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Robust building evacuation planning in a dynamic network flow model under collapsible nodes and arcs

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ABSTRACT

Rapid urbanization has caused various social problems. One typical example is the high population density of a building, particularly in a commercial building or a mega-mall. When an emergency, such as a natural or human-made disaster, occurs in a building with a high population, establishing a proper evacuation plan is required to minimize casualties. Accordingly, the evacuation planning problem, which determines optimal routes for evacuees from disaster-prone areas to safe areas, has been actively studied in various fields. However, research considering the possibility of further collapse of a specific area or intermediate route in the building has been overlooked. We propose a robust evacuation planning problem based on a dynamic network flow model that determines the optimal routes for evacuees from a building that has the potential to collapse. Computational results show that routes passing through areas with the potential to collapse may or may not be optimal for evacuees, depending on the given timeframe. If the timeframe is sufficient, detouring around the collapsible areas could be the optimal plan; however, if the timeframe is insufficient, passing through collapsible areas, with taking the risk, could be the optimal plan.

1. Introduction

As we enter the modern world, the phenomenon of high population density in a specific city, also known as a *metropolis*, has occurred throughout the world. To accommodate the high population density, the buildings have become high-rise and complicated. This phenomenon naturally leads to a concentration of population in specific buildings. Accordingly, the number of people staying or traveling on each floor of a building has increased. In such a situation, one of the significant issues is to respond to an emergency appropriately. If an emergency, such as a natural disaster, fire, or terrorist attack, transpires in a building, the damage may be more critical than it occurs in structures with low population density [1]. Since evacuees habitually head to specific exits while ignoring other available exits or follow the direction of the majority of other victims, some areas become crowded during egress [2,3]. Although such escape behavior could be the best route choice from an individual's perspective, the overall evacuation time for every evacuee could be delayed. These inadequate responses have resulted in preventable casualties. Therefore, research on evacuation planning is required such that evacuees are allocated appropriately to safe areas to establish a plan that is made from a global perspective.

As an emergency is an unpredictable and unintended event, it is crucial for decision makers to plan for disruptions and to prepare for them in advance, to reduce the potential losses of life and property. To mitigate further damage, the locations of all exits are provided on a floor plan. However, providing the floor plan is only prior information before an emergency occurs. Also, the plan does not incorporate the number of occupants distributed in each area at the time of crisis. When evacuation routes are determined based on prior and static information, they may be more congested in certain areas than routes that are based on real-time and dynamic data [4]. To alleviate this problem, egress routes should be provided in real-time, based on the number of occupants and the pertinent risk information available after an emergency has started [5]. The *evacuation planning problem* (EPP) is one of the research areas for disaster management incorporating the related information in real-time. It provides an evacuation plan for people threatened by a hazard, particularly in a building or other structure, from each disaster-prone location to a safe area. By identifying the optimal egress routes, bottlenecks that delay the overall evacuation time can be minimized. With the recent development of information technology, gathering real-time information in a building, such as an emergency, the number of people in each area, and real-time motion

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analysis, has become possible. Furthermore, mobile apps that provide real-time evacuation routes to people located in specific areas have been available [6]. With the development of these information technologies, evacuation planning can create synergy in reality, suggesting that much attention must be paid to the study of an EPP.

Most prior research was conducted on the assumption that emergency conditions do not change over time. Additional hazards, such as a structural collapse of a specific area or obstruction of an evacuee's passage, can occur after the emergency. In other words, we should not overlook the uncertainty that is inherent in an emergency. Although various types of uncertainty can manifest during an emergency, the instability of the building structure can be critical [7]. The collapse of a specific area in the building can block the route to the safe area, and casualty numbers may increase among those who have passed through the collapsible areas. When establishing an evacuation plan, additional collapses should be taken into account because of the potential hazards inside the building. Accordingly, we started the research with the following research questions:

(i) What approach is needed to incorporate the uncertainty of the building structure in the EPP?
(ii) In an emergency, fast decision-making is essential. Can the mathematical model retain the tractability even if the uncertainty is incorporated in the EPP?
(iii) How will the EPP, in light of the uncertainty, be varied according to the given time available?

When the uncertainty is considered on the collapse of a section of the building, it is critical to respond to the worst-case scenario of the damage, rather than the expected damage, concerning post-disaster management. It is also difficult to estimate the probability distribution practically or to design all possible scenarios of collapses at a particular location. In order to incorporate the uncertainty of the building structure, which can be collapsed over time, we used a *robust optimization* (RO) approach instead of estimating the distribution. From a network perspective, additional collapses can be interpreted as nodes and arcs being broken down. Generally, RO has been actively undertaken to solve an optimization problem dealing with uncertain parameters, such as demand, capacity, or travel time in the network model. RO provides an optimal solution by considering the worst-case scenario associated with uncertainty data. By applying the concept of RO to the EPP, the optimal routes for evacuees under any possible scenario, including the worst case of collapses in the building, can be considered. A stochastic optimization model that offers another way to deal with uncertainty also could include the worst-case scenario, but this focuses on optimizing the expected value. On the other hand, the RO that we consider in this study focuses on optimizing the problem based on the worst-case scenario. To sum up, we applied RO to the EPP to deal with the uncertainty of the network structure.

As the occurrence and consequences of an emergency cannot be precisely predicted, disasters are considered to have high levels of uncertain properties [8]. Therefore, the RO approach has been applied in various ways to disaster management. [9] considered the uncertainty in determining the number of injured people, the demand for commodities, and the available capacities of suppliers and hospitals during earthquake response planning. [10] assumed supply, demand, and the cost related to the procurement and transportation as uncertainty factors under relief logistics planning. [11] applied uncertainty factors to the amounts of demand, supply, and shortage cost.

According to the above-mentioned studies, a robust optimization related to emergency or disaster management has been studied by incorporating the uncertainty associated with various parameters. Despite possible uncertainties, we considered the risk of further collapse of the building to be critical in case of a building emergency. When considering evacuation planning as a network flow model, it can be applied as a robust flow decision problem on a collapsible network. [12] considered the maximum flow problem with collapsible nodes and arcs. In detail, they developed a *robust maximum flow problem* (RMFP) and an adaptive maximum flow problem with a static network flow problem. They also developed RMFP with the path-based formulation and expanded the adaptive maximum flow over time-horizon. This study was primarily influenced by the paper of [12]. We expanded the static RMFP to the discrete-time RMFP that they did not consider in their work. Although [13] used a robust optimization approach to the evacuation problem, which incorporates a loss of inflow to a specific node on a network flow problem, they did not cover the time-expanded network discussed in this study. To our knowledge, RO had not been considered for the EPP. We increased the dimension of RMFP from static network flow to the discrete-time dynamic network flow and transformed it into the form of EPP. To sum up, the main contributions of this study are as follows:

- We applied a robust optimization approach to the EPP. By expanding the network over a discrete-time horizon, robust optimal evacuation routes are presented, based on real-time assumptions.
- Rather than rely on uncertain input parameters such as demand or travel time, we focused on the collapsible network structure. In this way, the uncertain network structure, including nodes and arcs, is incorporated into the network model.
- By developing a mathematical model with a linear program and retaining tractability with a robust optimization approach, the robust model could efficiently solve the problems even though the size of input data increased.

The remainder of the paper is as follows: We summarize the previous studies which are related to the EPP in Section 2. We introduce the problem description of the EPP in Section 3. Section 4 deals with the mathematical models of the deterministic EPP and the robust counterpart of the EPP. Computational experiments and analyses are described in Section 5. In Section 6, the findings of this research are summarized.

2. Literature review

The EPP has been studied in various fields, including operations research, management science, civil engineering, industrial engineering, and computer science [14,15]. According to [16], EPP can be divided into macroscopic and microscopic models. The macroscopic model focuses on an optimization problem providing optimal egress routes for evacuees. The main notion of the macroscopic model is that the individual optimal route is only a local optimum, while evacuation planning should be considered from a global perspective by considering the routes of all evacuees simultaneously. If each evacuee seeks the nearest exit from an individual's perspective, they may experience crowds in particular areas, which delays the average evacuation time of all evacuees or the final evacuation time of the last evacuee. In macroscopic models, the crowd is treated as a continuum that can be described by averaged quantities such as density and pressure. Meanwhile, the microscopic model concentrates on a simulation study by focusing on the behavior of each evacuee. In addition, it represents pedestrian behavior as a reaction to forces and potentials, influenced not only by the surrounding environment but also by the pedestrians' motivations and desires internally [17]. In other words, the crucial difference between the two approaches is that the microscopic model analyzes the motion of each individual agent [18]. We mainly explored the optimization problem from a macroscopic point of view, which is related to this research.

Most problems from the macroscopic perspective are defined on a network [19]. An optimal path for each evacuee is obtained by considering the movement of an evacuee as a flow in the network. By accommodating several types of objective functions and constraints, various methodologies have been studied to solve the problem efficiently [20,21]. Research on the optimal routes of evacuees in a building has been carried out since the 1980s. [22] proposed a "uniform principle" of the evacuation problem in a building. Although this study has a limitation that the principle is only applied under the uncapacitated route, it is a meaningful study as it presents the mathematical analysis of the minimum evacuation time from a building. [23] transformed the structure of a building into a network and developed a mathematical model to obtain the optimal route for the evacuees. [24] developed EVACNET+, which is a computer program designed to determine the optimal evacuation route in the building. They also contended that the evacuation plan should be considered from a global perspective by considering all evacuees rather than from the perspective of each individual looking to evacuate. [25] considered the EPP with linear and non-linear side constraints by adding variable arc capacities in the network flow problem. They also proposed an algorithm to solve the problem efficiently. The EPP from this study has been extended into various forms. More recently, [26] proposed a two-step approach to finding the evacuation route efficiently. In their model, routes are found with an uncapacitated model in the first step, and schedules are established with a capacitated model in the second step. [27] considered crowd movement management as a possible solution to the evacuation planning problem. By considering the problem with bi-level optimization, both perspectives, including the network design problem and the pedestrian assignment problem, could be taken into account. In the upper level, a reconfiguration of the layout is considered to minimize the total travel time. Based on the decision made at the upper level, the pedestrian assignment problem is concerned with minimizing inefficiency for all pedestrians at the lower level. By incorporating the lower-level problem into the constraints of the upper-level problem, the problem is reduced to a single-level problem. They also developed the solution procedure based on the Tabu search algorithm to overcome the intractability of the proposed model, which features the nonlinear mixed integer program. [28] also considered the crowd management problem. Their study examined the redesign of the network through a bi-level optimization model that optimizes utility maximization from the perspective of the individual as well as network capacity from the viewpoint of the manager. Due to the non-linear nature of the proposed model, the authors of this study also employed the interior point algorithm with the aid of a heuristic algorithm to solve the problem efficiently. [29] showed the importance of optimal routes for evacuation from a building by conducting a case study based on realistic data. They also reported that the optimal evacuation plan could reduce the final evacuation time of the last evacuee. [30] proposed a heuristic algorithm for evacuation routing and scheduling by incorporating the centralized hybrid approach. Their algorithm solved the discrete optimization problems efficiently in terms of memory storage. [31] considered a lexicographical evacuation plan, which minimizes the overall evacuation time. They developed an algorithm that can solve the problem in pseudo-polynomial time. For verification of their algorithm, they conducted a case study. [4] developed a bi-level programming model to solve the evacuation planning problem by incorporating an upper-level model that minimizes the total travel time based on exit sign direction, as well as a lower-level model that is a dynamic pedestrian assignment model that satisfies the dynamic user optimum principle. Due to the NP-hard nature of the proposed bi-level programming model, it is difficult to obtain the optimal solution within the polynomial time. To overcome the limitation of the model, they developed the heuristic algorithm to solve the problem efficiently. They developed a heuristic algorithm to solve the problem in a more efficient manner in order to overcome this limitation of the model.

3. Problem description

To incorporate an emergency in a building, we considered the discrete-time network flow problem. Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ as a finite directed graph, with a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$. As shown in Fig. 1, the components of the building can be transformed into a network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ [32]. The areas, such as rooms or shops, where occupants stay in the building, are represented as nodes. The route from one area to an adjacent area in a building, such as corridors, is represented by an arc. When an evacuee moves to an adjacent node through an arc, travel time is required. Each node and arc has a capacity that corresponds to the maximum number of occupants in the area and passes through the corridor, respectively. To incorporate the number of moving evacuees over time, the dynamic network $\mathcal{G}^T = (\mathcal{N}^T, \mathcal{A}^T)$ is defined by expanding the static network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ over timeframe $t \in \{0, 1, \dots, T\}$. Throughout the paper, we will use $\mathcal{T}$ to represent the timeframe $\{0, 1, \dots, T\}$ for brevity.

Areas where occupants are initially distributed in the building correspond to the source nodes $i \in \mathcal{S}$ ($\mathcal{S} \subset \mathcal{N}$) of the network. Occupants in the source nodes will escape to safe areas within the given timeframe $t \in \mathcal{T}$. The safe areas correspond to the sink nodes $i \in \mathcal{D}$ ($\mathcal{D} \subset \mathcal{N}$) of the network. To handle the multiple source and sink nodes in the network, an artificial *super source node*, s, and *super sink node*, d, were introduced. By connecting artificial arcs with all source and sink nodes to s and d with travel time as zero and arc capacity as infinite at time $t = 0$, respectively, the discrete-time dynamic network flow problem can be solved as the static network flow problem [16]. For more efficient computation, the capacity of each arc connected with s could be substituted with the number of initialized occupants at each node. Meanwhile, the infinite capacity of an arc connected to d could be substituted with the total number of evacuees. The discrete-time dynamic network flow problem is illustrated in Fig. 2. Suppose that there is a potential for further collapse after an emergency in the building. For collapses of certain parts that happen after the initial crisis is over, a plan established without considering further collapses may not ameliorate damages or limit casualties. By contrast, a plan based on interdicting all routes through collapsible areas might be too conservative. Therefore, we adopted a robust optimization approach to the EPP in which additional collapses were taken into account.

4. Mathematical model

In the EPP, various objective functions can be specified based on the discrete-time dynamic network flow [16,32]. Typically, problems are formulated with an upper bound, which is a predetermined time-bound T. However, setting an inappropriate time-bound T leads to cumbersome results. An underestimated T could make the problem infeasible, whereas an overestimated T requires more computation time than is otherwise necessary. When calculating the minimum time required for all evacuees to escape, which is the golden time that yields the complete evacuation time, it can be done by recursively solving the static maximum flow problem until all evacuees have escaped or by using the dichotomous search method. The process of determining the time-bound T, therefore, can also be regarded as an optimization problem when the decision-maker has to look only at golden times for solutions. In practice, an evacuation plan should be developed according to the available time. Evacuation times may vary based on various circumstances, such as a collapse of the building or the release of a hazardous gas. We assumed the time-bound T as the golden time, which refers to the given time available for evacuees to evacuate a building safely in an emergency. We viewed the available time in an emergency as the golden time, which is when evacuees have sufficient time to safely evacuate a building within the given time. On the basis of the golden time, T, we developed a mathematical model based on the maximum flow problem. When the time-bound T is sufficient, a plan in which all flows are outbound is established. When the time-bound is insufficient, it may be necessary to maximize outbound flows within the time-bound. With an EPP based on the maximum flow problem, the troublesome consequences of an incorrectly determined time-bound T can be avoided. Thus, the maximum flow problem was adopted as the basic structure of our mathematical model.

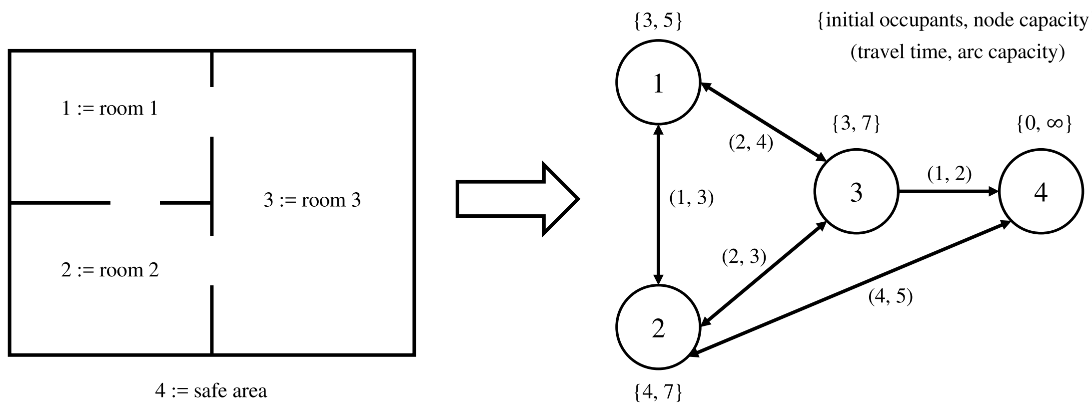

Fig. 1. Example of converting the structure of a building to a network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ [16].

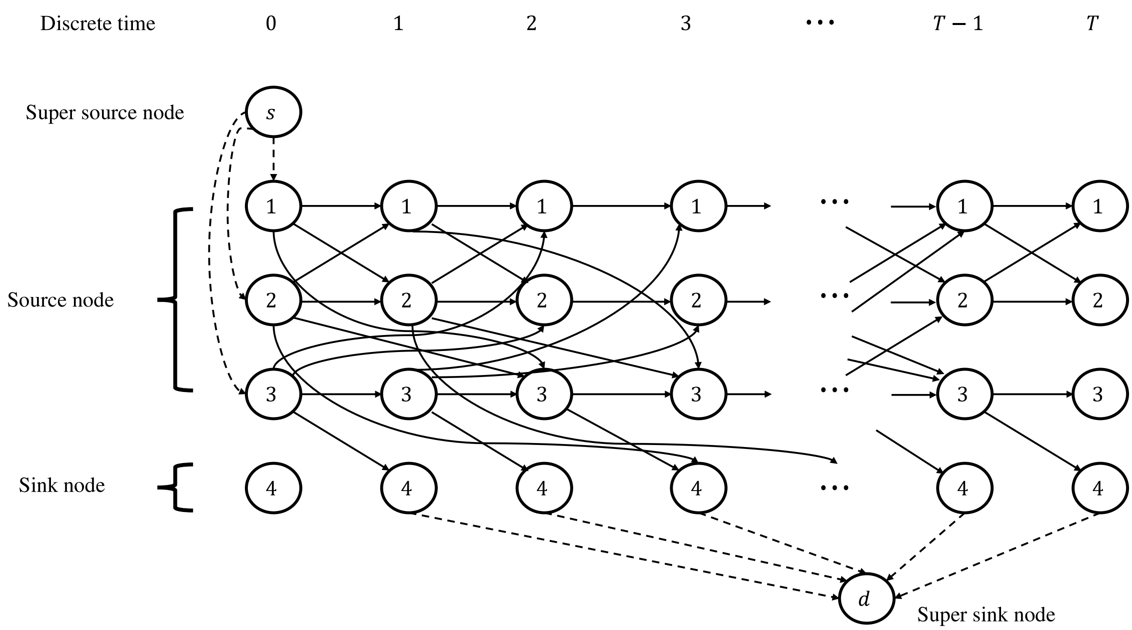

Fig. 2. Example of converting a static network into a dynamic network by introducing a super source node and a super sink node [16].

The maximum dynamic flow problem considered in this study is from the model of [16] and adjusted slightly. The model maximizes the number of evacuees reaching the sink nodes from the source nodes within the timeframe while satisfying the relevant constraints, including flow conservation and capacity constraints. Thus, in an optimal evacuation plan, the maximum flow indicates evacuating as many people as possible to safe areas within a specific timeframe. Taking into account the perspective of evacuees, this plan might not always involve the shortest route for every individual, but it is an optimal plan from an overall perspective.

For the mathematical formulation, indices, sets, parameters, and decision variables are defined in Table 1. We used indices i, j and (i, j) which represent the nodes and arcs, respectively. In the case of indices of nodes, they are elements of the finite set $\mathcal{N}$. To represent the initial locations of the evacuees, we used set of source nodes $\mathcal{S} \subset \mathcal{N}$ where occupants are initialized. For the representation of the safe areas where the evacuees are heading to, we used the set $\mathcal{D} \subset \mathcal{N}$. In the case of the disaster areas where the fire or terrorism attack occurs, the set of nodes $\mathcal{R} \subset \mathcal{N}$ is used. By introducing the super source node s and the super sink node d, we could expand the static network to the dynamic network, as illustrated in Fig. 2. Decision variables $x_{ij}(t)$ and $y_i(t)$ provide the direction to evacuees as to where to go or whether to wait or not at each point in time. Suppose that three, four, and three occupants are distributed in Nodes 1, 2, and 3, respectively, of the network illustrated in Fig. 1. If the solution of the evacuation plan at $t = 1$ is obtained as $x_{13}(1) = 2$, $x_{24}(1) = 3$, $x_{34}(1) = 2$, $y_1(1) = 1$, $y_2(1) = 1$, and $y_3(1) = 1$, then two, three, and two occupants in Nodes 1, 2, and 3, respectively, are directed to move. Meanwhile, three occupants distributed in Nodes 1, 2, and 3 are instructed to stay in their location at the time. When the evacuees move from Node i to Node j, the travel time λ_{ij} is required. We assumed that the capacity of a Node i, which restricts the number of evacuees that could stay in the Node i at the same time, is a_i. We also assumed that the capacity of an Arc (i, j), which restricts the number of evacuees that could move from Node i to Node j at the same time, is b_{ij}. To represent the initial number of evacuees located in each area, we used q_i. In this manner, each evacuee can be informed of the complete path from his or her initial location to the exit. It is an evacuation plan that is established through the optimization model from a macroscopic perspective.

For the simplicity of the analysis and computational tractability, we made the following assumptions:

Assumption 1. All evacuees are considered as a homogeneous group. That is, they move at the same travel time through an arc (i, j).

Assumption 2. The emergency occurs at $t = 0$ and all evacuees start evacuation from this time.

Table 1
Indices, sets, parameters, and decision variables for the mathematical model.

Indices	
i, j	Nodes
s	Super source node
d	Super sink node
Sets	
$\mathcal{N}$	Finite set of nodes $\mathcal{N} = \{1, 2, \ldots, N\}$
$\mathcal{S}$	Finite set of source nodes where occupants are initialized $\mathcal{S} = \{1, 2, \ldots, S\}$
$\mathcal{D}$	Finite set of sink nodes which represent safe areas $\mathcal{D} = \{1, 2, \ldots, D\}$
$\mathcal{R}$	Finite set of nodes which represent disaster areas $\mathcal{R} = \{1, 2, \ldots, R\}$
$\mathcal{A}$	Finite set of arcs $\mathcal{A} = \{1, 2, \ldots, A\}$
Parameters	
λ_{ij}	Travel time between nodes i and j
a_i	Capacity of a node i
b_{ij}	Capacity of an arc (i, j)
q_i	The number of evacuees distributed at each source node i
Decision variables	
$x_{ij}(t)$	Number of moving evacuees through arc (i, j) at time t
$y_i(t)$	Number of staying evacuees in node i at time t

Assumption 3. No evacuees head to the disaster areas.

Assumption 4. Upon arriving at initial locations, all evacuees go directly to safe areas.

Assumptions 1 and 2 are common assumptions in the evacuation planning from the macroscopic perspective [16,32,33]. In spite of the strong assumption and its disadvantage of macroscopic models, a realistic model can be achieved if the arc capacity and the travel time of the network are configured properly. Assumptions 3 and 4 support the reason that this study has considered a finite directed graph. By restricting all arcs connected to source, sink, and disaster area nodes as a one-way arc, we limited the abnormal behavior of the evacuee, such as heading to disaster areas or does not move to safe areas.

4.1. Evacuation planning model considering no further collapse

We begin with the mathematical model of an EPP without taking into account any further collapse in the building. A mathematical model that does not include the uncertain parameter is presented in this subsection. The objective function and constraints of the EPP based on a linear program (LP) are as follows:

$$\max \sum_{t\in\mathcal{T}} \sum_{i\in D} x_{id}(t) \tag{1}$$

$$\text{s.t.} \quad x_{si}(0) = q_i \qquad \forall i \in \mathcal{S}; \tag{2}$$

$$\sum_{k\in pred(i)} x_{ki}(t-\lambda_{ki}) - \sum_{j\in succ(i)} x_{ij}(t) = y_i(t+1) - y_i(t) \qquad \forall i \in \mathcal{N}; \forall t \in \mathcal{T}; \tag{3}$$

$$y_i(0) = 0 \qquad \forall i \in \mathcal{N}; \tag{4}$$

$$y_i(t) = 0 \qquad \forall i \in \mathcal{D}; \forall t \in \mathcal{T}; \tag{5}$$

$$y_i(t) \le a_i \qquad \forall i \in \mathcal{N} - \mathcal{D}; \forall t \in \mathcal{T}; \tag{6}$$

$$x_{ij}(t) \le b_{ij} \qquad \forall (i,j) \in \mathcal{A}; \forall t \in \mathcal{T}; \tag{7}$$

$$x_{ij}(t), y_i(t) \ge 0 \qquad \forall i \in \mathcal{N}; \forall (i,j) \in \mathcal{A}; t \in \mathcal{T}; \tag{8}$$

where $pred(i) \triangleq \{k | (k,i) \in \mathcal{A}\}$ and $succ(i) \triangleq \{j | (i,j) \in \mathcal{A}\}$

Objective function (1) maximizes the number of evacuees reaching safe areas within the timeframe $t \in \mathcal{T}$. Constraints 2 initialize the distribution of evacuees in each source node i at $t = 0$. Constraints (3) represent the flow conservation (balance equation) constraints among the number of evacuees entering into a node, leaving out a node, and staying in a node (See Fig. 3). Constraints (4) restrict the number of evacuees staying at $t = 0$ to zero according to Assumption 2. Constraints (5) also restrict the number of evacuees staying in sink nodes to zero according to Assumption 4. Constraints (6) mean that the number of evacuees staying in node i at time t cannot exceed the capacity of node i. Constraints (7) limit the capacity of the arc between node i and j in terms of the number of evacuees passing through an arc (i, j). Constraints (8) mean that variables $x_{ij}(t)$ and $y_i(t)$ are non-negative real variables.

4.2. Robust evacuation planning model under a collapsible building

When an emergency occurs in a building where many occupants are scattered, it may cause instability for certain parts of the building. Under this circumstance, any evacuation plan that ignores the possibility of further collapse could not prevent casualties caused by these collapses. We propose a robust optimization model for which the uncertainty can be countered by cardinality constraints on the number of collapsible inflow arcs to a node. Existing research on RO has concerned the structure of an *uncertainty set*, which is the set of values that can be possibly realized. The concept of RO was first proposed by [34]. By considering a *box uncertainty set*, which includes the full range of uncertain parameters, the model guarantees the feasibility for all possible realizations of uncertainty [35]. However, the optimal solution from the box uncertainty set was too conservative. Accordingly, further research was conducted to supplement the limitations. The next breakthrough was achieved by [36,37]. They considered an *ellipsoidal uncertainty set* defined by the covariance structure of the uncertain parameters, which led to a less conservative solution than that of the previously existing RO model. The structural form of the uncertainty set covered in this study is the *polyhedral uncertainty set* proposed by [38]. Consider the uncertain parameter that is symmetrically distributed around the mean $\hat{\mu}$ on the bounded support, such as $\mu \in [\hat{\mu} - \overline{\mu}, \hat{\mu} + \overline{\mu}]$ where $\hat{\mu} < \infty$ and $\overline{\mu} < \infty$. When the scaled deviations of parameters are limited to the *budget of uncertainty* Γ, the level of robustness can be adjusted as follows: $\sum \left(\mu - \overline{\mu}\right) / \hat{\mu} \le \Gamma$. By controlling the parameter Γ, the solution of the model can be less conservative than that of previous studies. Another benefit of taking this approach is that the robust formulation also features a linear optimization problem when the primal formulation is a linear optimization problem.

Denote by the binary variable $\mu_{ki}(t)$ representing the collapse of the arc between the nodes k and i at time t, when $\mu_{ki}(t) = 1$; otherwise, $\mu_{ki}(t) = 0$. The number of collapsible incoming arcs to node i at time t is restricted by the integer parameter $\Gamma_i(t)$ $(0 \le \Gamma_i(t) \le |pred(i)|, i \in \mathcal{N})$ which is called the *level of conservatism* [38]. The role of this parameter is to control the degree of robustness. Depending on the value of the parameter, the trade-off occurs between the probability of violation and the loss of objective value compared to the nominal problem. In terms of network flow problems, the parameter can be interpreted as the upper bound of the number of collapsing arcs entering a node [12]. Accordingly, the robust model provides egress routes for evacuees by accounting for further collapses. To make a more conservative decision about the risk of collapse, the decision-maker can set a larger gamma value, and vice versa.

Remark 1. In regards to the collapse of a node, it can be considered as the collapse of an arc. By adding a dummy node and a dummy arc, the collapsible node can be divided into two nodes.

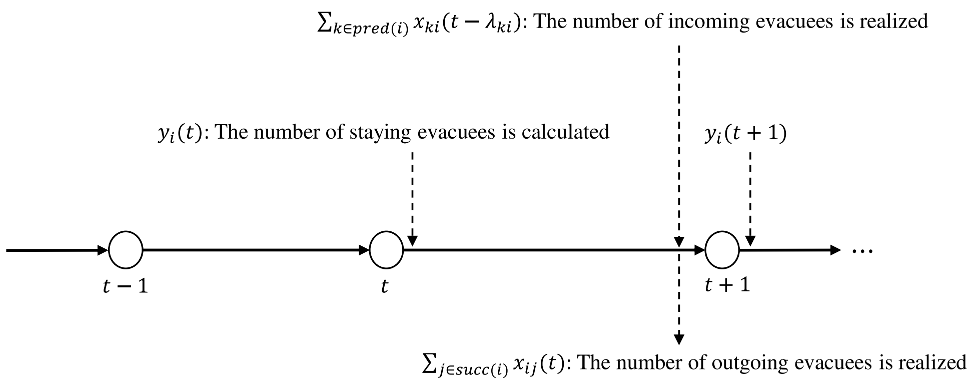

Fig. 3. Flow conservation among staying, incoming and outgoing evacuees based on the discrete time planning horizon.

For example, v is denoted as a collapsible node in a network. If node v is divided into v' and v'', and they are connected by the collapsible arc e, a collapsible node could be concerned in the same manner with the collapsible arc [12]. In this manner, the generalized mathematical model incorporating the collapsibility of nodes and arcs can be developed.

As [12] pointed out, Constraints (3) must be relaxed as weak flow conservation constraint as follows:

$$\sum_{k\in pred(i)} x_{ki}(t-\lambda_{ki}) - \sum_{j\in succ(i)} x_{ij}(t) \geq y_i(t+1) - y_i(t) \quad \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \tag{9}$$

In general network flow problems, it is necessary to meet strict flow conservation constraints in which the amount flowing into and out of a node must be equal. Nevertheless, a node adjacent to a collapsible arc cannot leave equally as much flow amount as it introduced. Accordingly, the incoming amount must satisfy the weak inequality by being greater than or equal to the amount leaving. To incorporate the collapse of the arc, the binary variable $\mu_{ki}(t)$ is introduced to Constraints (9) as follows:

$$\sum_{k\in pred(i)} \left(1-\mu_{ki}(t)\right)\cdot x_{ki}(t-\lambda_{ki}) - \sum_{j\in succ(i)} x_{ij}(t) \geq y_i(t+1) - y_i(t) \quad \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \tag{10}$$

Constraints (10) can be rearranged as Constraints (11) by shifting the term relevant to $\mu_{ki}(t)$ to the right-hand side and pushing all the other terms to the left-hand side. By defining $Z_i(x,t) := \sum_{k\in pred(i)} \mu_{ki}(t)\cdot x_{ki}(t-\lambda_{ki})$ as the maximum value obtained from the realization of the uncertain parameter $\mu_{ki}(t)$, we derived the following inequality.

$$\sum_{k\in pred(i)} x_{ki}(t-\lambda_{ki}) - \sum_{j\in succ(i)} x_{ij}(t) - y_i(t+1) + y_i(t) \geq Z_i(x,t) \quad \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \tag{11}$$

Since the value of the left-hand side must be larger than the maximum value of $Z_i(x,t)$ of the right-hand side, an inner optimization problem for $Z_i(x,t)$ for all (i,t)th pair should be considered as follows:

$$Z_i(x,t) \triangleq \max \sum_{k\in pred(i)} \mu_{ki}(t)\cdot x_{ki}(t-\lambda_{ki}) \tag{12}$$

$$\text{s.t.} \quad \sum_{k\in pred(i)} \mu_{ki}(t) \leq \Gamma_i(t) \tag{13}$$

$$0 \leq \mu_{ki}(t) \leq 1 \quad \forall k\in pred(i); \tag{14}$$

Remark 2. The binary variable $\mu_{ki}(t)$ can be linearly relaxed as shown in (14).

It is well known that the optimization problem $\max\{cx : Ax \leq b, x\in R_+\}$ provides the integer solution when the matrix A is totally unimodular and all instances of vector b are integers [39]. $\mu_{ki}(t)$ was modeled as the binary variable in Inequality (9), but it can be linearly relaxed due to the integer parameter $\Gamma_i(t)$. Consequently, the optimal solutions of $\mu_{ki}(t)$ are 0 or 1 in the inner optimization problem $Z_i(x,t)$. By strong duality, all (i,t)th pairs of the inner optimization problem (12)–(14) can be reformulated to a dual linear program as follows:

$$\min \sum_{k\in pred(i)} \theta_{ki}(t) + \Gamma_i(t)\cdot\delta_i(t) \tag{15}$$

$$\text{s.t.} \quad \theta_{ki}(t) + \delta_i(t) \geq x_{ki}(t-\lambda_{ki}) \quad \forall k\in pred(i); \tag{16}$$

$$\theta_{ki}(t) \geq 0 \quad \forall k\in pred(i); \tag{17}$$

$$\delta_i(t) \geq 0 \tag{18}$$

where $\theta_{ki}(t)$ and $\delta_i(t)$ are dual variables.

By replacing the $Z_i(x,t)$ with Objective function (15) and adding Constraints (16), (17), and (18) as for the all (i,t)th pair, a robust counterpart of EPP (REPP) can be developed as follows:

$$
\begin{aligned}
\max \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{D}} x_{id}(t) & \\
\text{s.t.} \quad & x_{si}(0) = q_i & \forall i\in\mathcal{S}; \\
& \sum_{k\in pred(i)} [x_{ki}(t-\lambda_{ki}) - \theta_{ki}(t)] - \sum_{j\in succ(i)} x_{ij}(t) & \\
& -\ y_i(t+1) + y_i(t) - \Gamma_i(t)\cdot\delta_i(t) \geq 0 & \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \\
& y_i(0) = 0 & \forall i\in\mathcal{N}; \\
& y_i(t) = 0 & \forall i\in\mathcal{D};\forall t\in\mathcal{T}; \\
& y_i(t) \leq a_i & \forall i\in\mathcal{N}-\mathcal{D};\forall t\in\mathcal{T}; \\
& x_{ij}(t) \leq b_{ij} & \forall (i,j)\in\mathcal{A};\forall t\in\mathcal{T}; \\
& \theta_{ki}(t) + \delta_i(t) \geq x_{ki}(t-\lambda_{ki}) & \forall k\in pred(i); \\
& & \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \\
& \theta_{ki}(t) \geq 0 & \forall k\in pred(i); \\
& & \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \\
& \delta_i(t) \geq 0 & \forall i\in\mathcal{N};\forall t\in\mathcal{T}; \\
& x_{ij}(t), y_i(t) \geq 0 & \forall i\in\mathcal{N}; \\
& & \forall (i,j)\in\mathcal{A};t\in\mathcal{T};
\end{aligned}
$$

Decision variables $x_{ij}(t)$ and $y_i(t)$ can be regarded as integer variables because they represent the number of moving and staying evacuees, respectively. Although the model proposed in Section 4.1 is developed as LP, it provides an optimal solution as an integer value when all the input data are integer values. In the same manner as the general network flow model dealt with a single commodity flow, the solution of the LP is the same as that of the integer program (IP). However, REPP provides a fractional solution [12]. Because this study deals with a relatively large evacuees group, not much difference is found between the fractional solutions from the LP and IP. In an emergency, rapid decision-making is required, but developing the problem based

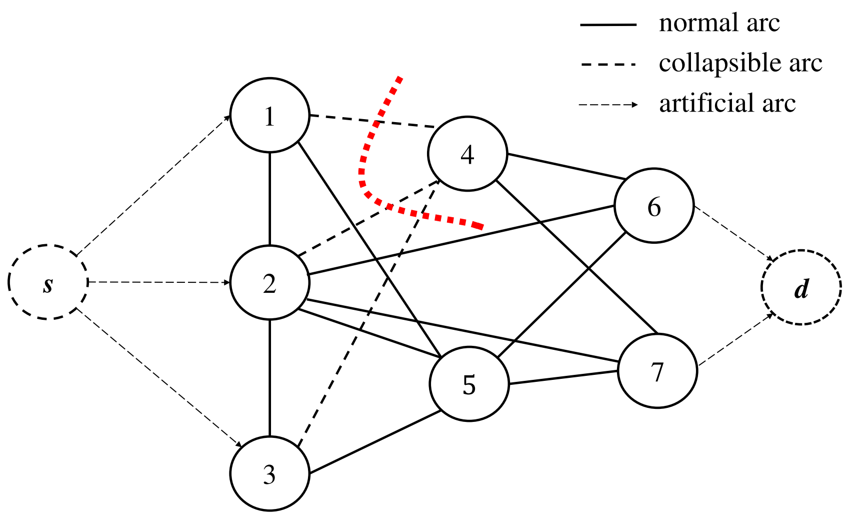

Fig. 4. Network structure for the numerical experiments.

on the IP model can cause complexity issues. However, under the LP model, fast decisions incorporating uncertain further collapses can be established. Therefore, we developed the model as an LP, which can expedite the decision and rounded the fractional solutions up to the integer values. Recall that the parameter $\Gamma_i(t)$ restricts the maximum number of collapsible inflow arcs to node i at time t. During an emergency in a building, the possibility of partial building collapses could increase over time. To incorporate these possibilities, $\Gamma_i(t)$ was assumed to be a non-decreasing function over t. If the probability of collapse does not vary, then $\Gamma_i(t)$ can be modeled as a constant value.

5. Computational experiments

The central question in this study was how the uncertainty of further collapse in the building structure affects the optimal routes of evacuees. To answer the question, we conducted three types of computational experiments; (i) small-size network, (ii) artificial data, (iii) and case studies. We describe the background of each experiment and the results in Sections 5.1–5.3. We conducted computational experiments with FICO XPRESS-IVE version 7.2 with an Intel® Core™ i5-7400 CPU @ 3.0 GHz. We first conducted numerical experiments on small problems.

5.1. Numerical experiments

For the numerical experiments, we made the network with nodes ($\forall i \in N, N = s, 1, 2, \ldots, 7, d$) and relevant arcs ($\forall (i,j) \in A$). We assumed that occupants were initially distributed on Nodes 1–3 which are source nodes (elements of set S) and escaped to safe areas which are Nodes 6 and 7 (elements of set D) via Nodes 4 and 5. The network structure for the numerical experiments is illustrated in Fig. 4. We also assumed that the arcs entering Node 4 have the possibility of further collapses at timeframe $t \in \mathcal{T}$ which is controlled by the parameter $\Gamma_4(t)$. To figure out how the results have a tendency depending on time-bound and the number of occupants, experiments were conducted with Instance 1 under the condition that the value of $\Gamma_4(t)$ did not vary within the timeframe $t \in \mathcal{T}$. Table 2 represents the summary of numerical experiments of the REPP by varying the time-bound T, $\Gamma_4(t)$, and an initial number of occupants in Instances 1.1–1.6. $Inbound$ and $Outbound$ in Table 2 represent the number of evacuees entering and leaving Node 4, respectively and $Total$ represents the total number of evacuees arriving in the safe areas within the timeframe. In one such case, in Instance 1.1 with $\Gamma_4 = 1$, 20 evacuees entered and 13 evacuees were able to come out from Node 4, respectively, and 143 out of 150 evacuees could escape to safe areas within the given timeframe.

Table 2
Results of numerical experiment 1.

	Number of evacuees reaching to safe areas			
	$\Gamma_4 = 3$	$\Gamma_4 = 2$	$\Gamma_4 = 1$	$\Gamma_4 = 0$
Instance 1.1	$T = 8$, $(q_1, q_2, q_3) = (50, 50, 50)$			
Inbound	20	19	20	45
Outbound	0	6	13	45
Total	130	136	143	150
Instance 1.2	$T = 9$, $(q_1, q_2, q_3) = (50, 50, 50)$			
Inbound	0	0	0	45
Outbound	0	0	0	45
Total	150	150	150	150
Instance 1.3	$T = 10$, $(q_1, q_2, q_3) = (50, 50, 50)$			
Inbound	0	0	0	60
Outbound	0	0	0	60
Total	150	150	150	150
Instance 1.4	$T = 8$, $(q_1, q_2, q_3) = (70, 70, 70)$			
Inbound	30	54	60	45
Outbound	0	8	15	45
Total	150	158	165	195
Instance 1.5	$T = 9$, $(q_1, q_2, q_3) = (70, 70, 70)$			
Inbound	17	15	18	45
Outbound	0	5	12	45
Total	193	198	204	210
Instance 1.6	$T = 10$, $(q_1, q_2, q_3) = (70, 70, 70)$			
Inbound	0	0	0	105
Outbound	0	0	0	105
Total	210	210	210	210

As shown in Table 2, if there is no possibility of collapse, $\Gamma_4(t) = 0$, the results are the same as those of the deterministic model where no collapse is assumed. On the contrary, for the case where all the arcs can collapse, $\Gamma_4(t) = 3$, the results are the same as the deterministic model for the data that there is no inflow arc to Node 4. In other words, excluding the possibility of collapses means planning from a risk-taker perspective with safety ignorance. Instead, overestimating the situation by considering all arcs to be collapsible makes the plan too conservative. Since the optimal value under $\Gamma_4(t) = 3$ showed the largest value in all instances, the following question can be raised: *Would it be better to follow the plan of $\Gamma_4(t) = 0$?* These results can be interpreted as indicating that the collapsible arcs represent conservative plans rather than actual needs. Consider the results of Instance 1.1. If collapses of all the incoming arcs for Node 4 are realized with the worst-case scenario, 105 people will survive under the plan of $\Gamma_4(t) = 0$,

Table 3
Results of numerical experiment 2.

Instance 2	$T = 8$, $(q_1, q_2, q_3) = (70, 70, 70)$			
	Instance 2.1 $\Gamma_4(t) = (0,0,1,2,3,3,3,3,3)$	Instance 2.2 $\Gamma_4(t) = (0,0,1,1,2,2,2,3,3)$	Instance 2.3 $\Gamma_4(t) = (0,0,1,1,1,1,1,2,2)$	Instance 2.4 $\Gamma_4(t) = (0,0,0,0,1,1,2,2,2)$
Inbound	30	64	60	45
Outbound	0	18	15	30
Total	150	158	165	180

but 130 would survive under the plan of $\Gamma_4(t) = 1, 2$, or 3. The results of the experiment show that REPP provides an optimal solution in the worst-case scenario of network structure uncertainty.

As can be observed in Instances 1.2, 1.3, and 1.6 of Table 2, when a given time-bound T is sufficient for all people to escape, it was suggested not to pass an arc that could collapse. In contrast, if time-bound T is insufficient, it is better to evacuate through collapsible arcs. Whenever possible, even if moving through a collapsible arc results in casualties who cannot get out, it is the most effective method of evacuating as many people within the time-bound T. In other words, REPP also takes into account whether or not all occupants can escape without passing through the collapsed areas within a predetermined period of time T.

Another research question of this study would be *whether detouring the collapsible arcs is optimal routes or not for evacuees*. Consider the changes of the optimal evacuation planning by varying the time-bound T. As shown in Instances 1.2, 1.3, and 1.6, if sufficient time-bound T is given to all evacuees going to safe areas, all evacuees can exit to their destination without going through the collapsible area. When the time limit T is not enough to allow for all evacuees to reach safe areas, it may be more feasible for some evacuees to pass through the collapsible area. As shown in Table 2, while inbound and outbound numbers are not equal at Node 4, computational experiments have shown that the REPP is a robust evacuation plan capable of saving more people within a given amount of time.

We also conducted the experiment by varying the value of $\Gamma_i(t)$ according to the time t. Recall that $\Gamma_i(t)$ represents the upper bound of the number of collapsible arcs entering node i at time t. It was assumed that the number of collapsible arcs will not decrease over time in a disaster situation. Therefore, $\Gamma_i(t)$ was set as a non-decreasing function with respect to t. Numerical experiments were conducted with Instance 2, which has a different value of $\Gamma_4(t)$ from Instances 2.1–2.3. The results of the experiments are described in Table 3. Based on the Table 3, it appears that the evacuation plan that leads more evacuees to reach the safe areas was established when $\Gamma_4(t)$ was set to a relatively small value. There was an increasing tendency in the ratio of outbound to inbound as the number of collapsible arcs decreased. We also noted that when the increase rate of $\Gamma_4(t)$ has a large value with time t, a small number of evacuees could make it safely out. Thus, the experiment for Instance 2 addresses the fact that modeling $\Gamma_4(t)$ as a non-decreasing function for t works correctly, and it can be modeled as a time-varying function like $\Gamma_4(t)$.

5.2. Artificial data

To further analyze the REPP, we generated artificial data. On the basis of the 100 nodes, two types of networks were randomly generated, each with *sparse* or *dense* arc connections. Despite the fact that the networks generated by artificial data are not as realistic as the networks generated from the case studies presented in subsection /refsec 4.3, the total number of arcs or collapsible arcs can be easily adjusted. We made 20 source nodes and 10 sink nodes from the 100 nodes. At the same time, randomly generated collapsible areas were created. We also varied the time-bound with 18, 20, and 22. With these data, we conducted computational experiments and illustrated the results in Fig. 5.

As Fig. 5 indicates, more evacuees were able to escape within a given time when the network density was low because a greater number of routes were available for those evacuees. Moreover, we observed that the difference in evacuee numbers reaching safe areas between dense and sparse networks decreased when the number of collapsible arcs increased. This means that the number of routes and collapsible areas affected the number of evacuated people reaching the safe areas rather than the number of areas. Furthermore, the number of evacuees reaching safe areas was more affected by the number of collapsible arcs within the dense network than within the sparse network. In the dense network, the number of evacuees successfully escaping decreased steadily as the number of collapsible areas increased. An overall decreasing tendency was noted in the sparse network, however, an inversion occurred and a significant decline rate was not observed. Since the sparse network has fewer paths for evacuees to take, the number of collapsed areas did not have a significant impact on the number of people who were able to flee.

Table 4
Results of Case study 1.

Case study 1	Number of evacuees succeed in reaching safe areas		
T	ID	REPP	NC
10	141	141	178
15	431	533	743
20	721	994	1363
25	1011	1167	1480
30	1267	1267	1480
35	1367	1367	1480

5.3. Case study

We conducted a case study based on data gathered from [32,33]. The background of the case study was the mega-mall Central City located in Seoul, Korea. Fig. 6 shows an illustration of the floor plan of Central City. We transformed it into a network form by considering each facility as a node and each corridor as an arc. Fig. 7 shows an illustration of the transformed network presented as a floor plan in Fig. 6.

As illustrated in Fig. 7, we assumed that the seven areas marked in orange were in emergency situations. The areas where occupants were initially distributed (source nodes) are shaded in orange and gray. The safe areas (sink nodes) are marked in green. The capacity of the nodes and arcs and the travel times between adjacent nodes were estimated based on data from [32,33]. We selected the collapsible arcs from the white nodes connected with three or more arcs and assumed that a random number of arcs connected to white nodes could collapse. For two types of case study, *Case studies* 1 and 2, we determined the collapsible arcs connected to the white nodes that serve as hubs through which many evacuees pass.

For Case study 1, Nodes 25, 28, and 36, which are the hubs of the evacuation path, were assumed to be collapsible. Additionally, we conducted Case study 2 in which arcs connecting Nodes 24, 27, and 35 are assumed to be collapsible arcs through which relatively few evacuees pass. The parameter $\Gamma_i(t)$ was assumed to increase with timeframe $t \in \mathcal{T}$. To analyze the characteristics of REPP, we also conducted experiments based on *interdiction* (ID), where all the paths to collapsible areas were blocked, and *no collapse* (NC), where all paths could not possibly collapse. Table 4 presents a summary of the results of computational experiments for Case study 1 conducted by varying the time-bound T.

It is evident from Table 4, results of ID, which restricted the evacuation route through potential collapsed areas, were too conservative.

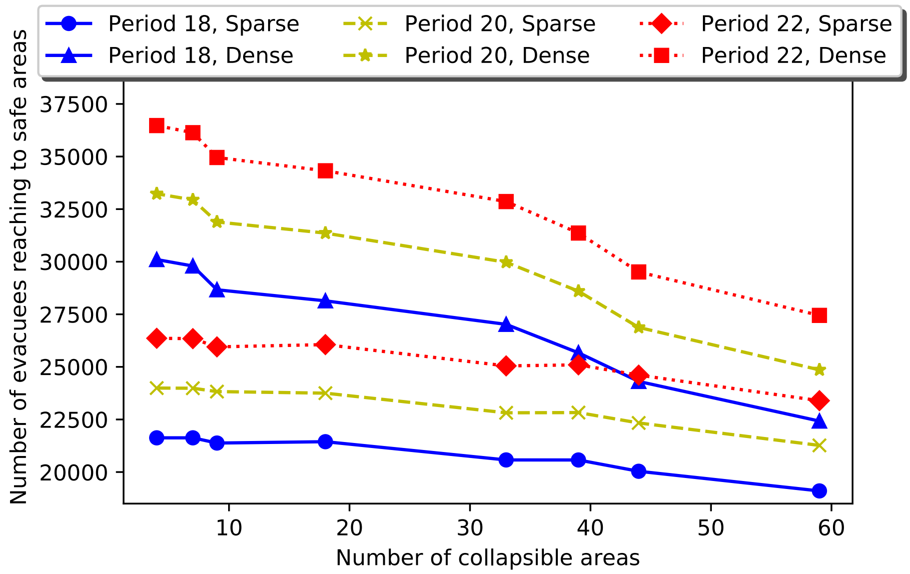

Fig. 5. Results of the computational experiments based on the artificial data.

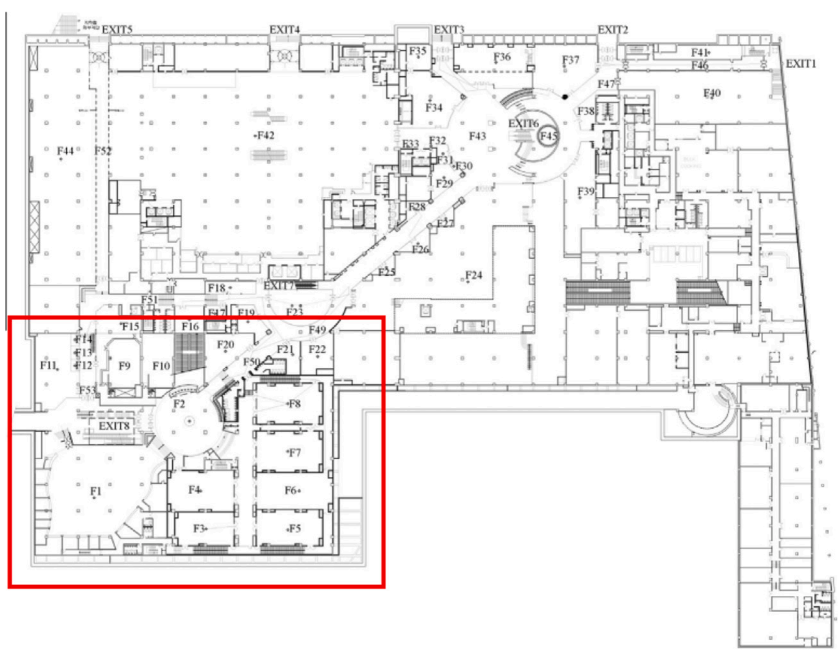

Fig. 6. Floor plan of Central City [32,33].

Contrary to this, NC, which was founded without considering the possibility of a collapse, underestimated the risk. This would have resulted in many casualties and blocked the route if the collapse had occurred. In this case, the evacuation plan had to be reestablished, which was not an optimal approach considering the entire timeframe. We conducted further experiments on Case Study 2 to discern the results of the plan with collapsible pathways through which relatively few evacuees passed. The summary of the results of Case study 2 is provided in Table 5.

According to Table 5, the number of occupants who could evacuate in a given timeframe was generally higher than those from the previous experiments. According to these two studies, the damage was more severe if the collapse occurred in an area where many people travel.

Table 5
Results of Case study 2.

Case study 2	Number of evacuees succeed in reaching safe areas		
T	ID	REPP	NC
10	178	178	178
15	653	685	743
20	1128	1170	1363
25	1480	1480	1480
30	1480	1480	1480
35	1480	1480	1480

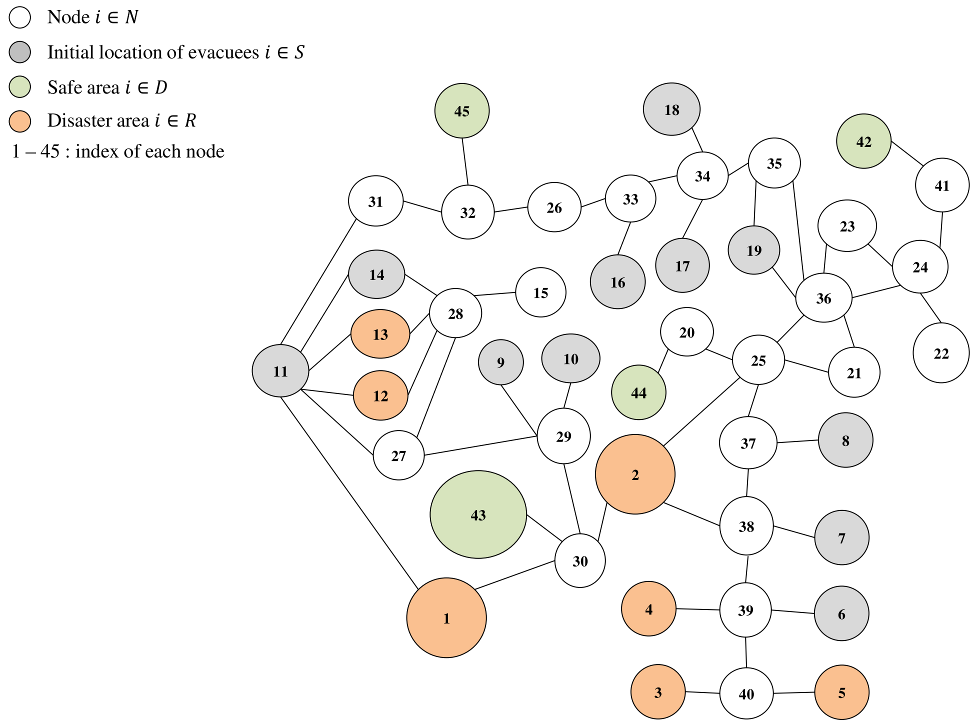

Fig. 7. Network form of the Central City.

Table 6
Results of Case study 1.1.

Case study 1	Number of evacuees succeed in reaching each safe area			
T	Node 42	Node 43	Node 44	Node 45
10	0	87	0	54
15	44	242	58	186
20	73	397	200	324
25	0	552	156	459
30	0	651	56	560
35	0	931	0	536

Table 7
Results of Case study 2.1.

Case study 2	Number of evacuees succeed in each safe area			
T	Node 42	Node 43	Node 44	Node 45
10	0	50	74	54
15	32	242	222	189
20	42	372	432	324
25	159	458	515	348
30	0	573	523	384
35	0	613	520	347

In this case, the worst-case scenario is the same as when all routes were restricted. Within a reasonable period of time, occupants could bypass collapsible areas. In other words, the REPP determines whether to bypass or pass through collapsible arcs based on a timeframe. We examined the characteristics of the REPP more thoroughly by analyzing the number of evacuees reaching each safe area. Tables 6 and 7 show how many evacuees used each safe area in Case studies 1 and 2. As shown in Tables 6 and 7, when the time-bound was sufficient, the evacuees should not head for safe areas by going through collapsible areas. When the time-bound was not sufficient, a relatively larger number of evacuees headed to safe areas by passing through areas that were in danger of collapsing. In other words, the REPP maximizes the number of evacuees escaping within a given timeframe while minimizing the risk of collapse.

6. Conclusions

This study establishes a robust evacuation plan in response to an emergency in a complex building with a collapsible structure. When an emergency occurs in the building, the uncertainty of further collapse of certain structural parts becomes a concern. If the evacuation plan is established without regard to the possibility of collapse, there could be extremely heavy casualties. Due to the difficulty of estimating the probability distribution function for all collapse scenarios in the building structure, the RO approach was used to come up with an EPP. By considering the problem as a discrete-time network flow problem, the collapsibility of the arc incoming to a node can be controlled by a parameter under an RO approach. As a result, a robust counterpart of the model has been developed as an LP, which makes the model computationally tractable for a large problem.

We conducted computational experiments and analyses to identify the characteristics of the model. Through numerical experiments, we analyzed whether optimal routes for evacuees would pass through collapsible areas, depending on the time available. Hence, our research led to the following results.

- If time allows, detouring around the collapsed areas is the most effective way to evacuate the most people. In other words, the model allows for detours around collapsible areas if sufficient time is available.
- When there is not enough time to complete a route, the model takes a risk and provides a route that includes collapsible areas.
- The model presented in this study actually takes a look at whether a risk should be taken based on a given time period. Moreover, it is possible for the decision-maker to choose how much risk-taking to incur by controlling the level of conservatism associated with this model.

This study features limitations in terms of measuring collapsible arcs. Although not as difficult to determine as the exact estimation of the probability distribution function or design of all possible scenarios, setting a threshold for the number of collapsible arcs was required. Furthermore, we only incorporated the uncertainty of the network structure. In an emergency in a building, various uncertainties would manifest, including the travel times between nodes, how people move, and whether or not fires spread or bombs explode late. Although it is difficult to construct a mathematical model which considers all possible uncertainties simultaneously, the uncertainties that can affect the EPP can be analyzed individually.

CRediT authorship contribution statement

Youngchul Shin: Conceptualization, Software, Methodology, Investigation, Data curation, Writing – Original Draft, Writing – review & editing, Visualization. **Ilkyeong Moon:** Conceptualization, Methodology, Validation, Writing – review & editing, Supervision.

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