)S_2. \qquad (2)\end{aligned}$$

It is easy to verify that $C^F(S_1,S_2)$ is strictly convex in S_1 and S_2. Upon setting $\partial C^F(S_1,S_2)/\partial S_1 = 0$ and $\partial C^F(S_1,S_2)/\partial S_2 = 0$, we get

$$F(S_1) = \frac{c_2+m-c_1}{m+v_2-v_1}, \qquad (3)$$

$$F(S_1+S_2) = 1 - \frac{c_2-v_2}{p-m-v_2}. \qquad (4)$$

The equations are due to Eynan and Rosenblatt [5], and we can find the optimal (S_1^F, S_2^F) satisfying the above equations using a line search.

Since $F(\cdot)$ is nonnegative and smaller than 1 and $c_1 < c_2 + m$, $v_2 < c_2$, we have $m + v_2 - v_1 \geqslant 0$ and $p - m - v_2 \geqslant 0$. Thus, we can rewrite Eq. (2) as follows:

$$\begin{aligned}C^F(S_1,S_2) = {} & aE[D-S_1-S_2]^+ + bE[D-S_1]^+ \\ & +(c_1-v_1)S_1 + (c_2-v_2)S_2, \qquad (5)\end{aligned}$$

where $a = p - m - v_2$, $b = m + v_2 - v_1$.

Now we consider the distribution free approach. We make no assumption on the distribution F of D other than saying that it belongs to the class $\mathcal{F}$ of distribution functions with mean μ and variance σ^2. Since the distribution F of D is unknown we want to minimize (5) against the worst possible distribution in $\mathcal{F}$. To this end, we need the following proposition [10].

Proposition 1.

$$E[D-S]^{+} \leqslant \frac{\sqrt{\sigma^2+(S-\mu)^2}-(S-\mu)}{2}. \tag{6}$$

Moreover, the upper bound (6) is tight. That is for every S, there exists a distribution $F^* \in \mathcal{F}$ where the bound (6) is tight.

Proof. First note that

$$[D-S]^{+} = \frac{|D-S|+(D-S)}{2}.$$

Eq. (6) follows by taking expectations and using the following Cauchy–Schwarz inequality:

$$E|D-S| \leqslant [E(D-S)^2]^{1/2} = [\sigma^2+(S-\mu)^2]^{1/2}.$$

Now we prove the tightness of the upper bound. For every S, consider two point distribution F^* assigning weight

$$\alpha = \frac{\sqrt{\sigma^2+(S-\mu)^2}+(S-\mu)}{2\sqrt{\sigma^2+(S-\mu)^2}}$$

to

$$\mu - \sigma\sqrt{\frac{1-\alpha}{\alpha}} = S - \sqrt{\sigma^2+(S-\mu)^2}$$

and weight

$$1-\alpha = \frac{\sqrt{\sigma^2+(S-\mu)^2}-(S-\mu)}{2\sqrt{\sigma^2+(S-\mu)^2}}$$

to

$$\mu + \sigma\sqrt{\frac{\alpha}{1-\alpha}} = S + \sqrt{\sigma^2+(S-\mu)^2}.$$

Clearly (6) holds with equality and it is easy to verify that $F^* \in \mathcal{F}$. □

The distribution free approach for this model is to find the most unfavorable distribution in $\mathcal{F}$ for S_1 and S_2 and then minimize over S_1 and S_2. Our problem is now to minimize the upper bound

$$C^W(S_1,S_2) = a\frac{\sqrt{\sigma^2+(S_1+S_2-\mu)^2}-(S_1+S_2-\mu)}{2} + b\frac{\sqrt{\sigma^2+(S_1-\mu)^2}-(S_1-\mu)}{2} + (c_1-v_1)S_1 + (c_2-v_2)S_2. \tag{7}$$

The expected cost function is strictly convex as shown in the following property.

Proposition 2. *$C^W(S_1,S_2)$ is strictly convex in both S_1 and S_2.*

Proof.

$$\frac{\partial^2 C^W(S_1,S_2)}{\partial S_1^2} = \frac{a}{2}\left\{[\sigma^2+(S_1+S_2-\mu)^2]^{-1/2} \times\left[1-\frac{(S_1+S_2-\mu)^2}{\sigma^2+(S_1+S_2-\mu)^2}\right]\right\} + \frac{b}{2}\left\{[\sigma^2+(S_1-\mu)^2]^{-1/2} \times\left[1-\frac{(S_1-\mu)^2}{\sigma^2+(S_1-\mu)^2}\right]\right\} > 0,$$

$$\frac{\partial^2 C^W(S_1,S_2)}{\partial S_2^2} = \frac{a}{2}\left\{[\sigma^2+(S_1+S_2-\mu)^2]^{-1/2} \times\left[1-\frac{(S_1+S_2-\mu)^2}{\sigma^2+(S_1+S_2-\mu)^2}\right]\right\} > 0,$$

$$\frac{\partial^2 C^W(S_1,S_2)}{\partial S_1 \partial S_2} = \frac{a}{2}\left\{[\sigma^2+(S_1+S_2-\mu)^2]^{-1/2} \times\left[1-\frac{(S_1+S_2-\mu)^2}{\sigma^2+(S_1+S_2-\mu)^2}\right]\right\} > 0.$$

We can easily check that the Hessian matrix is positive definite. Consequently, $C^W(S_1,S_2)$ is strictly convex in S_1 and S_2. □

Upon setting

$$\frac{\partial C^W(S_1,S_2)}{\partial S_1} = 0 \quad\text{and}\quad \frac{\partial C^W(S_1,S_2)}{\partial S_2} = 0,$$

we get

$$\frac{S_1-\mu}{\sqrt{\sigma^2+(S_1-\mu)^2}} = \frac{v_1-v_2-2c_1+2c_2+m}{v_2+m-v_1}, \tag{8}$$

$$\frac{a}{2}\frac{S_1+S_2-\mu}{\sqrt{\sigma^2+(S_1+S_2-\mu)^2}} + \frac{b}{2}\frac{S_1-\mu}{\sqrt{\sigma^2+(S_1-\mu)^2}} = \frac{a+b}{2} - (c_1-v_1). \tag{9}$$

Solving (8) for S_1, we obtain a closed-form optimal inventory level of the finished product against the worst distribution:

$$S_1^W = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{m+c_2-c_1}{c_1-c_2+v_2-v_1}} - \sqrt{\frac{c_1-c_2+v_2-v_1}{m+c_2-c_1}}\right) \quad (10)$$

By substituting (10) into (9) and solving for S_2, we obtain a closed-form optimal inventory level of raw material against the worst distribution:

$$S_2^W = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{p-m-c_2}{c_2-v_2}} - \sqrt{\frac{c_2-v_2}{p-m-c_2}}\right) - S_1^W. \quad (11)$$

If we use the quantity (S_1^W, S_2^W) instead of (S_1^F, S_2^F), the expected loss is equal to

$$\pi^F(S_1^F, S_2^F) - \pi^F(S_1^W, S_2^W).$$

This is the largest amount that we would be willing to pay for the knowledge of F. This quantity can be regarded as the *Expected Value of Additional Information* (EVAI) [10].

Example 1. This example is taken from Eynan and Rosenblatt [5]. Let the demand for the finished product be uniformly distributed, $U(0, 1000)$, and let $p = \$97, c_1 = \$51, c_2 = \$40$ and $m = \$14.6$ $(v_1 = v_2 = 0)$. Note that a unit which is purchased, or made in advance, costs \$51; while a unit which is made to order costs \$54.6.

We compare the performance of (S_1^W, S_2^W) with (S_1^U, S_2^U) where $U \in \mathcal{F}$ represents the uniform distribution. The results are $(S_1^W, S_2^W) = (330, 178)$ and $(S_1^U, S_2^U) = (246, 268)$, and the worst case expected profit $\pi^W(S_1^W, S_2^W)$ is \$9,295. The EVAI is

$$\pi^U(S_1^U, S_2^U) - \pi^U(S_1^W, S_2^W) = \$11,352 - \$11,300 = \$52.$$

Next we compare the procedure for the worst-case distribution with that for the normal distribution. The mean and the standard deviation of the demand are 500 and 288.7, respectively. The other data are the same as before. The results are $(S_1^N, S_2^N) = (302, 208)$ where $N \in \mathcal{F}$ represents the normal distribution. The EVAI is

$$\pi^N(S_1^N, S_2^N) - \pi^N(S_1^W, S_2^W) = \$12,187 - \$12,180 = \$7.$$

From the results of the above examples, we can conjecture the robustness of the distribution free approach.

We now test the profit differences between MIA, MTO and composite model. When $S_2 = 0$, the composite model reduces to MIA model, and the expected profit can be written as

$$\pi^W(S_1) = (p-v_1)\mu - (p-v_1) \times \frac{\sqrt{\sigma^2 + (S_1-\mu)^2} - (S_1-\mu)}{2} - (c_1-v_1)S_1. \quad (12)$$

Upon setting the derivative to zero, we obtain

$$\frac{S_1-\mu}{\sqrt{\sigma^2+(S_1-\mu)^2}} = \frac{(p-c_1)-(c_1-v_1)}{p-v_1}. \quad (13)$$

Solving (13) for S_1 results in the following closed-form solution:

$$S_1^W(\text{MIA}) = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{p-c_1}{c_1-v_1}} - \sqrt{\frac{c_1-v_1}{p-c_1}}\right). \quad (14)$$

When $S_1 = 0$, the composite model reduces to MTO model, and the expected profit can be written as

$$\pi^W(S_2) = (p-v_2-m)\mu + (v_2+m-p) \times \frac{\sqrt{\sigma^2+(S_2-\mu)^2} - (S_2-\mu)}{2} - (c_2-v_2)S_2. \quad (15)$$

Upon setting the derivative to zero, we obtain

$$\frac{S_2-\mu}{\sqrt{\sigma^2+(S_2-\mu)^2}} = \frac{(p-m-c_2)-(c_2-v_2)}{p-m-v_2}. \quad (16)$$

Solving (16) for S_2 results in the following closed-form solution:

$$S_2^W(\text{MTO}) = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{p-m-c_2}{c_2-v_2}} - \sqrt{\frac{c_2-v_2}{p-m-c_2}}\right). \quad (17)$$

Table 1
Optimal inventory levels and expected profits

Model	MIA	MTO	Composite model
S_1^W	485	–	330
S_2^W	–	508	178
π^W	\$9,018	\$8,747	\$9,295

From (11) and (17), we can obtain the following relationship. It means that the sum of the optimal inventory level of raw material and the optimal inventory level of the finished product under composite policy is equivalent to the optimal inventory level of raw material under make to order policy:

$$S_1^W + S_2^W = S_2^W(\text{MTO}). \tag{18}$$

The above relationship is quite intuitive. An optimal decision under MTO is to stock the same amount of the finished product under composite policy as raw material since the finished product cannot be inventoried. A similar equation for MIA (i.e. $S_1^W + S_2^W = S_1^W(MIA)$) does not hold since it requires higher inventory level of the finished product than that of composite policy and it is assumed that $c_2 - v_2 < c_1 - v_1$ (i.e. the risk of overstocking the finished product is higer than that of raw material).

Example 2. We use the same data as in Example 1. And, in this example we are going to compare the expected profit of MIA and MTO with that of the composite model. The optimal inventory levels against the worst distribution and their corresponding expected profits under the three models are summarized in Table 1.

3. A budget constraint

In the previous section, we have determined the optimal inventory levels without considering its budgetrary implications. However, there is often a space or budget constraint on the group of items [14]. Now we consider the composite model in the presence of a budget constraint on the investment in inventory.

In this section, we want to find the optimal inventory levels that maximize the expected profit against the worst possible distribution of the demand without exceeding the budget limit, say B. The problem can be formulated as follows:

$$\text{Min}_{S_1,S_2}\ C^W(S_1,S_2)$$
$$\text{subject to } c_1S_1 + c_2S_2 \leqslant B. \tag{19}$$

We form the Lagrangian function

$$L(S_1,S_2,\lambda) = a\frac{\sqrt{\sigma^2 + (S_1+S_2-\mu)^2} - (S_1+S_2-\mu)}{2} + b\frac{\sqrt{\sigma^2+(S_1-\mu)^2} - (S_1-\mu)}{2} + (c_1 - v_1)S_1 + (c_2 - v_2)S_2 - \lambda[c_1S_1 + c_2S_2 - B],$$

where λ is a Lagrange multiplier associated with the budget constraint. The Lagrange multiplier, λ, has an intersting economic interpretation; it is the value (in terms of increased total expected profit) of adding one more dollar to the available budget, B. By computing $\partial L/\partial S_1 = \partial L/\partial S_2 = 0$, we see that the solution is of the form

$$S_1^W(\lambda) = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{m + (c_2-c_1)(1-\lambda)}{(c_1-c_2)(1-\lambda)+v_2-v_1}} - \sqrt{\frac{(c_1-c_2)(1-\lambda)+v_2-v_1}{m+(c_2-c_1)(1-\lambda)}}\right), \tag{20}$$

$$S_2^W(\lambda) = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{p-m-c_2(1-\lambda)}{c_2(1-\lambda)-v_2}} - \sqrt{\frac{c_2(1-\lambda)-v_2}{p-m-c_2(1-\lambda)}}\right) - S_1^W(\lambda). \tag{21}$$

The problem is to find the smallest nonnegative λ such that $S_1^W(\lambda)$ and $S_2^W(\lambda)$ satisfies (19). A line search algorithm can be used to find the optimal value of λ. Clearly, we first need to check whether the unconstrained solution obtained from the previous section is optimal or not.

Table 2
Distributions for randomly generated data

Problem data	p	c_1	c_2	m
Range	[100, 110]	[50, 60]	[25, 35]	$[c_1 - c_2 + 5, c_1 - c_2 + 15]$

Example 3. We continue Example 1 with the modification that there is a budget limit $B = \$25,000$. Using a line search algorithm, the optimal order quantities are $(S_1^W, S_2^W) = (366, 159)$ and $(S_1^U, S_2^U) = (293, 252)$. The optimal Lagrange multiplier values are 0.0566 for the worst distribution and 0.0615 for the uniform distribution. Expected profit is \$9266 for the worst distribution and \$11 300 for the uniform distribution. The value of the distributional information when demand is uniformly distributed is

$$\pi^U(293, 252) - \pi^U(366, 159) = \$11,300 - \$11,245 = \$55.$$

4. Computational results

In order to investigate the robustness of the distribution free approach, 1000 test problem instances were generated randomly from uniform distributions on the given intervals. Table 2 shows the distributions for the data set. The mean and the standard deviation are fixed as 500 and 288.7, respectively. Salvage values are assumed to be zero.

Table 3 shows the comparative results using several different distributions including normal, uniform, t, and triangle distributions. We have reported the minimum, mean, and maximum ratios of $[\pi^F(S_1^F, S_2^F)]/[\pi^F(S_1^W, S_2^W)]$ for the 1000 instances. Most of the ratios are quite close to 1 which enables us to use the distribution free ordering rule in the absence of the specific form of the distribution function. It can be seen that the rule works best for the normal distribution. Also, we know from the statistical property that the normal distribution maximizes entropy subject to a fixed mean and variance (see Cozzolino and Zahner [15]). We conjecture that there are some kinds of connections between these two facts.

Table 3
Results of comparative examples

Distribution	Ratio	Minimum ratio	Mean ratio	Maximum ratio
Normal	$\frac{\pi^N(S_1^N, S_2^N)}{\pi^N(S_1^W, S_2^W)}$	1.00004	1.00127	1.00207
Uniform	$\frac{\pi^U(S_1^U, S_2^U)}{\pi^U(S_1^W, S_2^W)}$	1.00291	1.00997	1.01706
t	$\frac{\pi^t(S_1^t, S_2^t)}{\pi^t(S_1^W, S_2^W)}$	1.00097	1.00290	1.00477
Triangle	$\frac{\pi^{TR}(S_1^{TR}, S_2^{TR})}{\pi_1^{TR}(S_1^W, S_2^W)}$	1.00004	1.00211	1.00391

5. Concluding remarks

We have derived the optimal production policy for the composite model where only the mean and the variance of the demand are known. Based on the numerical examples and computational experiments we conjecture that the distribution free approach is robust. Further theoretical investigation on robustness of Scarf's ordering rule might be an interesting research problem. The model developed here extends to the lost sales case such that only a fraction of the customers will be waiting until the product is made from a raw material. We hope that this paper will help disseminate Scarf's minmax distribution free approach.

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