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Scheduling-location problem with drones

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Abstract

Drone operation, a new driving force for logistics innovation, is struggling to overcome practical challenges. One of the concerns for drone utilization is limited flight ranges, and different concepts of facilities are continually developed to support drone delivery. These new facilities prompt the need to integrate decision-making across different phases. In particular, the deployment of facilities that complement the physical limitations of drones and the scheduling of drones to perform delivery tasks are closely related. Therefore, we developed a scheduling-location problem with drones, a new methodology for integrating operational and strategic planning decisions. The integrated decision-making determines the location of the drone facilities by not only considering the critical distance of facilities but also by taking into account whether the delivery schedule is implemented. In our model, additional drone facilities are sometimes opened considering available drones due to the feasibility of the delivery schedule. An extended formulation and a restricted master heuristic are proposed to solve problems time-efficiently. Computational results show that the restricted master heuristic outperforms the mathematical model in finding solutions for large-scale instances. The developed model and heuristic algorithm provide drone delivery services even in areas that are not easily reachable by drones due to being located far from the warehouse and can be effectively applied to humanitarian logistics.

Keywords: facility location problem; scheduling; drone; last mile delivery

1. Introduction

Drones are playing a growing role in a variety of business environments, including logistics, creating significant market potential. The reasons why drone technology has emerged are manifold. First, drones can fly, so they are less restricted in movement and are not affected by traffic congestion. This is the main strength of drones in time-sensitive industries such as medical logistics and food delivery. Second, many industries can benefit from unmanned operations because labor costs can be saved. In addition, drones can be adopted in dangerous industries that are difficult

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for humans to perform. A third important advantage is that drone operation is environmentally friendly. Drones are battery-powered and use electricity as their power source, emitting less carbon than traditional means of transportation. Considering that sustainability is an issue that many industries are paying attention to now and in the future, the rapid market growth of drones is understandable. Accordingly, major logistics companies struggle to apply drone technology and commercialize drone delivery.

Drones appear to be a key technology for logistics innovation, but the technology still needs to be advanced to overcome practical challenges. Especially, limited battery capacity is a major concern for drone utilization. Except for tethered drones that receive energy through a power cord, most drones use a relatively small capacity battery (Muttin, 2011; Gu et al., 2016). Therefore, relatively few customers can benefit from drone-delivery services because most existing distribution centers have been built far from the central city. For this reason, leading retailers such as Amazon and Wal-Mart are working to build more distribution centers near cities, but the cost of building distribution centers is still a big obstacle. An emerging methodology to overcome the short range of drone delivery is the use of a cargo truck equipped with a drone docking system (Murray and Chu, 2015; Kim and Moon, 2018; Wang and Sheu, 2019). Plans to use streetlights, gas stations, or church steeples as drone docking stations are also being studied as alternatives. Moreover, different concepts of drone facilities are continually proposed to address these issues (Saad et al., 2014; Gentry et al., 2016; Jeong et al., 2020). In this paper, we study the methodology for determining the location of drone stations, which are simple facilities placed between large warehouses and the central city, along with planning drone–delivery schedules.

A challenging issue of utilizing drone stations is the need for integration of decision phases. Decision problems in the supply chain can be categorized into three types: strategic, tactical, and operational, based on the time planning horizon (Misni and Lee, 2017). Strategic, tactical, and operational decision problems in implementing drone–delivery operations have been independently well-studied (Kim et al., 2019; Chen et al., 2021; Saleu et al., 2022). As the advancement of drone operation complicates and diversifies supply chain decisions, many researchers have attempted to integrate decision problems at different phases. Nevertheless, the number of academic research investigating the integrated problem of facility location and scheduling, the ScheLoc problem, is yet limited. Our contribution is to propose a new variant of the ScheLoc problem, the scheduling-location problem with drones (ScheLoc-D), a methodology for integrating operational planning decisions with strategic planning decisions to implement drone–delivery services. Specifically, an advanced logistics system is formulated in which drone stations are installed at optimal locations to provide direct delivery services.

As can be guessed from the order of terms called "ScheLoc," existing studies that attempted to integrate the scheduling and location problems focused more on scheduling (Hamacher and Hennes, 2007; Elvikis et al., 2009; Kalsch and Drezner, 2010; Rajabzadeh et al., 2016; Li et al., 2022). Most studies aim to minimize the makespan from a traditional scheduling perspective, and the economic outlook was less considered. Only a few studies investigated the ScheLoc problem considering drone operation from the perspective of the facility location problem (FLP). The ScheLoc-D developed in this study is FLP-oriented rather than scheduling-oriented, and the objective function is set to minimize economic cost, not makespan. Therefore, our model can cost-efficiently provide drone delivery services even in areas that are not easily reachable by drones due to being located far from the warehouse. Moreover, the ScheLoc-D can be effectively applied to humanitarian logistics and disaster management, sending relief, or medical supplies to points of need that are difficult to access due to infrastructure damaged by disasters.

The remainder of this study is organized as follows. The related literature is reviewed in Section 2. Section 3 provides the problem description and the mathematical model. In Section 4, the problem was reformulated to the extended formulation and solved by the restricted master heuristic, which provides a time-efficient high-quality solution. The computational experiments and their results are summarized in Section 5. The difference between traditional FLP and the ScheLoc-D has been verified through numerical experiments. Finally, concluding remarks on this study are provided in Section 6.

2. Literature review

Continuous technological advances have led to the commercialization of drone operations. Drone research has become mainstream in several areas, such as logistics (Asadi and Pinkley, 2021; Lemardelé et al., 2021), disaster management (Park et al., 2020; Zhang et al., 2021), surveillance (Panadero et al., 2020), public security (He et al., 2017), traffic monitoring (Barmpounakis and Geroliminis, 2020), and agriculture (Tokekar et al., 2016). Other civil applications of drones are summarized in Otto et al. (2018). In addition, new variants of routing problems with drones are being developed continuously (Wang and Sheu, 2019; Schermer et al., 2019; Murray and Raj, 2020; Campbell et al., 2021; Nguyen et al., 2022; Vu et al., 2022), An extensive overview of drone-aided routing problems can be found in Chung et al. (2020), Macrina et al. (2020), and Rojas Viloria et al. (2021). Unlike the studies above, this study conducts research on the location of drone stations, which are auxiliary facilities for the direct delivery of drones, not the routing of drones.

The main challenges associated with drone delivery are limited shipping range and restricted payload in weight and volume. In addition, uncertainty exists in decision-making, as the reliability of drone delivery has not yet been verified. Therefore, most studies on the facility location problem with drones considered the limited range and payload and dealt with the uncertainty of drone delivery. Chauhan et al. (2019) developed a coverage-based facility location problem with drones, taking into account the battery and weight constraints of drones. Because of the battery constraints of drones, Chauhan et al. (2019) assumed that drones make multiple one-to-one deliveries from the depot locations based on real cases with deliveries of blood supplies. Kim et al. (2019) developed stochastic programming to determine the locations of drone facilities considering the uncertain flight range of drones. Kim et al. (2019) assumed that the flight distance of drones follows a probability distribution because uncertainty in the flight distance of drones can be estimated through performance tests. Shavarani et al. (2019) studied the fuzzy multi-level facility location model concerned with the uncertain distance capacity of the drones. Other existing research also studied the drone facility location problem considering uncertainty, but they considered the uncertainty of demand rather than the uncertainty of drone operation (Ghelichi et al., 2022; Zhu et al., 2022). As such, several past researchers dealt with the uncertainty of drone delivery based on the FLP. However, uncertainties inherent in drone delivery can be stabilized and managed, as noted in Kim et al. (2019). Therefore, we focused on the development of a new variant of drone-based delivery systems and the operational aspects of the system.

Scheduling and location problems are each an important area of operations research, as evidenced by their rich research history. Many papers on these topics have been published since their respective problems were established, but only a few focused on studying these problems from an integrated perspective. In the classic ScheLoc problem, the tactical decision, selecting locations for machines, and the operational decision, scheduling of the jobs, are integrated. One of the applications comes from the mining industry, where the minerals should be moved to the crushing machines. The best positions for crushing machines and the optimal schedule of minerals should be determined. The usage of movable machines in the production system can also be considered as an application of the ScheLoc problem (Kalsch, 2009). Another application can be found in a container harbor, where the containers should be loaded onto ships (Kalsch and Drezner, 2010). In this application, the decisions on the positions of ships on the berth (location problem) and the sequence for loading containers (scheduling problem) should be determined simultaneously.

A ScheLoc problem has been mathematically formulated and studied since the 21st century. Hamacher and Hennes (2007) first proposed an integrated model of scheduling and location problems, in which the release times for jobs are determined based on the locations of the machine to which the job is assigned. They considered a single-machine ScheLoc problem and proposed a polynomial algorithm in which the schedule of jobs is determined by the earliest release date (ERD) rule. Elvikis et al. (2009) considered a single machine planar ScheLoc problem, and three polynomial algorithms based on the ERD rule were developed. Kalsch and Drezner (2010) investigated a single machine ScheLoc in the plane to minimize the makespan and the total completion time. Based on the properties of models, a branch-and-bound approach is developed. Rajabzadeh et al. (2016) proposed the mathematical model for the parallel machine ScheLoc problem in discrete and continuous spaces to minimize the makespan. Heßler and Deghdak (2017) investigated the parallel machine ScheLoc problem, in which the candidate locations for machines are discrete. An integer programming (IP) model and different versions of clustering heuristics, in which jobs are split into clusters, are proposed. Liu and Liu (2019) proposed a parallel machine ScheLoc problem under stochastic processing times with only partial distributional information to minimize the cost of operating machines and control the service level. The service level is measured by the probability of ensuring an on-time schedule. Krumke and Le (2020) studied a robust single-machine ScheLoc problem with uncertain edge lengths of a given tree. They considered the concept of gamma robustness and proposed a polynomial time algorithm. Wang et al. (2020) and Kramer and Kramer (2021) investigated the discrete parallel machine ScheLoc problem. Wang et al. (2020) designed two heuristic procedures and a polynomial-time algorithm. Kramer and Kramer (2021) applied a column generation approach and developed three heuristic procedures. Recently, Li et al. (2022) developed three versions of the discrete parallel machine scheduling and location problems that outperform state-of-the-art formulations and a new logic-based Benders decomposition algorithm to solve practical instances. Their objective is to minimize the maximum completion time of all jobs, as in most studies in this field. Unlike the existing research above, we provide a formulation and a solution approach in a different way because our model minimizes the economic cost from the FLP perspective.

In summary, some excellent research is leading the field of the ScheLoc problem. However, a relatively small number of existing studies indicate that the history of research on the ScheLoc problem is not long and that the field has not flourished enough. Given that the decisions for facility location and delivery schedules are inherently interrelated in logistics with drones, it is promising to study the ScheLoc problem considering drones. Studies that seem most closely related to our study are Ghelichi et al. (2021) and Gentili et al. (2022). Their studies considered the ScheLoc and drone delivery concurrently. Ghelichi et al. (2021) developed an optimization model that schedules a set of trips to serve medical items in humanitarian and healthcare logistics. Their problem determines locations for charging stations and schedules the trips such that the total completion time to serve all demand points is minimized. Gentili et al. (2022) studied the problem of locating the platforms and determining the order that the platform serves demand points considering the perishability of items. Both studies used a discrete-time approximation to propose a more efficient and tractable decision-making tool. On the other hand, our model, the ScheLoc-D, is developed based on continuous-time models while existing studies developed time-slot formulation where flight duration and charging time are expressed as discretized time slots. A discussion of the temporal perspective of the ScheLoc will be further analyzed in Section 3.2. Above all, no research has been conducted to develop the ScheLoc problem considering drones with time window constraints, and our study can lead to a practical incentive to mature the field.

3. Problem description and mathematical model

This section provides a detailed definition of the ScheLoc-D. It is assumed that drones can only deliver to one customer in one flight due to physical limitations. The assumption is used in most drone research, including the problem of coordinated delivery of drones and trucks (Agatz et al., 2018; Chauhan et al., 2019; Wang and Sheu, 2019). The ScheLoc-D consists of determining the location of the facilities and the scheduling of delivery of drones deployed at each facility. Figure 1 presents an overview of the ScheLoc-D. Since the ScheLoc-D involves decision-making at the operational stage, it designs a practical network differently from the classical FLP. As can be seen in Fig. 1, the problem not only aims to cover all customers but also to determine the order of delivery. In order to understand the problem clearly, the customers in the illustrative example are temporarily classified into three categories. In this example, an urgent customer is someone who has a time window that closes to the beginning of the planning horizon, and a generous customer is someone who has a very wide time window that spans the entire planning horizon. The remaining customers are classified as general customers. In most FLPs, customer allocation is determined according to the capacity of the facility and critical distance, and the operational decision-making on how to plan the delivery schedule is omitted. However, the ScheLoc-D takes into account the feasibility of delivery schedules to determine the coverage of the facility. If only one drone is deployed in each facility to perform the delivery mission when two customers feature the same time window of urgency and tightness, an additional facility should be established to deliver both customers.

3.1. Mathematical model

The developed model makes decisions simultaneously, which are generally classified into strategic and operational levels. The objective of the ScheLoc-D is to find the optimal locations of drone facilities and feasible delivery schedules according to the minimization of total relevant costs. The

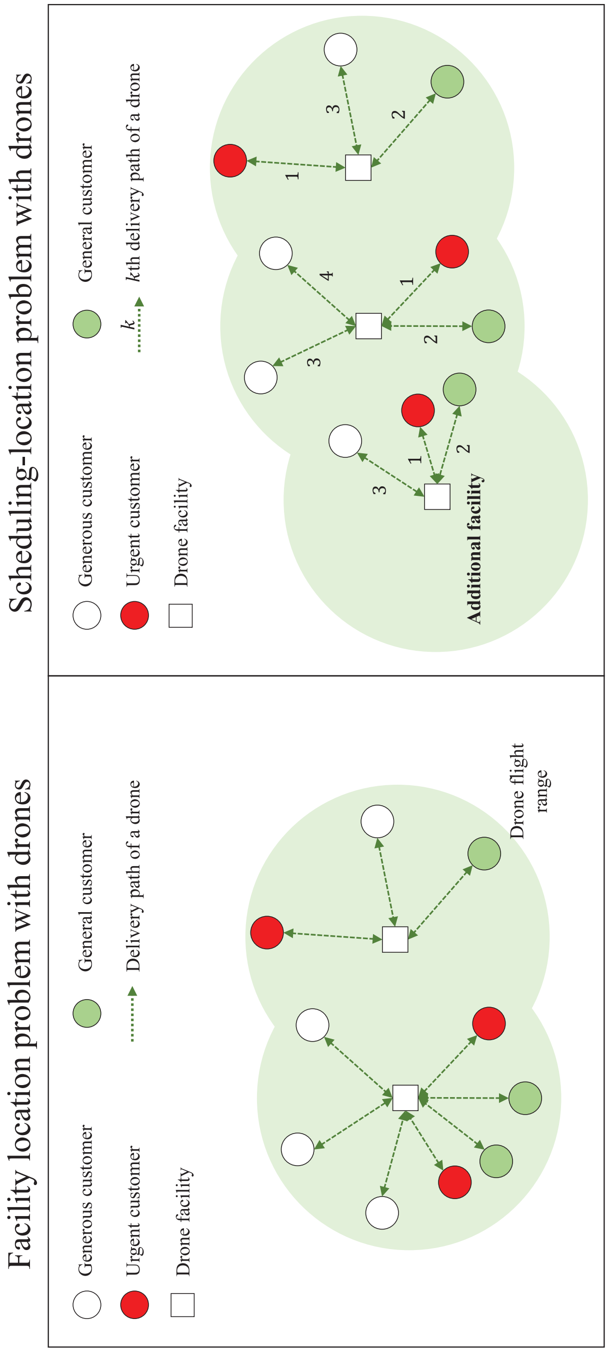

Fig. 1. Overview of the ScheLoc-D.

developed drone facility location problem involves four given sets: potential locations of facilities, customer zones, available drones, and the departures of the drone. $|J| = n$ is the number of customers to be satisfied. $|D| = m$ is the number of drones available at each facility, and $|R|$ is an upper bound of the repeated departures of each drone. $|R|$ can be roughly set to $n - m + 1$, considering the case that a drone at a single facility should serve all remaining demands while other drones at the same facility serve only one demand. Based on the following sets, parameters, and decision variables, the mathematical model of the ScheLoc-D is developed.

Sets

I set of candidate locations at which facilities can be sited.
J set of customer's locations
D set of available drones in each operating facility
R set of repeated departures of each drone from the facility for delivery

Parameters

τ_{ij} travel time between candidate location $i \in I$ and customer zone $j \in J$
γ maximum travel time of a drone (shipping range)
s_j service time required to meet the demand of customer $j \in J$
f_i opening cost of a drone facility at location $i \in I$ (fixed cost)
ρ cost factor for travel time (parameter for variable cost)
e_j earliest time that customer i can receive delivery
l_j latest time that customer i can receive delivery
M sufficiently large constant

Decision variables

$$x_{ij}^{dr} = \begin{cases} 1, & \text{if drone } d \in D \text{ deployed at facility } i \in I \text{ covers customer } j \in J \text{ with } r \in R\text{th shipment} \\ 0, & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if a drone facility is operated at site } i \in I \\ 0, & \text{otherwise} \end{cases}$$

T_i^{dr} time when drone $d \in D$ departs $r \in R$th shipment from facility $i \in I$

[Standard formulation, SF]

$$\min \quad \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}\sum_{d\in D}\sum_{r\in R} 2\rho\tau_{ij}x_{ij}^{dr}, \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in I}\sum_{d\in D}\sum_{r\in R} x_{ij}^{dr} = 1, \qquad \forall j \in J, \tag{2}$$

$$\sum_{j\in J}\sum_{d\in D}\sum_{r\in R} x_{ij}^{dr} \le n y_i, \qquad \forall i \in I, \tag{3}$$

$$\sum_{j\in J} x_{ij}^{dr} \leq 1, \qquad \forall i \in I, \forall d \in D, \forall r \in R, \tag{4}$$

$$\sum_{j\in J} x_{ij}^{d,r+1} \leq \sum_{j\in J} x_{ij}^{d,r}, \qquad \forall i \in I, \forall d \in D, \forall r \in R \setminus \{|R|\}, \tag{5}$$

$$2\tau_{ij} x_{ij}^{dr} \leq \gamma, \qquad \forall i \in I, \forall j \in J, \forall d \in D, \forall r \in R, \tag{6}$$

$$T_i^{d,r} + \sum_{j\in J} \{(2\tau_{ij} + s_j) x_{ij}^{dr}\} \leq T_i^{d,r+1}, \quad \forall i \in I, \forall d \in D, \forall r \in R \setminus \{|R|\}, \tag{7}$$

$$e_j x_{ij}^{dr} \leq T_i^{dr} + \tau_{ij} x_{ij}^{dr} \leq l_j + M(1 - x_{ij}^{dr}), \quad \forall i \in I, \forall j \in J, \forall d \in D, \forall r \in R, \tag{8}$$

$$T_i^{dr} \in \mathbb{R}^+ \quad \forall i \in I, \forall d \in D, \forall r \in R, \tag{9}$$

$$x_{ij}^{dr} \in \mathbb{B}, \qquad \forall i \in I, \forall j \in J, \forall d \in D, \forall r \in R, \tag{10}$$

$$y_i \in \mathbb{B}, \qquad \forall i \in I. \tag{11}$$

The objective function (1) minimizes the sum of the total relevant costs comprising the fixed costs of the facilities and the variable costs of serving demand from these facilities. Constraints (2) guarantee that each customer is assigned to delivery schedules. In other words, all demands are covered. Constraints (3) are linking constraints of the decision for opening a facility. Constraints (4) and (5) support defining a feasible delivery schedule for a drone. Constraints (6) limit the shipping range of each facility in consideration of the battery capacity of the drone. Constraints (7) and (8) ensure the schedule feasibility with respect to time windows. M should be large enough that the potentially optimal solution does not violate the constraints. In this case, M can be set to $\max_{i\in I, j\in J}\{l_j + s_j + \tau_{ij}\}$. Constraints (9) demonstrate the integer nature of the decision variables, and Constraints (10) and (11) demonstrate the binary nature of the decision variables.

3.2. Discussion of the ScheLoc-D

The factors that make the ScheLoc-D more difficult compared to a traditional FLP are the availability of drones and the time window of customers. The customer distribution and the number of candidate locations also affect the difficulty of the problem, but they affect similarly in the FLP. The biggest difference between the developed problem and the FLP is that the capacity of a facility is not simply a given constant but includes the decision-making of whether a delivery schedule that the facility can cover is planned, given its customers (i.e., demand). When a sufficiently large number of drones can deliver anywhere and the all time windows are wider than the planning horizon, the ScheLoc-D is an equivalent problem to the uncapacitated FLP. The uncapacitated FLP on general graphs is $\mathcal{NP}$-hard, by reduction from the set covering problem, one of Karp's 21 $\mathcal{NP}$-complete problems (Karp, 1972). Therefore, the ScheLoc-D also belongs to the class of $\mathcal{NP}$-hard optimization problem. In order to highlight the originality that the ScheLoc-D differs from the related models found in the existing literature, it is necessary to discuss several features of the model.

This study covers different aspects of the FLP involving time-dependent decision variables. Although most of the existing facility-location models focused on a discrete setting, the emphasis for the ScheLoc-D is put on the time window constraints defined over a continuous-time planning horizon. In the continuous-time models, there are no given moments for making decisions. The best time to make a decision is only known after finding the optimal solution. These features complicate the problem, but a continuous-time model allows for better scheduling and, thus, greater flexibility of drone delivery than discrete-time model. Although several works by Drezner and Wesolowsky (1991), Orda and Rom (1991), Puerto and Rodríguez-Chía (1999), and Farahani et al. (2009) explore the features, continuous-time facility location problems are less covered. Therefore, the ScheLoc-D can provide a new breath in this field.

Features in terms of model structure are as follows. Linear programming (LP)-relaxed bound for the ScheLoc-D can be very weak due to time window constraints. In addition, the SF of the ScheLoc-D is formulated based on a four-index formulation. Since the index contains the number of drone departures, which can be known after finding a solution, a large number of variables should be generated by setting $|R|$ close to n to deal with the worst case. Furthermore, the ScheLoc-D has symmetric solution space. Different solutions to the model can correspond to the same objective value. For example, swapping the entire delivery missions assigned to any two drones in the same facility to each other generates different solutions with the same objective function value. Considering the case where all facilities and drones are used, up to $(m!)^n$ identical delivery plans with different indices can be generated. That is, equivalent $(m!)^n$ solutions may exist in the worst case. Therefore, solving the SF of the ScheLoc-D with a commercial solver is challenging, and an efficient solution approach is necessary to solve large instances. To deal with the computational complexity, in Section 4, we developed a restricted master heuristic that can solve the ScheLoc-D.

4. Pattern-based solution approach for the ScheLoc-D

4.1. Set-covering reformulation

Minkowski's theorem proves that a polyhedron can be represented by its extreme points and extreme rays instead of the original variables (Conforti et al., 2010). In other words, a vector in a polyhedron can be represented as a summation of a convex combination of the extreme points and a conic combination of the extreme rays of the polyhedron. Minkowski's theorem enables the Dantzig–Wolfe decomposition. Dantzig–Wolfe decomposition reformulates the original problem by decomposing the block-diagonal structure of the constraint into smaller subproblems and the extended formulation (Dantzig and Wolfe, 1960). By definition, even when the extended formulation is LP-relaxed, tighter bounds than the LP-relaxed bound of the original formulation can be provided. This is because the LP relaxation of the extended formulation is the dual of the Lagrangian subproblem. Thus, the LP-relaxed bound of the extended formulation has the same value as the Lagrangian dual bound.

The ScheLoc-D is difficult because the FLP and the scheduling, which are difficult to determine, even for each, are combined. If scheduling-related decisions are included in variables, the problem could be solved relatively easily based on the improved LP-relaxed bound. A decomposition-based approach can handle the relationship between the decision variables implicitly. Garfinkel et al. (1974) applied Dantzig–Wolfe decomposition to an uncapacitated facility location problem successfully, and since then Dantzig–Wolfe decompositions have been widely employed for various types of facility location problems (Garfinkel et al., 1974; Barahona and Jensen, 1998; Klose and Drexl, 2005; Wu et al., 2020; Ryu and Park, 2022). Thanks to the superiority of the technique proven in existing studies, we applied Dantzig–Wolfe decomposition to the ScheLoc-D. The solution of the ScheLoc-D consists of individual decisions about each facility, which constructs a set-covering structure. Based on the Dantzig–Wolfe decomposition, the SF can be reformulated with pattern-based decisions. Each variable in the extended formulation defines a set of customers covered by a facility. In other words, several feasible allocations of the set of demand points for each facility are given in advance. Ω_i is a set of feasible columns for a facility at site i. The parameters and the decision variables of the extended formulation model of the ScheLoc-D are presented as follows:

Parameters

c_{ik} cost associated withpattern k of facility i

$$a_{ij}^k = \begin{cases} 1, & \text{if pattern } k \text{ of facility } i \text{ covers customer } j \\ 0, & \text{otherwise} \end{cases}$$

Decision variables

$$z_{ik} = \begin{cases} 1, & \text{if pattern } k \text{ of facility } i \text{ is used} & \forall i \in I, \\ 0, & \text{otherwise} & \forall k \in \Omega_i \end{cases}$$

The cost of each column is defined as $c_{ik} := f_i + 2\rho \sum_{j \in J} \tau_{ij} a_{ij}^k$. The set covering model of the ScheLoc-D is represented in the following integer program:

[Set covering model for the master problem]

$$\min \quad \sum_{i \in I} \sum_{k \in \Omega_i} c_{ik} z_{ik}, \tag{12}$$

$$\text{s.t.} \quad \sum_{i \in I} \sum_{k \in \Omega_i} a_{ij}^k z_{ik} \geq 1 \qquad \forall j \in J, \tag{13}$$

$$\sum_{k \in \Omega_i} z_{ik} \leq 1 \qquad \forall i \in I, \tag{14}$$

$$z_{ik} \in \mathbb{B} \qquad \forall i \in I, \forall k \in \Omega_i. \tag{15}$$

The extended formulation only remains the set-covering structure, while the scheduling-related constraints are considered implicitly in the column. Objective function (12) is equivalent to Objective function (1) and seeks to minimize total cost. Set covering constraints (13) impose that all customers be covered. Constraints (14) restrict that each facility can be opened at most once at a site. The binary requirements on the pattern-choice variables are expressed by Constraints (15). By Minkowski's theorem, every solution of the compact formulation can be represented in the extended formulation. If Ω_i contains every feasible pattern for every facility i, then the solution set of the master problem defines the convex hull of the ScheLoc-D. However, this requires an exponential number of patterns. The pattern (column)-generating technique can be implemented to solve the problem. Let π_j and σ_i be dual prices associated with Constraints (13) and (14). To generate patterns, the subproblem of the ScheLoc-D is defined as follows:

[Subproblem, SP_i]

$$\min \quad f_i + \sum_{j \in J} \sum_{d \in D} \sum_{r \in R} (2\rho\tau_{ij}x_j^{dr} - \pi_j) - \sigma_i, \tag{16}$$

$$\text{s.t.} \quad \sum_{j \in J} x_j^{dr} \leq 1, \qquad \forall d \in D, \forall r \in R, \tag{17}$$

$$\sum_{j \in J} x_j^{d,r+1} \leq \sum_{j \in J} x_j^{d,r}, \qquad \forall d \in D, \forall r \in R \setminus \{|R|\}, \tag{18}$$

$$2\tau_{ij}x_j^{dr} \leq \gamma, \qquad \forall j \in J, \forall d \in D, \forall r \in R, \tag{19}$$

$$\sum_{j \in J} \{(2\tau_{ij} + s_j)x_j^{dr}\} \leq T^{d,r+1} - T^{d,r}, \qquad \forall d \in D, \forall r \in R \setminus \{|R|\}, \tag{20}$$

$$e_j x_j^{dr} \leq T^{dr} + \tau_{ij}x_j^{dr} \leq l_j + M(1 - x_j^{dr}), \quad \forall j \in J, \forall d \in D, \forall r \in R, \tag{21}$$

$$x_j^{dr} \in \mathbb{B}, \qquad \forall j \in J, \forall d \in D, \forall r \in R, \tag{22}$$

$$T^{dr} \in \mathbb{R}^+ \qquad \forall d \in D, \forall r \in R. \tag{23}$$

Constraints (17)–(23) correspond to Constraints (4)–(10) of any specific facility i, and index i is omitted in decision variables x_j^{dr} and T^{dr}. Objective function (16) minimizes the reduced cost of allocation and ensures that a negative reduced cost pattern is found when one exists. Given the difficulty of solving the subproblem, a large part of the computing time is devoted to solving the subproblem. Therefore, maintaining a reasonable number of variables is essential to solving the problem efficiently. As is well known for various applications, heuristics can be used to solve the subproblem as long as they succeed in generating negative reduced cost columns. In the following sections, a heuristic approach to solve the ScheLoc-D is proposed.

4.2. Restricted master heuristic

Heuristics based on exact methodologies have gained some recognition from both researchers and practitioners. To solve the large-sized problem in a reasonable computing time, a *restricted master heuristic* (RMH), one of the most widely used heuristics related to column generation, is developed. In this approach, the pattern-based formulation of the master problem is restricted to a subset of variables, and it is solved as a static IP. A large part of the computing time is usually devoted to solving the subproblem and generating columns. To deal with the issue, the restricted set of columns can be generated heuristically, and the ScheLoc-D can be solved with the extended formulation using optimization solvers.

The subproblem in Section 4.1 is equivalent to the parallel machine scheduling problem given by considering deliverable customers as a set of jobs. Therefore, the attractive column can be generated by the following heuristics. Each facility contains $|N_i| = n_i$ jobs (transportation requests), $\mathcal{J} =< J_1, \ldots, J_{n_i} >$, and m identical machines (drones). Each job J_j is characterized by the quadruple (e'_j, l'_j, p_j, w_j). The interpretation is that job J_j is available at time e'_j, *release time* (earliest time that customer j can receive a delivery); it must be delivered by time l'_j, *deadline* (latest time that customer j can receive a delivery); its *processing time* (sum of round-trip travel time and service time at customer j) is p_j; and w_j is the *weight* (variable cost converted to a negative value) associated with the job. In order to satisfy the customer's time window, e'_j and l'_j that must depart from facility i are newly defined, and the detailed equations are as follows.

$$e'_j(l'_j) = e_j(l_j) - \tau_{ij} - s_j.$$

A feasible scheduling of job J_j on machine $d \in D$ at time t, $e'_j \le t \le l'_j$, is referred to as a *job instance*, denoted by $J_{jd}(t)$. A job instance can be represented by an interval on the timeline. Interval $J_{jd}(t) = [t, t + p_j]$ belongs to job J_j, and many intervals may belong to a job. Job instances $J_{1d}(t_1), \ldots, J_{hd}(t_h)$ are a feasible schedule on a machine if the corresponding intervals do not overlap, and they belong to distinct jobs.

Based on the work by Bar-Noy et al. (2001), a dispatching rule for drones in facility i (Algorithm 1) is developed to maximize the throughput of all schedules. At each time step t, the algorithm schedules the job instance that finishes first among all jobs that can be scheduled at t or later. Procedure $Next(t, d, \mathcal{J})$ needs to be defined to execute Algorithm 1. The procedure determines the job instance $J_{jd}(t')$, $t' \ge t$, such that $t' + p_j$ is the earliest among all job instances of jobs in $\mathcal{J}$ that start at time t or later on machine (drone) d. If no such a job exists, the procedure returns *null*. Note that k^d jobs are scheduled on machine (drone) $d \in D$ without loss of generality, and tie breaks arbitrarily. A pseudocode of the algorithm is given in Algorithm 1.

The following properties of Algorithm 1 are used in the analysis of the developed algorithms. Based on Proposition 1, Proposition 2 ensures the performance of the Algorithm 1.

Proposition 1 (Proposition 3.1 in Bar-Noy et al. (2001))**.** *Let S be the schedule found by Algorithm 1 for a job $\mathcal{J}$, and F be any feasible schedule on a machine (drone) among the jobs in $\mathcal{J} \setminus S$. Then,* $|F| < |S|$

Proof. For each job instance in F, there exists an interval in S that overlaps with it and terminates earlier. Otherwise, Algorithm 1 would have chosen this interval. The proposition follows from the feasibility of F, since at most one interval in F can overlap with the endpoint of any interval in S. □

Proposition 2 (applied from Theorem 3.2 in Bar-Noy et al. (2001))**.** *Algorithm 1 generates a pattern within an approximation factor of 2.*

Proof. Let $S(m) = S^1 \cup \cdots \cup S^m$ be the output of Algorithm 1 and let $\mathrm{OPT}(m) = O^1 \cup \cdots \cup O^m$ be the sets of intervals scheduled on the m machines (drones) by an optimal solution OPT. Let $R(m) = R^1 \cup \cdots \cup R^m$, where $R^d = O^d \setminus S(d)$ is the set of all jobs scheduled by OPT on machine (drone) d that Algorithm 1 did not schedule on any machine. Let $OS = OPT(m) \cap S(m)$ be the set of

Algorithm 1. Greedy algorithm for maximizing throughput

Input: $\mathcal{J} =< J_1, \ldots, J_n >$
Output: Schedules for drone delivery

Initialization
$t^d = 0, \forall d \in D$
if $Next(\min_{j \in N_i}\{e'_j\}, 1, \mathcal{J}) \neq null$ **then**
  $J_{j_1,1}(t^1) = Next(\min_{j \in N_i}\{e'_j\}, 1, \mathcal{J})$
  $\mathcal{J} := \mathcal{J} \setminus J_{j_1}$
  $t^1 = \min_{j \in N_i}\{e'_j\} + p_{j_1}$
  Current time, $ct = \min_{d \in D} t^d$
  Current machine (drone), $cm = \operatorname{argmin}_{d \in D} t^d$
else
  return empty schedule
end if

Main loop
while $Next(ct, cm, \mathcal{J}) \neq null$ **do**
  for kth iteration of machine (drone) cm,
  $J_{j_k,cm}(t^{k_{cm}}) = Next(ct, cm, \mathcal{J})$
  $\mathcal{J} := \mathcal{J} \setminus J_{j_k}$
  $t^{cm} \leftarrow t^{k_{cm}} + p_{j_k}$
  Current time, $ct = \min_{d \in D} t^d$
  Current machine (drone), $cm = \operatorname{argmin}_{d \in D} t^d$
end while
return schedules $\{J_{j_1,d}(t^1), \ldots, J_{j_{k_d},d}(t^{k_d})\} \quad \forall d \in D$

jobs scheduled by both Algorithm 1 and OPT. It follows that $\mathrm{OPT}(m) = OS \cup R(m)$. Proposition 1 implies that $|R^d| \leq |S(d)|$. This is true since R^d is a feasible schedule on machine (drone) d among the jobs that were not picked by Algorithm 1 while constructing the schedule for machine (drone) d. Since sets R^m are mutually disjoint and the same holds for sets S^m, $|R(m)| \leq |S(m)|$. Since $|OG| \leq |S(m)|$, we get that $|\mathrm{OPT}(m)| \leq 2|S(m)|$ and the theorem follows. □

Algorithm 2 was developed to maximize the sum of weights. The algorithm is inspired by online call admission algorithms in Baruah et al. (1992), Albers et al. (2000), and Bar-Noy et al. (2001). Job instances (or intervals) are checked one by one, and, for each job instance, it is determined whether to be scheduled. Even if a job instance is scheduled, the decision may change in the future, while previously unscheduled jobs are no longer considered. Similar to procedure $Next(t, d, \mathcal{J})$, defined in Algorithm 1, job instances are reordered based on when they finish early. Sets $\mathcal{S}$, the set of currently scheduled intervals, and $\mathcal{U}$, the set of unprocessed job instances, are defined. When a new job instance, $J_{jd}(t)$, is considered according to the sorted order, it is immediately scheduled if it does not overlap with any other interval in $\mathcal{S}$. If $J_{jd}(t)$ overlaps with one or more job instances in $\mathcal{S}$, it is accepted in schedules only if its weight is less than β times the sum of the weights of all overlapping job instances. $J_{jd}(t)$ is added to $\mathcal{S}$ and discards all the overlapped job instances from $\mathcal{S}$. The process ends when there are no more job instances to check. A pseudocode of the algorithm is

Algorithm 2. Greedy algorithm for maximizing the sum of weights

Input: $\mathcal{J} = \{J_1, \cdots, J_{n_i}\} \setminus \bigcup_{k \in P} \mathcal{S}^k$
Output: Schedules for drone delivery

Initialization
$\mathcal{S}^d = \varnothing$
Let $\mathcal{U}$ be the set of unchecked job instances,
$\mathcal{U} = \{J_{1d}(t_1), \ldots, J_{n_i d}(t_{n_i})\}$

Main loop
while $\mathcal{U} \neq \varnothing$ **do**
  Let $I \in J_i$ be the job instance that finishes earliest among all instances in $\mathcal{U}$, and let w be its weight.
  Let W be the sum of the weights of all instances $I_1, \ldots, I_h$ in $\mathcal{S}^d$ that overlap I.
  $\mathcal{U} := \mathcal{U} \setminus \{I\}$.
  if $J_i \cap \mathcal{S}^d \neq \varnothing$ **then**
    Discard I.
  else if $W = 0$ **then**
    Schedule I, $\mathcal{S}^d := \mathcal{S}^d \cup \{I\}$
  else if $\frac{w}{W} < \beta$ **then**
    Schedule I, $\mathcal{S}^d := \mathcal{S}^d \cup \{I\} \setminus \{I_1, \ldots, I_h\}$.
  else
    Discard I.
  end if
end while
return schedules $\{J_{j_1,d}, \ldots, J_{j_{k_d},d}\}$

given in Algorithm 2. For the case of multiple machines (drones), Algorithm 2 is repeated machine by machine, each time updating the set $\mathcal{J} = \{J_1, \cdots, J_{n_i}\} \setminus \bigcup_{d \in P} \mathcal{S}^d$ of jobs to be scheduled, where P is the set of processed machines (drones).

In general, restricted master heuristic finds the good primal bound of the master problem if the column-generating procedure maintains good column-wise decisions. Proposition 2 already proved the quality of column-wise decisions. The main drawback of the RMH is that the resulting restricted master integer problem is often infeasible. To deal with feasibility, the shortest-processing-time rule is used to generate additional columns. If all customer nodes are considered candidate locations for the facility, the column generated by the rule prevents an infeasible solution. A pseudocode of the restricted master heuristic is given in Algorithm 3.

5. Computational experiments

We conducted computational experiments and sensitivity analysis, and the results are provided in this section. Section 5.1 describes what data set was utilized and how the experiment was carried out. Experiments demonstrating the significance of the ScheLoc-D are presented in Section 5.2. The sensitivity analysis of drones set to various maximum travel times and cost factors can be found in Section 5.3. In Section 5.4, the performance of two solution approaches, the mixed-integer linear programming (MILP) and the RMH, is discussed.

Algorithm 3. Restricted master heuristic for the ScheLoc-D

Input: All sets, parameters defined in the ScheLoc-D
Output: Solution of the ScheLoc-D

Initialization
Create jobs for the subproblem by adjusting parameters
$\mathcal{J} =< J_1, \ldots, J_n >$

Procedures for the subproblem
Call Algorithm 1
Call Algorithm 2
Generate columns according to shortest-processing-time rule

Procedures for the master problem
Solve the restricted master problem with generated columns.
Reallocate customers to opened facilities

return Feasible heuristic solution for the ScheLoc-D

5.1. Description of experiments

We utilized Solomon benchmark (Solomon, 1987) instances to verify the performance of the developed heuristic, the RMH. Although the Solomon benchmark was originally generated for the vehicle routing study, using the information it has, we can generate a data set that can be used for the ScheLoc-D. Problem sets were generated in which customers are located randomly (the R-problems) or in clusters (the C-problems). The RC-problems include a mixture of two geographic features. The instances in the same set differ in the tightness and positioning of the time windows. In the generated data set, the data on x–y coordinates, demands, time windows, and service times for customers are set the same as the values provided by the Solomon benchmark. The information on the depot is fixed to the first candidate site of the facility in the generated instance. The remaining candidate sites are randomly selected from the customer's location. Parameter γ is set to twice $\overline{\tau}$, the average value of all $\tau_{i\in I, j\in J}$, and τ_{ij} is calculated as the following equation to use values with one decimal point.

$$\tau_{ij} = \frac{\lfloor 10\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \rfloor}{10},$$

$$\gamma = 2 * \overline{\tau} = 2 * \frac{\sum_{i\in I, j\in J} \tau_{ij}}{|I| \cdot |J|}$$

where (x_i, y_i) and (x_j, y_j) denote the coordinates for candidate location of facility i and customer j, respectively. Facility setup costs were generated using a uniform distribution on the interval [1500,2300] based on the cost values reported in Akca et al. (2008) and Ponboon et al. (2016). A variable cost is the same as the travel time (i.e., $\rho = 1$). The ScheLoc-D as an MILP and restricted master problems were solved with FICO Xpress version 8.5 (http://www.fico.com), and

Table 1
Comparison of objective function values between the FLP and the ShceLoc-D

Class	NP	NC	NS	Solved	ScheLoc-1	ScheLoc-2	ScheLoc-3	FLP
R1	12	10	11	12	10,951.88	7,050.55	5,593.93	3,866.40
		15	16	12	15,034.43	9,112.22	7,225.72	3,741.60
		20	21	12	19,627.13	11,224.18	8,313.50	3,965.20
R2	11	10	11	11	4,704.35	3,866.40	3,866.40	3,866.40
		15	16	11	5,422.18	3,883.60	3,748.00	3,741.60
		20	21	10	6,791.36	4,457.47	3,967.15	3,965.20
RC1	8	10	11	8	7,151.25	4,931.45	4,322.08	3,341.80
		15	16	8	10,402.95	6,501.15	4,750.53	3,265.20
		20	21	8	14,003.20	8,445.03	7,039.30	4,979.80
RC2	8	10	11	8	3,341.80	3,341.80	3,341.80	3,341.80
		15	16	8	3,290.23	3,265.20	3,265.20	3,265.20
		20	21	8	5,209.38	4,979.80	4,979.80	4,979.80
C1	9	10	11	9	6,772.31	3,608.36	3,299.20	3,299.20
		15	16	9	10,488.27	5,382.71	4,796.80	3,215.00
		20	21	9	14,372.29	8,500.51	6,772.60	5,244.00
C2	8	10	11	8	5,806.50	4,596.65	3,845.80	3,845.80
		15	16	8	8,214.93	6,011.95	5,429.60	5,429.60
		20	21	8	9,561.03	6,689.15	5,645.88	5,454.00

the heuristics for column generation in RMH were implemented in JAVA SE 8. When solving the ScheLoc-D as an MILP, the computing time of the solver was limited to 1800 seconds. Computational experiments were performed using an AMD Ryzen 7 2700X eight-core 3.7 GHz processor with 16 GB RAM in the Microsoft Windows 10 operating system.

5.2. Comparing the ScheLoc-D to the FLP

The main difference between the two models is a scheduling decision to satisfy time window constraints. To show the difference between the ScheLoc-D and the FLP clearly, in Section 5.2, we tightly set the time windows of the Solomon benchmark by multiplying them by 0.25. Due to the computational complexity of the ScheLoc-D, we experimented with instances consisting of 10, 15, and 20 customers. In larger instances, unlike the FLP, the ScheLoc-D cannot find the optimal solution within 1800 seconds. Table 1 reports the class name (Class), the number of instances in each problem class (NP), the number of customers (NC), the number of candidate locations for a facility (NS), the number of instances solved optimally (Solved), the average objective function value of the solutions found by the ScheLoc-D considering one drone (ScheLoc-1), the average objective function value of the solutions found by the ScheLoc-D considering two drones (ScheLoc-2), the average objective function value of the solutions found by the ScheLoc-D considering three drones (ScheLoc-3), and the average objective function value of the solutions found by the uncapacitated FLP (FLP).

Table 1 shows that the ScheLoc-D incurs more cost than the method of determining the location of a facility by simply checking whether demand is located within a critical distance. Even if the

Table 2
Comparison of solutions between the FLP and the ScheLoc-D

Class	NF-ScheLoc-1	NF-ScheLoc-2	NF-ScheLoc-3	NF-FLP
R1	11.33	6.50	4.75	2.00
R2	3.80	2.27	2.00	2.00
RC1	8.38	5.13	4.25	3.00
RC2	3.13	3.00	3.00	3.00
C1	8.44	5.00	4.00	3.00
C2	5.50	3.75	3.13	3.00

delivery network planned by the ScheLoc-D costs more, a solution with a viable delivery schedule is more practical. The problem sets with number 1 have a tighter time window than the problem sets with number 2. Therefore, the difference between the ScheLoc-D and the FLP is clearly observed in the problem sets with number 1. As the number of customers to be serviced increases, the total cost generally increases. However, the cost incurred rather decreased as the range of choices for cost-efficient locations of facilities increased, which can be seen in the results of 15 customer instances.

The main reason for the higher cost is that the solution found by the ScheLoc-D opens more facilities. For a more detailed analysis, Table 2 shows the number of facilities opened for 20 customer instances. Table 2 reports the class name (Class), the number of facilities that should be built in the solution found by the ScheLoc-D considering one drone (NF-ScheLoc-1), the number of facilities that should be built in the solution found by the ScheLoc-D considering two drones (NF-ScheLoc-2), the number of facilities that should be built in the solution found by the ScheLoc-D considering three drones (NF-ScheLoc-3), and the number of facilities that should be built in the solution found by the uncapacitated FLP (NF-FLP). Instances for which an optimal solution was not found were excluded from the table.

In the uncapacitated FLP scheme, two or three facilities could cover all customers, but in the ScheLoc-D dealing with the feasibility of the delivery schedule, a small number of facilities could not cover all customers, and the construction of additional facilities is inevitable. Obviously, using more drones, which are relatively inexpensive, can reduce the number of facilities that need to be opened. Moreover, Tables 1 and 2 show that the ScheLoc and the FLP provide the same solutions when a sufficient number of drones are deployed in each facility. This is because the deployment of a sufficient number of drones at each facility alleviates the complexity of scheduling drone delivery. In this case, the ScheLoc-D and the uncapacitated FLP are considered equivalent problems.

5.3. Sensitivity analysis with drone features

To validate the influence of drone features clearly, we assumed that only one drone was deployed at each facility and set the other parameters to the same values as in previous experiments, except for the features we wanted to analyze and the time windows of customers. First, we investigated the impact of the cost factor of drones on designing networks. Table 3 reports the class name and the number of facilities that should be built in the solution on different cost factors. The number of

Table 3
Results of sensitivity analysis for different variable costs

Class	$\rho = 10$	$\rho = 20$	$\rho = 30$	$\rho = 40$	$\rho = 50$
R1	3.22	3.33	3.33	3.56	4.22
R2	3.00	4.00	4.00	4.00	5.00
RC1	4.55	4.67	5.17	7.25	8.17
RC2	2.00	4.00	4.00	7.00	8.00
C1	3.63	3.63	3.75	3.75	3.75
C2	3.00	3.00	3.00	3.00	3.00

Table 4
Results of sensitivity analysis for different shipping ranges

Class	NP	Solved	$\gamma = \overline{\tau}$	$\gamma = 2\overline{\tau}$	$\gamma = 3\overline{\tau}$	$\gamma = 4\overline{\tau}$	$\gamma = 5\overline{\tau}$
R1	12	1	15,636.80	10,805.80	10,739.40	10,739.40	10,739.40
R2	11	11	12,168.80	3,965.20	3,870.06	3,870.06	3,870.06
RC1	8	5	6,314.92	6,014.20	5,356.92	5,356.92	5,356.92
RC2	8	8	4,979.80	4,979.80	3,721.20	3,579.80	3,579.80
C1	9	7	6,544.26	5,797.29	4,090.89	4,051.06	4,051.06
C2	8	6	8,958.80	5,454.00	2,530.83	2,369.20	2,369.20

facilities opened was reported because it is meaningless to directly compare the objective function values consisting of the changed cost factors. The table shows that the number of facilities opened increases as the variable cost factor increases. If the impact of variable costs on the objective function increases, a reasonable network design is to reduce the total travel distance by opening additional drone facilities. This trend is more clearly confirmed when customers are randomly distributed rather than geographically clustered because the impact of opening a new facility that affects total travel distance is significant when customers are located sparsely. A smaller variable cost factor means that the cost required to travel the same distance decreases, which denotes that more efficient drones are used. Therefore, the results highlight the importance of the cost-effectiveness of drones and technological advances in drones to validate the efficiency of drone delivery easily and to commercialize it quickly.

We also investigated the impact of the shipping range of drones on cost efficiency. The maximum travel time of drones was originally set to double the average value of all distances between candidate locations of facilities and customers. In the following experiments, we varied the maximum travel times of the drone, γ, changing them from 1 to 5 times the average value of all distances between customers and facilities. As γ changed, an optimal solution was not found in some instances of 20 customers. To better observe the difference in the total cost, we only analyzed cases where the optimal solution was found in each set within the time limit. Table 4 reports the class name, the number of instances in each problem class, the number of instances solved optimally, and the average objective function values of the optimal solutions found with different shipping ranges. The table clearly shows that the cost savings increased as drones have a wider shipping range. In particular, when drones with highly limited shipping ranges are used, a larger number of facilities are necessary to satisfy the demands of customers. Moreover, short-range drones may

Table 5
Results on small-sized instances: summary

Class	NP	NC	NS	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_S
R1	12	10	11	12	3,866.40	0.41	3,866.40	0.18	0.00%
		15	16	12	3,872.30	1.20	3,872.30	0.18	0.00%
		20	21	11	4,240.73	46.38	4,240.73	0.18	0.00%
R2	11	10	11	11	3,866.40	0.56	3,866.40	0.17	0.00%
		15	16	11	3,748.00	1.20	3,748.00	0.17	0.00%
		20	21	11	3,965.20	11.86	3,965.20	0.17	0.00%
RC1	8	10	11	8	3,341.80	0.27	3,341.80	0.18	0.00%
		15	16	8	3,265.20	0.88	3,265.20	0.18	0.00%
		20	21	8	4,797.80	7.94	4,979.80	0.18	0.00%
RC2	8	10	11	8	3,341.40	0.22	3,341.80	0.18	0.00%
		15	16	8	3,265.20	1.04	3,265.20	0.18	0.00%
		20	21	8	4,979.80	7.69	4,979.80	0.18	0.00%
C1	9	10	11	9	3,299.20	0.67	3,299.20	0.17	0.00%
		15	16	9	3,215.00	1.70	3,215.00	0.18	0.00%
		20	21	9	5,244.00	6.27	5,267.00	0.18	0.002%
C2	8	10	11	8	3,845.80	0.18	3,845.80	0.17	0.00%
		15	16	8	5,429.60	1.04	5,429.60	0.18	0.00%
		20	21	8	5,454.00	9.56	5,454.00	0.18	0.00%

lead to failures in the design of delivery networks. On the other hand, increasing the shipping range is not a panacea for efficient delivery networks in the scheme of the ScheLoc-D. Total cost decreases to a certain value as the shipping range of drones increases. Even if a drone can deliver to every customer, delivering to a remote customer means giving up the opportunity to deliver to multiple nearby customers. The solution found in the FLP might open only one facility in the cheapest location when drones can deliver to all customers and the variable cost factor is small enough. However, in our model, if the number of available drones is not sufficient, the model will find different solutions because delivery to all customers within a planning horizon is impossible. Therefore, this insight is a unique finding, resulting from the difference between the FLP and the ScheLoc-D.

5.4. Comparing the RMH to an MILP formulation

To assess the solution quality of the RMH, we compared heuristic solutions to the optimal solutions obtained by solving the mathematical model for small-sized instances. We assumed that three drones were deployed at each facility and that $\beta = 0.2$. In the experiments with small-sized instances, only the results of the instances in which the optimal solution was found were compared to verify the performance of the RMH clearly.

Table 5 reports the class name (Class), the number of instances in each problem class (NP), the number of customers (NC), the number of candidate locations for a facility (NS), the number of instances solved optimally (Solved), the average objective function value of the solution found by the MILP (Obj-M), the average computing time taken to find a solution with the MILP in seconds

Table 6
Results on large-sized instances: summary

Class	NP	NC	NS	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_L
R1	12	50	26	0	8,846.48	1,800.00	6,988.65	0.22	–12.18%
			51	0	67,069.26	1,800.00	6,581.22	0.30	–90.98%
R2	11	50	26	1	6,936.31	1,793.52	6,960.49	0.25	1.02%
			51	0	66,622.80	1,800.00	6,679.80	0.27	–89.65%
RC1	8	50	26	0	7,508.73	1,800.00	6,598.55	0.23	–10.07%
			51	0	66,351.53	1,800.00	6,420.90	0.28	–88.98%
RC2	8	50	26	4	6,547.35	1,623.04	6,551.65	0.24	0.06%
			51	0	22,628.05	1,800.00	6,172.20	0.25	–53.80%
C1	9	50	26	3	5,364.04	1,666.58	5,391.29	0.21	0.66%
			51	0	16,318.40	1,800.00	4,824.40	0.27	–70.00%
C2	8	50	26	7	6,539.78	1,445.09	6,662.93	0.23	1.89%
			51	0	62,056.00	1,800.00	6,450.20	0.26	–89.61%

(Time-M), the average objective function value of the solution found by the RMH (Obj-H), the average computing time taken to find a solution with the RMH in seconds (Time-H), and the average gap between two solutions in percentages (Δ_S). We compute the optimality gap as,

$$\Delta_S = \frac{obj.\,value\,(heuristic) - optimal(MILP)}{optimal(MILP)},$$

where $obj.value(heuristic)$ is the objective function value of the heuristic solution and $optimal(MILP)$ represents the optimal solution value of an instance.

The computing times for the RMH are faster than the time needed to solve the MILP model by a commercial solver. The RMH quickly obtained a solution, with costs lying within 0.1% of those of the optimal solutions. The solution quality of the RMH can be verified through a small gap for all tested instances. Experiments were conducted on large-sized instances to prove the performance of the heuristic algorithm clearly. The size of the instances used in the experiment consisted of 50 customers, because it was observed that 100-customer instances could not even be loaded to the commercial solver within the given time limit. The descriptions in each column in Table 6 are similar to those in Table 5. We compute the average gap, Δ_L, between the two solutions in percentages as the following.

$$\Delta_L = \frac{obj.\,value\,(heuristic) - obj.\,value\,(MILP)}{obj.\,value\,(MILP)},$$

where $obj.\,value\,(MILP)$ represents the objective function value of the solution that the mathematical model found in the given time limit.

Table 6 shows that heuristic solutions are generally much better than the solutions that a commercial solver finds in given computing time limits. A negative value of Δ_L is computed if the objective function value of the heuristic solution is better than the solution found by a given time limit. The results show that the time efficiency of heuristics is validated clearly in the larger instance. In addition, it was also observed that the lower bound found by the solver generating the cutting plane was very weak. A good solution was found for some instances but could not converge due to the weak bound. To sum up, the RMH outperforms the performance of the commercial solver as instances grow in size.

6. Conclusions

The advancement of drone technology is accelerating the introduction of drones into various elements of the logistic system. A drone-integrated system leads to new operation problems that integrate different levels of decision-making. In particular, facilities such as drone stations require decision-making simultaneously at the operational level when determining their location. This is because supporting facilities are installed to plan feasible delivery schedules for drones. Therefore, we proposed a scheduling-location problem with drones (ScheLoc-D), a new methodology for integrating operational planning decisions with strategic planning decisions. The developed model can design a more practical network for locating the facilities needed for drone delivery than the traditional FLP.

Because of the highly fractional solution of the LP relaxation, the standard formulation of the ScheLoc-D has a weak LP-relaxed bound. Despite the weak LP-relaxed bound, in the small-sized problems, the SF could find the optimal solution in a short time. However, in larger problems, the SF could not be solved within a given time limit. Therefore, an extended formulation and a restricted master heuristic were proposed. To generate attractive patterns, the subproblem is considered a parallel machine scheduling problem, and the approximation algorithms based on a simple dispatching rule are used. The computational results showed that the restricted master heuristic outperforms the commercial solver in finding solutions for large-sized instances. As a result of conducting a sensitivity analysis by changing the features of the drone, we demonstrated that the longer flight distance and cost-efficiency of the drone are key factors. Therefore, technological advances and innovations of drones themselves must accompany such findings in order to prove the feasibility of drone delivery easily and to commercialize it quickly.

Despite the rapid improvement in the computing power of computers, optimization techniques have not yet been applied in the field. Therefore, further research on the exact algorithm is promising. Polytope-associated approaches and cutting plane algorithms could be utilized because the structure of the models based on the FLP is relatively simple. Alternatively, dynamic programming can be used to efficiently solve the pricing subproblem to develop a branch and price algorithm. The solution structure and the progress of the column generation can be investigated more for insightful research.

Lastly, we addressed the limited shipping range of drone delivery and did not consider other features of drones. Limited payload capacity is another critical aspect of drone delivery. Because our model considers homogeneous drones and does not utilize other types of vehicles, such as trucks, the payload capacity of drones only determines whether the demand is satisfied in the preprocessing stage. In future studies, the ScheLoc problem can be extended to consider heterogeneous vehicles and the different payload capacities of drones or to deal with uncertainties in drone delivery systems. Another extension of the ScheLoc problem can allow locating drone stations anywhere on a given plane, not in a set of discrete candidate locations.

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Appendix

We provided the opening costs of a facility at each location in Table A.1 for future research. We also provided the selected location numbers in large-sized problems as follows.

Table A.1
Opening costs of facilities

Locations	Opening costs	Locations	Opening costs	Locations	Opening costs	Locations	Opening costs
0	1,942	13	1,688	26	2,197	39	2,261
1	1,585	14	1,531	27	1,547	40	1,949
2	1,721	15	1,731	28	1,693	41	1,831
3	1,790	16	1,683	29	1,924	42	1,997
4	1,569	17	1,809	30	2,057	43	2,210
5	1,944	18	1,761	31	1,645	44	2,134
6	1,920	19	1,670	32	1,672	45	2,081
7	2,252	20	1,985	33	2,123	46	2,041
8	1,967	21	2,120	34	1,888	47	2,189
9	1,683	22	1,771	35	1,567	48	1,779
10	2,090	23	1,540	36	2,126	49	2,193
11	1,931	24	1,712	37	1,563	50	1,724
12	2,048	25	1,800	38	2,073		

Candidate location numbers:
[0, 1, 2, 3, 5, 7, 10, 13, 14, 15, 16, 18, 19, 22, 23, 24, 26, 28, 33, 34, 36, 37, 41, 43, 48]