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# Pricing, warranty, and shelf space decisions for the supply chain with non-symmetric market and warranty-period dependent demand

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## Abstract

This article presents a pricing, warranty, and shelf-space-size decision problem associated with a two-echelon supply chain composed of one retailer and two competing manufacturers. The products (belonging to two distinct manufacturers) come through a non-symmetric market. Demand for them is price sensitive and depends on the warranty period. As the sole leader, the retailer specifies the size of the available shelf space and the retail price for each product. At the same time, the manufacturers determine wholesale prices and warranty periods for their respective products. Mathematical models are developed to derive optimal solutions. Moreover, a sensitivity analysis for the models is performed to analyze how differences in market potential, production cost, warranty service cost, warranty-competition factors, and cross-price sensitivity parameters of products influence optimal solutions. Experimental results show that manufacturers can offset increased warranty service costs by decreasing the warranty competition factor. They can also adopt mixed strategies to reach a pre-specified target profit by considering their capability to either drop the warranty service cost or increase the warranty-competition factor. In addition, the retailer can modify the impact of reduced market potential or increased production cost (the warranty service cost) by using shelf-space cost management strategies.

**Keywords** Game theory · Competitive supply chain · Pricing · Warranty · Shelf-space allocation

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## 1 Introduction

Pricing, warranty, and shelf space are three critical decisions that affect the profitability and competitiveness of the supply chain members. Pricing determines the products' revenue and market share, as well as their perceived value and quality. In recent decades, rapid technological advancements have significantly changed the role of supply chains. Intensified business competition might contribute to a given company's avoidance of traditional strategies that focus only on price (Zhao et al. 2020, Liu et al. 2021). However, not all customers have the same preferences, needs, and willingness to pay for a product or service (Parvasi and Taleizadeh 2021). This attitude can be redirected to service and product quality to build brand loyalty in customers (Chen et al. 2012, Liu et al. 2023). For instance, IBM and HP enjoy excellent reputations for their customer service support (Lu et al. 2011). Major concerns for end-users include accessibility to low and affordable prices and the appropriateness of product liability related to support services (Giri et al. 2015).

A warranty promises or guarantees that a product or service will meet specific standards or perform as expected. Warranty influences customer satisfaction and loyalty, warranty service cost, and product reliability (Murthy and Djamaludin 2002). Warranties have become well-known measures for encouraging market demands because they decrease risks for consumers. They are contracts for restoration administered during sales (Tinic and West 1979). In fact, warranties offer protection for both manufacturers and customers. Warranties provide insurance of recovery for customers if a product does not perform to the promised quality or functional features (Taleizadeh et al. 2017). The pre-defined situations and specific time periods of coverage detailed in warranties save manufacturers the trouble and expense of needing to compensate customers who misuse or overuse products covered under warranties (Murthy and Djamaludin 2002). Warranties have received considerable attention in both economics and operations management literature. From an economic viewpoint, three conventional theories have been presented to explain product warranties: insurance, signaling, and incentive (Chen et al. 2012). Insurance theory reveals that consumers are more risk-averse than sellers to product damage (Heal 1977). The signaling theory specifies that a comprehensive warranty usually indicates high product quality (Spence 1977). Incentive theory suggests that responsibility for moral risks lies with both manufacturers and consumers (Cooper and Ross 1985). Although a warranty is considered a competitive strategy, in cases of customer unwillingness to pay, the manufacturer chooses to set lower prices for products and not necessarily offer a warranty. As a result, the warranty can be provided at additional cost to keep prices at desired levels. In this situation, customers may or may not be willing to pay for the warranty (Taleizadeh et al. 2017). Non-symmetric market characteristics and warranty-period dependent demand are common phenomena in many markets, such as the markets for smartphones, automobiles, electronics, and appliances. For instance, in the smartphone market, Apple and Samsung are two dominant players that offer different models with varying features, prices, warranties, and market shares (Counterpoint Research 2023). Therefore, the demand for each model is determined not only by its own price and warranty period but also by those of the competing models. Similarly, in the automobile market, customers may prefer a car with a more extended warranty period over a shorter one, even if they have comparable prices and features. Toyota and Honda are two leading competitors in this market that offer different models with various features, prices, warranties, and market shares (Statista 2023). Therefore, there is a need to investigate how these factors influence the optimal decisions of the supply chain members in a non-symmetric market with warranty period-dependent demand.

In retailing, the shelf-space allocation (SSA) problem is essential, and many articles have addressed the issues relevant to determining optimal SSA. Shelf space is the area or volume that a product occupies on a retailer's shelf (Urban 1998). Shelf space affects product visibility and availability, as well as the demand and retail price of the products. Corstjens and Doyle (1981, 1983) suggested models that considered the problem of allocating products to limited shelf space. According to Li et al. (2013), retailers aim to allocate shelf space because of three significant factors: profitability, customer satisfaction, and competition. Because these elements seem contradictory, many researchers have developed models to explore the effects of limited shelf space on these factors, and the results have provided valuable insights for managers to deal with SSA problems in real-life scenarios.

This study addresses a pricing, warranty, and shelf-space-size decision problem for a supply chain composed of a common retailer and two manufacturers who offer warranty period assurance. The goods produced by the manufacturers have non-symmetric market potential, along with warranty service costs, warranty competition factors, unit production costs, and cross-price sensitivity parameters. In this way, demands for the products are price sensitive and depend on the warranty period. The retailer determines the size of allocated shelf space and the retail price for each product, while each manufacturer decides on the wholesale price and warranty period for the products they make. To deal with the problem, the linear demand function proposed by Wang et al. (2015), which considered the cross-price sensitivity parameters, is extended to incorporate warranty period dependent demands used in Chen et al. (2012). The main contribution of this research is the evaluation of retailer's shelf-space decisions and pricing strategies along with manufacturers' pricing and warranty strategies on decisions throughout the supply chain, which makes a unique contribution due to providing such a connection. Furthermore, the results of our study can provide useful insights for both manufacturers and retailers who face similar challenges in real-world problems.According to the explanations given, the research questions are as follows:

- How do the non-symmetric market characteristics and the warranty-period dependent demand affect the supply chain members' optimal pricing, warranty, and shelf space decisions?

- How do differences in market potential, production cost, warranty service cost, warranty-competition factors, and cross-product price parameters affect profitability and optimal decisions of supply chain members?

The rest of this paper is organized as follows. A review of the related literature is presented in Sect. 2. Problem assumptions and the model formulation are explained in Sect. 3. To check the validity of the model, numerical solutions and sensitivity analyses are described in Sect. 4. Finally, some managerial insights and conclusions are given in Sect. 5.

## 2 Literature review

Considering that this research examines the influence of pricing, warranty, and SSA on the decision-making process of supply chain members, this section thoroughly reviews past research in three specific areas, focusing on identifying gaps in the existing literature.

### 2.1 Products' pricing in supply chains

Pricing problems between substitutable products in supply chains are commonly considered in the literature. Zhao et al. (2012) studied a pricing problem of substitutable products in a supply chain with one manufacturer and two competitive retailers in which consumer demands and manufacturing costs were assumed to be fuzzy variables. In this case, one centralized and three decentralized pricing models were developed and solved using a game theory approach. Ma et al. (2012) considered the effects of a dominant manufacturer (trying to maintain the dominant position) on the members of an entire supply chain. The channel system (operating substitutable products) was comprised of two manufacturers, one retailer, and several consumers. Their study showed that only the dominant manufacturer could enjoy a wholesale price dominance strategy. Zhao et al. (2014) developed a two-echelon supply chain consisting of two manufacturers and one retailer with substitutable products. They analyzed the influence of different competitive strategies and channel members′ various power structures on the optimal pricing decisions through one centralized and seven decentralized pricing models. Wei and Zhao (2016) extended the research of Zhao et al. (2014) and investigated other scenarios of horizontal interaction, such as the Stackelberg game and cooperation between manufacturers. Taleizadeh and Sadeghi (2019) and Zhou et al. (2022) studied pricing strategies in a competitive supply chain from a game-theoretical approach. Chen et al. (2021) investigated different scenarios in which the pricing decisions of manufacturer and retailer took place in different time sequences. Moreover, they explored the effect of the manufacturer, being risk-averse or neutral, lead time, and exchange rate on pricing decisions. Fang (2020) proposed a non-cooperative duopoly model to explore the interaction between two competing manufacturers' pricing and warranty period decisions. Taleizadeh et al. (2021) studied the consequences and conflicts between the members of a cooperative supply chain, which competitive advantages will result in, and sought optimal decisions in a cost-sharing contract to alleviate them. Sun et al. (2021) studied the joint problem of pricing and replenishment of seasonal and non-seasonal products in a three-echelon supply chain. Parvasi et al. (2023) proposed a pricing model to examine the competitive behavior of domestic and international companies in a supply chain context. They formulated the problem as a bi-level optimization model, in which the domestic company determined the optimal prices of its products to maximize its profit, and the retailer decided the optimal quantities to purchase from each manufacturer to minimize its purchasing cost. The retailer's demand for each product depends on the prices and the quality of the products, which is modeled by the multinomial logit model.

Many studies have explored different aspects of pricing decisions for substitutable products in supply chains, such as demand functions, game models, contract mechanisms, and profitability analysis. However, they have paid little attention to the role of warranties in supply chains. Moreover, they have often assumed that the market potential is identical for different products. Most of these studies have also focused on developing price-dependent demand functions. However, the purpose is to make aggregate pricing decisions and use price as leverage to induce market demand.

### 2.2 Warranty and product quality in supply chains

Warranty is considered as one of the critical factors in our research. Many studies have investigated the different aspects of warranty reviewed by Murthy and Djamaludin (2002). Chukova et al. (2005) also provided a brief survey of the literature by introducing some statistical models and methods used to analyze warranty claim data. Wu (2012) summarized previously presented researches conducted in warranty data analysis from several different perspectives on the basis of the models, methods, and applications. In another survey, Wu (2013) reviewed articles on the analysis of coarse warranty data. Coarse data arises from the situation in which the information is aggregated, delayed, censored, missed, or vague. Li et al. (2011) emphasized that unobservable quality information could be demonstrated using signals, such as warranty and brand reputation. Developing a cooperative duopoly model, they discussed a supply chain with two competing manufacturers and interactions between an offered warranty, brand reputation, and product quality that impressed the consumers' purchase decisions. Dai et al. (2012) utilized Nash equilibrium to investigate the way that interaction between product quality and warranty influenced the performance of a supply chain comprised of a supplier (controlling the order quality) and a manufacturer (controlling the order quantity). They also compared the results of centralized and decentralized systems to determine the optimal conditions leading to better product quality and more extended warranty periods. Lan et al. (2014) developed a supply-chain contract problem in which buyer's pricing and warranty decisions were considered to maximize the expected payoff. According to the buyer's perspective, each supplier's product quality was unobservable and contained vague information, and thus, quality factors were considered fuzzy variables in the model. Tsao et al. (2014) proposed a joint model for high technology products under a replacement warranty policy to simultaneously determine pricing and inventory decisions. Using two-stage game theory methodology, Wei et al. (2015) developed two cooperative and three non-cooperative decision models to find the optimal price and warranty period in a duopoly supply chain with two manufacturers and one common retailer as determined through firms' different bargain powers. They also considered complementary products and different pricing measures between manufacturers. Modak et al. (2015) proposed a two-echelon supply chain with one manufacturer and one retailer for a single product type. They assumed that the customers' demands depended on the product's warranty, quality, and sales price. They maximized the profit functions of retailers and the manufacturer under two centralized and decentralized scenarios. Taleizadeh et al. (2017) introduced warranty as a competitive factor for decreasing the effects of gray markets. They considered two markets with different levels of willingness to pay in which the manufacturer offered the same product with distinct prices on the basis of consumer willingness to pay. Shang et al. (2018) studied optimal design of warranty and post-warranty required measures under stochastic degradation. Sarada and Sangeetha (2021) developed a game-theoretical model for a remanufactured product. They aimed to carry out warranty for remanufactured products under stochastic circumstances. Giri et al. (2018) investigated the impact of warranty period along with selling price and greening level of the supply chain on demand. Tang et al. (2020) studied warranty, offered for two categories of products, new and remanufactured, and the effect of the warranty period on pricing decisions. Cai et al. (2020) explored how different warranty policies affect the supply chain under information and cost-sharing structure in the existence of uncertain demand and demand forecasting. Cui et al. (2023) explore how a manufacturer's recycling strategy affects the extended warranty service provided by an e-commerce platform in a closed-loop supply chain context. They formulate the problem as a bi-level optimization model, where the manufacturer and the e-commerce platform determine their optimal prices for selling and recycling products under three different recycling strategies: low, high, and discriminated recycling prices.

Studies on warranty service in supply chains have explored warranty data analysis, quality signals, and contract mechanisms. However, these studies mostly assume a fixed warranty for all products, which may not reflect reality since product quality, consumer preference, and competition intensity can affect warranty factors.

### 2.3 Shelf space allocation in supply chains

The final area that needs attention in this study involves the SSA problem. Numerous marketing and supply chain management studies have investigated pricing and SSA simultaneously for strategic decision-making. Among the related works in the literature, Urban (1998), Yang and Chen (1999), Wang and Gerchak (2001), and Balakrishnan et al. (2004) focused on the single shelf space constrained retailer SSA problem. Martin-Herran et al. (2006) investigated the impact of manufacturers' wholesale prices on a retailer's shelf-space through a Stackelberg game between two competing manufacturers (as leaders) and a retailer (as a follower) playing a simultaneous and non-cooperative game. They showed that the manufacturers could influence the retailer's SSA through the wholesale price. Chen et al. (2011) studied the problem of coordination between a supplier and a manufacturer under a revenue-sharing contract. They modeled the problem as a Stackelberg game in which the retailer decided on the revenue-sharing percentage and the slotting fee, and the manufacturer decided on the retail price and the size of shelf-space for both the centralized and decentralized scenarios. Kurtuluş and Toktay (2011) developed a model with two competing manufacturers and a common retailer who decided on SSA under two categories of management mechanisms. Using a game theory approach, Li et al. (2013) explored the effect of competition among supply chain members on product demand that depended on the product pricing and SSA. To achieve optimal strategies, Cournot competition and Stackelberg game were applied, and Nash equilibriums were achieved by optimizing the profit as a function of demand and price. Recently, Wang et al. (2015) developed a pricing and shelf-space decision problem for a supply chain comprised of two manufacturers and a common retailer. The products produced by the manufacturers featured non-symmetric market potentials, unit production costs, and cross-price sensitivity parameters. The retailer determined the size of the shelf space and the retail price for each product, while manufacturers decided on wholesale prices. Reisi et al. (2019) compared the merits of an integrated and non-integrated supply chain in case of the existence of competition on shelf-space and price. Zhao et al. (2020) developed a joint model of shelf space and pricing for a supply chain under consignment and revenue sharing contract. Kim and Moon (2021) developed an integrated model for shelf-space allocation, product selection, and replenishment to maximize the retailer's profit. Karki et al. (2021) investigated the Joint optimization problem of rack layout and shelf-space allocation. Akkaş (2019) developed an infinite horizon Markov chain model for a single product to optimize the shelf-space allocation problem regarding the expiration of the perishable inventory.

Previous research has explored various elements that impact the choices made in the supply chain's SSA, such as pricing models, demand functions, and competition effects. However, these studies have often overlooked the significant influence that warranty services provided by the manufacturer or retailer can have on shelf space allocation. In reality, the warranty service may affect the demand and profitability of the products and influence the shelf-space allocation decisions.

The review of the relevant literature in these three areas (pricing of substitutable products, warranty, and SSA) reveals their strong interdependence throughout the supply chain. However, most studies have only considered two of these problems (Wang et al. 2015; Chen et al. 2011). Moreover, most existing studies have assumed that the market is symmetric, meaning that the products from different manufacturers or suppliers have the same characteristics that affect their demand. They have also assumed that the demand is independent of the warranty period offered by the manufacturer or the retailer. These assumptions may not capture the reality of many markets, where products have different market potentials, production costs, cross-price sensitivities, and warranty policies. Therefore, there is a need to investigate how these factors influence the optimal decisions of the supply chain members in a non-symmetric market. Our study takes a unique approach by simultaneously considering retailer shelf-space and pricing decisions, as well as manufacturer pricing and warranty strategies. This connection provides a valuable contribution to the field.

## 3 Model formulation

This model represents a two-echelon supply chain comprised of one shelf-space-constrained retailer and two competing manufacturers producing similar products of different brands that they sell through the common retailer. Depending on the amount of available shelf space, the retailer orders products from the manufacturers and stores all the inventory on the allocated shelf space. In addition, a warranty period is provided by both manufacturers, which is a common practice. This study focuses on determining the wholesale prices and warranty periods for both manufacturers, as well as the retailer's retail price and shelf-space-allocation

**Table 1** Notation

| | |
|---|---|
| *Parameters* | |
| $\theta_A$ | Cross-price sensitivity parameter for product A |
| $\theta_B$ | Cross-price sensitivity parameter for product B |
| $a$ | Market potentials for product A |
| $b$ | Market potentials for product B |
| $c_A$ | The unit production costs of product A |
| $c_B$ | The unit production costs of product B |
| $k$ | The unit shelf-space cost |
| $\lambda_A$ | A multiplier of the warranty that shows the effect on demand for product A |
| $\lambda_B$ | A multiplier of the warranty that shows the effect on demand for product B |
| $\gamma_A$ | A multiplier of the warranty competition factor that influences the demand for product A |
| $\gamma_B$ | A multiplier of the warranty competition factor that influences the demand for product B |
| $cr_A$ | Warranty service cost for product A |
| $cr_B$ | Warranty service cost for product B |
| *Decision variables* | |
| $q_A$ | Demand for product A |
| $q_B$ | Demand for product B |
| $p_A$ | The retail price of product A |
| $p_B$ | The retail price of product B |
| $wp_A$ | The wholesale price of product A |
| $wp_B$ | The wholesale price of product B |
| $S$ | Retailer shelf space |
| $W_A$ | Warranty period offered by manufacturer A |
| $W_B$ | Warranty period offered by manufacturer B |

decisions. We consider the price and warranty period as dependent demand functions. Table 1 presents definitions of the notations used.

We assume that the ordered quantity of each product is equal to the demand for it. The demand rate for the product $i$ is dependent on both the warranty period ($W_i$) and retail price ($p_i$). The non-symmetric demand function used in Wang et al. (2015) is extended to incorporate warranty-period-dependent demands used in Chen et al. (2012).

The following linear demand functions can determine the demand for each product:

$$q_A = a - p_A + \theta_A\left(p_B - p_A\right) + \lambda_A \cdot W_A - \gamma_A \cdot W_B \tag{1}$$

$$q_B = b - p_B + \theta_B\left(p_A - p_B\right) + \lambda_B \cdot W_B - \gamma_B \cdot W_A \tag{2}$$

In these functions, $a$, $b$, $\lambda_i$, and $\gamma_i$ are positive constants and $\lambda_i > \gamma_i$, which appears reasonable because the demands are relatively more sensitive to warranty period at a manufacturer's outlet(s) than at the competing manufacturer's outlet(s).

Non-symmetric cross-price sensitivity parameters were introduced by Wang et al. (2015) to explain the situation in which the level of product substitution is not the same for both items where $\theta_A, \theta_B \in [0, 1]$. The market potentials also show the primary consumer demand for products without the effect of the price.

The problem described in this study can be specified as a three-stage decision problem. We assume that the retailer is the central leader of the supply chain and manufacturers are followers. The retailer first declares the retail prices for both manufacturers' products. The two manufacturers then compete against each other to offer the best wholesale price and warranty period through a simultaneous-move Nash game. Finally, the retailer decides on the amount of shelf space to maximize its profit.

The mathematical calculations are done by backward induction; first, the amount of shelf space set by the retailer is determined, and then the optimal wholesale prices and warranty periods set by manufacturers are found. Finally, the retail price set by the retailer is calculated as shown in the following three subsections.

### 3.1 The retailer's problem

The retailer allocates $S$ units of shelf space to both items. In this stage, it is assumed that the shelf space $S$, wholesale prices, and warranty periods are given. The retailer determines the retail prices for two manufacturers' products to maximize the profit function, which is defined as follows:

$$\Pi_r = \Pi_{r,A} + \Pi_{r,B} = (p_A - wp_A) \cdot q_A + (p_B - wp_B) \cdot q_B \tag{3}$$

In this function, $\Pi_{r,A}$ and $\Pi_{r,B}$ denote the retailer's profits from products A and B, respectively. Due to retail pricing decisions being made subject to $q_A + q_B \leq S$, the retailer must solve a constrained maximization problem with a non-linear objective function. The optimal retail prices are determined by satisfying the

Karush–Kuhn–Tucker (KKT) conditions similar to the approach used in Wang et al. (2015).

**Lemma 1.** *Given S, $wp_A$, $wp_B$, $W_A$, and $W_B$, the optimal retail prices $p^*_A$ and $p^*_B$ are as follows*:

(a) If $S > S_1$

$$p_A^* = \frac{2a(1+\theta_B) + b(\theta_A+\theta_B) + wp_A E_1 + wp_B E_3 + 2W_A E_5 + W_B E_7}{E_9} \tag{4}$$

$$p_B^* = \frac{a(\theta_A+\theta_B) + 2b(1+\theta_A) + wp_A E_2 + wp_B E_4 + W_A E_6 + 2W_B E_8}{E_9} \tag{5}$$

(b) If $S \le S_1$

$$p_A^* = \frac{a(3+\theta_A+3\theta_B) + b(1+\theta_B+3\theta_A) + wp_A F_1 + wp_B F_3 + SF_5 + W_A F_7 + W_B F_9}{4\left(\theta_A + \theta_B + 1\right)} \tag{6}$$

$$p_B^* = \frac{a(1+\theta_A+3\theta_B) + b(3+\theta_B+3\theta_A) + wp_A F_2 + wp_B F_4 + SF_6 + W_A F_8 + W_B F_{10}}{4\left(\theta_A + \theta_B + 1\right)} \tag{7}$$

The value of $S_1$ and auxiliary expressions, *E1* to *E9* and *F1* to *F10* is presented in Appendix A in supplementry material.

***Proof.*** (Please see Appendix A in supplementry material).

### 3.2 The manufacturers' problem

In the second stage, each manufacturer decides on the wholesale price and warranty period of the product to maximize the profit for given shelf space *S*. In this section, the model with two manufacturers competing against each other through a simultaneous-move Nash game was developed. The profit functions for the manufacturers are as follows:

$$\Pi_A = (wp_A - c_A)\cdot q_A - cr_A W_A^2 \tag{8}$$

$$\Pi_B = (wp_B - c_B)\cdot q_B - cr_B W_B^2 \tag{9}$$

**Lemma 2.** *Given S, the optimal wholesale prices, $wp^*_A$ and $wp^*_B$, and offered warranty periods, $W^*_A$ and $W^*_B$, can be determined as follows*:

a. If $S > S_2$

$$wp_A^* = \frac{2acr_A G_1(2\,cr_B G_3) + 2cr_A G_1 b\left(2cr_B G_5 - c_B G_7\right) - c_A\left(G_9 \times (2cr_B G_{11} - 2\,cr_B G_{13}\right)}{\left(2\,cr_A\,(2\,cr_B G_{15})\right) + G_{16}(2\,cr_B G_{17}) - 2\,cr_B G_{18} - G_{19}} \tag{10}$$

$$wp_B^* = \frac{2bcr_B G_2(2cr_A G_4) + 2cr_B G_2 a\left(2cr_A G_6 - c_A G_8\right) - c_B\left(G_{10} \times (2cr_A G_{12} - 2cr_A G_{14}\right)}{\left(2\,cr_A(2cr_B G_{15})\right) + G_{16}(2cr_B G_{17}) - 2cr_B G_{18} - G_{19}} \tag{11}$$

$$W_A^* = \frac{\left(\left(2bcr_B H_1 + c_B H_3 - 4c_B cr_B H_5 + c_A H_7 + aH_9 + 2acr_B H_{11} + bH_{13} - 2c_A cr_B H_{15} + 2\,c_B H_{17}\right) H_{19}\right)}{\left(2\,cr_A(2\,cr_B G_{15})\right) + G_{16}(2\,cr_B G_{17}) - 2cr_B G_{18} - G_{19}} \tag{12}$$

$$W_B^* = \frac{\left(\left(2acr_A H_2 + c_A H_4 - 4c_A cr_A H_6 + c_B H_8 + b\,H_{10} + 2bcr_A H_{12} + aH_{14} - 2c_B cr_A H_{16} + 2c_A H_{18}\right) H_{20}\right)}{\left(2\,cr_A(2\,cr_B G_{15})\right) + G_{16}(2cr_B G_{17}) - 2cr_B G_{18} - G_{19}} \tag{13}$$

b. If $S \leq S_2$

$$wp_A^* = \frac{8(a-b)cr_A cr_B + 4cr_A(cr_B(c_B I_1 + SI_2) - SI_4) - cr_B I_5 + c_A cr_A(cr_B I_6 - I_7)}{cr_A(cr_B I_9 - I_7) - cr_B I_8} \tag{14}$$

$$wp_B^* = \frac{8(b-a)cr_A cr_B + 4cr_B(cr_A(c_A I_1 + SI_3) - SI_5) - cr_A I_4 + c_B cr_B(cr_A I_6 - I_8)}{cr_A(cr_B I_9 - I_7) - cr_B I_8} \tag{15}$$

$$W_A^* = \frac{J_1(c_B cr_B - c_A cr_B) + cr_B SJ_3 - SJ_5 + cr_B J_7}{cr_A(cr_B I_9 - I_7) - cr_B I_8} \tag{16}$$

$$W_B^* = \frac{J_2(c_A cr_A - c_B cr_A) + cr_A SJ_4 - SJ_6 + cr_A J_8}{cr_A(cr_B I_9 - I_7) - cr_B I_8} \tag{17}$$

The value of $S_2$ and auxiliary expressions, $G_1$ to $G_{19}$, $H_1$ to $H_{20}$, $I_1$ to $I_8$, and $J_1$ to $J_8$ can be found in Appendix B in supplementry material.

***Proof.*** Please see Appendix B in supplementry material.

There is a crucial point about the non-negativity of the warranty periods which needs to be considered. According to the numerical studies, as far as the retailer allocates a part of his shelf space to a product, the value of warranty period for the corresponding manufacturer is greater than or equal to zero; which means that, if there are any demands for a product, it is always profitable for the manufacturer to provide the warranty for it ($W_i \geq 0$).

Based on the optimal wholesale prices and warranty periods determined in Lemma 2, the optimal demands for every product in the different cases calculated in Lemma 2 are as follows:

a. If $S > S_2$

$$q_A^* = \frac{4cr_AK_1(2\,bcr_BK_3 + 4c_Bcr_BK_5 - c_BK_7 - 2acr_BK_9 + 2c_Acr_BK_{11} + aI_{13} + bK_{15} - c_AK_{17} + c_BK_{19})}{(2\,cr_A\,(2\,cr_BK_{21})) + K_{22}(2\,cr_BK_{23}) - 2\,cr_BK_{24} - K_{25}} \tag{18}$$

$$q_B^* = \frac{4cr_BK_2(2\,acr_AK_4 + 4c_Acr_AK_6 - c_AK_8 - 2bcr_AK_{10} + 2c_Bcr_AK_{12} + bK_{14} + aK_{16} - c_BK_{18} + c_AK_{20})}{(2\,cr_A\,(2\,cr_BK_{21})) + K_{22}(2\,cr_BK_{23}) - 2cr_BK_{24} - K_{25}} \tag{19}$$

The auxiliary expressions $K_1$ to $K_{25}$ in the above equations can be found in Appendix C in supplementry material.

b. If S $\leq S_2$

$$q_A^* = \frac{cr_A\left(c_Bcr_BL_1 - c_Acr_BL_1 + cr_BSL_2 - SL_4 + cr_B(a-b)L_6\right)}{cr_A(cr_BL_7 - L_8) - cr_BL_9} \tag{20}$$

$$q_B^* = \frac{cr_B\left(c_Acr_AL_1 - c_Bcr_AL_1 + cr_ASL_3 - SL_5 + cr_A(b-a)L_6\right)}{cr_A(cr_BL_7 - L_8) - cr_BL_9} \tag{21}$$

The auxiliary expressions $L_1$ to $L_9$ in the above equations can be found in Appendix C in supplementry material.

In this section, it should be noted that ordering products to the manufacturers are profitable for the retailer as long as $\pi_{r,i} \geq 0$; which means that in the case of $\pi_{r,i}$ being under zero ($q_i \leq 0$), it is not reasonable for the retailer to allocate any shelf space to this product, which turns the problem into a monopoly.

### 3.3 Retailer shelf-space-size optimization

In the initial stage, the retailer determines shelf space $S$. The retailer's profit function is defined as follows:

$$\Pi_r = \Pi_n - k \times S^2 \tag{22}$$

$k \times S^2$ is an increasing convex function with regard to $S$, indicating the shelf-space cost, and $k$ is a positive constant denoting the shelf-space cost parameter. Kurtuluş and Toktay (2011) first introduced this shelf-space-price function. Based on the results from Lemma 2, the optimal shelf space is determined as in Proposition 1. The proof of Proposition 1 is provided in Appendix D in supplementry material.

In managerial terms, there may be combinations of parameter values under which it is not profitable for the retailer to allocate any shelf space to the products ($\pi_r \leq 0$). When this happens, the optimal value of $S$ should be zero.

**Proposition 1.** *The optimal shelf space S can be calculated as follows*:

$$S = \frac{8cr_A^2 cr_B\left((c_B - c_A)M_1 + (a-b)M_2\right) + (c_A - c_B)M_3 + (b-a)M_4 + c_B M_5 + aM_6 - c_A M_7}{\left(2\,cr_A^2\,(8\,cr_B^2\,M_8 + M_9 - 8cr_B M_{10} + M_{11}\right) + cr_B M_{12} - cr_A M_{13}\right)} \tag{23}$$

The auxiliary expressions $M_1$ to $M_{13}$ can be found in Appendix D in supplementry material.

***Proof.*** Please see the proof in Appendix D in supplementry material.

By substituting the optimal retailer shelf space into Eqs. (14)–(17), (20) and (21), the optimal wholesale prices and warranty periods for manufacturers and the optimal demand for every product can be determined, respectively. In addition, using optimal values of retailer shelf space, wholesale prices, and warranty periods, the optimal retailer prices can be calculated by Eqs. (6) and (7).

## 4 Computational and practical results

In Sect. 4, we demonstrate the developed model through numerical analysis. According to Wang et al. (2015), accounting for the effects of non-symmetric market potential, production costs, and cross-price sensitivity parameters makes the solution process very complicated. By incorporating the influence of the warranty service cost and the warranty competition factor into our study, the model we propose is more complicated than that of Wang et al. Wang et al. (2015). In this section, we analyze computational and practical results from different perspectives.

### 4.1 Impact of product's market potential

In this section, we considered the impact of market potential on the profit of the manufacturers, and the retailer, in tandem with decision variables. Table 2 presents optimal results for different $a/b$ values (with $b=10,\ k=1,\ \theta_A=\theta_B=1,\ \lambda_A=\lambda_B=0.8,\ \gamma_A=\gamma_B=0.2,\ c_A=c_B=1,$ and $cr_A=cr_B=0.2$). As shown in Table 2, the retail price of each product and the retailer's shelf space allocation for the product both become greater with the increased market potential ratio ($a/b$). Indeed, with this setup, a product's market potential has a more intense effect on its optimal retail price than does the other product's market potential.

Therefore, as can be seen in Fig. 1, the increase of the market potential of a product leads to a considerable rise in the retailer's profit because the increase in profit from selling this product is more noticeable than that of the other product. For example, if the market potential of product A increases by 60 percent ($\frac{a}{b} = 1.6$), compared to $\frac{a}{b} = 1$, the retailer's profit increases on product A by 132 percent (9.67 vs. 22.4) and on product B by 25 percent (9.67 vs. 12.1). And the overall profit of the retailer also increases by 80 percent. On the other hand, if the market potential of product A decreases by 60 percent ($\frac{a}{b} = 0.4$), the profitability of both products A and

**Table 2** Numerical results based on product's market potential

| | | Retailer | | | | | | Manufacturer A | | | | Manufacturer B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S$ | $p_A$ | $p_B$ | $\pi_{r,A}$ | $\pi_{r,B}$ | $\pi_r$ | $wp_A$ | $W_A$ | $q_A$ | $\pi_A$ | $wp_B$ | $W_B$ | $q_B$ | $\pi_B$ |
| *a/b* | 0.2 | 1.76 | 4.26 | 6.85 | 0.33 | 6.03 | 4.81 | 1.14 | 0.09 | 0.11 | 0.01 | 3.20 | 1.38 | 1.65 | 3.27 |
| | 0.4 | 2.11 | 5.50 | 7.43 | 1.84 | 6.97 | 6.57 | 1.63 | 0.39 | 0.47 | 0.27 | 3.18 | 1.36 | 1.63 | 3.21 |
| | 0.6 | 2.47 | 6.73 | 8.02 | 3.90 | 7.89 | 8.74 | 2.13 | 0.70 | 0.84 | 0.86 | 3.16 | 1.35 | 1.62 | 3.14 |
| | 0.8 | 2.82 | 7.97 | 8.61 | 6.51 | 8.79 | 11.32 | 2.62 | 1.01 | 1.21 | 1.77 | 3.14 | 1.33 | 1.60 | 3.07 |
| | 1.0 | 3.17 | 9.20 | 9.20 | 9.67 | 9.67 | 14.29 | 3.11 | 1.32 | 1.58 | 3.01 | 3.11 | 1.32 | 1.58 | 3.01 |
| | 1.2 | 3.53 | 10.44 | 9.79 | 13.37 | 10.52 | 17.67 | 3.61 | 1.63 | 1.96 | 4.58 | 3.09 | 1.30 | 1.57 | 2.95 |
| | 1.4 | 3.88 | 11.67 | 10.38 | 17.62 | 11.36 | 21.45 | 4.10 | 1.94 | 2.32 | 6.47 | 3.07 | 1.29 | 1.55 | 2.88 |
| | 1.6 | 4.23 | 12.90 | 10.97 | 22.42 | 12.18 | 25.63 | 4.59 | 2.25 | 2.70 | 8.69 | 3.05 | 1.28 | 1.53 | 2.82 |
| | 1.8 | 4.58 | 14.14 | 11.56 | 27.77 | 12.97 | 30.22 | 5.09 | 2.55 | 3.07 | 11.24 | 3.02 | 1.26 | 1.52 | 2.75 |
| | 2.0 | 4.94 | 15.37 | 12.15 | 33.67 | 13.75 | 35.21 | 5.58 | 2.86 | 3.44 | 14.12 | 3.00 | 1.25 | 1.50 | 2.69 |

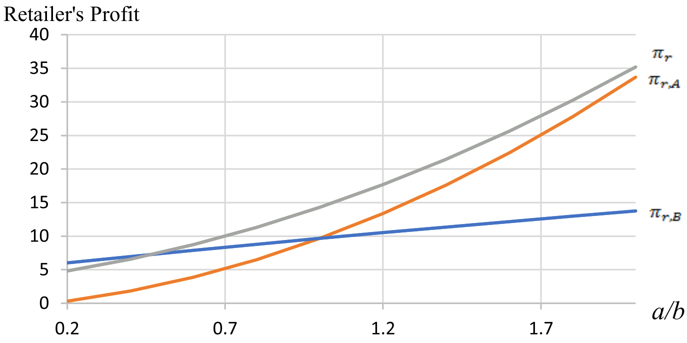


**Fig. 1** Retailer's profit versus a/b

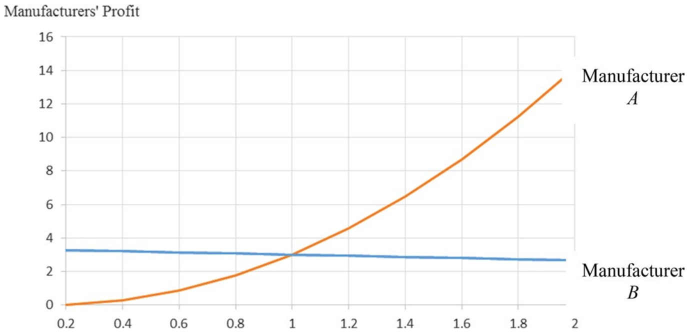


**Fig. 2** Manufracturer's profit versus a/b

B decreases by 81 percent and 28 percent, respectively, while the overall profit for the retailer decreases by 54 percent.

From Table 2, we also find that the increase in the market potential of a product causes a significant increase in the wholesale price and length of the warranty period associated with it. Still, taking into account that the manufacturer of product B is a competitor, this manufacturer's wholesale price and warranty period are slightly reduced, which leads to a remarkable climb in the profits of the manufacturer of product A, which makes the product B less profitable and causes a slow decline in the profit of its manufacturer (Fig. 2). These results make sense because the market potential of a product reflects the maximum possible demand for that product in a given market. Therefore, a higher market potential implies that more consumers are willing to buy the product at a certain price level. This shifts the demand function upward, indicating more demand for the product at any given price, and creates an opportunity for both the manufacturer and the retailer to increase their profits by raising their prices. The manufacturer can set a higher wholesale price for the retailer. Also, the retailer can set a higher retail price for the consumers as long as the demand remains positive and elastic. However, higher prices also increase the risk of losing some consumers to the competing product. To prevent this, the

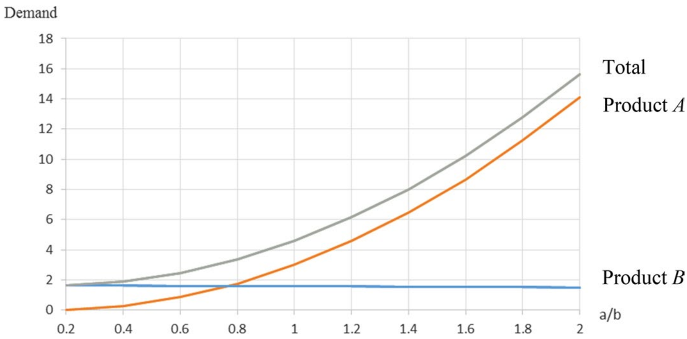


**Fig. 3** Demand versus a/b

**Table 3** Numerical results based on production cost of each unit

| | | Retailer | | | | | | Manufacturer A | | | | Manufacturer B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S$ | $p_A$ | $p_B$ | $\pi_{r,A}$ | $\pi_{r,B}$ | $\pi_r$ | $wp_A$ | $W_A$ | $q_A$ | $\pi_A$ | $wp_B$ | $W_B$ | $q_B$ | $\pi_B$ |
| $c_A/c_A$ | 0.2 | 2.45 | 9.37 | 9.39 | 10.54 | 7.03 | 11.56 | 2.14 | 1.21 | 1.45 | 2.54 | 2.32 | 0.83 | 0.99 | 1.18 |
| | 0.4 | 2.42 | 9.38 | 9.40 | 9.897 | 7.29 | 11.30 | 2.25 | 1.15 | 1.38 | 2.29 | 2.38 | 0.86 | 1.03 | 1.29 |
| | 0.6 | 2.40 | 9.39 | 9.40 | 9.26 | 7.54 | 11.05 | 2.35 | 1.09 | 1.31 | 2.06 | 2.44 | 0.90 | 1.08 | 1.40 |
| | 0.8 | 2.37 | 9.40 | 9.41 | 8.64 | 7.79 | 10.80 | 2.46 | 1.03 | 1.24 | 1.85 | 2.50 | 0.94 | 1.13 | 1.52 |
| | 1.0 | 2.34 | 9.41 | 9.41 | 8.04 | 8.04 | 10.56 | 2.56 | 0.98 | 1.17 | 1.64 | 2.56 | 0.98 | 1.17 | 1.64 |
| | 1.2 | 2.32 | 9.42 | 9.41 | 7.44 | 8.27 | 10.33 | 2.67 | 0.92 | 1.10 | 1.45 | 2.62 | 1.01 | 1.21 | 1.77 |
| | 1.4 | 2.29 | 9.43 | 9.42 | 6.86 | 8.51 | 10.11 | 2.77 | 0.85 | 1.03 | 1.27 | 2.68 | 1.05 | 1.26 | 1.90 |
| | 1.6 | 2.27 | 9.44 | 9.42 | 6.30 | 8.74 | 9.892 | 2.88 | 0.80 | 0.96 | 1.10 | 2.74 | 1.09 | 1.30 | 2.04 |
| | 1.8 | 2.24 | 9.45 | 9.42 | 5.75 | 8.96 | 9.682 | 2.98 | 0.74 | 0.89 | 0.94 | 2.80 | 1.12 | 1.35 | 2.18 |
| | 2.0 | 2.21 | 9.46 | 9.43 | 5.21 | 9.18 | 9.480 | 3.09 | 0.68 | 0.81 | 0.80 | 2.86 | 1.16 | 1.39 | 2.33 |

manufacturer may offer a more extended warranty period for his product, enhancing the perceived quality and reliability and reducing the risk for the consumers. However, a more extended warranty period can also increase the switching costs for consumers, making them less likely to switch to another product in the future. Therefore, by increasing both the wholesale price and the length of the warranty period, the manufacturer can capture more of the increased market potential and maximize his profit. The retailer can also benefit from this strategy, as he can charge a higher retail price and earn a higher margin per unit sold.

In a manner similar to the price and warranty, the market potential of a product has a prominent positive influence on the optimal demand for it and a slightly negative influence on the demand for the competitor's product (Fig. 3), which is the reason that the retailer needs to improve the market potential of only one product to create more sales volume, in total.

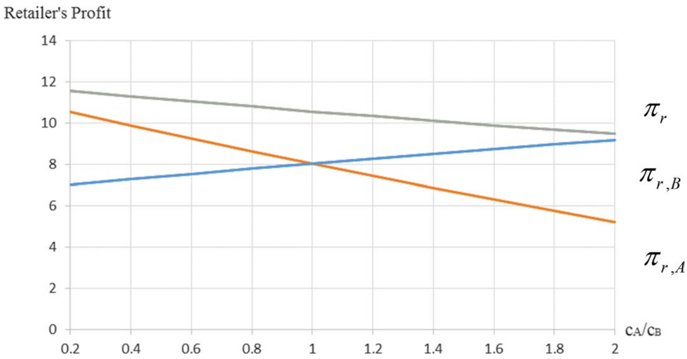


**Fig. 4** Retailer's profit versus $c_A/c_B$

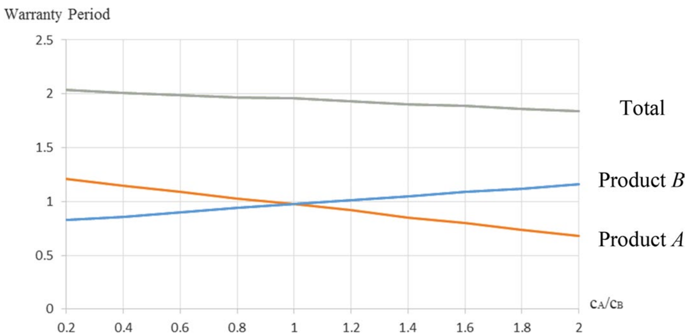


**Fig. 5** Warrenty period versus $c_A/c_B$

## 4.2 Impact of product's production cost

In this section, we examined the effect of unit production costs. Table 3 shows the optimal results for different $c_A/c_B$ values (with $a=b=10,\ k=1,\ \theta_A=\theta_B=1,\ \lambda_A=\lambda_B=0.8,\ \gamma_A=\gamma_B=0.2,\ c_B=1$, and $cr_A=cr_B=0.2$).

According to Table 3, the optimal shelf space decreases with the upward trend of production costs of the product. Moreover, the increase in the production cost of a product causes the retail and wholesale prices for both products to go up slightly. Furthermore, if the production cost of a product increases, the demand for this product decreases while the demand for the competing product rises. Indeed, the demand decline of this product is greater than the demand growth of the competing product, which indicates a drop in total demand. Similar to the effects on shelf allocation and price, an increase in the production cost of a product decreases the retailer's profit for this product. Still, it increases the retailer's profit for the competing product (Fig. 4). For instance, if the production cost of product A increases by 60 percent ($\frac{C_A}{C_B}$ = 1.6), compared to a/b = 1, the retailer's profit will decrease by 21.6 percent in product A and increase by 8.7 percent in product B, and the overall profit of the retailer will be 6.3 percent less. On the other hand, if the production cost of product A decreases by 60 percent ($\frac{C_A}{C_B}$ = 0.4), the profitability of product A increases by 23 percent, and product B decreases by 9.3 percent. In this way, the overall profit of the retailer increases by 7 percent.

It should be noted that the sum of $\pi_{r,A}$ and $\pi_{r,B}$ is less than $\pi_r$, which is derived from the additional shelf space cost. In actuality, the profit decrement of the higher-cost product is more significant than the profit increment of the competing product; that is, a decrease in the total profit of the retailer results.

Additionally, regarding the warranty period, we find similar demand results to those found for the retailer's profit. As shown in Fig. 5, if the production cost increases, the warranty period for this product decreases, but the warranty period for the competing products increases; however, the total warranty periods for both products decrease. This is a logical output because a higher production cost means a lower profit margin for the manufacturer, which may reduce its willingness to offer a longer warranty period for the product.

### 4.3 Impact of warranty service cost

In this section, we investigated the influence of warranty service costs on channel members, which is significant point. Table 4 presents the optimal results for various $\frac{Cr_A}{Cr_B}$ values (with $a = b = 10,\ k = 1,\ \theta_A = \theta_B = 1,\ \lambda_A = \lambda_B = 0.8,\ \gamma_A = \gamma_B = 0.2,\ c_A = c_B = 1$, and $cr_B = 0.2$). According to Table 4, one can see that the optimal shelf space decreases with increased warranty service costs of products. Moreover, the increase of the warranty service cost for a product causes the retail prices for both products to go down considerably. Although the rise in the warranty service cost significantly decreases the wholesale price of the product, it assures the wholesale price for the competing product moves up slowly. A real-world example of this is how Apple adjusts its retail prices for different products, based on the warranty service cost. If it becomes more expensive to repair an iPhone screen, Apple may reduce the iPhone's price to appeal to customers who are ready to pay for the warranty service or purchase AppleCare +, which is an extended warranty service that offers extra benefits and coverage for a certain time period. Conversely, Apple may raise the price of another product that has a lower warranty service cost, such as an iPad, to attract customers who care more about the product quality and performance. By doing this, Apple can balance the market potential and cross-price sensitivity of both products (Zhao et al. 2019).

Furthermore, if the warranty service cost of a product increases the demand for this product decreases, while the demand for the competing product rises. Indeed, the demand decline for the higher-cost warranty is more substantial than the demand growth for the competing product, which means a drop in total demand (see Fig. 6).

Figure 7 clearly shows that an increase in the warranty service cost of a product decreases the retailer's profit for this product dramatically. In contrast, the retailer's profit for the competing product stays constant, resulting in a collapse in the retailer's total profit. For example, as seen in Table 4, if the warranty service cost of

**Table 4** Numerical results based on warranty service costs

| | | Retailer | | | | | | Manufacturer A | | | | Manufacturer B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S$ | $p_A$ | $p_B$ | $\pi_{r,A}$ | $\pi_{r,B}$ | $\pi_r$ | $wp_A$ | $W_A$ | $q_A$ | $\pi_A$ | $wp_B$ | $W_B$ | $q_B$ | $\pi_B$ |
| $cr_A/cr_B$ | 0.2 | 3.94 | 13.49 | 10.36 | 25.06 | 8.24 | 17.75 | 4.88 | 12.13 | 2.91 | 5.41 | 2.37 | 0.86 | 1.03 | 1.27 |
| | 0.4 | 2.64 | 10.20 | 9.59 | 10.83 | 8.07 | 11.91 | 3.00 | 3.13 | 1.50 | 2.23 | 2.52 | 0.95 | 1.14 | 1.55 |
| | 0.6 | 2.46 | 9.718 | 9.48 | 9.08 | 8.05 | 11.07 | 2.73 | 1.80 | 1.30 | 1.86 | 2.54 | 0.96 | 1.16 | 1.60 |
| | 0.8 | 2.38 | 9.52 | 9.43 | 8.40 | 8.04 | 10.74 | 2.62 | 1.27 | 1.21 | 1.72 | 2.55 | 0.97 | 1.17 | 1.63 |
| | 1.0 | 2.34 | 9.41 | 9.41 | 8.04 | 8.04 | 10.56 | 2.56 | 0.98 | 1.17 | 1.64 | 2.56 | 0.98 | 1.17 | 1.64 |
| | 1.2 | 2.32 | 9.34 | 9.39 | 7.81 | 8.03 | 10.45 | 2.52 | 0.79 | 1.14 | 1.60 | 2.57 | 0.98 | 1.17 | 1.65 |
| | 1.4 | 2.30 | 9.29 | 9.38 | 7.66 | 8.03 | 10.37 | 2.50 | 0.67 | 1.12 | 1.56 | 2.57 | 0.98 | 1.17 | 1.66 |
| | 1.6 | 2.29 | 9.26 | 9.37 | 7.54 | 8.03 | 10.32 | 2.48 | 0.58 | 1.11 | 1.54 | 2.57 | 0.98 | 1.18 | 1.66 |
| | 1.8 | 2.28 | 9.24 | 9.37 | 7.46 | 8.03 | 10.27 | 2.47 | 0.51 | 1.10 | 1.52 | 2.57 | 0.98 | 1.18 | 1.66 |
| | 2.0 | 2.27 | 9.22 | 9.36 | 7.39 | 8.03 | 10.24 | 2.46 | 0.45 | 1.09 | 1.51 | 2.57 | 0.98 | 1.18 | 1.67 |

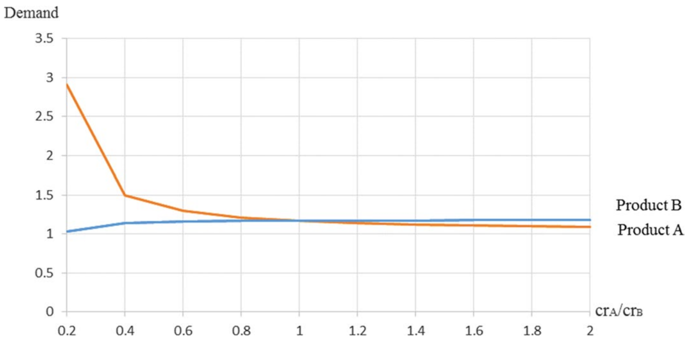


**Fig. 6** Demand versus $cr_A/\ cr_B$

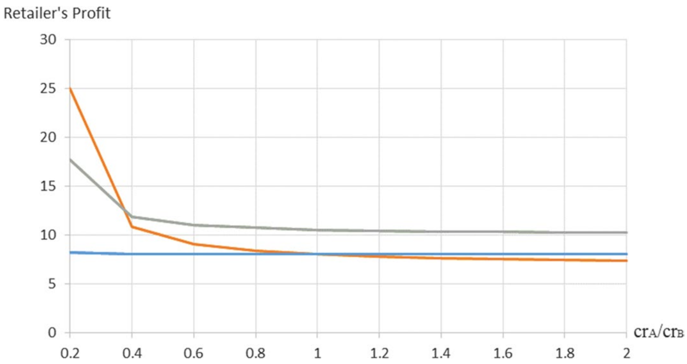


**Fig. 7** Retailer's profit versus $cr_A/\ cr_B$

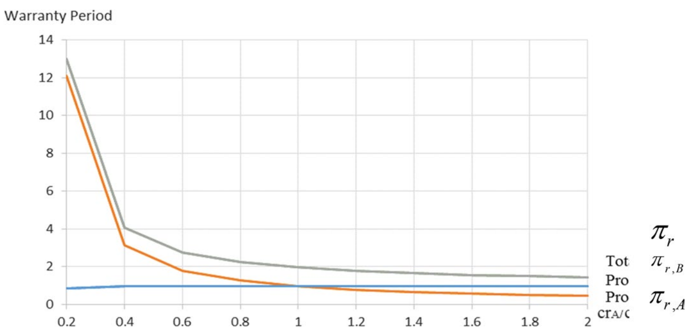


**Fig. 8** Warrenty period versus $cr_A/\ cr_B$

product A increases by 60 percent ($\frac{Cr_A}{Cr_B} = 1.6$), compared to a/b = 1, the retailer's profit on product A will decrease by 6.2 percent, and it is almost constant for product B. In this case, the total profit of the retailer will decrease by 2.2 percent.

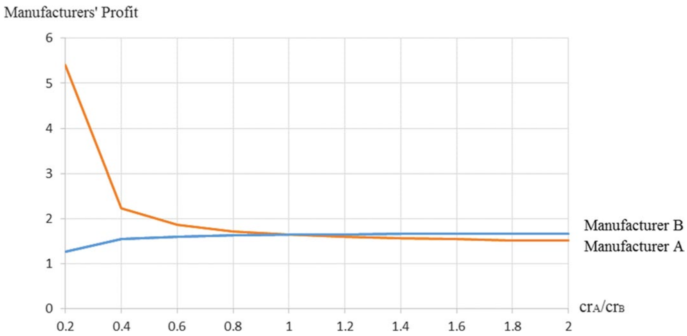


**Fig. 9** Manufracturer's profit versus $cr_A$/ $cr_B$

**Table 5** Numerical results based on various values of parameters

| | | Retailer | | | | | | Manufacturer A | | | | Manufacturer B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S$ | $p_A$ | $p_B$ | $\pi_{r,A}$ | $\pi_{r,B}$ | $\pi_r$ | $wp_A$ | $W_A$ | $q_A$ | $\pi_A$ | $wp_B$ | $W_B$ | $q_B$ | $\pi_B$ |
| $\theta_A/\theta_A$ | 0.2 | 1.95 | 9.89 | 9.95 | 5.94 | 6.71 | 8.81 | 3.22 | 1.39 | 0.89 | 1.59 | 3.67 | 1.66 | 1.07 | 2.29 |
| | 0.4 | 2.00 | 9.85 | 9.89 | 6.23 | 6.82 | 9.02 | 3.20 | 1.37 | 0.93 | 1.68 | 3.51 | 1.57 | 1.07 | 2.19 |
| | 0.6 | 2.04 | 9.81 | 9.83 | 6.50 | 6.90 | 9.21 | 3.17 | 1.36 | 0.97 | 1.76 | 3.37 | 1.48 | 1.06 | 2.09 |
| | 0.8 | 2.08 | 9.77 | 9.78 | 6.76 | 6.96 | 9.38 | 3.14 | 1.34 | 1.02 | 1.83 | 3.24 | 1.40 | 1.06 | 1.99 |
| | 1.0 | 2.11 | 9.73 | 9.73 | 7.00 | 7.00 | 9.52 | 3.11 | 1.32 | 1.05 | 1.89 | 3.11 | 1.32 | 1.05 | 1.89 |
| | 1.2 | 2.14 | 9.69 | 9.69 | 7.24 | 7.03 | 9.66 | 3.08 | 1.30 | 1.09 | 1.94 | 3.00 | 1.25 | 1.05 | 1.79 |
| | 1.4 | 2.17 | 9.65 | 9.65 | 7.46 | 7.04 | 9.78 | 3.05 | 1.28 | 1.13 | 1.99 | 2.89 | 1.18 | 1.04 | 1.69 |
| | 1.6 | 2.19 | 9.62 | 9.61 | 7.68 | 7.04 | 9.89 | 3.02 | 1.26 | 1.16 | 2.04 | 2.79 | 1.12 | 1.03 | 1.60 |
| | 1.8 | 2.22 | 9.58 | 9.58 | 7.90 | 7.02 | 9.99 | 2.99 | 1.24 | 1.19 | 2.08 | 2.70 | 1.06 | 1.02 | 1.51 |
| | 2.0 | 2.24 | 9.55 | 9.55 | 8.10 | 6.99 | 10.08 | 2.97 | 1.23 | 1.23 | 2.12 | 2.61 | 1.00 | 1.00 | 1.42 |

Nevertheless, if this cost is reduced by 60 percent ($\frac{Cr_A}{Cr_B} = 0.4$), the profit from product A for the retailer will increase by 34.3 percent, and the total profit will increase by 12.7 percent.

Furthermore, with regard to the warranty period, we find that if the warranty service cost of a product increases, the warranty period for this product is sharply reduced, and the warranty period for the competing product increases slightly; however, the total warranty periods for both products decrease (Fig. 8).

Considering the manufacturers' profits, as presented in Fig. 9, the increase in the warranty service cost of product $A$ decreases this manufacturer's profit dramatically. In contrast, the profit for the manufacturer of product $B$ goes up slightly with the rise of the warranty service cost of product $A$.

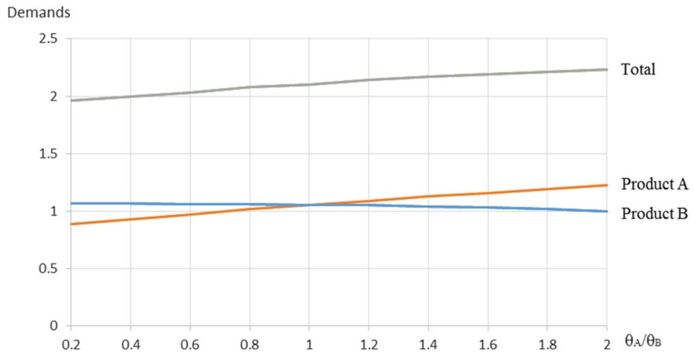


**Fig. 10** Demand versus θA/θB

**Table 6** Numerical results based on warranty competition factors

| | | Retailer | | | | | | Manufacturer A | | | | Manufacturer B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S$ | $p_A$ | $p_B$ | $\pi_{r,A}$ | $\pi_{r,B}$ | $\pi_r$ | $wp_A$ | $W_A$ | $q_A$ | $\pi_A$ | $wp_B$ | $W_B$ | $q_B$ | $\pi_B$ |
| $\gamma_A/\gamma_B$ | 0.2 | 2.35 | 9.47 | 9.38 | 8.28 | 7.88 | 10.61 | 2.60 | 1.00 | 1.20 | 1.73 | 2.53 | 0.80 | 1.15 | 1.63 |
| | 0.4 | 2.35 | 9.46 | 9.39 | 8.23 | 7.92 | 10.60 | 2.59 | 0.99 | 1.19 | 1.71 | 2.54 | 0.84 | 1.15 | 1.64 |
| | 0.6 | 2.35 | 9.44 | 9.39 | 8.17 | 7.96 | 10.59 | 2.58 | 0.99 | 1.19 | 1.69 | 2.55 | 0.89 | 1.16 | 1.64 |
| | 0.8 | 2.35 | 9.43 | 9.40 | 8.10 | 8.00 | 10.58 | 2.57 | 0.98 | 1.18 | 1.67 | 2.55 | 0.93 | 1.16 | 1.64 |
| | 1.0 | 2.34 | 9.41 | 9.41 | 8.03 | 8.03 | 10.56 | 2.56 | 0.97 | 1.17 | 1.64 | 2.56 | 0.97 | 1.17 | 1.64 |
| | 1.2 | 2.34 | 9.39 | 9.41 | 7.96 | 8.05 07 | 10.54 | 2.55 | 0.96 | 1.16 | 1.61 | 2.57 | 1.02 | 1.17 | 1.64 |
| | 1.4 | 2.33 | 9.37 | 9.42 | 7.87 | 8.10 | 10.52 | 2.53 | 0.96 | 1.15 | 1.58 | 2.58 | 1.06 | 1.18 | 1.64 |
| | 1.6 | 2.33 | 9.34 | 9.42 | 7.78 | 8.14 | 10.49 | 2.52 | 0.95 | 1.14 | 1.55 | 2.58 | 1.11 | 1.19 | 1.64 |
| | 1.8 | 2.32 | 9.31 | 9.42 | 7.69 | 8.17 | 10.46 | 2.50 | 0.94 | 1.12 | 1.52 | 2.59 | 1.15 | 1.19 | 1.64 |
| | 2.0 | 2.31 | 9.29 | 9.43 | 7.58 | 8.20 | 10.42 | 2.48 | 0.93 | 1.11 | 1.48 | 2.60 | 1.20 | 1.20 | 1.63 |

### 4.4 Impact of cross-price sensitivity parameters

In this section, we study the impact of cross-price sensitivity parameters on different supply chain members. Table 5 presents the optimal results for different $\frac{\theta_A}{\theta_B}$ values (with a = b = 10, k = 1, $\theta_B=0.5$, $\lambda_A=\lambda_B=0.8$, $\gamma_A=\gamma_B=0.2$, $c_A=c_B=1$ and $cr_A=cr_B=0.2$). This section shows the negative relationship between the cross-price sensitivity of a product and the retail price; that is, the retailer sets a lower retail price for products with relatively high cross-price sensitivity. The retailer wants to attract more customers who are sensitive to price changes and increase the demand for those products. This also helps the retailer to increase its total profit by selling more units of products. So, with the increase in price sensitivity, the retail profit caused by product A and its total profit also increases. For instance, Table 5 indicates that if the cross-price sensitivity of product A increases from $\frac{\theta_A}{\theta_B}=1$ to $\frac{\theta_A}{\theta_B}=1.6$, the retailer's profit on product A rises by 9.7

percent, and the retailer's total profit goes up by 3.8 percent. If it decreases from $\frac{\theta_A}{\theta_B} = 1$ to $\frac{\theta_A}{\theta_B} = 0.4$, the retailer's profit on product A drops by 11 percent, and its total profit falls by 5.2 percent.

Furthermore, the manufacturer of the product with higher cross-price sensitivity will set a higher wholesale price and longer warranty period, thereby leading to greater profit than that earned by the competition.

Figure 10 shows that the product with higher cross-price sensitivity will induce more demand than will the product with lower cross-price sensitivity.

### 4.5 Impact of warranty competition factors

In this section, we analyze the effect of the warranty competition factors on the system. Table 6 shows the optimal results for different $\frac{\gamma_A}{\gamma_B}$ values (with $a = b = 10$, $k = 1$, $\theta_A = \theta_B = 1$, $\lambda_A = \lambda_B = 0.8$, $\gamma_B = 0.2$, $c_A = c_B = 1$, and $cr_A = cr_B = 0.2$).

Table 6 shows the negative relationship between the warranty competition factor of a product, and its retail price, which means that the product's retail price will decrease if the warranty competition factor of the product increases. Indeed, this pattern is found when the retailer sets a higher retail price for a product with a lower value for the warranty competition factor. Furthermore, the manufacturer of the product with the higher value warranty competition factor will set a lower wholesale price and shorter warranty period, which leads to less profit than that earned by the competition. Moreover, the product with a lower warranty competition factor will provide more demand than will the product with a higher warranty competition factor. In fact, the manufacturer with the higher value warranty competition factor chooses to set a lower wholesale price to incentivize the retailer to devote more shelf space and effort to its product. This enables it to enhance its demand and market share and mitigate the adverse effect of having a lower profit margin. Nevertheless, this strategy exposes it to price and warranty competition from the other manufacturer, who can offer a higher wholesale price and a longer warranty period and capture more profit from both the retailer and the consumers. Therefore, the warranty competition factor is found to be significant in determining which manufacturer will be the winner when they both sell products through a single retailer.

Table 6 shows the competition between two manufacturers with the impact of warranty competition factors. With the increase of the warranty competition factor of product A, the retailer's profit decreases for product A and increases for product B. This can be seen, for example, if we assume that the warranty competition factors of product A change by 60 percent. If they increase from $\frac{\gamma_A}{\gamma_B} = 1$ to $\frac{\gamma_A}{\gamma_B} = 1.6$, the retailer's profit from product A declines by 3.11 percent, and its profit from product B rises by 1.36 percent. If they decrease from $\frac{\gamma_A}{\gamma_B} = 1$ to $\frac{\gamma_A}{\gamma_B} = 0.4$, the retailer's profit from product A grows by 2.5 percent, and its profit from product B falls by 1.36 percent.

**Table 7** Numerical results based on shelf-space cost

| | Retailer | | | | | | Manufacturer A | | | | Manufacturer B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $S$ | $p_A$ | $p_B$ | $\pi_{r,A}$ | $\pi_{r,B}$ | $\pi_r$ | $wp_A$ | $W_A$ | $q_A$ | $\pi_A$ | $wp_B$ | $W_B$ | $q_B$ | $\pi_B$ |
| $k$ 2 | 1.54 | 9.61 | 9.61 | 5.85 | 5.85 | 6.94 | 2.03 | 0.64 | 0.77 | 0.71 | 2.03 | 0.64 | 0.77 | 0.71 |
| 1.8 | 1.65 | 9.58 | 9.58 | 6.19 | 6.19 | 7.45 | 2.10 | 0.69 | 0.82 | 0.82 | 2.10 | 0.69 | 0.82 | 0.82 |
| 1.6 | 1.78 | 9.55 | 9.55 | 6.58 | 6.58 | 8.04 | 2.19 | 0.74 | 0.89 | 0.95 | 2.19 | 0.74 | 0.89 | 0.95 |
| 1.4 | 1.94 | 9.51 | 9.51 | 7.01 | 7.01 | 8.74 | 2.29 | 0.80 | 0.97 | 1.12 | 2.29 | 0.80 | 0.97 | 1.12 |
| 1.2 | 2.12 | 9.47 | 9.47 | 7.50 | 7.50 | 9.56 | 2.41 | 0.88 | 1.06 | 1.35 | 2.41 | 0.88 | 1.06 | 1.35 |
| 1 | 2.34 | 9.41 | 9.41 | 8.04 | 8.04 | 10.56 | 2.56 | 0.97 | 1.17 | 1.64 | 2.56 | 0.97 | 1.17 | 1.64 |
| 0.8 | 2.62 | 9.34 | 9.34 | 8.64 | 8.64 | 11.80 | 2.74 | 1.09 | 1.31 | 2.05 | 2.74 | 1.09 | 1.31 | 2.05 |
| 0.6 | 2.97 | 9.26 | 9.26 | 9.31 | 9.31 | 13.35 | 2.98 | 1.24 | 1.48 | 2.63 | 2.98 | 1.24 | 1.48 | 2.63 |
| 0.4 | 3.41 | 9.14 | 9.14 | 10.02 | 10.02 | 15.38 | 3.28 | 1.42 | 1.70 | 3.48 | 3.28 | 1.42 | 1.70 | 3.48 |
| 0.2 | 4.03 | 8.99 | 8.99 | 10.69 | 10.69 | 18.13 | 3.69 | 1.68 | 2.01 | 4.85 | 3.69 | 1.68 | 2.01 | 4.85 |

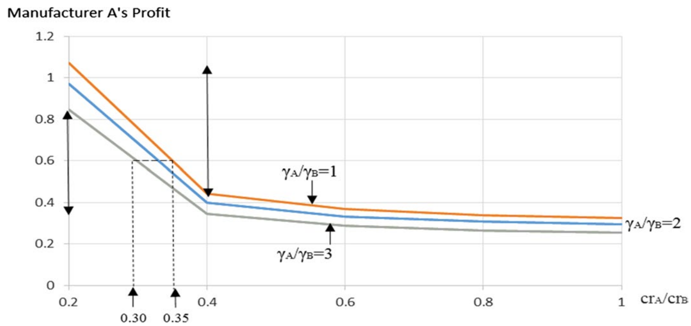


**Fig. 11** Manufracturer A's profit versus $cr_A$/ $cr_B$

## 4.6 Impact of shelf space cost

In this section, we take into consideration the impact of shelf-space cost on the supply chain members. Table 7 contains the optimal results for different $k$ values (with $a=b=10$, $\theta_A=\theta_B=1$, $\lambda_A=\lambda_B=0.8$, $\gamma_A=\gamma_B=0.2$, $c_A=c_B=1$, and $cr_A=cr_B=0.2$). This section shows that the retailer can increase the total demand by lowering the shelf space cost per unit. Additionally, by reducing the shelf-space cost per unit, the retailer sets lower retail prices for products and can earn more profit. Furthermore, from the manufacturers' perspectives, lowering the shelf space cost per unit results in higher wholesale prices and extended warranty periods, leading to more profit for both manufacturers. Table 7 demonstrates the impact of $k$ on the profits of the retailer. Increasing $k$ by 60 percent (from k = 1 to k = 1.6) results in a significant decrease of 18.1 percent in profits for products A and B, as well as an overall decrease of 23.8 percent in the retailer's total profit. Conversely, decreasing $k$ by 60 percent (from k = 1 to k = 0.4) leads to a notable increase of 24.3 percent in profits

**Table 8** Numerical results for various values of shelf-space cost (with $b=10$, $\theta_A=\theta_B=1$, $\lambda_A=\lambda_B=0.8$, $\gamma_A=\gamma_B=0.2$, $c_B=1$, and $cr_B=0.2$)

| $c_A$ | Total demand | | | Retail price of product A | | | Retailer's profit | | |
|---|---|---|---|---|---|---|---|---|---|
| | $k=1$ | $k=2$ | $k=3$ | $k=1$ | $k=2$ | $k=3$ | $k=1$ | $k=2$ | $k=3$ |
| 0.4 | 2.42 | 1.59 | 1.18 | 9.38 | 9.59 | 9.69 | 11.30 | 7.43 | 5,54 |
| 0.8 | 2.37 | 1.56 | 1.16 | 9.40 | 9.60 | 9.70 | 10.80 | 7.10 | 5.28 |
| 1.2 | 2.32 | 1.52 | 1.13 | 9.42 | 9.62 | 9.72 | 10.33 | 6.79 | 5.06 |
| 1.6 | 2.27 | 1.49 | 1.11 | 9.44 | 9.63 | 9.73 | 9.89 | 6.50 | 4.85 |
| 2 | 2.21 | 1.45 | 1.085 | 9.46 | 9.65 | 9.74 | 9.48 | 6.25 | 4.66 |
| $cr_A$ | | | | | | | | | |
| 0.1 | 2.53 | 1.62 | 1.19 | 9.89 | 9.93 | 9.95 | 11.38 | 7.29 | 5.35 |
| 0.2 | 2.34 | 1.54 | 1.15 | 9.41 | 9.61 | 9.71 | 10.56 | 6.94 | 5.17 |
| 0.3 | 2.29 | 1.52 | 1.14 | 9.28 | 9.52 | 9.64 | 10.34 | 6.84 | 5.11 |
| 0.4 | 2.27 | 1.51 | 1.13 | 9.22 | 9.48 | 9.61 | 10.24 | 6.80 | 5.09 |
| 0.5 | 2.26 | 1.50 | 1.12 | 9.18 | 9.45 | 9.59 | 10.18 | 6.77 | 5.07 |
| $a$ | | | | | | | | | |
| 6 | 1.82 | 1.2 | 0.89 | 6.89 | 7.05 | 7.13 | 6.49 | 4.30 | 3.22 |
| 8 | 2.08 | 1.37 | 1.02 | 8.15 | 8.33 | 8.42 | 8.37 | 5.51 | 4.11 |
| 10 | 2.34 | 1.54 | 1.15 | 9.41 | 9.61 | 9.71 | 10.56 | 6.94 | 5.17 |
| 12 | 2.60 | 1.71 | 1.27 | 10.67 | 10.89 | 11.00 | 13.06 | 8.59 | 6.40 |
| 14 | 2.86 | 1.88 | 1.40 | 11.92 | 12.17 | 12.29 | 15.88 | 10.47 | 7.82 |

for products A and B and a substantial overall increase of 45.6 percent in the retailer's total profit.

As explained earlier, an increase in the warranty service cost will decrease the corresponding manufacturer's profit. Figure 11 shows that the manufacturer of product *A* can offset the impact of the increase in the warranty service cost by decreasing $\gamma_A/\gamma_B$. As can be seen in Fig. 11, when the warranty-service cost ratio increases from 0.2 to 0.4, the manufacturer of product *A* will experience a more significant drop in profits when $\gamma_A/\gamma_B=1$ than when $\gamma_A/\gamma_B=3$.

From Fig. 11, one can discern that the manufacturer can meet the specific target of 0.6 by lowering the warranty service cost ratio from 0.35 to 0.30 and by raising $\gamma_A/\gamma_B$ from 1 to 3 simultaneously. In other words, manufacturers can adopt mixed strategies to reach a pre-specified profit when considering their capability to decrease either the warranty service cost or to increase the warranty-competition factor. Three strategies might enhance a retailer's profit: increasing the market potential, reducing the production cost, and reducing the warranty service cost. Suppose the retailer cannot improve the market potential of products or help manufacturers reduce production and warranty service costs. In that case, the manufacturer can use shelf space cost management, which is done by decreasing the shelf space cost, thereby resulting in lower retail prices and more profit. Therefore, we investigated how to control the shelf-space cost that will lead the retailer to improve its profit. For this purpose, Table 8 presents the optimal results for different values of shelf-space cost. They show that the retailer can use shelf space cost management as a strategy to modify the impact of reduced market potential or increased production cost (the warranty service cost) on the total demand, a product's retail price, and the retailer's profit.

A glance at Tables 2, 3, 4, 5, 6 and 7 reveals that when two products have the same market potential, production cost, cross-price sensitivity, warranty service cost, and warranty competition factor parameters the retailer sets the same retail price for both products, and the manufacturers set the same wholesale prices and warranty periods for both products. Furthermore, both products will share shelf space equally. This conclusion is similar to that of Wang et al. (2015).

## 5 Managerial insights and conclusions

In this study, we presented a pricing, warranty, and shelf-space-size decision problem associated with a two-echelon supply chain composed of one shelf-space-constrained retailer and two competing manufacturers producing similar products of different brands and selling them through a common retailer. The products made by manufacturers had non-symmetric market and warranty-period-dependent demands. The retailer determined the amount of shelf space allocation, and the retailer set the price for each product. At the same time, each manufacturer decided on wholesale prices and offered warranty periods through a three-stage decision problem. Compared with other studies, our research concentrated on evaluating the impact of a retailer's shelf space decisions and pricing strategies, along with the impact of two manufacturers' pricing and warranty strategies, on supply chain decisions. We believe that this research offers a unique contribution because few analyses of similar studies have made such connections. Our analysis led us to identical conclusions with Wang et al. (2015). However, we went above and beyond by incorporating the parameters associated with the warranty. We obtained the following results from our model.

### 5.1 Market potential

We found that increasing the market potential of a product significantly increases the demand for the product but slightly reduces the demand for the competing product. This results in an increase in the total demand and the need for additional shelf space. Therefore, the retailer can benefit from enhancing the market potential of one product to boost the total sales volume (Wang et al. 2015). This can be done by using marketing strategies such as advertising, promotion, or branding. However, the retailer should also consider the trade-off between the increased revenue and shelf space cost. Moreover, the manufacturer of the product with higher market potential can charge a higher wholesale price and offer a longer warranty period, which leads to more profit than its competitor can achieve. Therefore, the manufacturers should also try to improve their products' market potential to gain a competitive edge.

### 5.2 Production cost

We found that increasing the production cost of a product decreases the demand for the product while increasing the demand for the competing product. However, the decrease in demand for the product with a higher production cost is more than the increase in demand for the product with a lower production cost, which indicates a drop in the total demand. Therefore, the retailer suffers from higher production costs for either product, as it reduces the total sales volume and increases the retail prices (Wang et al. 2015). Furthermore, the manufacturer of the product with higher production costs faces lower wholesale prices and shorter warranty periods, which leads to lower profit than its competitor achieves. Therefore, manufacturers should also seek to lower production costs by adopting efficient technologies or processes.

### 5.3 Warranty service cost

We found that increasing the warranty service cost of a product causes retail prices for both products to go down considerably. This is because the retailer tries to stimulate demand by lowering prices when facing higher warranty service costs. Moreover, increasing the warranty service cost of a product significantly decreases the demand for the product while slightly increasing the demand for the competing product. This results in a drop in the total demand that comports with a need for less shelf space. Therefore, the retailer suffers from higher warranty service costs for either product, as it reduces the total sales volume and lowers its profit margin.

Additionally, increasing the warranty service cost of a product decreases its warranty period sharply while slightly increasing the warranty period of the competing product. This results in a decrease in the total warranty periods for both products. Therefore, the manufacturer of the product with the higher warranty service cost faces lower wholesale prices and a shorter warranty period, which leads to lower profit than its competitor achieves. Therefore, manufacturers should also seek to lower their warranty service costs by improving their products' reliability and durability.

### 5.4 Cross-price sensitivity

We found that increasing the cross-price sensitivity of a product decreases its retail price, as well as increases its demand and decreases the demand for the competing product (Wang et al. 2015). This means that the retailer sets a lower retail price for a product if it is more sensitive to the price change of the competing product, and vice versa. Therefore, the retailer can benefit from higher cross-price sensitivity of either product, as it increases the total sales volume and allows more flexibility in pricing. However, the retailer should also consider the trade-off between the increased revenue and the decreased profit margin. Moreover, increasing the cross-price sensitivity of a product increases its wholesale price and warranty period, as well as increases its profit and decreases the profit of the competing product. This means that the manufacturer of a product with higher cross-price sensitivity can charge a higher wholesale price and offer a more extended warranty period, as it faces less competition from the other product, and vice versa. Therefore, the manufacturers should also increase their products' cross-price sensitivity to gain a competitive advantage. This can be done by using differentiation strategies such as innovation, design, or features.

### 5.5 Warranty competition factor

We found that increasing the warranty competition factor of a product decreases its retail price, as well as decreases its demand and increases the demand for the competing product. This means that the retailer sets a lower retail price for a product if it has a lower value of warranty competition factor than the competing product, and vice versa. Therefore, the retailer suffers from a lower warranty competition factor of either product, which reduces the total sales volume and lowers its profit margin. Furthermore, increasing a product's warranty competition factor decreases its wholesale price and warranty period, decreases its profit, and increases the profit of the competing product. This means that the manufacturer of a product with a lower warranty competition factor faces a lower wholesale price and a shorter warranty period, as it faces more competition from the other product, and vice versa. Therefore, the manufacturers should also seek to increase their warranty competition factors by improving their products' warranty service quality and coverage.

Manufacturers can offset the increase in warranty service cost by decreasing the warranty competition factor ratio. The manufacturers can adopt mixed strategies to reach a pre-specified profit by considering their capabilities to diminish the warranty service cost or increase the warranty competition factor.

### 5.6 Shelf space cost

We found that decreasing the shelf space cost increases the total demand by lowering the retail prices for both products. This means the retailer can stimulate demand by reducing the shelf space cost per unit. Moreover, decreasing the shelf space cost increases the retailer's profit by increasing the sales volume. Therefore, the retailer can benefit from reducing the shelf space cost by optimizing its shelf space allocation. Additionally, decreasing the shelf space cost increases the wholesale prices and warranty periods for both products and the profits for both manufacturers. This means that the manufacturers can benefit from lower shelf space costs by charging higher wholesale prices and offering longer warranty periods.

Based on Wang et al. (2015), the retailer also can modify the impact of reduced market potential or increased production cost (the warranty service cost) on total demand, a product's retail price, and the retailer's profit by using shelf space cost management strategies.

Similar to other studies previously published in this field, the present model was based on a set of assumptions. For example, we assumed that the ordered quantity of each product is equal to the demand for that product. The demand rate was deterministic and dependent on both the product's warranty period, *Wi*, and the product's retail price, *pi*. An important extension of this study would include the stochastic demand in the model. Incorporating cooperative decisions between manufacturers and the retailer might be another interesting extension of this research. Furthermore, the problem would become more challenging by considering quality assurance measures alongside a model that also takes into account pricing, warranty, and shelf space factors. This research studied a supply chain comprising two manufacturers and one retailer. The problem becomes more complicated if multiple retailers are in the supply chain. The retailers may compete with each other on price and shelf space allocation, influencing the product demand and the manufacturers' profits. The manufacturers may also have different bargaining power with different retailers, depending on their market share, product differentiation, and cost structure. We suggest two possible ways to model the interactions among the retailers and the manufacturers: a game-theoretic framework and a collaborative approach. The game-theoretic framework captures the actual competition and conflict among the supply chain members, while the collaborative approach aims to optimize the overall performance of the supply chain.

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## Declarations

**Conflict of interest** No potential conflict of interest was reported by the authors.

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