Transportation Research Part E 175 (2023) 103149

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

A facility location problem in a mixed duopoly on networks

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ARTICLE INFO

Keywords:
Facility location
Charging station location
Mixed market
Multiple decision makers
Competition
Cooperation

ABSTRACT

This paper studied a facility location problem (FLP) between a public firm and a private firm on a network space. Diverse situations are presented, and the corresponding mathematical formulations are modeled by implementing the optimization approach. The relationships between the models are then analyzed mathematically, showing the existence of complementarity and dominance. Considering the increasing interest in electric vehicle (EV) charging stations and the relevance to our models, computational experiments for a charging station location problem (CSLP) follow this in order to validate the presented models and verify the analyses. The trade-off between the stakeholders' objectives is demonstrated, providing policy implications for the public sector and managerial insights for private investors.

1. Introduction

The facility location problem (FLP) has received much attention after the work of Alfred Weber, which considered the location of a warehouse to minimize the total travel distance between the warehouse and the customers (Weber, 1913). Many researchers have dealt with FLPs, and naturally, extensive reviews and surveys were inevitable. Because of insufficient publications in the past, comprehensive review papers were published that covered the widespread use of FLPs, such as papers by Brandeau and Chiu (1989) or Owen and Daskin (1998). However, as the number of publications has increased significantly in the last few decades, there has been a tendency to narrow the scope of reviews or surveys.

Farahani et al. (2012) presented a literature review for set covering problems in facility locations, which the models we present in this paper originate from. Set covering location problems have been used to identify the optimal locations of facilities to serve demand points within a previously defined distance of time.

The problem may be completely different depending on the purpose of the facility. In particular, Revelle et al. (1970) distinguishes the public firm from the private firm and demonstrates the difference between the two. The public sector usually focuses on non-economic benefits (e.g., social welfare), whereas the private firms typically focus on monetary gain. This distinction mainly appears in the objectives. Current et al. (1990) reviewed the studies that examined the multi-objective aspects of FLPs, as well as classified the objectives most frequently used for FLPs. They considered the most popular 23 objectives categorized into four types: cost objectives, demand-oriented objectives, profit objectives, and environmental objectives. Furthermore, Farahani et al. (2010) investigated multi-criteria decision-making problems in the location analysis, where multi-criteria decision-making problems are composed of multi-objective decision-making problems and multi-attribute decision-making problems.

Beyond problems that focus on a single decision maker, there are also problems that focus on multiple decision makers. When more than one decision maker exists, interactions inevitably occur between them, which take the form of either competition or cooperation. The competitive location problem originated with the study of Hotelling (1990). He considered a case in which two

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https://doi.org/10.1016/j.tre.2023.103149
Received 12 September 2022; Received in revised form 3 May 2023; Accepted 6 May 2023
Available online 22 May 2023
1366-5545/

Table 1
Categorization of the problem.

	Sequential		Simultaneous
	Public ⟶ Private	Private ⟶ Public	
Competition	Model 1	–	–
Cooperation	–	Model 2	Model 3 (3–1 & 3–2)

firms simultaneously made decisions on a finite linear space with uniformly distributed customers. After this groundbreaking study, much work has been carried out on the competitive location problem. Aboolian et al. (2007) thoroughly investigated simultaneous situations, whereas Kress and Pesch (2012) presented a rich literature review on the sequential case setup, especially on networks. On the other hand, research into cooperation has also been extensively conducted. Goemans and Skutella (2004) studied the fair cost allocation of several variants of FLPs based on the cooperative game theory.

It is easily observable that the multiple decision makers of the previously mentioned papers have the same objectives, respectively. Unfortunately, there may be multiple decision makers with different or conflicting objectives, as we show in this paper through our consideration of both a single public firm and a single private firm that pursued different purposes. A market in which both public and private firms participate is called a mixed market, which is challenging to analyze because of the complex situation caused by several different objectives. Due to the complexity inherent in the setup, studies dealing with the FLP in a mixed market were limited to a linear space (Matsumura and Matsushima, 2003; Ogawa and Sanjo, 2007; Heywood and Ye, 2009a,b; Sanjo, 2009a,b; Bárcena-Ruiz and Casado-Izaga, 2012; Zhang and Li, 2013; Fousekis, 2015; Matsumura and Tomaru, 2015; Zennyo, 2017; Fernández-Ruiz, 2020; Hehenkamp and Kaarbøe, 2020; Ma et al., 2021; Heywood et al., 2022; Michelacakis, 2023) or a circular space (Li, 2006; Matsushima and Matsumura, 2006). Extensions to more complex spaces should be extensively studied.

To the best of our knowledge, no attempts have been made to integrate a mixed market into the FLP on a network. The motivation for this study is to tackle such issue. In this paper, we focus on an FLP between a public firm and a private firm. We present several situations with corresponding mathematical models. Mathematical analyses are performed based on the optimization of the presented models. Considering the increasing interest in electric vehicle (EV) charging stations and the relevance to our models, numerical experiments for a charging station location problem (CSLP) are conducted.

The remainder of this study is organized as follows: In Section 2, we describe our problem, including the assumptions. The mathematical formulations are presented in Section 3. Based on these formulations, some research questions are raised in Section 4. In addition, mathematical analyses based on four propositions are provided to answer the research questions. Extensions for the capacitated environment are introduced separately in Section 5. Computational experiments are reported in Section 6, and conclusions are offered in Section 7.

2. Problem description

We consider an FLP at a network formed market, based on the mixed duopoly model for a single period. As we handle a network space, the optimal solutions could not be expressed in a closed form, or generally. Therefore, we implement the mixed-integer linear programming optimization with mathematical analyses instead of the game theoretic approach, which the previous works have applied to a linear or circular space. The duopoly consists of a public firm attempting to maximize the total coverage within a given budget and a private firm that maximizes its own profit. The firms, hereafter "public" and "private" respectively, are willing to locate ordinary service facilities (Farahani et al., 2019; Celik Turkoglu and Erol Genevois, 2020). The demands are assumed to be deterministic. In detail, the number of demands contained in each node is fixed, and it does not deliver any information about customer arrivals or individual demands. The facilities are located at the nodes of the network, and only one facility, at most, can exist at any node.

For now, we assume that every facility is uncapacitated. More precisely, the capacities are set to be sufficient to satisfy the predicted demands. As we assumed deterministic demands, the demands that a facility should cover in maximum are fixed once its location is decided. Moreover, we can determine the maximum demand load for every potential facility location. Then, the number of servers could be prepared before choosing the locations, and a scheduling problem could be conducted afterward additionally to manage all the demands to be served appropriately (Bansal et al., 2022; Park and Moon, 2022). Therefore, as all the demands could be satisfied regardless of their volumes, we can assume infinite capacity. In addition, we assume that every facility is identical. Combined with the infinite capacity assumption, it is easily known that every customer visits only the nearest facility. However, if the identity assumption does not hold, the problem gets more complicated. Such an extension will be discussed in Section 5, considering the preferences over heterogeneous facilities. The problem can be categorized into six cases, as shown in Table 1.

First, simultaneous competition can take place between the two firms. However, considering that an FLP is being handled at a mixed market, simultaneous competition is not likely to occur in the real world.

Next, the competition can arise sequentially, branching again into two cases: the private decided first, followed by the public, and vice versa. For the former case, the public might intrude into the coverage by the private to maximize its own objective. Then the private might lose some of its revenue and might file a civil complaint, which the public would not countenance, because of its publicity. Thus, the public would attempt to preserve the coverage of the private. This situation is more likely to represent cooperation and will be introduced again later. The latter situation seems to be more realistic. After the public chooses its locations myopically, the private can enter the market only for profitable spots. Because the entry of the private allows only the total coverage to benefit, the public has no reason to inhibit it.

Cooperation can take place instead of competition, leading to another three cases. Among them, the private leading in cooperation corresponds to the previous case. The private decides first to maximize its profit, and then the public chooses the locations while maintaining the market share of the private. As a result, the public will cover the lonesome nodes left by the private. Public facilities located in the countryside, with negligible populations are an example. When the public decides preemptively, the situation barely shows any characteristics of cooperation.

The two firms could decide simultaneously as well. Cooperation, then, can be regarded as a bi-objective decision-making problem. Classical approaches for solving the multi-objective optimization problem, including the bi-objective problem, try to convert such problems into a single-objective problem. One of the most popular approaches is to modify all except one objective as a constraint. To apply such an approach to this case, we consider one of the firms as the main decision maker and optimize its objective function while guaranteeing the other for a certain level as a constraint. As a result, we can consider two separate models regarding which firm has the main decision: the model with the main decision maker of the private and the model with the public having the main decision.

Consequently, we consider the more realistic three cases: (i) sequential competition starting from the public (Model 1), (ii) sequential cooperation starting from the private (Model 2), and (iii) simultaneous cooperation (Model 3 (3–1 & 3–2)).

3. Mathematical formulation

3.1. Notations

The model sets and parameters are defined as follows:

I : set of customer zones

J : set of potential facility locations

B : annual budget allocated to the public firm

r : fixed coverage radius

h_i : annual demand of customer zone $i \in I$

d_{ij} : distance between customer zone $i \in I$ and candidate location $j \in J$

a_{ij} : 1 if $d_{ij} \le r$, 0 otherwise

f_j : annual amortized total cost to open, operate and maintain a facility at candidate location $j \in J$

α : annual earnings gained by serving a unit demand

P : assurance level of the private's profit

β : assurance level of the total coverage

The opening of facilities usually costs a big lump sum, but this expense arises only once. However, operating costs and maintaining costs occur regularly but are relatively small. We assume the opening costs to be annually amortized, to consider all costs together. Consequently, the total cost, including operating costs, maintenance costs, and amortized opening costs, is assumed to arise annually. The interest rate is not introduced because we are not handling multiple periods. Because we assume deterministic demands, the profit is predictable and can be estimated if the coverage is specified. Hence, we deal with the annual earnings gained by serving a unit demand instead of dealing with the fee imposed for a one-time service. The assurance levels should be given as parameters for guaranteeing the secondary firm's objective function to be not less than a certain level in Model 3. For some practical situations, these values may be given explicitly. Elsewhere, for a general situation, we will later discuss in Section 4 how to set the assurance levels.

Also, four decision variables are used to construct the mathematical models, as follows:

x_j : 1 if a public facility is located at candidate location $j \in J$, 0 otherwise

y_{ij} : the fraction of demand of customer zone $i \in I$ served by public facility $j \in J$

z_j : 1 if a private facility is located at candidate location $j \in J$, 0 otherwise

w_{ij} : the fraction of demand of customer zone $i \in I$ served by private facility $j \in J$

3.2. The sequential competition model (Model 1)

The sequential competition model (Model 1) is as follows:

$$\max \quad \sum_j \sum_i h_i y_{ij}$$

$$\text{s.t.} \quad y_{ij} \le a_{ij} x_j \qquad i \in I,\ j \in J \quad (1)$$

$$\sum_j y_{ij} \le 1 \qquad i \in I \quad (2)$$

$$y_{ik} \le 2 - (a_{ij}x_j + a_{ik}x_k) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \quad (3)$$

$$\sum_j f_j x_j \le B \quad (4)$$

$$x_j \in \{0,1\} \qquad j \in J \quad (5)$$

$$y_{ij} \ge 0 \qquad i \in I,\ j \in J \quad (6)$$

$$\max \quad \alpha \sum_j \sum_i h_i w_{ij} - \sum_j f_j z_j$$

$$\text{s.t.} \quad y_{ij} \le a_{ij} x'_j \qquad i \in I,\ j \in J \quad (7)$$

$$w_{ij} \le a_{ij} z_j \qquad i \in I,\ j \in J \quad (8)$$

$$\sum_j (y_{ij} + w_{ij}) \le 1 \qquad i \in I \quad (9)$$

$$x'_j + z_j \le 1 \qquad j \in J \quad (10)$$

$$y_{ik} + w_{ik} \le 2 - (a_{ij}(x'_j + z_j) + a_{ik}(x'_k + z_k)) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \quad (11)$$

$$z_j \in \{0,1\} \qquad j \in J \quad (12)$$

$$y_{ij}, w_{ij} \ge 0 \qquad i \in I,\ j \in J \quad (13)$$

$$\max \quad \sum_j \sum_i h_i (y_{ij} + w'_{ij})$$

$$\text{s.t.} \quad y_{ij} \le a_{ij} x'_j \qquad i \in I,\ j \in J \quad (14)$$

$$\sum_j (y_{ij} + w'_{ij}) \le 1 \qquad i \in I \quad (15)$$

$$y_{ik} + w'_{ik} \le 2 - (a_{ij}(x'_j + z'_j) + a_{ik}(x'_k + z'_k)) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \quad (16)$$

$$y_{ij} \ge 0 \qquad i \in I,\ j \in J \quad (17)$$

Model 1 consists of three stages. The first and the second stages represent the decision of the public and the profit-seeking choice of the private with the decision of the public given, respectively. The objective function of Stage 1 maximizes the public's coverage. Constraints (1) prohibit a customer from being covered by a facility that has not been opened, or that is not within a given radius, r. Constraints (2) state that customers can be disregarded. Note that altering the inequality to strict equality necessitates covering every customer, which might be infeasible because of the budget constraint. Constraints (3) represent the customers' preferences for the nearest facility. In detail, if facilities j and k are open and j is relatively closer, customers will not visit facility k. Constraint (4) indicates the budget limitation over the costs. Constraints (5) define the domain of variable x, and Constraints (6) require variable y to be non-negative. Restricting x to a binary state satisfies the assumption that, at most, only one facility can exist at any given node. Note that customers always visit the nearest facility, and all facilities are uncapacitated. Therefore, despite y being defined as continuous, there always exists an optimal solution in which $y_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J$. In fact, it is reasonable to designate y as being continuous rather than binary, because the customers of a single node can be partitioned into several facilities at the same distance. Moreover, considering the capacitated environment in Section 5, it is better to define y as a continuous variable. The public's decision in Stage 1 (x) is fixed as a parameter in Stage 2 (x'), but the coverage is still represented by a variable (y), because it can change by the private's choice.

The objective function of the second stage maximizes the private's profit, which is composed of the revenue earned from covering the customers and the costs. Note that the private's revenue is proportional to its coverage. The variables z and w of the private correspond to the public's variables x and y, respectively. Constraints (10) restrict any node from having more than one facility, regardless of the owner. Other constraints are comparable to the first stage.

The last stage has no conceptual meaning but guarantees the maximum total coverage among several optimal solutions retrieved from the second stage. In detail, as the objective function of the second stage is independent of y, the actual coverage may not be fully reflected in y. Regardless of the actual coverage, it can be feasible and optimal even if y is all zero in the second stage. The third stage prevents such a case.

3.3. The sequential cooperation model (Model 2)

The sequential cooperation model (Model 2) is as follows:

$$\max \quad \alpha \sum_j \sum_i h_i w_{ij} - \sum_j f_j z_j$$

$$\text{s.t.} \quad w_{ij} \le a_{ij} z_j \qquad i \in I,\ j \in J \quad (18)$$

$$\sum_j w_{ij} \le 1 \qquad i \in I \quad (19)$$

$$w_{ik} \le 2 - (a_{ij} z_j + a_{ik} z_k) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \quad (20)$$

$$z_j \in \{0,1\} \qquad j \in J \quad (21)$$

$$w_{ij} \ge 0 \qquad i \in I,\ j \in J \quad (22)$$

$$\max \quad \sum_j \sum_i h_i (y_{ij} + w'_{ij})$$

$$\text{s.t.} \quad y_{ij} \le a_{ij} x_j \qquad i \in I,\ j \in J \quad (23)$$

$$\sum_j (y_{ij} + w'_{ij}) \le 1 \qquad i \in I \quad (24)$$

$$x_j + z'_j \le 1 \qquad j \in J \quad (25)$$

$$y_{ik} + w'_{ik} \le 2 - (a_{ij}(x_j + z'_j) + a_{ik}(x_k + z'_k)) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \quad (26)$$

$$\sum_j f_j x_j \le B \quad (27)$$

$$x_j \in \{0,1\} \qquad j \in J \quad (28)$$

$$y_{ij} \ge 0 \qquad i \in I,\ j \in J \quad (29)$$

Model 2 consists of two stages. The first stage represents the decision of the private with a profit-maximizing objective function. All the constraints of Model 2, including the second stage, have been described previously.

Stage 2 represents the public's decision with the choice given by the private. The objective function maximizes the total coverage. The private's decision of Stage 1 (z) is fixed as a parameter in Stage 2 (z'), as in Model 1. The coverage (w) is also fixed (w'), because Model 2 handles the situation of the public that preserves the private's market share. Therefore, the private's coverage should remain static. Integration of Constraints (24) and (26) inhibits the public's intrusion and renders any solution permitting the private's loss as infeasible.

3.4. The simultaneous cooperation model (Model 3)

The simultaneous cooperation model with the main decision maker of the private (Model 3–1) is as follows:

$$\max \quad \alpha \sum_j \sum_i h_i w_{ij} - \sum_j f_j z_j$$

$$\text{s.t.} \quad \sum_j \sum_i h_i (y_{ij} + w_{ij}) \ge \beta \sum_i h_i \quad (30)$$

$$y_{ij} \le a_{ij} x_j \qquad i \in I,\ j \in J \quad (31)$$

$$w_{ij} \le a_{ij} z_j \qquad i \in I,\ j \in J \quad (32)$$

$$\sum_j (y_{ij} + w_{ij}) \le 1 \qquad i \in I \quad (33)$$

$$x_j + z_j \le 1 \qquad j \in J \quad (34)$$

$$y_{ik} + w_{ik} \le 2 - (a_{ij}(x_j + z_j) + a_{ik}(x_k + z_k)) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \quad (35)$$

$$\sum_j f_j x_j \le B \quad (36)$$

$$x_j, z_j \in \{0,1\} \qquad j \in J \quad (37)$$

$$y_{ij}, w_{ij} \ge 0 \qquad i \in I,\ j \in J \quad (38)$$

Table 2
Results of the small-size problem.

Model	Total coverage	The private's profit
1	79.5%	73.3265
2	82.9%	219.9895
3–1	82.9%	219.9895
3–2	100.0%	115.9871

On the other hand, the simultaneous cooperation model with the public having the main decision (Model 3–2) is as follows:

$$\max \quad \sum_j \sum_i h_i (y_{ij} + w_{ij})$$

$$\text{s.t.} \quad \alpha \sum_j \sum_i h_i w_{ij} - \sum_j f_j z_j \geq P \tag{39}$$

$$y_{ij} \leq a_{ij} x_j \qquad i \in I,\ j \in J \tag{40}$$

$$w_{ij} \leq a_{ij} z_j \qquad i \in I,\ j \in J \tag{41}$$

$$\sum_j (y_{ij} + w_{ij}) \leq 1 \qquad i \in I \tag{42}$$

$$x_j + z_j \leq 1 \qquad j \in J \tag{43}$$

$$y_{ik} + w_{ik} \leq 2 - (a_{ij}(x_j + z_j) + a_{ik}(x_k + z_k)) \qquad i \in I,\ j,k \in J : d_{ik} > d_{ij} \tag{44}$$

$$\sum_j f_j x_j \leq B \tag{45}$$

$$x_j, z_j \in \{0, 1\} \qquad j \in J \tag{46}$$

$$y_{ij}, w_{ij} \geq 0 \qquad i \in I,\ j \in J \tag{47}$$

Both versions of Model 3 contain a single stage each, with each showing the simultaneous circumstance. Note that no variables are fixed and are considered as parameters because of the simultaneousness of the model. The objective function of Model 3–1 maximizes the private's profit, because the private is the main decision maker, whereas the objective function of Model 3–2 maximizes the total coverage considering mainly the public.

The only difference in the constraints between the two models is Constraints (30) and (39). These two constraints represent the guarantee for the secondary firms. For example, the main decision maker of Model 3–1 is the private, which makes the public the secondary participant. Therefore, the total coverage is guaranteed to be higher than a certain level, β. The private's profit is guaranteed in Model 3–2 because the public carries the main decision. All the other constraints have been described previously.

Note that the objectives of the secondary firms for each model are not guaranteed as being maximized. In other words, several optimal solutions can exist with the same objective function value but with different total coverage (Model 3–1) or with different profit outcomes of the private (Model 3–2) because both models contain a single stage, unlike Model 1. This latent problem will be handled in the next section.

4. Problem analysis

To validate the models presented in Section 3, a simple experiment has been conducted. The data were taken from Snyder and Shen (2019), which was named "10-node facility instance". The setting for the parameters and the detailed analysis will be introduced in Section 6. Model 1 and Model 2 were applied to the instance in advance. Then the results of Model 1, the total coverage and the private's profit, were used as the assurance levels for Model 3–1 and Model 3–2.

Based on the summarized results shown in Table 2, four research questions have been raised and investigated. They are as follows:

- Will Model 3–1 and Model 3–2 retrieve the same results, and if so, under which conditions?
- Will Model 2 and Model 3–1 always retrieve the same results?
- Is there a trade-off between the total coverage and the private's profit?
- Will Models 2, 3–1, and 3–2 dominate Model 1?

The optimal results are denoted as $(\tilde{P}, \tilde{\beta})$, (P^*, β^*), $(P_1^*(\beta), \beta_1)$ and $(P_2, \beta_2^*(P))$ for Models 1, 2, 3–1, and 3–2, respectively. Before answering the above questions, we will clarify the definitions of the terms "dominate" and "Pareto optimal".

Definition 1. Model A strictly dominates Model B if the outcomes of Model B are all worse than Model A. Model A weakly dominates Model B if the outcomes of Model A are all at least as good as the outcomes for Model B. Model A dominates Model B if Model A either strictly dominates or weakly dominates Model B.

Definition 2. A situation is called "Pareto optimal" if some improvements of an outcome always lead to a strict decline of any other outcome.

Proposition 1. *Assuming feasibility of Model 3–1 for a given $\bar{\beta}$,*

(1) $\beta_2^*(P_1^*(\bar{\beta})) \geq \bar{\beta}$ *and* $(P_1^*(\bar{\beta}), \beta_2^*(P_1^*(\bar{\beta})))$ *is Pareto optimal*

(2) *For* $P < P_1^*(\bar{\beta});\ \beta_2^*(P) \geq \beta_2^*(P_1^*(\bar{\beta}))$ *and* $\beta_2^*(P) > \beta_2^*(P_1^*(\bar{\beta})) \implies P_2 < P_1^*(\bar{\beta})$

(3) *For* $P > P_1^*(\bar{\beta});\ \beta_2^*(P) < \beta_2^*(P_1^*(\bar{\beta})),\ P_2 > P_1^*(\bar{\beta})$ *for every feasible solutions of Model 3–2*

Proposition 2. *Assuming feasibility of Model 3–2 for a given $\bar{P}$,*

(1) $P_1^*(\beta_2^*(\bar{P})) \geq \bar{P}$ *and* $(P_1^*(\beta_2^*(\bar{P})), \beta_2^*(\bar{P}))$ *is Pareto optimal*

(2) *For* $\beta < \beta_2^*(\bar{P});\ P_1^*(\beta) \geq P_1^*(\beta_2^*(\bar{P}))$ *and* $P_1^*(\beta) > P_1^*(\beta_2^*(\bar{P})) \implies \beta_1 < \beta_2^*(\bar{P})$

(3) *For* $\beta > \beta_2^*(\bar{P});\ P_1^*(\beta) < P_1^*(\beta_2^*(\bar{P})),\ \beta_1 > \beta_2^*(\bar{P})$ *for every feasible solutions of Model 3–1*

Corollary 1. $P_1^*(\beta_2^*(P_1^*(\beta))) = P_1^*(\beta),\ \beta_2^*(P_1^*(\beta_2^*(P))) = \beta_2^*(P)$

All the proofs are provided in the online appendix. The first question can be answered by Corollary 1 and Propositions 1 and 2. Models 3–1 and 3–2 will retrieve the same results when $\bar{\beta} = \beta_2^*(P_1^*(\beta))$ for Model 3–1 and $\bar{P} = P_1^*(\beta)$ for Model 3–2 with a given β or $\bar{\beta} = \beta_2^*(P)$ for Model 3–1, and $\bar{P} = P_1^*(\beta_2^*(P))$ for Model 3–2 with a given P.

As mentioned previously, Models 3–1 and 3–2 may have several optimal solutions. Note that $(P_1^*(\bar{\beta}), \beta_2^*(P_1^*(\bar{\beta})))$ and $(P_1^*(\beta_2^*(\bar{P})), \beta_2^*(\bar{P}))$ are Pareto optimal each, which indicates that these results are the best among those multiple optimal solutions. Consequently, using the two models sequentially is the key to ensuring the best outcome for the secondary firm in a given circumstance. This demonstrates the complementarity of the two models, despite the fact that they are presented to describe different situations.

Proposition 3. *Assuming feasibility of Model 2,*

(1) $P^* \geq P_1^*(\beta),\ \forall\beta$

(2) $\beta \leq \beta^* \iff P_1^*(\beta) = P^*$

(3) *For* $\beta = \beta^*;\ (P_1^*(\beta), \beta_1) = (P^*, \beta^*)$

(4) *For* $\beta < \beta^*;\ \beta_2^*(P_1^*(\beta)) = \beta^*$

(5) *For* $\beta > \beta^*;\ P_1^*(\beta) < P^*,\ \beta_1 > \beta^*$ *for every feasible solutions of Model 3–1*

The second question can be answered "no" by Proposition 3. It is easily shown that Model 2 is a relaxation of Model 3–1. Therefore, the two models are not precisely equivalent and may not retrieve the same results. In detail, Model 3–1 will always result in the same outcome as the outcome for Model 2 if the given β is equal to β^*. If the given β is smaller than β^*, the private's profit in Model 3–1 is the same as that of Model 2, but the total coverage is not guaranteed to be the same because of the existence of multiple optimal solutions. For this case, additionally applying Model 3–2 after Model 3–1 will ensure that the results of Model 2 are achieved. The two models cannot have the same results if β is given as being bigger than β^*.

By combining the three propositions, we can conclude that a trade-off between the total coverage and the private's profit exists in cooperative situations, and we can answer the third question. The trade-off, hereafter, is only considered in a cooperative environment. In particular, Proposition 3 shows that the maximum profit among any circumstances is achieved by Model 2 and gives the bound for the trade-off curve. In detail, the result of Model 2 (P^*, β^*) will be placed at one end of the curve, having the highest profit of the private and the lowest total coverage within the curve. The other points of the curve can be found by increasing the input β of Model 3–1 starting from β^* until 100 percent, and applying Model 3–2 consecutively. Decreasing the input P of Model 3–2 starting from P^* could be another way to achieve this end. For this case, P decreases until the resulting total coverage of the subsequent model, Model 3–1, reaches 100 percent. Either approach will give the same results in the same order, and we implement the first procedure for the experiments. However, although the trade-off curve is organized equally regardless of which model is applied first, the sequence clearly distinguishes the main decision maker. The first model determines the main decision maker, but it does not imply that the following model shifts the main decision maker. It is somewhat related to the purpose of the third stage of Model 1. Utilizing the other model consecutively just implies caring for the secondary firm even for a little while the objective value of the main decision maker of the primary model is already ensured. The complementarity of the models is enhanced by accompanying Model 2, considering that it sets the starting point when drawing the trade-off curve. The result of Model 2, which is one end of the trade-off curve, can also be obtained by giving the value of β as 0 for Model 3–1 and again applying Model 3–2. Thus, the employment of Model 2 is actually not mandatory for drawing the trade-off curve. Yet, as Model 2 is one of the starting points of the curve, it offers computational advantages.

Note that the private's profit also contains a part of the coverage, given that the private's revenue is expressed as a linear function of its own coverage. The interesting part of the third question is that the total coverage and the private's profit present a trade-off despite the common factor. In fact, it is quite apparent mathematically, because Propositions 1 and 2 guarantee the Pareto optimality. However, it is slightly more complex logically. Recall that locating more facilities never drops the total coverage. Model 3–1 does not particularly inhibit the overlapped demands heading to the public, whereas immediately following Model 3–2 assures the private's profit to be achieved. Considering a solution having the Pareto optimality, there are only two cases that can increase the total coverage: (i) the public locating additional facilities, regardless of however the private changes its decision, and (ii) the private locating more facilities while the public does not add more.

The first case denotes that there was enough left in the budget to place more facilities. However, the public did not utilize the remaining budget, although Model 3–2 maximizes the total coverage, indicating that the private's profit would have suffered. On the other hand, the private covers the most profitable nodes after Model 3–1, while assuring a certain level of the total coverage. If any valuable nodes remained, the private should have already covered them. Consequently, all remaining nodes are genuinely unprofitable, signifying that the private has to take a loss to locate more facilities. Therefore, either case necessitates cutting off the private's profit in order to increase the total coverage.

Proposition 4. *Assuming feasibility of Model 1,*

(1) $P^* \geq P_1^*(\tilde{\beta}) \geq \tilde{P}$

(2) $\beta_2^*(\tilde{P}) \geq \tilde{\beta},\ \beta_2^*(\tilde{P}) \geq \beta^*$

The dominance of Models 3–1 and 3–2 over Model 1 can be easily established by Proposition 4. There always exists a case in which the two models weakly dominate Model 1. One of the objectives is guaranteed to be no worse than Model 1 by Constraints (30) and (39), while the other is ensured by Proposition 4 for both models. Furthermore, Model 1 could be strictly dominated in practice, as shown in Section 6. However, the dominance of Model 2 is uncertain. The private's profit is always at least better than Model 1, as shown in Proposition 4, but the total coverage is not assured. The total coverage can either be higher or lower, which is also demonstrated in Section 6.

We can conclude that the first three propositions indicate the main contribution of this study, which shows that a trade-off between the total coverage and the private's profit exists. Meanwhile, the last proposition implies that cooperation always guarantees a better solution than competition. These findings are illustrated and verified via computational experiments in Section 6.

5. Extensions for capacitated facilities

In Sections 3 and 4, we considered uncapacitated identical facilities. However, in practice, it is more common where the capacities are limited, and thus, the capacitated environment should also be considered. We assume that the capacities are not necessarily the same, while the other characteristics, except for the capacity, remain identical. Unlike in the previous uncapacitated situation, customers no longer simply visit the nearest facility. As the facilities are heterogeneous now, preference over them must be considered. Zhang et al. (2023) investigated the optimal locations of charging stations considering users' preferences and waiting time. Our models now reflect customer choice by employing the gravity model with exponential decay (Küçükaydın et al., 2012; Drezner and Drezner, 2016). Several notations have been added to indicate the capacitated environment, and based on this, modified models are introduced.

5.1. Additional notations

The additional parameters are defined as follows:

c_j : potential capacity of candidate location $j \in J$

b_{ij} : utility of the facility, if open, at candidate location $j \in J$ for customer zone $i \in I$

M : sufficiently large number

As we apply the gravity model with exponential decay to demonstrate the preferences over different facilities, the utility of a facility, b_{ij}, is given by $c_j \exp(-d_{ij})$. The customers will regard a facility with more capacity as more attractive, which is true as well for a closer facility.

Also, several variables are additionally introduced to present the capacitated settings, as follows:

Y_{ijk}^x : equals to $a_{ik}x_k y_{ij},\ i \in I,\ j,k \in J$

Y_{ijk}^z : equals to $a_{ik}z_k y_{ij},\ i \in I,\ j,k \in J$

W_{ijk}^x : equals to $a_{ik}x_k w_{ij},\ i \in I,\ j,k \in J$

W_{ijk}^z : equals to $a_{ik}z_k w_{ij},\ i \in I,\ j,k \in J$

p_j : equals to $\min\left\{\sum_i h_i y_{ij}, c_j\right\},\ j \in J$

s_j : auxiliary binary variable to utilize $p_j,\ j \in J$

q_j : equals to $\min\left\{\sum_i h_i w_{ij}, c_j\right\},\ j \in J$

t_j : auxiliary binary variable to utilize $p_j,\ j \in J$

Considering that customers in the same customer zone are now bound to be diverted to various facilities following the gravity model, and facilities now have limited capacities, the definition of the coverage should be redefined. Unlike FLPs handling traditional problems with production and logistics, we are dealing with service facilities, and, thus, customers with free will. Consequently, forcing the allocations of the demands is impossible. Hence, though we are captured in a capacitated environment, the number of customers visiting facility j is still calculated as $\sum_i h_i y_{ij}$ or $\sum_i h_i w_{ij}$, depending on the type of facility. The difference appears in the additional constraints limiting the values of ys and ws due to the employment of the gravity model. Nevertheless, this number of customers may exceed the capacity, which is plausible in practice. Therefore, we redefined the coverage of a facility as the smaller value among the number of its visiting customers and its capacity, notated as p and q.

All modified models are based on the models presented in Section 3. Therefore, the new models and detailed explanations for the additional variables are provided in the online appendix, and only the core modifications in comparison with the original models will be highlighted in this section. The new models are numbered following the corresponding original models, with an apostrophe attached. For example, the capacitated sequential competition model and the capacitated sequential cooperation model are numbered as Model 1′ and Model 2′, respectively.

5.2. Modifications

First, the customers' preferences for the nearest facility have been replaced with the implementation of the gravity model. Customers now visit all the open facilities within the coverage radius but are divided by attractiveness. Inequality (48) stands for the situation in which only the private's facilities are placed. The other cases, as well as the linearization process of Inequality (48), are offered in the online appendix.

$$y_{ij} \le \frac{b_{ij}a_{ij}x_j}{\sum_k b_{ik}a_{ik}x_k} \qquad i \in I,\ j \in J \quad (48)$$

The preservation of the private's market share in the second stage of Model 2 has also been modified. In Model 2, such preservation was represented as fixing the values of w. However, the employment of the gravity model forces w to be changed, which violates the original logic. Therefore, a more intuitive way to preserve the private's coverage has been applied in Model 2′, directly constraining the coverage of Stage 2 to be no less than in Stage 1.

The objective functions of the two firms have also changed, following the previously described altered definition of the coverage. The coverage of a firm is calculated as $p_j = \min\{\sum_i h_i y_{ij}, c_j\}$ or $q_j = \min\{\sum_i h_i w_{ij}, c_j\}$, depending on its type. Similarly, Constraint (30) and Constraint (39) were modified. The linearization process of the modified coverage is also offered in the online appendix.

6. Computational experiments

6.1. Charging station location problem (CSLP)

In the past, EVs emerged as a solution to the depletion of fossil fuels, and interest in them has risen again because of environmental concerns. While interest in EVs has been growing for quite some time, it dipped temporarily because of technical problems and the lack of available charging stations. As EV-related technology has developed, however, tremendous progress has been made in recent years, and interest in EVs has emerged again. According to Bloomberg NEF (https://about.bnef.com/electric-vehicle-outlook/), the global EV market has proliferated and will continue to do so. However, more technical developments for EVs are necessary.

One of the most critical considerations relates to EV batteries. Conventional internal combustion engine vehicles (ICEVs), with their gas tanks, only take a few minutes to refuel, even if the tank is empty. Contrary to this, EVs require about 30 minutes to recharge if quick chargers are supported, and take hours to recharge with standard chargers. An additional drawback to EVs is that, even with this long charging duration, EVs have shorter driving ranges than ICEVs. While longer driving ranges, mostly made possible by bigger battery capacities, and faster charging options are continuously being researched, advances are still insufficient to spur consumers to completely replace their ICEVs with EVs. Given this, the greatest concern for EV users appears to be the 'charging capabilities for EVs'.

Many studies on EVs have emphasized the importance of charging stations (Kurani et al., 2008; Graham-Rowe et al., 2012; Bunce et al., 2014; Ahmad et al., 2022). These studies showed that the availability of public charging stations, such as gas stations which can be used by anyone, play a significant role not only in boosting convenience for EV owners but also in inducing potential customers to purchase EVs. On the other hand, some studies argue that public charging stations are actually not crucial to the EV users (Turrentine et al., 2011; Vilimek et al., 2012; Franke and Krems, 2013; Bunce et al., 2014; Lee et al., 2019). Unlike ICEVs, which are difficult to refuel at home, EVs can be recharged at home whenever as long as a charger is installed. In surveys of actual EV users, including trial participants, public charging stations were not used much, and most of the charging events took place at home. These results obviously assume that a private charger is equipped at home. It is easy to provide private chargers in a residential environment composed of houses with garages. However, the availability of home charging is a highly valued attribute in cities with a high percentage of residents living in multi-units without garages (for example, Seoul, South Korea). In the end, public charging stations are essential not only for EV users but also for prospective owners (Carroll and Walsh, 2010; Turrentine et al., 2011; Park et al., 2021). For this reason, governments worldwide are allocating a considerable budget for the expansion of charging facilities, while automakers themselves are also actively investing in such facilities.

As the significance of the charging stations has been emphasized, related studies, including studies for the CSLP, have increased in practical and academic importance. Asamer et al. (2016), Huang and Kockelman (2020), and Kchaou-Boujelben (2021) have solidly introduced the characteristics of charging stations. Except for offering battery swapping (Yang et al., 2017; Quddus et al., 2019; Hu et al., 2023), charging stations can be broadly classified into three categories according to their technology: level 1, level 2, and level 3. The charging time decreases as the level gets higher, while the installation cost for the station increases. In particular, levels 1 and 2 chargers require hours for a complete charge, while a level 3 charger will not take even an hour. Hence, it is reasonable that levels 1 and 2 chargers are preferable in locations with long dwell times or for private purposes (e.g., home charging), while level 3 chargers are normally used for long-distance trips.

One major part of the CSLP that has been extensively studied is the flow-based model (Kchaou-Boujelben, 2021). EV drivers taking long-distance trips must recharge the battery on their way, and indeed, on the return trip, too. To satisfy such demands, charging stations, mostly level 3, should be located adequately to make sure that the distance between two consecutive stations is within the driving range, taking into account the origin–destination trips (Lam et al., 2014; You and Hsieh, 2014; Li et al., 2016; Efthymiou et al., 2017; Guo et al., 2018; He et al., 2018; Yang, 2018; Xie et al., 2018; Chen et al., 2020; Csiszár et al., 2020; Kınay et al., 2021; Liu et al., 2021; Tran et al., 2021; Li et al., 2022; Song et al., 2023).

Another part considers the node-based models, which are highly related to the set covering problems (Kchaou-Boujelben, 2021). These are the cases in which the demands simply arise at the nodes. The node-based models are usually used for demonstrating the charging events that take hours while the users are resting at home, working, or shopping (Cavadas et al., 2015; Asamer et al., 2016; Zhu et al., 2016; Cui et al., 2019; Vazifeh et al., 2019; Hu et al., 2020; Kim et al., 2022). As mentioned previously, home charging is a highly valued attribute, not only in Seoul but also in most of the cities of South Korea in general (Park et al., 2021). This problem has also been a challenge for other countries (Asamer et al., 2016; Huang and Kockelman, 2020; Zhou et al., 2022). To this point, the South Korean government has announced that by 2025, it will build more than 500,000 levels 1 and 2 chargers in areas within 5 minutes' walking distance from residences or workplaces. Given this, we consider the situation of locating publicly accessible charging stations with level 2 chargers in areas near the demand that offer services similar to home charging.

In contrast to previous studies handling the node-based models, we study the case in which a public firm and a private firm participate with different objectives. Only a few studies have been published that integrate multiple decision makers into a CSLP. Even papers dealing with multiple decision makers only dealt with competition among profit maximizers (Luo et al., 2015; Bernardo et al., 2016; Guo et al., 2016; Zhao et al., 2020; Crönert and Minner, 2021) or with a game composed of a charging station builder and the users (He et al., 2013; Bernardo et al., 2016; Guo et al., 2016; Chen et al., 2016; Li et al., 2018; Lin and Lin, 2018). To the best of our knowledge, no attempts have been made to handle a CSLP with multiple facility builders with different objectives.

In practice, on a wide range, the CSLP may involve multiple private firms. However, considering the rapid expansion of charging stations, if the range is narrowed, it can be assumed to be single. For general facilities, such as gas stations, which are placed naturally over a long period, it is rather natural for several private firms to exist in one area. However, the current expansion of charging stations is a project promoted by the government over a concentrated period. In particular, it often happens that a local government and a single firm collaborate to cover an entire region. In this respect, we can assume a single private firm, as the South Korean government promised the expansion of charging stations. In the case of competitive situations, recall that this study deals only with cases in which the public makes the decision first. As the private targets only profitable spots afterward, a single private firm can be assumed again if the range is narrowed. Therefore, we can assume a mixed duopoly.

Recall that we consider the situation of locating publicly accessible charging stations with level 2 chargers in areas near the demand that offer services similar to home charging. The users of these chargers nearby residences or workplaces are highly likely to be the living population in the vicinity. In South Korea, it is common to install chargers in parking lots due to the highly valued attribute of home charging. Therefore, as EV users have to park their vehicles anyway, transportation costs are negligible. For this reason, applying a set covering model or a node-based model is reasonable.

According to Bunce et al. (2014), 49 percent of drivers recharged at regular intervals, usually at home overnight or at work during the day. In addition, Langbroek et al. (2017) found that 60 percent of EV owners charge every day, rather than only when it is necessary. Therefore, we can assume that the majority of people charge their EVs regularly, which leads to deterministic demands and predictable profits, considering that the users of the chargers are highly likely to be the living population in the vicinity. Moreover, as we are regarding level 2 chargers offering similar services to home charging and assuming that the majority of people charge their EVs regularly, the charging capacity and the charging speed are insignificant to the customers under the assumption of deterministic demands. Thus, we can additionally assume uncapacitated and identical facilities. However, we will also investigate the impact of considering capacitated facilities and, thus, heterogeneous facilities.

Taking into account the significance of publicly accessible charging points, along with the previously made assumptions, we found that the proposed models are suitably applicable to the CSLP. Therefore, computational experiments based on the CSLP have been conducted. Two types of problems, small and big, have been considered. The small-size problem was originally employed just to validate the presented models. However, as the results provided the basis for the research questions, the small-size problem is also reported. Furthermore, several settings were retained to generate the big-size problem. Considering that no assumptions or qualifications are raised for the dimensions of the network in this paper, expanding the size of the problem is unrestricted. The big-size problem is conducted to illustrate the theoretical results established in Section 4. Moreover, the impacts caused by changing the values of B and r are examined. Finally, an additional experiment considering capacitated facilities based on the big-size problem is carried out. The terms "facility" and "station" are used interchangeably throughout this section.

Table 3
Data of the small-size problem.

Index	x-coordinate	y-coordinate	Annual demand	Annual amortized total cost
1	2	1	60	200
2	9	7	27	200
3	2	4	29	200
4	9	2	26	200
5	5	9	33	200
6	6	3	15	200
7	8	4	17	200
8	5	3	97	200
9	3	6	97	200
10	2	6	19	200

Table 4
Assumptions for financial parameters.

	Assumption
Average charging duration	30 min/charge
Charging price per minute	\$ 0.125/min
Average charging price	\$ 3.75/charge
Amortization period	5 years
Land acquisition cost	\$ 300,000 ~ \$ 500,000
Cord installation cost	\$ 20,000
Variable cost including maintenance	\$ 10,000/year
Annual amortized total cost	\$ 74,000/year ~ \$ 114,000/year

6.2. Experiments for the small-size problem

The mathematical models were solved with FICO Xpress version 8.12. The set of customer zones and potential facility locations are equal (i.e., $I = J$). The coordinate values of the nodes have been scaled up 10 times, and the distances between nodes are calculated as the Euclidean distance, assuming a complete graph. The fixed costs were substituted to amortized total costs. Table 3 summarizes the data.

Other parameters have been generated on the basis of work by Chu et al. (2019), Franke and Krems (2013), and Huang and Kockelman (2020). Chu et al. (2019) noted that EV users in South Korea charged their vehicles 14.07 times per month, while Franke and Krems (2013) figured out that users in Berlin, Germany, charged their EVs 3.1 times per week. Consequently, we assumed that people charge their EVs around 161~169 times per year, on average. The financial parameters refer to Huang and Kockelman (2020) and are organized in Table 4.

Multiplying the charging frequency with the charging price, the revenue gained by serving one demand per year (α) is between \$603.75 and \$633.75. Because the amortized total cost (f) is in the range of \$74,000 to \$114,000, f/α varies from 116 to 189. Therefore, we assumed the value of f/α to be fixed as 150, the middle of the range. Because the costs are all equal to 200, α will take the value of $4/3$. In addition, the public's budget, B, is assumed to be 10 percent of the sum of the costs, while the coverage radius, r, is set so that 2.8 nodes, on average, are within r, resulting in a B of 200 and an r of 4, respectively. As the values of B and r are set arbitrarily, the impacts caused by the change of these values will be investigated in the following subsection with a big-size problem. The numerical results were summarized in Table 2.

Fig. 1 visualizes the results of the first stage and the third stage of Model 1, respectively. Black and red represent the public and the private, respectively. The big circles show the coverage radius, and the green crosses indicate the nodes uncovered. The small circles denote the locations where the facilities are placed, and the dots signify the covered nodes.

The public can locate exactly one station, because all costs are equal to 200, and the budget is also 200. To maximize its own coverage, the public set its placement at (5, 3), which offers the most coverage. After the public made its decision, the private located a facility at (2, 4). Because Model 1 describes a competitive situation, we might have found that the private felt free to intrude into the public's coverage. The result also indicates that other points are not profitable enough for the private to place additional facilities, considering that it placed only one.

Fig. 2 shows the results of the two stages of Model 2. The private also chose the point that the public chose in Model 1. However, the public could not make the same decision as the private made in Model 1. Points (5, 9) and (9, 7) were the only feasible nodes the public could have chosen in order to preserve the private's coverage, thereby maintaining the cooperation. The public placed its facility at (5, 9), which garners more demand. The overlapping point (3, 6) is located at the same distance from the two facilities. Still, the private fully covers that point because the capacities are infinite, and the public will not care, given that the total coverage is the same, no matter who covers it.

Fig. 3 demonstrates the results of Model 3–1 and Model 3–2. As shown in Table 2, the results of Model 3–1 are the same as those for Model 2, and it is not surprising that the solutions are also the same. Model 3–2 shows that even 100 percent coverage could be achieved while still guaranteeing that the profit is higher than it is in Model 1.

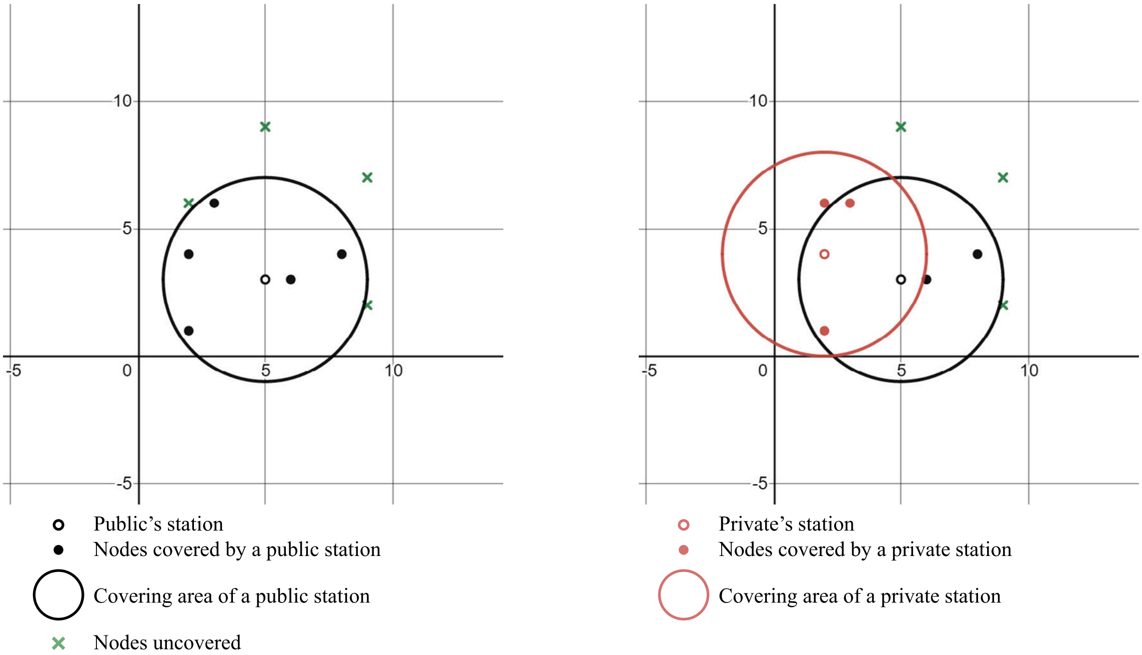

Fig. 1. Results of Model 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

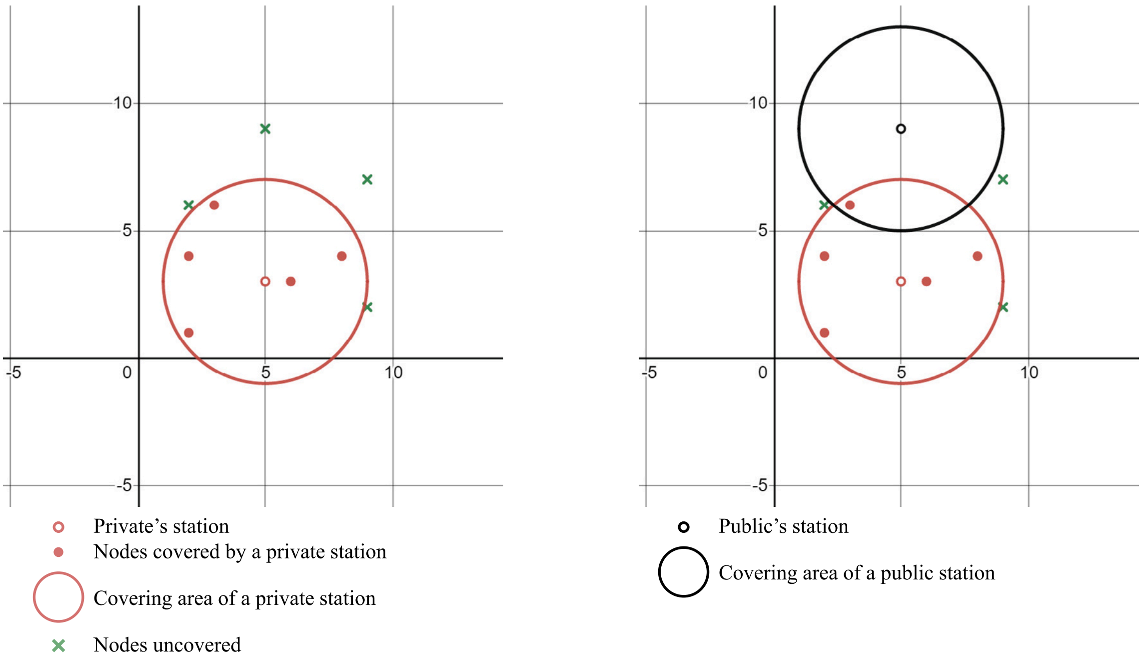

Fig. 2. Results of Model 2.

6.3. Experiments for the big-size problem

A bigger-size experiment also has been conducted to illustrate the theoretical results established in Section 4. The big-size problem was generated according to the instance on the Euclidean plane of simple location problems from the Benchmark Library (http://www.math.nsc.ru/AP/benchmarks/english.html). Because our contribution is neither algorithmic nor based on computations, we have not attempted to solve large or various instances. Instead, a single instance from the Benchmark Library, Code 111, was

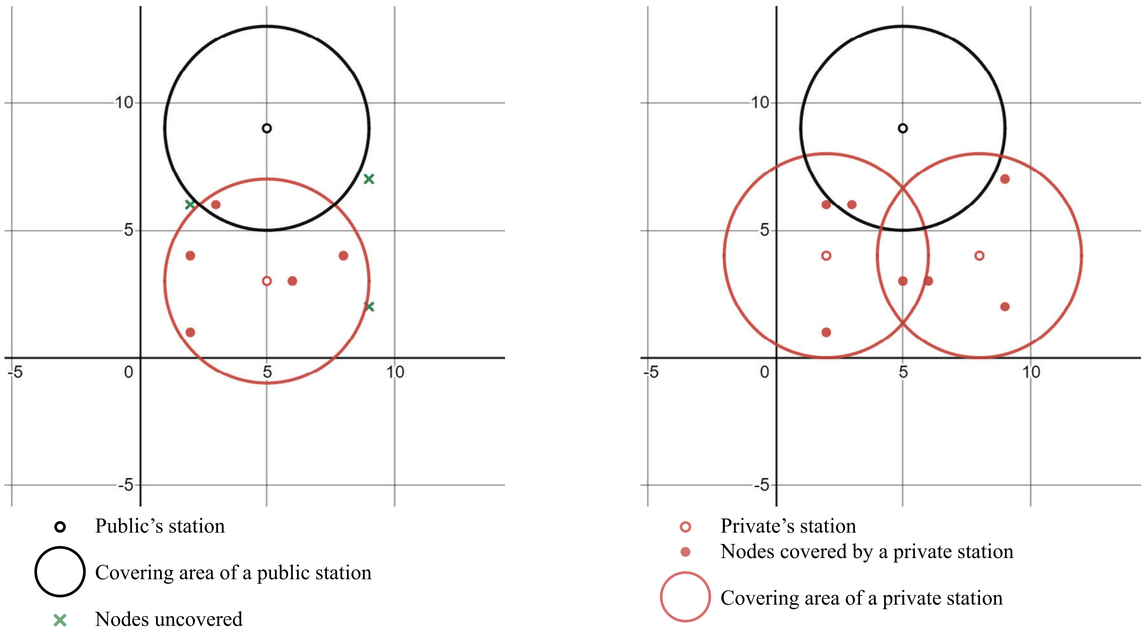

Fig. 3. Results of Model 3.

implemented to verify the theoretical results and the answers to the research questions. The given transportation costs between nodes in the instance were substituted with distances between nodes.

The demands and the costs were generated following the small-size problem. The mean and the standard deviation of the demands in the small-size problem were 42 and 31.6, respectively. Excluding the biggest and the smallest demand for each, the mean and the standard deviation become 38.5 and 27.09. The demands (h_j) were randomly generated from a normal distribution with the parameters of 38.5 and 27.09.

$$f_j = \left(\frac{H_j + 2}{\text{Amortization period}(= 5)} + 1 \right) \times 100 \text{ (unit: \$100)} \tag{49}$$

The costs were calculated as Eq. (49), where H_j corresponds to the land acquisition cost in Table 4. Considering a node with a bigger demand as being more profitable, we assumed the cost to be affected by the demand. To implement such influence, every H_j is randomly generated from a normal distribution with mean h_j and standard deviation $(h_j/5)^2$ (i.e., $N(h_j, (h_j/5)^2)$), respectively.

$$\mathbb{E}\left[f_j\right] = \frac{\mathbb{E}\left[H_j\right] + 7}{5} \times 100 = \frac{h_j + 7}{5} \times 100 \tag{50}$$

$$\mathbb{E}\left[\overline{\mathbb{E}\left[f_j\right]}\right] = \frac{\mathbb{E}\left[\overline{h_j}\right] + 7}{5} \times 100 = 910 \tag{51}$$

Eq. (50) shows the expectations of the costs, which are also random variables. Note that the expectation of the sample mean of the costs becomes 910, as calculated as Eq. (51), which moderately fits within 740 and 1,140, the range investigated previously (Table 4). As a result, costs with an average of 1,043 were generated, and this data was used throughout this section.

Other parameters nearly follow the assumptions of the small-size problem. The value of $\bar{f}/\alpha$ instead of f/α is approximated to 150, resulting in α having the value of 7. The public's budget, B, is again assumed to be 10 percent of the sum of the costs. The coverage radius, r, is set so that 2.82 nodes are within r, on average, because there was no r that carried out exactly 2.8 nodes, on average. Consequently, the standards were set to be B of 10,430 and r of 663. Based on these settings, five values of r and B each were considered, with B and r being fixed as the standards, respectively. In detail, the value of r was changed from 500 to 600, 663, 750, and 900, with B fixed as 10,430. Then the value of B was changed from 2,086 to 5,215, 10,430, 20,860, and 52,150, which corresponds to changing n of 1,043$\times n$ from 2 to 5, 10, 20, and 50, with r fixed as 663. Models 1 and 2 have been applied in each case, followed by an iteration of Models 3–1 and 3–2, with the input β increasing from β^* to 100 percent while only having integer percentages.

Fig. 4 shows the total coverage and the private's profit of the models for each r. Each graph consists of two parts: a line connecting two points and a curve passing through several points. The point located at the bottom of the line part, the isolated point, corresponds to the result of Model 1. The other point of the line part represents the result of Model 2, which clearly shows that the profit of Model 2 is the highest. The curve part starts from the point of Model 2 and continues via Model 3 until the total coverage reaches up to 100 percent. Except for the points of Model 1, it is clear that Models 2, 3–1, and 3–2 demonstrate the trade-off between the total

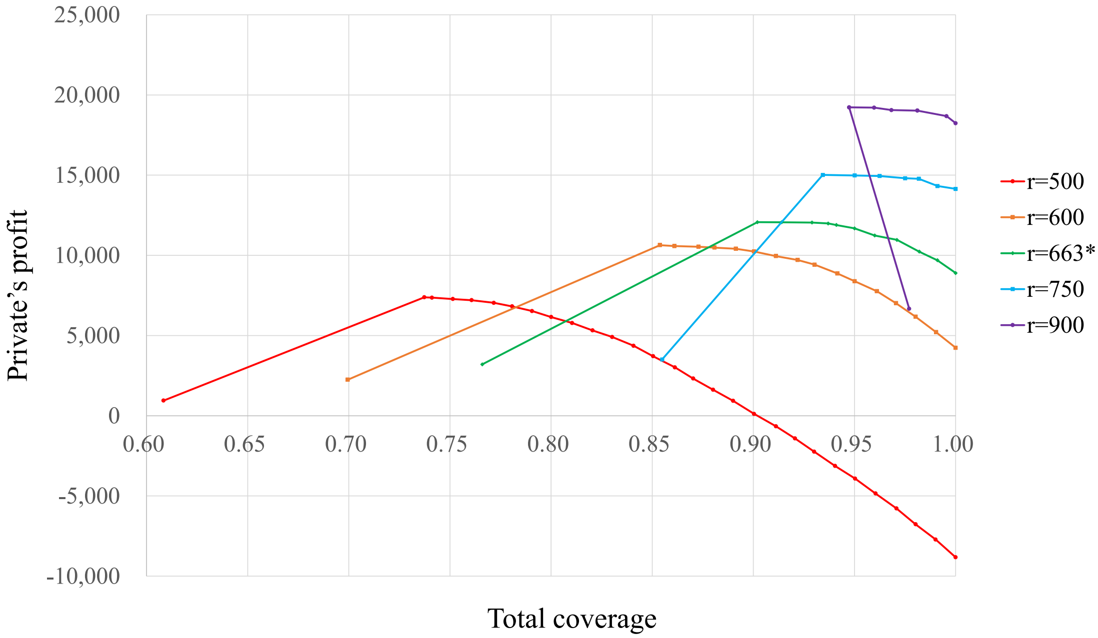

Fig. 4. Aggregated results of each r.

coverage and the private's profit, regardless of the value of r, and form a trade-off curve. Also, it is noticeable that only until $r = 750$ are the isolated points located at the bottom left of the trade-off curve. This indicates that for $r = 900$, the total coverage of Model 1 is higher than that of Model 2, which leads to the fact that Model 2 cannot dominate Model 1. The public locates its facilities within a given budget. For Model 1, when r gets bigger, the public could cover a bigger area by itself in the first stage. However, the facilities would be located sparsely to offer efficient coverage. This leads to more favorable circumstances for the private to intrude, resulting in higher coverage. On the other hand, the private will now locate its facilities sparsely in the first stage in Model 2. When r gets bigger, only a few nodes will remain feasible in the next stage for the public, because the private's coverage should be preserved. Consequently, the total coverage could not rise sufficiently in Model 2, while it rises steeply in Model 1, and thus, a reversal takes place. Note that the graphs generally move to the upper right. Fig. 5 demonstrates such movement by emphasizing the shifts of the points for Models 1 and 2, as well as the points representing total coverage of 100 percent, caused by the change of r.

Model 1 draws a gentle upward curve. Considering that the private faces an easier market in which to intrude and that the total coverage increases as r gets bigger, this still does not mean that the private will place many facilities. Arranging facilities sparsely may in fact increase the number of lost opportunities when competing on scales of distance with the public. In contrast, if the facilities are placed densely, the overlapping areas will increase, leading to inefficient and unprofitable coverage. As a result, the increase of the private's profit is insignificant compared to the total coverage, which indicates that a vulnerable market (i.e., a market that is easy to intrude upon), does not always guarantee high profits. Such a market will work positively if the private bears a cutthroat competition to secure more market share, but this is not the case, and thus, it would be more prudent to expand cautiously.

Model 2 also draws an upward curve. As r gets bigger, the private could cover the nodes more efficiently, resulting in higher profits and coverage. Consequently, the total coverage grows together but faces a wall when r gets excessive.

The private's profit also increases for the points having 100 percent of the total coverage. To cover all the demands when r is restricted, the private must take a loss, because it is responsible for all the nodes that the public could not cover due to the budget constraint. As r increases, the public could cover more nodes by itself. Considering that more of the unprofitable nodes are taken away by the public and that the private covers the nodes more efficiently, the private's profit grows as the burden transferred to it is reduced.

Fig. 6 presents the results that occur when the value of B gets changed. Note that the isolated points correspond to the results of Model 1 and that the trade-off curves are well illustrated, regardless of the value of B, as is the case when r is changed. It is again noticeable that the isolated point appears at the bottom right of the trade-off curve only for $B = 52,150$. Model 2 also failed to dominate Model 1 in the case of increasing the budget. The public only focuses on expanding its coverage within a given budget, irrespective of its profit. Therefore, the public would simply build more facilities as the budget increases, and it could achieve high-level coverage even by itself in the first stage of Model 1. This implies that the public fully utilizes the budget in Model 1, which is not happening in Model 2. In Model 2, the private moves first, and thus, always makes the same decision, because the only changing part is the budget, which belongs to the public. For this reason, all points corresponding to Model 2, the left side

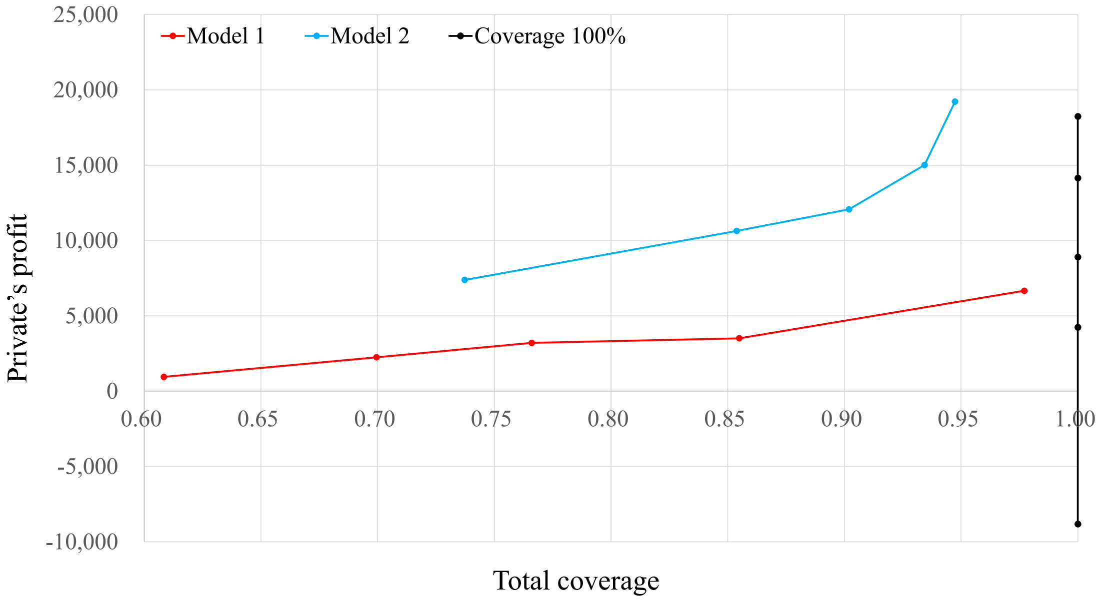

Fig. 5. Results of changing the value of r.

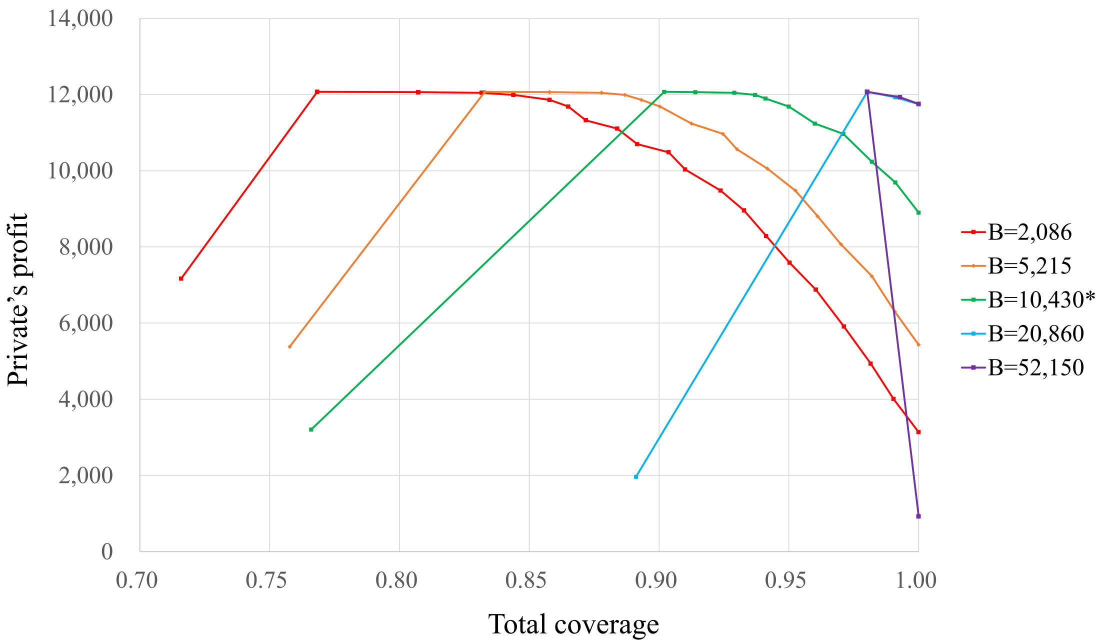

Fig. 6. Aggregated results of each B.

starting points of the trade-off curves, display the same level of the private's profit. However, the public must preserve the private's coverage. Eventually, the public should leave some of the budgets idle when they are given immoderately, while the results of the first stage remain, regardless of the budget. Consequently, a sufficient budget is underutilized in Model 2, and the total coverage could not rise enough compared with Model 1, resulting in an overtaking as was the case with r. Note that the graphs generally move to the right. Fig. 7 shows the results of Models 1 and 2, as well as the cases of the total coverage reaching 100 percent, similar to Fig. 5.

Model 1 draws a downward curve as opposed to the case of r. As the budget increases, the public simply builds more facilities. Accordingly, the achievable nodes for the private competing on the platform of distance against the public will decrease, and thus, the profit will reduce, too.

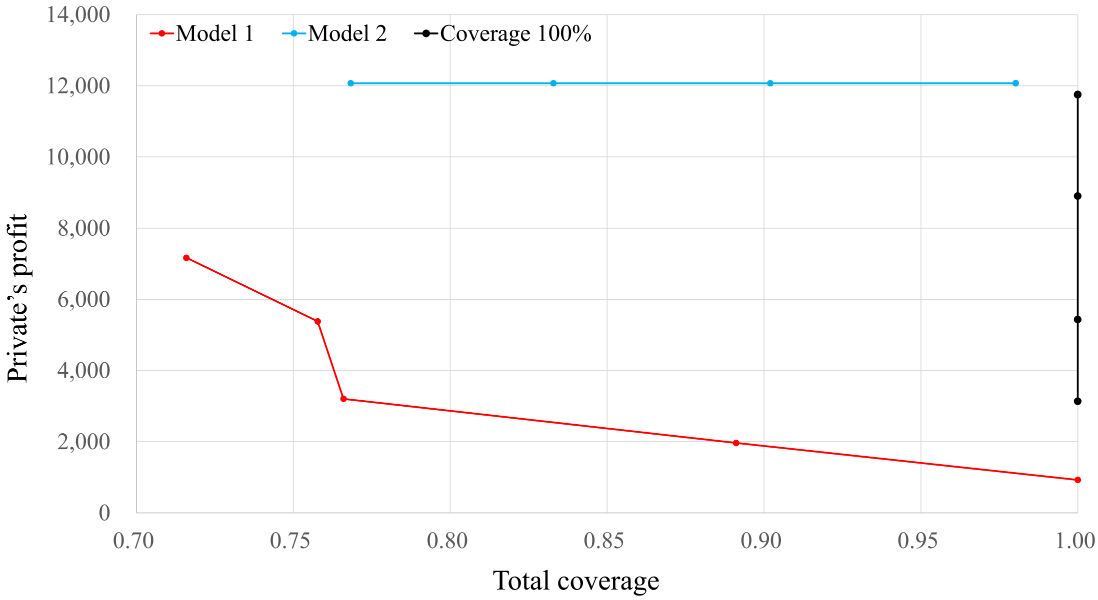

Fig. 7. Results of changing the value of B.

Model 2 presents a horizontal line, indicating constant profit and increasing total coverage as the budget grows. Note that only four points are illustrated on the line. The last two cases retrieve the same total coverage, magnifying the underutilization of a sufficient budget. It is also noticeable that the constant profit is less than the profit gained by a bigger r, because the facilities' covering capabilities are unchanged.

The private's profit increased again for the points with 100 percent of the total coverage, as in the case of r. To fully cover the demands for a fixed r and an insufficient budget, the private must give up its profit again. As the budget grows, the public could handle more of the nodes by itself, just as in the case of r. The difference is that the covering efficiency of the private is consistent. Consequently, the private retains gainful nodes and gets emancipated from covering the unprofitable ones as the budget increases. The profit eventually rises in general as the budget grows but ceases when only the most profitable nodes are left, specifically when the budget is excessive. It is evident that only four points are presented again for this reason, indicating that the last two points overlap. Additionally, it is again conspicuous that the profit gained from the most significant budget is lower than that from the biggest r, for the same reason as in Model 2.

6.4. Experiments for the capacitated problem

An additional experiment has been conducted to analyze the impact of considering capacities. The newly included parameters c and b were additionally generated. To generate c, we first defined two temporal parameters. First, parameter e_{jk} is defined similarly to parameter a_{ij}, except that it takes the value 1 if $d_{jk} \leq 663$ by the requirement of a parameter independent of r. Then, parameter g_j was randomly generated from a uniform distribution with a range of $\min\{h_j, H_j\}$ and $\max\{h_j, H_j\}$. Taking into account these two parameters, e and g, the capacities (c_j) were defined as Eq. (52).

$$c_j = \text{round}\left(\frac{\sum_k e_{jk} g_k}{1.41}\right) = \lfloor \frac{\sum_k e_{jk} g_k}{1.41} + \frac{1}{2} \rfloor \qquad j \in J \quad (52)$$

As 2.82 nodes, on average, are within the standard coverage radius (663), the capacities were generated, roughly reflecting the potential demands. Based on the values of c, b_{ij} was defined as $c_j \exp(-d_{ij})$. To balance the leverage between the two terms, d_{ij}s should have been normalized. However, considering that d_{ii}s are all equal to 0 and, thus, should have the strongest weight, standardization has been expelled. The min–max scaling, another popular normalization method, was also not considered because we are not necessitating values within 0 and 1. Accordingly, the distances have been scaled down by 663, the standard coverage radius. Similar to e, the scaling parameter is fixed to 663, being independent of r. As a result, b_{ij}s were calculated as $b_{ij} = c_j \exp(-\tilde{d}_{ij})$, where $\tilde{d}_{ij} = d_{ij}/663$. All other parameters are retained from Section 6.3, and the process is also the same.

Fig. 8 shows the total coverage and the private's profit of the capacitated environment for each r. Some resemblances to the uncapacitated identical case are apparently observable. The trade-off curves are again explicitly illustrated, regardless of the value of r, while Model 1′ is still strictly dominated. However, some distinctions are also evident. It failed to reach 100 percent of the total coverage for all the cases due to the limited capacity. In addition, it is noticeable that both the total coverage and the private's profit are reduced, compared to the uncapacitated identical setting.

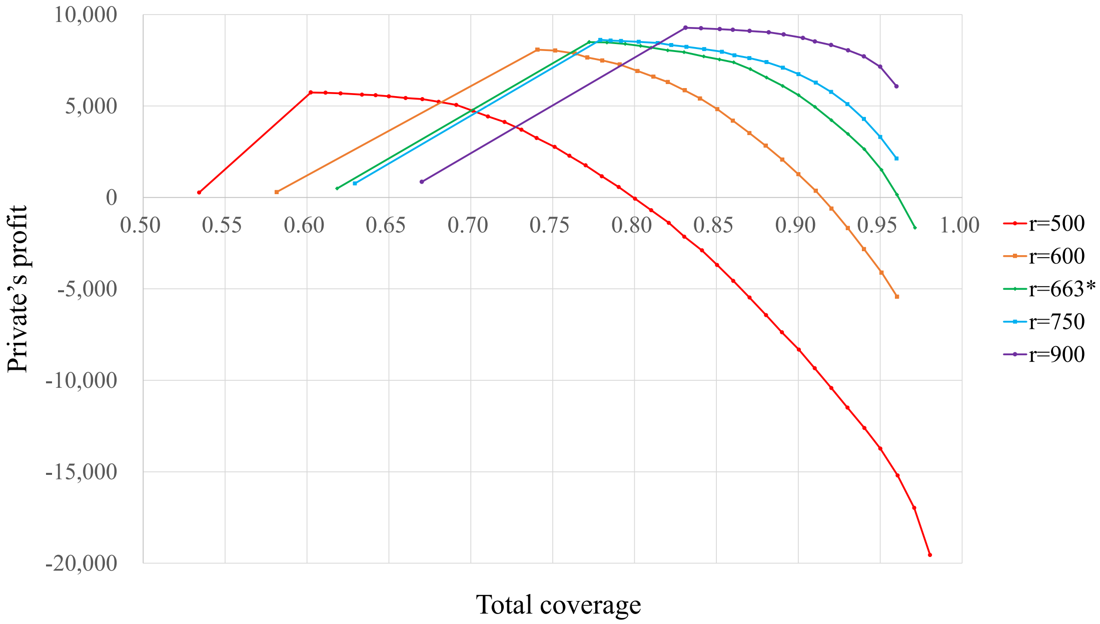

Fig. 8. Aggregated results of each r considering capacities.

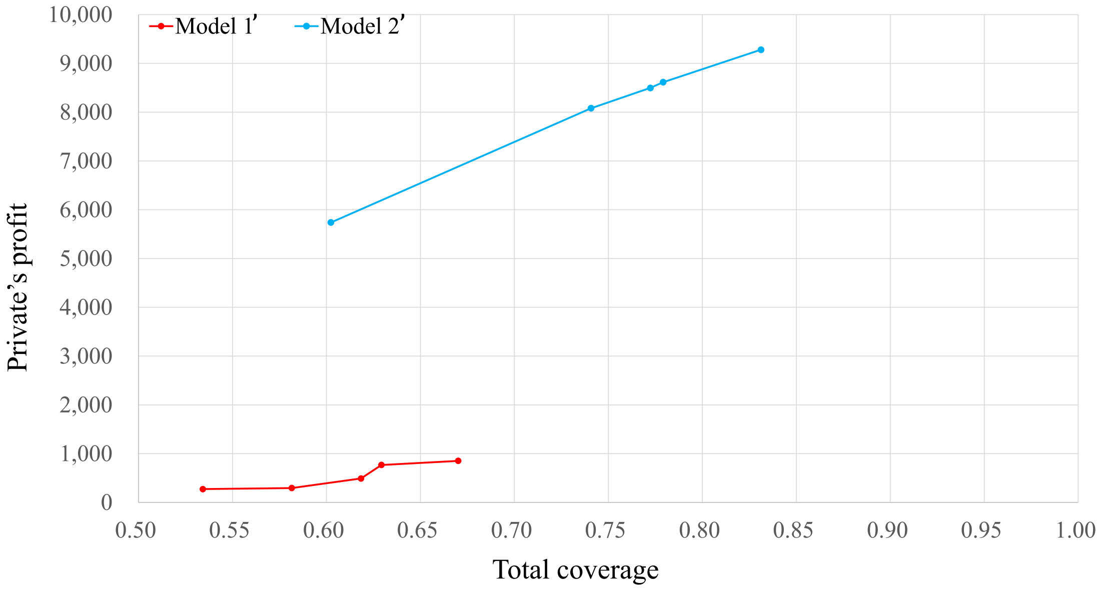

Fig. 9. Results of changing the value of r considering capacities.

Fig. 9 illustrates the results of Models 1′ and 2′. Note that the graph representing the points with total coverage of 100 percent is excluded, as it was unreachable. It is conspicuous that both graphs are still drawing upward curves. This shows that the impact of increasing the value of r is invariable regardless of the existence of capacities. However, the increment of the total coverage in Model 1′ is rather unsatisfiable. Considering that a bigger value of r does not increase the number of the public's facilities, a larger coverage area is worthless due to the existence of capacity limitations. On the other hand, it is noticeable that the profits in Model 1′ are lying on the floor. In the previous situation, the private could take away a customer node as a whole from the public by competing on distance, and thus, the market was more profitable. However, the private faced harsher circumstances as the environment changed, because the number of seizable customers decreased and the capacities directly limited the covering capability.

The relationships between Figs. 4 and 8 are also observable in Figs. 6 and 10. The trade-off curves are well demonstrated, and Model 1′ is strictly dominated. In addition, 100 percent of the total coverage is again unreachable, while both the total coverage and the private's profit are diminished.

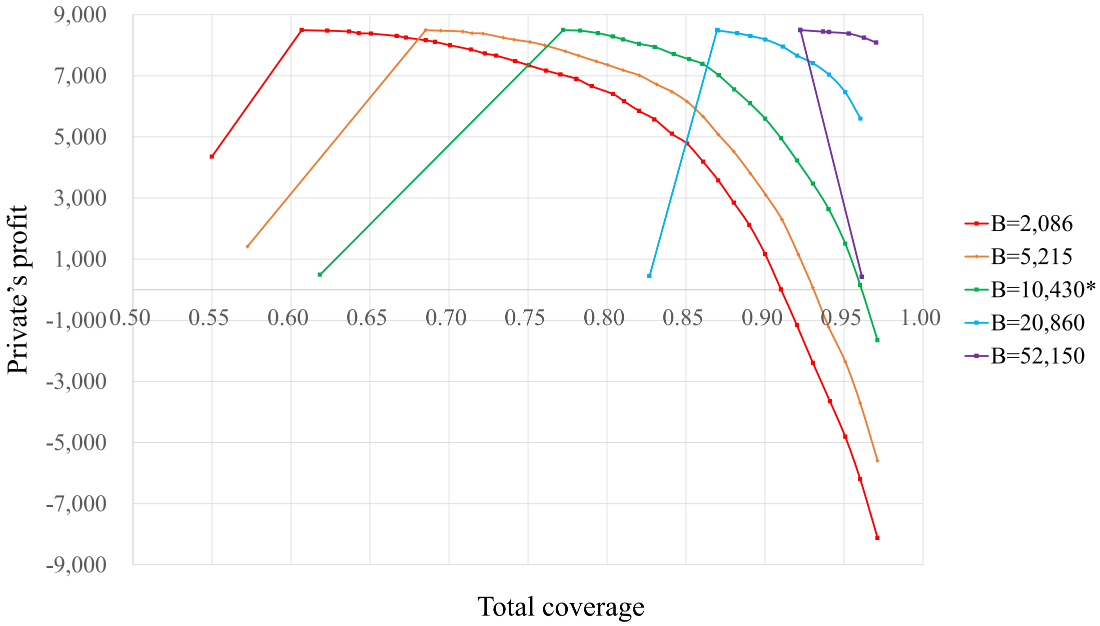

Fig. 10. Aggregated results of each B considering capacities.

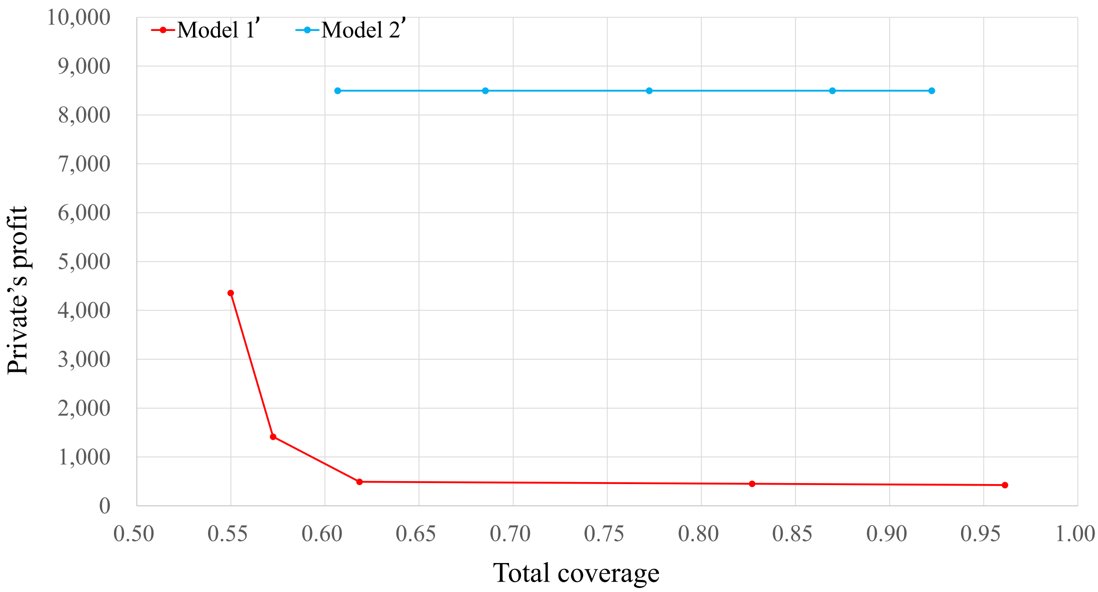

Fig. 11. Results of changing the value of B considering capacities.

Fig. 11 depicts the same situation as the one depicted in Fig. 10. The relationships between Figs. 5 and 9 similarly appear between Figs. 7 and 11. The graph demonstrating the points with 100 percent of the total coverage is removed, while the other graphs still show the same trends shown in Fig. 7. Although it is hardly recognizable, the latter three points of Model 1′ are decreasing. Therefore, regardless of the existence of capacities, the impact of increasing the budget is invariable, as it was for the case of r. The only difference is that, in the uncapacitated identical situation, changing the values of r or B both significantly affected the results, but for Model 1′ in the capacitated environment, increasing the budget shows much more dramatic change compared to increasing the value of r. When the budget is limited, the private finds it more profitable to intrude. However, although the total coverage rapidly grows as the budget increases from 10,430, the private's profit only decreases negligibly. Hence, we can deduce that the budget of 10,430 is sufficient to prevent the private from aggressively entering, and thus, results in a comparable level of the private's profit compared with that depicted in Fig. 9.

7. Concluding remarks

In this paper, we studied an FLP between a public firm and a private firm on a network space. We presented diverse situations and modeled the corresponding mathematical formulations by implementing the optimization approach. We mathematically analyzed the relationships between the models and showed that complementarity and dominance exist. We conducted computational experiments in order to validate the presented models and verify the analyses. Consequently, we demonstrated the trade-off between the total coverage and the private sector's profit, despite a common factor being shared.

To support the replacement of ICEVs with EVs and disseminate EVs harmoniously, not only technical developments but also policies should be carefully considered because publicly accessible charging stations are especially essential for cities with a high percentage of residents living in multi-unit housing without garages. For this reason, the public sector, as well as private firms, are actively investigating the expansion of charging stations. Considering the increasing interest in EV charging stations and the relevance to our models, numerical experiments for a CSLP are proposed. To the best of our knowledge, this study is the first to graft a mixed market to an FLP on a network or to consider multiple participants with different objectives in locating charging stations.

Our research supports the decision makers, regardless of their sectors. In detail, we provided policy implications in this paper for the public sector (e.g., the policymakers or the budget allocators), and we provided managerial insights for private investors. We suggest that cooperation among the public and private sectors is beneficial for both parties, compared to a competitive situation, and we supported this theory through analyses and verification with computational experiments. Moreover, we believe that the shown trade-off could shed light on how best to negotiate conflicting interests between stakeholders.

For researchers, we hope that our research will serve as a base for future studies of the FLP in a mixed market on a network or in the CSLP with multiple decision makers. The objectives of each sector vary widely, and there might be conflicting viewpoints about the objectives we proposed in this paper. In addition, some researchers may not accept the assumptions raised by our research (for example, the deterministic demands or the single period of focus). Applying a stochastic approach for probabilistic demands or implementing a game theoretic approach might be considered in future research. Further consideration for the extensions and variations of our work can yield meaningful conclusions and enrich the growing body of literature on the FLP.

CRediT authorship contribution statement

Junseok Park: Conceptualization, Methodology, Investigation, Software, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Ilkyeong Moon:** Conceptualization, Validation, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors are grateful for the valuable comments from the editor-in-chief, the associate editor, and three anonymous reviewers. This research was supported by the National Research Foundation of Korea, Republic of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning, Republic of Korea [Grant Number NRF-2019R1A2C2084616].

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.tre.2023.103149.

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