A Game Theoretic Approach to the Selection, Mentorship, and Investment Decisions of Start-Up Accelerators

Hamid Zarei, Morteza Rasti-Barzoki, and Ilkyeong Moon

***Abstract*—Start-up accelerators have attracted increasing attention from start-ups in recent years. However, little is known about how they interact with start-ups and investors, how they prioritize a wide variety of their services, and how macro-level factors affect their performance. We use a game-theoretic approach to study mutual effects between an accelerator, a venture capitalist (VC), and start-ups. In the model, the accelerator selects start-ups and invests in them. During its program, it provides intensive mentorship for the selected start-ups and screens them to determine whether or not they are successful. Finally, it introduces successful start-ups to the VC to obtain financing for their ideas. Our results indicate that 1) the most vital role of an accelerator is its screening services. 2) Given limited resources of accelerators, prioritizing their services is the key to their efficacy. Screening, mentorship, and seed investment services possess the first, second, and third priorities, respectively. 3) Although accelerators are anywhere beneficial, macro-level heterogeneity of entrepreneurial ecosystems is associated to their efficacy. Their efficacy is higher (lower) in less (more) developed regions. 4) Institutional reforms targeting mentoring or entrepreneurial communities are unexpectedly detrimental to the efficacy of accelerators, while they may enhance start-ups' equilibrium willingness to participate.**

***Index Terms*—Entrepreneurship, game theory, portfolio selection, start-up, start-up accelerator, venture capital.**

I. Introduction

In RECENT years, developed countries have transitioned from managed economies to entrepreneurial economies [1]. Acs and Szerb [2] described the main characteristics of an entrepreneurial economy. First, they believed that new firms play a significant role, if not the primary role, in this type of economy unlike in the U.S. economy after World War II, for example, where oligopolistic companies dominated due to the managed structure of the economy. Second, new firms in an entrepreneurial economy can react quickly to rapid changes in technology and market demand, while bureaucracy and mass production hinder large companies from responding appropriately to these changes. In addition, Audretsch *et al.* [3] believed that entrepreneurship can contribute to economic growth by allowing neglected ideas to be commercialized.

The financial growth cycle paradigm states that new firms pass several stages of growth until they become mature companies [4]–[6]. As nascent but susceptible start-ups, new firms require different types and levels of support at each stage of their growth. Similarly, the supply of such different types of support varies significantly during the stages of growth [7]. To decide whether they benefit from a type of support at each stage of growth, new firms should consider the availability and expense of that support in that stage. Broadly speaking, achieving key milestones by a new firm greatly increases the possibility and decreases the expense of attracting further support [8], which potentially contributes to its continuation of growth. However, an important question arises about very early stage start-ups, which have not reached such milestones yet. How do they receive support that is vital for their present growth and achievement of future milestones?

Traditionally, business incubators have been the most affordable and available organizations that have been providing some support for very early stage start-ups [9], [10]. The main idea of business incubators about them is clear. Business incubators believe very early stage start-ups to be so vulnerable that they cannot bear the challenges of a real business. Therefore, they shelter start-ups in a relatively long period, usually between one to five years, and help them become stronger before being independent and facing business challenges [11]. Science parks [12] and university incubators [13] are some variations on business incubators.

As mentioned above, start-ups need agile reaction to rapid changes in technology and market demand, especially in some sectors such as software development and service provision. Therefore, a long-term buffer by a business incubator may not be a suitable solution for some start-ups. Instead, they need to learn as much and quickly as possible from the market, and

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use feedback from the market to design valuable products for customers. In this case, the terms "agile software development" [14] and "rapid prototyping" [15] properly capture the strategies adopted by today's start-ups. Hence, a new business model is needed to help very early stage start-ups adopt such strategies.

Start-up accelerators (hereafter accelerators), a recent but growing variation on business incubators, accelerate interactions between start-ups and the market [11]. The main difference between accelerators and traditional incubators is the short-term duration of accelerators. This short duration stimulates entrepreneurs to learn more from the market, adapt quickly, and progress rapidly [16], [17]. Business incubators receive some rents from start-ups in exchange for providing a workplace [17]. Accelerators, in contrast, receive some share of the start-up's equity in exchange for covering living and workplace expenses. Therefore, accelerators seem more attractive for cash-strapped, early-stage start-ups compared to business incubators. Another advantage of accelerators over business incubators is the intensive mentorship and network development opportunities, business incubators provide at a much lower level [17]. Meeting with four or five mentors in each day in an accelerator program builds a powerful social network for start-ups [17]. In spite of such differences, accelerators and business incubators serve the same clients: very early stage start-ups. About 80% of applicants of start-ups have no prior funding, signaling that they are at very early stages of development [18]. Accelerators can help very early stage start-ups achieve key milestones [19], and their support is the bridge to accessing further future support [19], [20].

Accelerators provide three main types of financial and non-financial supports for start-ups. First, intensive mentorship is among the most important value-added services provided by accelerators [11], [21]. Mentorship is essential for start-ups because they cannot afford to use similar services offered by other providers such as consulting firms. Mentors are usually former entrepreneurs in the related fields, and accelerators employ them after full examination. They help start-ups define their own business model and communicate with customers and investors [16]. In some programs, start-ups can meet up to 75 different mentors during the first month [17], [22]. Second, accelerator programs provide seed investment services that help cover the workplace and living expenses of start-ups and their founders [16]. The third type of value-added service provided by accelerators is the screening processes. Start-ups want to know whether or not their ideas will be successful in gaining customer traction and profits. Screening selected ideas to determine whether they will be successful requires time and careful analysis. In this case, the exceptional screening abilities of accelerators distinguish them from other financial institutions, such as venture capitalists (VCs) [23].

Given their limited resources, provision of a variety of services in an intensive and time-limited manner is a challenge of accelerators. Therefore, they should prioritize their services and focus on services that are essential for subsequent performance indicators of their selected start-ups. To the best of our knowledge, the existing literature lacks of knowledge about how accelerators should optimize the usage of their limited resources and prioritize their services. According to the above-mentioned explanations, the first research question addressed in this article is as follows.

> *Question 1.* How should accelerators prioritize their services to provide their maximum efficacy for start-ups?

Because accelerators play the role of intermediary between start-ups and investors, it seems essential to study the interplay between accelerators, start-ups, and investors in order to investigate the efficacy of different accelerators [7]. For example, the survival and performance of start-ups that are selected for accelerator programs should be compared with the survival and performance of start-ups that apply, but are not selected [7]. Therefore, the second research question of this article is as follows.

> *Question 2.* How do the interactions between accelerators, investors, and start-ups affect all their performances?

The rising profile of accelerators has urged some researchers to investigate their efficacy for entrepreneurial ecosystems. Smith *et al.* [24] found that start-ups who participate in accelerator programs receive some milestones sooner and are more possible to fail or to be acquired. Fehder and Hochberg [25] and Hochberg and Fehder [26] found that the existence of accelerators in a region increases venture capital funds for all start-ups, even for start-ups who have not participated in those accelerator programs. They found that this result is irrespective of the rank of the accelerators in the annual seed accelerator rankings. Yu [27] found that accelerators reduce uncertainty about the quality of start-ups and help them not waste additional resources for ideas that will finally fail. Assenova [28] addressed the effects of institutional reforms by governments on the performance of accelerators. Regulatory and legal reforms facilitate the formation, growth, and exit of new start-ups in a region. The researcher found that such reforms increase the number, diversity, and quality of start-ups who apply for accelerator programs. The researcher also found that institutional reforms improve accelerators' capability to select high-quality start-ups, and increase start-ups' perceived benefits of participating in accelerator programs. Gonzalez-Uribe and Leatherbee [29] found no empirical evidence indicating that providing seed investment and workplace services affect the performance of selected start-ups in accelerator programs.

To some extent, the efficiency of accelerators has been statistically investigated by researchers. However, the effect of different dimensions of heterogeneity on their efficacy should be explored in future research [7], [30]. More recently, Cohen *et al.* [18] found that accelerators vary significantly in their *firm-level* processes and design features. They also argued that such variations are substantially associated with future performance of selected start-ups. For example, they found that external mentorship services provided by accelerators do not necessarily enhance subsequent outcomes of selected start-ups. Future research is needed to investigate how *macro-level* factors affect the efficacy of accelerators [30]. Therefore, the third research question addressed in this article is as follows.

Question 3. How does macro-level heterogeneity of accelerators affect their efficacy and performance?

With regard to the above-mentioned research questions, our article contributes to the literature on accelerators by increasing the current knowledge about accelerators at two levels. At the micro (firm) level, we focus on the mutual interaction between accelerators, start-ups, and investors. At the macro (regional) level, we focus on how accelerators contribute to entrepreneurial ecosystems and what factors affect their efficacy. It is worthy to note that these two levels are closely related, and a focus on the macro level is not possible without a proper investigation of the micro level; macrostructures are built on microprocesses [31]. In this case, we use a game-theoretic approach to study these mutual interactions at the micro level. Generally, game theory is an appropriate tool to study strategic interaction among rational decision makers [32].

In addition to its theoretical contributions, our article is also practically relevant. For example, understanding the mutual interactions between accelerators, start-ups, and investors will help us propose some strategies that enhance the outcomes of such interactions and ultimately benefit all these agents. Also, exploring the factors that affect the efficacy of accelerators at the macro level will help governments propose policies that have positive and effective impacts on entrepreneurial ecosystems.

The rest of the article is organized as follows. In Section II, we review the related literature. In Section III, the problem is defined, and the notations and feasibility conditions are provided. In Section IV, the model and the agents' decision making with respect to the related game theoretic framework are formulated. In Section V, we provide a parametric analysis and draw some managerial insights. In Section VI, we discuss our results and the contributions of this article. We also provide some real-world observations that support our results and present some practical suggestions for the different agents. Finally, Section VII, concludes this article.

II. Literature Review

This article is closely related to two streams of literature: the literature on accelerators and business incubators, and the literature on interactions between start-ups and investors. Therefore, this section reviews these two streams respectively.

A. Theoretical Foundation and the Literature on Accelerators

Accelerators have attracted increasing attention from start-ups in recent years. The first commercialized accelerator was Y Combinator, which was organized in Silicon Valley, the home of many IT entrepreneurs. The unique features of accelerators are as follows [16], [17], [33]: they offer time-limited programs to support start-ups with education, screening, networking, and intensive mentorship in exchange for an equity share. Accelerators are cohort-based programs, meaning that a group of start-ups is selected, grows, and graduates together in each accelerator program. Programs start with interviewing and selection of start-ups and end with a *demo day* in which start-ups present their ideas to outside investors. Accelerators may also provide seed funding and workplace facilities for their selected start-ups.

Mentorship is among the most important services offered by accelerators. Generally, mentors helps organizations achieve carrier and managerial success and become successful leaders [34]. Also, individuals that use mentors learn more from their organizations [35]. Specifically, a unique feature of mentorship in accelerator programs is transferring tacit knowledge, which refers to the accumulated experience of their mentors [36]. Accelerator programs enable start-ups to learn from such tacit knowledge, without a need to experience it directly [36].

Although theoretical research has suggested that the services offered by accelerators are of great importance for start-ups, some empirical evidence has pointed out that such services do not necessarily improve subsequent outcomes of selected start-ups. Gonzalez-Uribe and Leatherbee [29] found that providing financial funds and workplace services do not strongly affect the future performance of selected start-ups in the "Start-Up Chile" program. In addition, Cohen *et al.* [18] found that external mentorship services do not necessarily improve subsequent outcomes of selected start-ups. Given their generally positive impacts [24]–[26], it is of great importance to investigate how accelerators could provide their maximum benefits for entrepreneurial ecosystems.

Generally, firms provide several types of services for their customers. According to the theory of "service prioritization" [37], [38], every firm should first uncover how much each type of its services contribute to its customer satisfaction. Second, within the challenges of limited resources, organizations should focus on services that are vital for their role in satisfying their customers. Therefore, service prioritization is the key to the optimal usage of limited resources. The term "value proposition," coined by Osterwalder [39], also supports the idea that firms should focus on services that provide the most benefits for their customers. Specifically, service prioritization is of special importance for accelerators because they provide a variety of services in an intensive and time-limited manner.

What may happen if a firm does not appropriately determine the priority of its services? The firm may burden adverse effects of an increase in some types of its services, especially the ones with lower priorities. Suppose that a firm provides two types of services A and B. Service A has more priority than service B, but the firm dedicates most of its resources to service B. Because of limited resources, as providing service B increases, paying attention to the most vital service, service A, decreases, which may even worsen the performance of the firm and/or the utility of the customers. Similarly, a decrease in the provision of service B may lead to aggregately positive outcomes. The degree to which the firm burdens the adverse effects of service B depends on the degree of scarcity of its resources and how much the resources for providing service A are common with the ones for service B.

According to the above-mentioned arguments, it is essential for accelerators to determine the bundle of their services, i.e., to determine how much they should provide for each type of service. Also, they must decide on the price of those services,

i.e., their request for the equity share of start-ups. Finally, within the context of limited resources, they must decide on how many start-ups they select, i.e., their portfolio size [23]. The analytical model in this article tries to address these issues.

Given the importance of portfolio size in the performance of accelerators, research should be conducted to unravel their selection processes [7], [30]. Although many studies have investigated different dimensions of selection processes applied by business incubators [10], [40]–[43], research on accelerators' selection processes is scant. Yin and Luo [9] focused on the decision criteria applied by accelerators when selecting start-ups. The researchers found that the selection processes of accelerators may involve several stages. In addition, the selection criteria fall into the following three distinct categories:

1) "is it real?"
2) "is it worth doing?"
3) "can it win?"

At the initial selection stage, accelerators use eight "real" or "win" criteria. They involve: demand validation, customer affordability, market demographics, concept maturity, sales and distribution, product maturity, value proposition, and technology expertise. At the final selection stage, accelerators use four "win" or "worth" criteria. They involve: sustainable advantage, prior start-up experience, feedback mechanism, and growth strategy. In brief, Yin and Luo [9] addressed how accelerators evaluate the ex-ante quality of start-ups (the probability of success) before their selection. However, an important question is remained unanswered by the researchers: How does an accelerator determine its optimal portfolio size?

Kim and Wagman [23] argued that the main determinant of the optimal portfolio size of an accelerator is its limited resources. The researchers focused on the screening role of accelerators, in which high-quality start-ups are distinguished from low-quality ones during accelerator programs. Given the limited human resources of an accelerator to screen start-ups, a larger portfolio produces less precise signals about the quality of the selected start-ups, leading to a decrease in the start-ups' willingness to participate in the program. Therefore, the researchers addressed the optimal portfolio size of the accelerator, which maximizes its profit. However, the researchers have not investigated the effect of macro-level factors on the optimal portfolio sizes of accelerators. Also, they assumed that all start-ups are homogenous in their ex-ante quality (ex-ante probability of success). Therefore, an accelerator does not require a selection process and can select start-ups randomly. This assumption is in contrast with real-world observations in which accelerators use rigorous selection processes [9], [16]. Finally, the researchers have not investigated the variety of services offered by accelerators, and investigated only the screening service to provide signals about ex-post quality of the selected start-ups.

In this article, our perspective with respect to the selection processes of accelerators benefits from some innovations compared to the approaches taken by Yin and Luo [9] and Kim and Wagman [23]. In contrast to the two studies and given the potential significance of service prioritization in the efficacy of accelerators, we investigate the three types of services by accelerators: seed investment in the form of financial funds and workplace facilities, mentorship services, and screening capabilities. Consistent with Yin and Luo [9] and in contrast to Kim and Wagman [23], we consider selection processes for accelerators, which allow us better understanding the efficacy of accelerators. For example, Amezcua *et al.* [42] and Barrow [43] found that high performance of start-ups in some business incubators are mainly due to their own high qualities, and not due to their growth in the programs. Testing this hypothesis for accelerators will not be possible without addressing ex-ante heterogeneity in individual quality between start-ups and considering selection processes for accelerators. We will test this hypothesis for accelerators in Section VI-C.

B. Literature on Interactions Between Start-Ups and Investors

The work of Fairchild [44] described how a start-up can choose between a VC and an angel investor to finance its idea. The VC's competitive advantage is its ability to provide some value-added opportunities that increase the expected value of the project. In contrast, the angel investor's competitive advantage is its more trusting relationship with the entrepreneur, which decreases the incentives on both sides to expropriate the project. The researcher found that the relationship between the start-up and a VC always induces both elements of the dyad to seek to expropriate the project. The researcher found that when the VC's ability to provide value-added opportunities is great enough, the VC wins the bidding game and supplies the funds.

The work of De Bettignies and Brander [45] discussed a start-up's choice between a commercial bank and a VC. A bank finances the start-up's project in the form of debt financing. Hence, the start-up retains full project ownership and has full incentive to exert maximum effort. A VC, in contrast, finances the start-up's project in exchange for an equity share. The VC offers some value-added opportunities for the start-up, but it requests some amount of equity, which reduces the start-up's incentive to invest effort. The researchers found that when the VC's ability to provide value-added opportunities is great enough, the start-up will prefer to seek financing from the VC.

To sum up, VCs seem to be generally more attractive than other investors for start-ups. Real-world observations indicate that VCs benefit from extensive business knowledge and networks [46], which signals that the VCs' productivity ability to provide value-added opportunities is great. According to Fairchild [44] and De Bettignies and Brander [45], this will lead to the superiority of VCs to angel investors and commercial banks.

VCs are also superior to commercial banks and angel investors from a practical point of view. In the real world, commercial banks are often hesitant to provide loans to start-ups unless they provide personal, valuable collateral [47]. Because many start-ups have limited-liability formats [48]–[50] and/or they are in very early stages of growth, they cannot usually provide this collateral. Also, debt financing imposes more costs than what is expected [51]. Therefore, many start-ups prefer equity financing

over debt financing. In addition, VCs have some practical advantages over angel investors in equity financing for start-ups [46]. First, angel investors are former entrepreneurs and invest only in industries they know a great deal about. Therefore, they expect to have a certain amount of involvement in the decisions of the start-up founders, which some founders may not be comfortable with. Second, VCs can provide more funds than angel investors, and they can better help start-ups recruit the professionals they need, mainly because of their extensive business knowledge and networks.

Vergara *et al.* [52] considered three inputs in the development of an idea: capital, the efforts of the start-up and the efforts of the VC. The researchers assumed that both the start-up and the VC can provide some of the required capital, and they identified a complementary effect between the efforts of the two parties. They assumed that the start-up determines the VC's equity share and the VC determines the amount of capital it provides. They found that the start-up will have no incentive to reduce its equity share to stimulate the VC to exert more effort. Also, they show that full complementarity between the efforts of the two parties leads to an equal distribution of income between the start-up and the VC.

The work of Kim and Wagman [23] divided ideas of start-ups into two types. Good ideas yield a higher expected value of income than bad ideas. Also, good (bad) ideas are (are not) worth financing. The researchers focused on the screening role of accelerators. An accelerator distinguishes good ideas from bad ideas during its program and publicly announces the quality of the start-ups. It is assumed that the accelerator correctly identifies good ideas, but there can be error in discerning bad ideas. The amount of this error is positively related to the accelerator's portfolio size, due to its limited human resources to screen start-ups. VCs can also screen start-ups, but with substantial error in discerning bad ideas. Also, the researchers assumed that the accelerator cannot evaluate the ex-ante quality of start-ups by interviewing them. Therefore, the accelerator must choose among the start-ups randomly. This assumption is in contrast with real-world observations in which accelerators use a rigorous selection process [16].

Kim and Wagman [23] studied a perfectly competitive market for VCs. Therefore, investments in start-ups are in the form of debt financing because the competition between VCs makes their net payoff zero. Their underlying assumption is that the start-ups have the responsibility to return the total amount of investment to the investor. In other words, the start-ups are general partnership companies. Therefore, a start-up will not seek financing if it knows that it has a bad idea. Hence, the start-ups desire an accelerator program with a small portfolio size, because larger portfolio sizes increase the error in identifying bad ideas. Briefly, two factors determine the start-ups' participation in the accelerator program: its request for equity share and its portfolio size.

III. Problem Definition

This section provides the problem definition in two sections. Section III-A describes the problem, and Section III-B addresses the feasibility conditions of the problem. The definitions of the notations are provided in the Supplementary material.

A. Problem Description

We consider an entrepreneurial community composed of N penniless start-ups, one accelerator, and one VC. According to a common assumption in the literature, all the players are risk neutral, and they have no outside options offering a minimum attractive rate of return (MARR). Each start-up is engaged in bringing an idea to the market. Much like in the models used in other studies [23], [53], the entrepreneurial community is in an industry whose ideas require F monetary units to be realized. If an idea is successful and realized, R monetary units will be created. Otherwise, no income will be generated. The success of an idea depends on many factors, such as the importance of the invention, the radicalness of the idea, and the extent of the intellectual property protection [54]. Moreover, organizational legitimacy [55], organizational learning [56], network structure [57], and dynamic capabilities [58] of a company can affect its innovation and success.

In the real world, some accelerators focus on a specific industry. For example, the medical start-ups website[1] lists top accelerators in the field of medicine. As another example, the Impact Connected Car program[2] accepts start-ups in the car industry. If the industry is fixed, we can use an estimation of the parameters R and F. For example, Vajre [59], a start-up founder, points out that start-ups in the field of software-as-a-service gains \$1 million ($R = \$1$ million) if they can acquire 100 customers. Also, Caramela [60] points out that most microbusinesses cost around $\$3\,000 (F = \$3000)$ to launch.

Before initiating a program, an accelerator must make some decisions. It must announce the portfolio size (the number of start-ups it will accept), the amount of seed investment, the duration of the program, the amount of mentorship to be provided, and the equity share it will request. For example, Y Combinator announces on its website[3] that the portfolio size will be 105, it will invest \$120 000 in exchange for a 7% equity share, and the participants will benefit from the experience of 3000 domain experts.

Determining whether an idea is successful takes time and careful examination. As shown in the studies [53], [61], the entrepreneurs in a start-up cannot perform this examination or even estimate their probability of success. They only know that their probability of success (x) is a random variable with a uniform density function in the interval of $[0, 1]$. The situation in which the success probability is a random variable is called ambiguity [61]. However, the accelerator and the VC can assess the start-ups' success probabilities after interviews. Thus, this article identifies some selection criteria of the VC and the accelerator for the acceptance of a start-up.

It is obvious that this assessment ability will be activated only after the interviews. Therefore, if some decisions are to be made

[1]Online. Available: http://www.medicalstartups.org/top/accelerator/

[2]Online. Available: http://www.impact-accelerator.com/category/impact-connected-car/

[3]Online. Available: http://www.ycombinator.com/

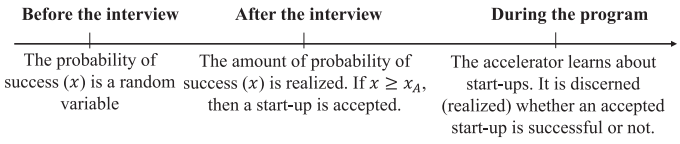

Fig. 1. Timing of the accelerator's events.

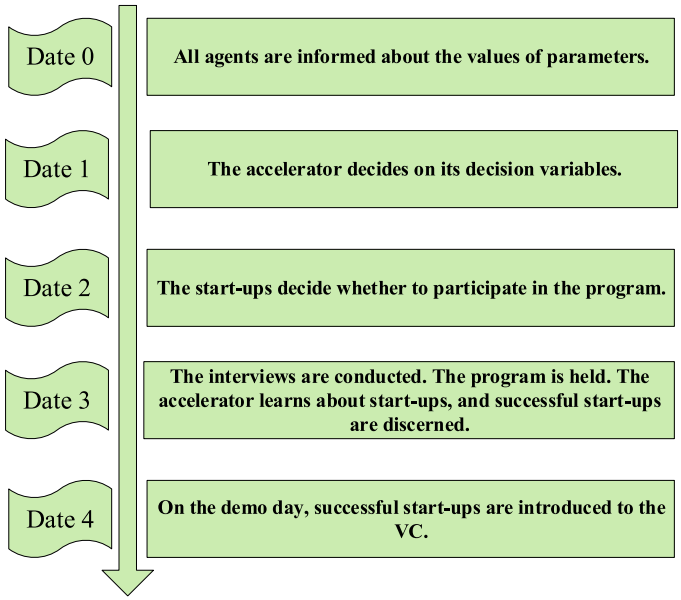

Fig. 2. Timeline of the problem.

before the interviews, these parties, much like the start-ups, will treat the probability of success as a random variable. After the interviews, the accelerator considers an acceptance threshold, denoted by x_A, for the success probability of the selected start-ups. The accelerator also learns more about the selected start-ups throughout the program and ultimately determines their types (successful or failed). This role is acknowledged as its screening ability distinguishes accelerators from other financial institutions [23]. The timing of the accelerator's events is depicted in Fig. 1.

Start-ups cannot afford to pay the required capital to realize their ideas because they are at the beginning stages of their growth. In this community, a VC has the responsibility to provide all this capital for its selected start-ups. The VC attempts to invest in start-ups whose success probabilities exceed a certain quality threshold, denoted by x_V. In exchange for financing a start-up, the VC will own $s_V \times 100\%$ of the final income of that start-up. Therefore, if the probability of success for a start-up is x, the expected income of the VC will be $s_V x$, and its expected profit will be $Rs_V x - F$. The VC aims to determine the value of x_V to maximize its total profit.

We first study a situation in which s_V is a parameter. The value of s_V implies the VC's bargaining power relative to the start-ups. This value can be calculated by referring to previous rounds of funding in similar industries. Also, some researchers indicate that a VC usually obtains a 50% equity share [52], [62]. Another scenario, in which the VC attempts to determine its equity share at the beginning of the game, will be studied as well.

The accelerator accepts the start-ups, provides intensive mentorship for them, and also assess their type of idea (successful or failed). Then, it introduces the successful start-ups to the VC on the demo day. The provided amount of mentorship is denoted by k_A. The unit of k_A can be the hours of mentorship that the accelerator provides for each start-up during its program. Also, the accelerator provides d_A monetary units for each selected start-up to help cover living and workplace expenses.

Intensive mentorship increases the expected value of income of start-ups through two paths. It increases both the probability of success and the income of successful start-ups. Because the players are risk neutral, they have no preference between these two paths and they consider only the net effect of mentorship on the expected value of income of the start-ups. This indifference is implicitly acknowledged in the literature on value-added services. Some papers assume that value-added services increase the income of successful start-ups [45], [52], while other studies assume that these services increase the start-ups' probability of success [44].

The accelerator pays ck_A^2 monetary units to provide k_A units of mentorship for one start-up. This quadratic function means that as the provided mentorship increases, finding more mentors with the necessary quality and productivity becomes extremely difficult, which indicates the scarcity of resources. In this article, a productivity parameter (μ) for mentorship services is considered. One unit of mentorship will lead to a $\mu \times 100\%$ increase in the expected value of income of the start-ups. Therefore, the expected value of income for an eligible start-up with the probability of success x is equal to $Rx(1 + \mu k_A)$ if it participates in the program. A similar type of cost and income structure is considered in the works of De Bettignies and Brander [45] and Elitzur and Gavious [63] for value-added services.

In exchange for participation in the program, the accelerator requests an equity share of s_A. Therefore, if the income of an idea is R', the accelerator's income will be $s_A(1 - s_V)R'$ and the start-up's income will be $(1 - s_A)(1 - s_V)R'$. As in the studies [23], [53], [64], investors who participate in the later stages of financing have higher priority when income is distributed. The accelerator aims to determine the values of k_A, s_A, x_A, and d_A to maximize its total profit.

Start-ups can choose among two alternatives: They can participate in the accelerator program, and be introduced to the VC, or they can approach the VC directly. Therefore, they will participate in the accelerator's program if it provides more expected profits for them compared to contacting the VC directly.

Finally, the model presented in this article is robust under any assumption about the nature of start-ups with respect to liability. In the Kim and Wagman [23] model, an accelerator program provides no benefit to limited-liability start-ups. According to the literature, the limited-liability format is common among start-ups [48]– [50]. Therefore, this robustness can be regarded as an important modeling innovation. The timeline of the problem is depicted in Fig. 2.

Interpreting the variables and parameters in a problem is generally of great importance. In this study, the random variable

x denotes the quality of a start-up on the individual (micro) scale. In contrast, the parameter R indicates the quality of a start-up on the entrepreneurial community (macro) scale. High values of x and R denote high quality. The indicator $\frac{\mu}{c}$ can also appropriately represent the quality of an accelerator. Higher values of $\frac{\mu}{c}$ mean that the accelerator can employ more experienced mentors at relatively low costs. Finally, according to the literature [18] and from the point of view of start-ups, the efficacy of an accelerator is equivalent to the degree to which participation of start-ups in the program increases their ability to raise funding from investors. When an accelerator program exists, the VC's portfolio size (n_V^A) represents how many graduates have raised funding from the VC. Therefore, n_V^A represents the performance of the accelerator, and the difference between n_V^A and n_V^{NA} (the VC's portfolio size when there is no accelerator program), the indicator $E = n_V^A - n_V^{NA}$ can properly indicate the efficacy of the accelerator. A positive value of E indicates that the accelerator has a positive impact on the ability of start-ups in raising funds from the investors.

B. Feasibility Conditions

The feasibility conditions of the problem are as follows.

Condition 1: The acceptance of successful start-ups satisfies the VC's MARR.

Condition 2: Start-ups that are rejected by the accelerator will have no chance to be accepted by the VC (i.e., $x_A \leq x_V$).

Condition 3: The value of x_V falls into the interval of $(0, 1)$.

Condition 4: The value of x_A falls into the interval of $(0, 1)$. The value of k_A falls into the interval of $(0, +\infty)$.

Condition 5: The value of s_A falls into the interval of $(0, 1)$.

IV. Model Formulation

In this section, the agents' objective functions are formulated. Also, their decisions are analyzed by backward induction, in which decisions at upper levels are made according to rational expectations about the agents' decisions at lower levels.

A. VC's Problem When There Is no Accelerator Program or the Start-Ups Decide Not to Participate in the Program

As stated in Section III, the VC has established an acceptance threshold, denoted by x_V, for the success probability of the selected start-ups. Therefore, its profit function will be equal to

$$\Pi_V^{NA}(x_V) = N\int_{x_V}^{1}(Rs_V x - F)\,dx = \frac{1}{2}N(1-x_V)(Rs_V(1+x_V) - 2F). \quad (1)$$

To calculate the optimal value of x_V, the first- and second-order conditions of optimality are considered

$$\frac{\partial \Pi_V^{NA}}{\partial x_V} = N(F - Rs_V x_V) = 0,\ \frac{\partial^2 \Pi_V^{NA}}{\partial {x_V}^2} = -NRs_V < 0. \quad (2)$$

Therefore, the equilibrium value of x_V will be equal to

$$x_V^* = \frac{F}{Rs_V}. \quad (3)$$

And x_V^* will maximize the VC's profit according to (2). In addition, according to Condition 3, the following relation holds:

$$x_V^* = \frac{F}{Rs_V} < 1 \Rightarrow Rs_V > F. \quad (4)$$

Relation (4) confirms Condition 1, so it is concluded that Condition 1 and Condition 3 are equivalent. The VC's expected profit by selecting a start-up with success probability of x_V^* will be equal to

$$Rs_V x_V^* - F = 0. \quad (5)$$

Equation (5) means that the VC sets x_V as low as it satisfies its MARR. The VC's equilibrium profit and portfolio size are equal to

$$\left(\Pi_V^{NA}\right)^* = \frac{N(F - Rs_V)^2}{2Rs_V},\ \left(n_V^{NA}\right)^* = N\int_{x_V^*}^{1} dx = N(1 - x_V^*) = N\left(1 - \frac{F}{Rs_V}\right). \quad (6)$$

B. VC's Problem When the Start-Ups Decide to Participate in the Accelerator Program

As stated in Section III, the income of successful start-ups (R') will be greater than or equal to R. According to relation (4), the acceptance of successful accelerator graduates satisfies the VC's MARR. Therefore, it is concluded that

$$Rs_V > F,\ R' \geq R \Rightarrow R's_V \geq Rs_V > F. \quad (7)$$

Hence, the VC selects all successful accelerator graduates.

C. Start-Ups' Problem

According to the problem definition in Section III, the accelerator selects a start-up, if its success probability (x) exceeds x_A. Therefore, the ex-ante expected value of income of a start-up is equal to

$$V(x_A, k_A) = \int_{x_A}^{1} R(1 + \mu k_A)\,x\,dx = \frac{1}{2}R(1 + \mu k_A)\left(1 - x_A^2\right) > 0. \quad (8)$$

Suppose that a start-up decides to approach the VC directly. According to relation (4), it will be selected if its success probability exceeds $\frac{F}{Rs_V}$. Therefore, the ex-ante expected value of its profit is equal to

$$I_V = \int_{\frac{F}{Rs_V}}^{1} R(1 - s_V)\,x\,dx = \frac{(1 - s_V)\left(R^2 s_V^2 - F^2\right)}{2Rs_V^2}. \quad (9)$$

In the case that the start-up decides to participate in the accelerator program, its ex-ante expected value of profit is

equal to

$$I_A = (1-s_V)(1-s_A)V(x_A,k_A) + d_A\int_{x_A}^{1} dx$$
$$= (1-s_V)(1-s_A)V(x_A,k_A) + d_A(1-x_A). \quad (10)$$

Condition 2 is fundamental to build the value of E_A. We will further examine the validity of this condition in Section IV-D. A start-up participates in a program if this participation creates a greater expected value of profit. Therefore, the following inequality is the participation constraint of the start-ups:

$$I_A \geq I_V \Rightarrow s_A \leq \widehat{s_A}(x_A,k_A,\ d_A)$$
$$= 1 - \frac{I_V - d_A(1-x_A)}{(1-s_V)V(x_A,k_A)}. \quad (11)$$

The function $\widehat{s_A}(x_A,k_A,\ d_A)$ represents the maximum amount of equity share that the start-ups are willing to cede to the accelerator in exchange for participation in its program. In other words, $\widehat{s_A}(x_A,k_A,\ d_A)$ measures the degree of the start-up's willingness to participate in the program. $\widehat{s_A}(x_A,k_A,\ d_A)$ can be rewritten as follows:

$$\widehat{s_A}(x_A,k_A,\ d_A)$$
$$= 1 - \frac{\left(R^2 s_V^2 - F^2\right)(1-s_V) - 2d_A R s_V^2 (1-x_A)}{R^2(1+\mu k_A)(1-s_V)s_V^2(1-x_A^2)}. \quad (12)$$

The first derivatives of $\widehat{s_A}(x_A,k_A,\ d_A)$ with respect to d_A, k_A, x_A, and R are equal to

$$\forall x_A,k_A,\ d_A:\ \frac{\partial \widehat{s_A}}{\partial d_A} = \frac{2}{R(1+\mu k_A)(1-s_V)(1+x_A)} > 0 \quad (13)$$

$$\forall x_A,k_A,\ d_A:\ \frac{\partial \widehat{s_A}}{\partial k_A}$$
$$= \frac{\mu\left(s_V^2 - \left(\frac{F}{R}\right)^2\right)(1-s_V) - \frac{2\mu d_A s_V^2(1-x_A)}{R}}{(1+\mu k_A)^2(1-s_V)s_V^2(1-x_A^2)} \quad (14)$$

$$\forall x_A,k_A,\ d_A:\ \frac{\partial \widehat{s_A}}{\partial x_A}$$
$$= -\frac{2x_A(1-s_V) + 2d_A R s_V^2(1-2x_A(1-x_A))}{R^2(1+\mu k_A)(1-s_V)s_V^2(1-x_A^2)^2} < 0 \quad (15)$$

$$\forall x_A,k_A,\ d_A:\ \frac{\partial \widehat{s_A}}{\partial R}$$
$$= -\frac{2\left(F^2(1-s_V) + R d_A s_V^2(1-x_A)\right)}{R^3(1+\mu k_A)(1-s_V)s_V^2(1-x_A^2)} < 0. \quad (16)$$

According to (14), if $\frac{d_A}{R} \approx 0$ is satisfied, then the derivative $\frac{\partial \widehat{s_A}}{\partial k_A}$ is positive

$$\forall x_A,k_A,\ d_A:\ \frac{\partial \widehat{s_A}}{\partial k_A} \approx \frac{\mu\left(s_V^2 - \left(\frac{F}{R}\right)^2\right)}{(1+\mu k_A)^2 s_V^2(1-x_A^2)} > 0. \quad (17)$$

Now, we aim to investigate the mutual interaction of start-ups with the VC and the accelerator. In other words, we investigate how different start-ups respond to the decisions of different accelerators. Other factors remaining constant, we investigate how heterogeneity in the accelerator's seed investment and mentorship services affects start-ups' willingness to participate (Managerial insight 1), and how heterogeneity in the characteristics of start-ups affects such willingness to participate (Managerial insight 2). It may seem that these insights study the responses of start-ups to the decisions of accelerators. However, the role of the VC is fundamental in these insights because the start-ups' willingness to participate indicates the relative preference for the accelerator over the VC.

Managerial Insight 1: Other factors remaining constant, the start-ups' willingness to participate enhances if it provides more seed investments. When the seed investment is much less than the income of successful start-ups, their willingness to participate increases if the accelerator provides more mentorship services.

According to (15), if the accelerator increases the value of x_A, the start-ups' willingness to participate will deteriorate. An increase in the value of x_A means the selection of start-ups with higher individual quality. In addition, as stated in Section III, high values of R mean that the start-ups have high quality on the community scale. Briefly, Managerial insight 2 is obtained.

Managerial Insight 2: Other factors remaining constant, start-ups with higher quality on both the individual and community scales are less willing to participate in the accelerator program.

D. Accelerator's Problem

The ex-ante accelerator's profit is equal to

$$\Pi_A(s_A,x_A,k_A,\ d_A)$$
$$= N\left(s_A(1-s_V)V(x_A,k_A) - \int_{x_A}^{1}\left(c{k_A}^2 + d_A\right)dx\right). \quad (18)$$

According to Section IV-B, the VC pays F monetary units if a start-up is successful. Therefore, the ex-ante VC's profit is equal to

$$\Pi_V^A(x_A,k_A) = N\left(s_V V(x_A,k_A) - \int_{x_A}^{1} F x dx\right)$$
$$= N\int_{x_A}^{1}\left(R s_V(1+\mu k_A) - F\right)x dx$$
$$= \frac{N\left(R s_V(1+\mu k_A) - F\right)\left(1-{x_A}^2\right)}{2}. \quad (19)$$

Condition 2 is fundamental to build the $\Pi_V^A(x_A,k_A)$ function. We will further examine the validity of this assumption in this section.

The ex-ante values of the portfolio sizes are as follows:

$$n_A = N\int_{x_A}^{1} dx = N(1-x_A),\ n_V^A = N\int_{x_A}^{1} x dx$$
$$= \frac{N\left(1-(x_A)^2\right)}{2}. \quad (20)$$

To calculate the equilibrium value of s_A, Proposition 1 is used. The related proofs for propositions are presented in the Supplementary material.

Proposition 1: It is optimal for the accelerator to set its equity share equal to the start-ups' willingness to participate

$$s_A^* = \widehat{s_A}\left(x_A, k_A,\ d_A\right). \tag{21}$$

After replacing s_A^* in the accelerator's profit function, it will be equal to

$$\begin{aligned}&\Pi_A\left(s_A^*, x_A, k_A, d_A\right)\\&= N\left(\left(1-\frac{I_V - d_A\left(1-x_A\right)}{\left(1-s_V\right)V\left(x_A,k_A\right)}\right)\right.\\&\quad\left.\times\left(1-s_V\right)V\left(x_A,k_A\right) - \left(ck_A^2 + d_A\right)\left(1-x_A\right)\right)\\&= N\left(\left(1-s_V\right)V\left(x_A,k_A\right) - I_V - ck_A^2\left(1-x_A\right)\right).\end{aligned} \tag{22}$$

According to (22), the variable d_A is not included in the accelerator's profit function after replacing s_A^* in it.

Managerial Insight 3: The amount of seed investment is irrelevant to the equilibrium values of the accelerator's profit and efficacy, portfolio size, and mentorship services.

Managerial insight 3 reveals an important fact about the interaction between the accelerator and start-ups. An increase in the seed investment only increases (decreases) the equilibrium value of the equity share requested by the accelerator and has no effect on the equilibrium values of the accelerator's profit, portfolio size, or mentorship services. Therefore, the ability of top accelerators to be successful intermediaries between start-ups and VCs is not due to their seed investment supports. Also, according to (22), Managerial insight 3 is robust under any formulation of the function $V(x_A, k_A)$.

Proposition 2 shows the equilibrium values of x_A and k_A.

Proposition 2: Under Condition 2, the equilibrium values of x_A^* and k_A^* are equal to

$$x_A^* = \frac{2\sqrt{\theta_1} - \theta_2 - 8c}{3\theta_2} \tag{23}$$

$$k_A^* = \frac{\sqrt{\theta_1} + \theta_2 - 4c}{6c\mu}. \tag{24}$$

In addition, these equilibrium values satisfy Condition 4.

After replacing x_A^* and k_A^* in (21), the value of s_A^* will be equal to

$$s_A^* = \widehat{s_A}\left(x_A^*, k_A^*\right) = 1 + \frac{\theta_3}{\theta_4}. \tag{25}$$

Condition 5 is equivalent to the following relation:

$$\begin{aligned}&0 \le s_A^* < 1 \Leftrightarrow 0 \le d_A\\&< \frac{\left(1-s_V\right)\left(R^2 s_V^2 - F^2\right)\left(4c + 2\theta_2 + \sqrt{\theta_1}\right)}{4Rs_V^2\left(4c+\theta_2\right)}.\end{aligned} \tag{26}$$

Relation (26) can be rewritten as follows:

$$\begin{aligned}0 \le \frac{d_A}{R} &< \frac{\left(1-s_V\right)}{4}\left(1-\left(\frac{F}{Rs_V}\right)^2\right)\\&\times\left(1+\frac{\sqrt{\theta_1}}{4c+\theta_2}+\frac{\theta_2}{4c+\theta_2}\right).\end{aligned} \tag{27}$$

We conclude that if the value of $\frac{d_A}{R}$ is close to zero (the seed investment is much less than the income of successful start-ups), then Condition 5 is satisfied.

Now, we aim to examine the validity of Condition 2. We must show that the value of x_A^* presented in (23) is less than $\frac{F}{Rs_V}$ and the accelerator has no incentive to violate Condition 2.

Proposition 3: If $Rs_V < 3F$ holds, then Condition 2 will be satisfied.

The condition presented in Proposition 3 is a mild condition; under this condition, the VC's optimal acceptance rate when there is no accelerator program is less than $\frac{2}{3}$; see (6). In the real world, this acceptance rate is usually less than 10% [65]. Based on (6) and (20) and Proposition 3, Managerial insight 4 is obtained.

Managerial Insight 4: The accelerator's optimal portfolio size is larger than a threshold value that is equal to the equilibrium portfolio size of the VC when there is no accelerator program.

After replacing the values of x_A^*, k_A^* and s_A^*, the accelerator's and the VC's equilibrium profits are equal to

$$\begin{aligned}\Pi_A^* = \frac{1}{54}N\left(\theta_5 + \frac{Rs_V\left(15c - 4R\mu^2 - 2\sqrt{\theta_1}\right)}{c}\right.\\\left.+\frac{\theta_6\sqrt{\theta_1}}{cR\mu^4\left(1-s_V\right)}\right)\end{aligned} \tag{28}$$

$$\left(\Pi_V^A\right)^* = \frac{N\theta_7\theta_8}{27c{\theta_2}^2}. \tag{29}$$

V. Parametric Analysis

In this section, we investigate how different types of heterogeneity affect the equilibrium values of the variables in the problem.

Proposition 4: Analytical sensitivity analysis

1) n_A^* is decreasing in μ and R and increasing in c and s_V;
2) k_A^* is increasing in μ and R and decreasing in c and s_V;
3) $\lim_{(\mu,\ d_A)\to(0,0)} s_A^* = (\frac{F}{Rs_V})^2$. The value of this limit is decreasing in R;
4) $\lim_{\mu\to 0}(\Pi_V^A)^* = \frac{N(Rs_V - F)}{2} > (\Pi_V^{NA})^*$;
5) if $Rs_V < 3F$ holds, then $\Pi_A^* > 0$. In particular $\lim_{\mu\to 0}\Pi_A^* = \frac{27NF^2(1-s_V)}{{Rs_V}^2} > 0$;
6) if $Rs_V < \frac{9}{5}F$ holds,[4] then $E^* = (n_V^A)^* - (n_V^{NA})^* > 0$, in particular, $\lim_{\mu\to 0}E^* = \frac{N}{2}(\frac{2F}{Rs_V} - 1) > \frac{N}{18} > 0$;
7) E^* and $(n_V^A)^*$ are decreasing in μ and R and increasing in c.

[4]According to Section IV-D, it is a mild condition. Under this condition, the VC's optimal acceptance rate in regions without the accelerator program is less than $\frac{4}{9}$. In the real world, this acceptance rate is less than 10%.

Based on Proposition 4, the following managerial insights are provided.

Managerial Insight 5: According to items 3 and 4, the screening role of accelerators is by itself valuable to the start-ups and the VC. Even when the quality of mentorship and seed investment services tend to be zero, the accelerator can force a positive equity share to start-ups and the VC's profit increases compared to the situation in which such an accelerator does not exist. This result indicates that providing screening services has more priority for accelerators than providing mentorship services.

Managerial Insight 6: Despite the priority of screening services for start-ups, lower-quality entrepreneurial communities appreciate the screening services of accelerators more than higher quality communities.[5]

Managerial insight 5 reveals some important facts about the impact of accelerators on start-ups and VCs. It indicates that the screening capabilities of top accelerators make them able to successfully accelerate start-ups and benefit VCs. Moreover, the mentorship support of accelerators is beneficiary but not vital to their success and impact. Nevertheless, Managerial insight 6 reveals that the start-ups' perception of the importance of screening services varies substantially between different types of entrepreneurial communities.

Managerial Insight 7: According to item 2, the accelerator provides more mentorship services for entrepreneurial communities with higher quality.

Managerial Insight 8: According to item 1, accelerators with higher quality (higher values of $\frac{\mu}{c}$ index) have smaller portfolio sizes, but provide more mentorship services.

Managerial Insight 9: According to item 1, entrepreneurial communities with higher qualities include accelerators with smaller portfolio sizes.

Managerial Insight 10: According to item 5, accelerators can earn positive profits even when the productivity of their mentorship services is very low.

Managerial Insight 11: According to item 6, more start-ups receive funding from the VC when there is an accelerator program, compared to the situation in which there is no program. This indicates the positive impact of accelerators on the performance of start-ups. Also, such positive impact holds even if the mentorship productivity of the accelerator is very low.

Managerial Insight 12: According to item 7, it is concluded that, despite the positive impact of accelerators everywhere, their efficacy is lower in entrepreneurial communities with higher qualities. Also, accelerator programs with higher productivity of mentorship services provide lower efficacy for start-ups.

Managerial insights 7–9 show how heterogeneity in accelerators and entrepreneurial communities affects the portfolio sizes and mentorship services offered by accelerators. Also, according to managerial insights 10 and 11, accelerators, even ones with low productivity of mentorship services, will survive in entrepreneurial ecosystems because their screening role is by itself valuable for start-ups and VCs. Managerial insight 12, while generally agrees with managerial insight 5 and the positive impact of accelerators, argues that heterogeneity in accelerators and entrepreneurial communities are significantly associated with the efficacy of accelerators.

[5]See the second section of item 3.

TABLE I
DEFAULT VALUES FOR THE PARAMETERS OF THE PROBLEM

Parameter name	Parameter value
μ	0.3
R	1000
s_V	0.4
F	250
c	4
d_A	0
N	1

A numerical analysis is also provided to investigate the effect of parameters on s_A^*, Π_A^*, $(\Pi_V^{NA})^*$, and $(\Pi_V^A)^*$. The default values for the parameters of the problem are presented in Table I. The decision variable d_A is considered as a parameter because it cannot influence the accelerator's profit function. The domains of the parameters are such that Condition 1 and Condition 2 are always satisfied, i.e., $F < Rs_V < 3F$.

Fig. 3 depicts the effect of parameters on the value of s_A^*, the start-ups' equilibrium willingness to participate.

Based on Fig. 3(a) and (b), Managerial insight 13 is obtained.

Managerial Insight 13: An increase in the quality of the accelerator (an increase in the $\frac{\mu}{c}$ index) enhances the start-ups' equilibrium willingness to participate.

Based on Fig. 3(c), Managerial insight 14 is obtained.

Managerial Insight 14: If the quality of the entrepreneurial community is low (high) enough, then an increase in such quality deteriorates (enhance) the start-ups' equilibrium willingness to participate.

Managerial insight 13 shows the total effect of the productivity of the mentorship services on the start-ups' equilibrium willingness to participate. There are two opposite effects for this productivity. According to managerial insight 8, an increase in the mentorship productivity leads to reduction in the portfolio size. Such a decrease signals that only high-quality start-ups, on the individual scale, are selected for the accelerator program. These start-ups have low willingness to participate (see managerial insight 2). However, managerial insight 8 also predicts that an increase in the mentorship productivity enhances the mentorship services provided by the accelerator. Therefore, such an increase in the mentorship services enhances the start-ups' willingness to participate (see managerial insight 1). According to managerial insight 13, the net effect of these two opposite effects is that an increase in the quality of the accelerator (an increase in the $\frac{\mu}{c}$ index) enhances the start-ups' equilibrium willingness to participate.

Similarly, two opposite effects are observable in an increase in the quality of the entrepreneurial community. On one hand, such an increase reduces the accelerator's portfolio size (see managerial insight 9), which deteriorates the start-ups' willingness to participate (see managerial insight 2). On the other hand, it enhances the accelerator's mentorship services (see managerial insight 7), which enhances the start-ups' willingness to participate (see managerial insight 1).

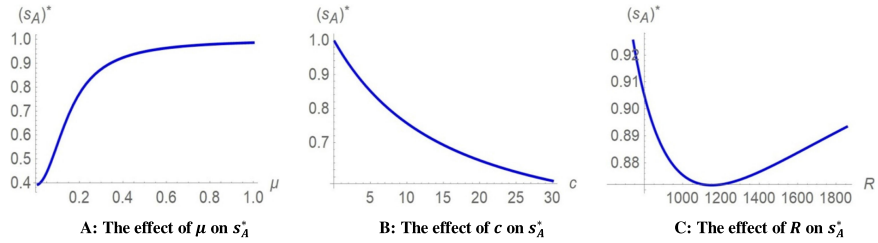

Fig. 3. Effect of parameters on $\boldsymbol{s_A^*}$. (a) Effect of μ on $\boldsymbol{s_A^*}$. (b) Effect of c on $\boldsymbol{s_A^*}$. (c) Effect of R on $\boldsymbol{s_A^*}$.

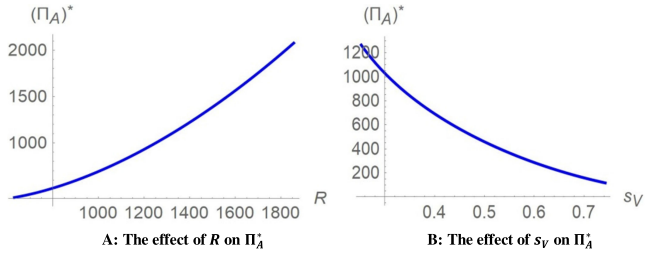

Fig. 4. Effect of parameters on $\boldsymbol{\Pi_A^*}$. (a) Effect of R on $\boldsymbol{\Pi_A^*}$. (b) Effect of s_v on $\boldsymbol{\Pi_A^*}$.

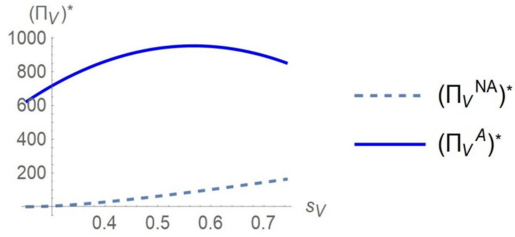

Fig. 5. Effect of $\boldsymbol{s_V}$ on $(\boldsymbol{\Pi_V^{NA}})^*$ and $(\boldsymbol{\Pi_V^{A}})^*$.

Managerial insight 14 suggests that the response of the start-ups' equilibrium willingness to participate toward these two opposite effects is not straightforward. It predicts that high-quality entrepreneurial communities perceive an increase in their quality to be beneficial for their participation in accelerators, while low-quality communities do not. The different perception for the two types of community originates from their different appreciation for the screening services. Low-quality communities appreciate the screening services more than high-quality communities (see managerial insight 6). They prefer the major portion of the accelerator's resources to be devoted to its screening services, not enhancing the mentorship services. Therefore, the increase in their willingness to participate because of enhancing the mentorship services cannot compensate the decrease in willingness to participate because of reduction in the portfolio size of the accelerator.

Fig. 4 depicts the effect of parameters on Π_A^*, the accelerator's equilibrium profit.

Based on Fig. 4(a), managerial insight 15 is obtained.

Managerial Insight 15: Accelerators gain more profits in higher quality entrepreneurial communities.

An increase in the quality of the entrepreneurial community, the parameter R, may have two opposite effects on the accelerator's profit. On one hand, such an increase may deteriorate the equilibrium amount of the equity share that the accelerator requests (see managerial insight 14). On the other hand, more initial qualities and more mentorship services (see managerial insight 7) lead to higher expected value of income for the start-ups. According to managerial insight 15, the net effect of these two opposite effects is that an increase in the quality of the entrepreneurial community enhances the accelerator's profit.

In the following, we aim to propose a strategy for the VC to increase the efficacy of the accelerator services and thus increase the VC's profit. Also, we investigate the effect of this strategy on the accelerator's profit.

Fig. 5 depicts the effect of the parameter s_V, the VC's equity share, on $(\Pi_V^{NA})^*$and $(\Pi_V^{A})^*$, the VC's equilibrium profit.

Suppose that the VC is the leader of our problem. Specifically, it can announce its equity share individually at date 0 in the timeline (see Fig. 2). According to Fig. 5, an equity share of 0.54 maximizes its profit. Therefore, if its previous equity share is above 0.54, it will have an incentive to reduce it to 0.54.

The existence of one extreme point for the value of s_V in this situation indicates a tradeoff phenomenon. According to (19), two decisions by the accelerator increase the VC's profit: enhancing the mentorship services and expanding the portfolio size. The latter decision increases the VC's profit because it accepts only successful start-ups. According to Proposition 4 (items 1 and 2), the parameter s_V has two opposite effects on the VC's profit. A decrease in the value of s_V stimulates the accelerator to enhance its mentorship services but also induces a reduction in its portfolio size. Such a decrease in the parameter s_V also enhances the accelerator's profit according to Fig. 4(b).

Managerial Insight 16: Suppose that the start-ups decide to participate in the accelerator program. If the VC is the leader, it might provide some efficiency wages to the accelerator, by reducing its own equity share. Although this decision would induce the accelerator to reduce its portfolio size, it would also increase its mentorship services, which would ultimately lead to an increase in the VC's profit. This efficiency wage also increases the accelerator's profit. Therefore, it properly aligns the profits of the accelerator and the VC.

VI. Discussion

A. Overview of the Results

We study the micro and macro effects of accelerators. For the micro effects, we study the mutual interactions among the accelerator, the start-ups, and the VC. For the macro effects, we study how accelerators contribute to entrepreneurial ecosystems by playing the intermediary role between start-ups and investors.

In particular, managerial insights 6, 7, 9–12, 14, and 15 concern the macro effects of accelerators, while the other managerial insights concern their micro effects.

At the micro level, managerial insights 1 and 2 investigate how the characteristics of different start-ups and different accelerators affect start-ups' willingness to participate. By studying the accelerator's equilibrium decisions, we find that the seed investment services is irrelevant to the equilibrium values of the accelerator's profit, efficacy, portfolio size, and mentorship services (see managerial insight 3). Also, by integrating managerial insights 3 and 5, we find that the accelerator's most vital contribution is its screening role. Managerial insights 4 and 8 show that although top accelerators are highly competitive, their portfolio sizes should not be too small. Despite the underlying tradeoff between the portfolio size and mentorship services, managerial insight 13 shows that when the productivity of mentorship increases, start-ups' equilibrium willingness to participate also enhances. Finally, managerial insight 16 proposes that the VC's offering some efficiency wages to the accelerator may increase profits for both parties.

At the macro level, higher quality entrepreneurial communities encourage accelerators to provide more mentorship services (see managerial insight 7) but reduce their portfolio sizes (see managerial insight 9). Also, accelerators can survive (see managerial insight 10) and play a successful intermediary role between start-ups and VCs (see managerial insight 11), with their screening capabilities alone. Although the screening services are important for all types of entrepreneurial communities, their significance is higher in lower-quality communities (see managerial insight 6). This result is consistent with managerial insight 9, which states that accelerators' portfolio sizes are larger in lower quality communities. A larger portfolio size encourages accelerators devote a larger portion of their resources to the screening of their selected start-ups to determine whether they will be successful ventures. This leads to an increase in the efficacy of the accelerators, consistent with managerial insight 12, and subsequently an increase in the start-ups' appreciation for the screening services of accelerators (see managerial insight 6) and equilibrium willingness to participate (see managerial insight 14). Despite the possible tradeoff between accelerators' equity shares and mentorship services, managerial insight 15 shows that higher quality entrepreneurial communities provide more profits for accelerators.

B. Response to Question 1

The literature on accelerators suggests that providing a variety of services is common between accelerators [16], [17], [33]. Given their limited resources [23] and the short duration of their programs, we found that service prioritization is of special importance for accelerators. The results indicate that their most vital service is their screening capabilities, which determine whether or not each selected start-up will be a successful venture. The second priority of services belongs to their mentorship services, while seed investment services possess the last priority. This suggests that an accelerator's differentiation and marketing strategies should focus on its screening and, not its mentorship capabilities and seed investment services. For example, according to the business model Canvas proposed by Osterwalder [39], value proposition by accelerators should focus on their screening services.

Our findings can explain the reason for the vital role of screening services for accelerators. Mutual interactions between them, VCs, and start-ups push accelerators toward choosing equilibrium portfolio sizes that must be always larger than a threshold value. A larger portfolio size requires special focus on screening services because a relatively big cohort of start-ups must be assessed to determine whether or not each start-up will be a successful venture. Therefore, it is of great importance for an accelerator to devote a major portion of its limited resources to the screening services.

In addition, the role of limited resources in providing screening and mentorship services are fundamental in accelerators. As can be seen from the findings, the effects of micro and macro factors on the two types of services are completely different; a negative association between a factor and an accelerator's mentorship services guarantees its positive association with the accelerator's portfolio size, which is equivalent to a positive association with its screening services. In other words, because of limited resources, an accelerator cannot increase both of its mentorship services and portfolio size simultaneously.

In Section II-A, the theory of service prioritization revealed adverse effects of low-priority services. Also, this theory mentions that the degree of such adverse effects for a low-priority service depends on how much its resources are common with the resources required by the most vital service. We found that the efficacy of accelerators is negatively associated with the productivity of mentorship, while it is irrelevant to the seed investment services. Therefore, the findings indicate that the severity of such adverse effect for mentorship services is higher than the one for seed investment services. Common resources for screening and mentorship services are human and financial resources, while only financial resources are common between screening and seed investment services. Hence, our findings and the theory of service prioritization are consistent with each other. Moreover, the findings can properly explain empirical evidence by Gonzalez-Uribe and Leatherbee [29] and Cohen *et al.* [18], which indicates that enhancing seed investment and mentorship services do not improve the subsequent fundraising of accelerators' graduates.

The irrelevance of seed investment services to the efficacy of accelerators reveal an important practical suggestion. Start-ups should not worry about their chances of selection and the mentorship services provided by accelerators that offer large seed investments. Their chances of selection and the mentorship services depend on their quality and on the productivity of the mentorship services.

C. Response to Question 2

Amezcua *et al.* [42] and Barrow [43] found that some business incubators select only high quality start-ups and waiting for their success in the market. Therefore, the better performance of start-ups in the incubation programs are mainly due to their own quality, and not their growth in the programs. "By targeting resources to young firms with a record of performance, sponsors

may increase the effectiveness of their limited resources by allowing market selection of viable organizations for further development," said Amezcua *et al.* [42]. However, these two studies do not explain why the start-ups are still willing to participate in these programs, and whether or not these programs are beneficial for VCs.

It is found that the above-mentioned hypothesis is valid for accelerators. The accelerators' efficacy to increase fundraising of their graduates is negatively associated with the quality of the entrepreneurial community. This suggests that, in higher quality entrepreneurial communities, attaining higher valuation of an accelerators' graduate is mainly of their individual-scale and community-scale qualities, not of the accelerators' contributions.

The findings reveal that although these programs select high-quality start-ups, the quality of some of the selected start-ups are not as high as they can approach VCs directly. This reason and the fact that the start-ups are unaware of their true qualities stimulate the willingness of all the start-ups to participate in these programs to benefit from the screening services. Also, the existence of the accelerator programs and such screening services are valuable for the VCs. This is because only successful start-ups, from an initial diverse portfolio of an accelerator program, are introduced to them. In contrast, if a VC was to invest in start-ups without an accelerator program, it would select a smaller portfolio, and there would be no guarantee that each selected start-up to be a successful venture.

Consistent with the literature on accelerators [24]–[26], our results generally support the positive impact of accelerators on start-ups and VCs. Nevertheless, our results contribute the literature on accelerators by capturing an interesting paradox about their efficacy. While enhancing the productivity of mentorship services involve an adverse effect on the efficacy of accelerators, it improves the start-ups' equilibrium willingness to participate. In other words, start-ups are highly willing to participate in an accelerator program that they know it does not provide substantial efficacy.

The reason is as follows: although the number of successful graduates that can raise funding is relatively small, each of them attain significant valuation. This is because of the substantial amounts of mentorship services and the participants' high quality in the individual-scale. Hence, the start-ups know that they have a little ex-ante chance of fundraising by VCs, which indicates low efficacy of the accelerator. However, they know that their ex-post fundraising and valuation conditional on their acceptance in the program are substantially significant. Therefore, significant ex-post fundraising and valuation and the start-ups' unawareness about their true quality lead to their high equilibrium willingness to participate. In other words, acceptance into such an accelerator serves as an excellent "launch pad" for a start-ups, and produces a positive, highly informative signal for VCs about the ex-post quality of that start-up, i.e., its success. This result is consistent with the previously mentioned result that better performance of accelerators' graduates in more-developed regions are mainly of their own qualities.

Some insights about the "launch pad" role of accelerators are also observable in the work of Kim and Wagman [23]. They argued that graduation from an accelerator program with a small portfolio generates highly informative signal for VCs about its success. We showed that producing such informative signals takes place very sooner, at the time the accelerator selects relatively high-quality start-ups. This suggests that the "launch pad" role of accelerators is more powerful than what has been previously thought in the literature.

We found that that top accelerators, which exist in more-developed region, are highly competitive, consistent with the literature on accelerators [9], [16], [33], [66]. However, our findings increase the knowledge about the accelerators' selection processes. Because of the essential role of their screening services, the portfolio sizes and acceptance rates of top accelerators should not be too small. Specifically, the accelerators' portfolio sizes are always larger than the VCs' portfolio sizes in regions that there is no accelerator program. This suggests that it is optimal for a VC not to select start-ups that have just been rejected from the selection process of an accelerator. Moreover, start-ups that are rejected by an accelerator program should not approach VCs directly or participate in accelerator programs with higher mentorship productivity. They should focus on and participate in another accelerator program with a lower productivity of mentorship. They can also improve their ideas individually and then approach a VC.

We propose a practical solution that facilitates the interaction between an accelerator and a VC. VC managers in the real world may think that they should not lose their equity shares in start-ups, that higher values of equity shares are always beneficial to them, and that "(accelerators) may also be in competition with other funding sources" [7]. This attitude should be modified. The VC's offering some efficiency wages to the accelerator may improve both of their profits. The efficiency wages for the accelerator are realized through a decrease in the VC's equity share in start-ups. Also, such efficiency wages indirectly promote collaborative actions between the VC and the accelerator because of the induced coordination between their profits.

D. Response to Question 3

According to the literature on accelerators, firm-level heterogeneity of accelerators is significantly associated with their efficacy [18]. In this section, some insights are provided to show significant association between macro-level factors and the efficacy of accelerators.

More (less) developed regions include higher (lower) quality entrepreneurial communities and accelerator with more (less) productivity of mentorship. It is found that that top accelerators, which exist in more developed region, are highly competitive, consistent with the literature on accelerators [9], [16], [33], [66].

Despite the positive impacts of accelerators anywhere, the results indicate that the efficacy of accelerators in less developed regions are higher than such efficacy in more developed regions. The reason originates from the special role of the screening services in less developed regions. The screening services of accelerators has the most priority in their successful intermediary role between start-ups and VCs. We found that focus of accelerators on providing screening services are higher than in less developed countries. Their higher efficacy in less developed regions can be explained with such special focus.

Governments regard accelerators as attractive targets for their interventions and supports to enhance entrepreneurial ecosystems [18]. In doing so, they usually initiate some institutional reforms by changing regulatory and legal infrastructures [28]. Our results contribute to the literature on accelerators by increasing our knowledge about the effect of all types of institutional reforms on accelerators.

The empirical analysis conducted by Assenova [28] addressed the effects of institutional reforms on accelerators. Some regulatory and legal reforms by governments facilitate the formation, growth, and exit of new start-ups in a region. Therefore, such reforms target the quality of entrepreneurial communities. The researcher found that such reforms enhance accelerators' capability to select high-quality start-ups, which is consistent with our results.

Assenova [28] found that reforms that target entrepreneurial communities always enhance the start-ups' perceived benefits of participating in accelerator programs. In contrast, we showed that the effect of these reforms on the start-ups' equilibrium willingness to participate is not straightforward. Specifically, these reforms may deteriorate the start-ups' equilibrium willingness to participate in less-developed regions. Moreover, we propose a practical suggestion to eliminate this effect. If reforms target mentoring communities, instead of entrepreneurial communities, then they are always beneficial to the start-ups' equilibrium willingness to participate. Finally, Assenova [28] does not investigate the effect of institutional reforms on the efficacy of accelerators. We found that reforms targeting entrepreneurial or mentoring communities are unexpectedly detrimental to the efficacy of accelerators, while they may enhance the start-ups' equilibrium willingness to participate. This is mainly because they encourage accelerators to reduce their attention to their most vital services, their screening services, and pay more attention to their lower priority services, their mentorship services. Instead, given the priority of screening services, the results imply that only institutional reforms that target screening services will enhance the efficacy of accelerators.

E. More Real-World Observations

The studies conducted by Pauwels *et al.* [16], Miller and Bound [66], Yin and Luo [9], and Barrehag *et al.* [33] investigated the decisions of accelerators in the real world. They found that although applications to accelerators are open to all, acceptance to the best-known programs is highly competitive. These programs are usually located in developed regions, such as the United States or Europe, which have transitioned to entrepreneurial economies. According to managerial insights 7 and 8, this high level of competition is the result of the fact that accelerators and entrepreneurial communities in these regions have high qualities.

The studies conducted by Pauwels *et al.* [16], Miller and Bound [66], and Barrehag *et al.* [33] also found that seed investments for accepted start-ups often range from £10 000 to £50 000 per company. Some accelerators provide co-located workplaces to encourage mutual learning and collaboration between start-ups. However, other accelerators suggest that the start-ups find suitable workplaces for themselves. Managerial insight 3 can explain these variations by noting that accelerators have no particular incentive pushing them toward any given level or type of seed investment services.

There is a well-known hypothesis of investment practitioners and researchers that states that accelerator programs are not only suitable for start-ups with successful ideas but are also beneficial for those with ideas that will fail. Participating in an accelerator program does not guarantee the success of a start-up. Instead, it speeds up its cycle [67], which means that the start-up will grow more quickly or will fail more quickly [16], [17], [68]. "With acceleration, companies do not waste time and money with a business plan that does not work. In failing sooner, they learn a lot about how to be a successful company" said Frank Vallese,[6] managing director of the NSIN Launch Startup Acceleration Program at New York University.

Managerial insight 5 supports the hypothesis mentioned above. An accelerator with zero mentorship productivity and zero seed investment cannot provide any mentorship or funding services. However, start-ups are still willing to participate in its programs. Also, the VC benefits from the screening role of the accelerator; because establishing an accelerator with zero mentorship productivity will increase the profit of the VC.

Some empirical studies note that regions with accelerators attract more VC funds compared to regions without accelerators [26], [69], [70]. Managerial insight 11 is consistent with this observation and notes that it is mainly due to the accelerators' screening capabilities, not their mentorship services.

VII. Conclusion

This article highlighted the screening and mentorship role of start-up accelerators. An accelerator can identify successful and failed ideas during its program. Also, intensive mentorship increases the expected value of income of start-ups. At the end of the accelerator program, successful start-ups were introduced to a VC. The VC has the responsibility to provide the required capital to realize a successful idea in exchange for some share of equity.

In the game theory model presented in this article, the accelerator was the leader and the VC and the start-ups were the followers. The accelerator chooses its portfolio size, equity share, mentorship services, and seed investment. Start-ups can participate in the accelerator program and approach the VC on the demo day or they can decide to approach the VC directly. This decision imposed a participation constraint on the accelerator's optimization problem. We found that the amount of seed investment offered was irrelevant to the accelerator's profit, portfolio size, and mentorship services. Also, another scenario was studied in which the VC was the leader of the accelerator, so it can set its own equity share individually, instead of by negotiation with start-ups on the demo day. In this scenario, the VC might provide some efficiency wages to the accelerator by reducing its own equity share to stimulate the accelerator to increase its mentorship services.

The following results contribute to the literature on accelerators.

[6]https://www.linkedin.com/pulse/accelerators-get-focused-better-assist-startups-frank-vallese

1) The most vital role of an accelerator for being a successful intermediary between start-ups and VCs is its screening services. This role pushes them toward selecting a portfolio of start-ups whose size is not too small.
2) Given the limited resources of accelerators, prioritizing their services is the key to their efficacy. Screening, mentorship, and seed investment services possess the first, second, and third priorities of start-ups, respectively. In accelerators, a large portion of resources for providing mentorship is common with resources for providing screening services. Such resources include human and financial resources. Therefore, enhancing mentorship services deteriorate the efficacy of accelerators since it limits accelerators to provide their most vital services, their screening services.
3) Although accelerators are anywhere beneficial, macro-level heterogeneity of entrepreneurial ecosystems is significantly associated to their efficacy. In more developed regions, the existence of high-quality entrepreneurial communities and high-productivity mentorship services push accelerators toward increasing their mentorship services, which worsen their efficacy. In contrast, the existence of low-quality entrepreneurial in less developed regions leads to high appreciation of start-ups for screening services, and push them toward concentrating on their most vital services, which enhances their efficacy.
4) Low efficacy of accelerators in developed regions is mainly due to focusing on high-quality start-ups and providing low priority services, i.e., mentorship services. Therefore, better performance of start-ups in such programs is mainly of their own quality and the quality of their entrepreneurial community, not accelerators' contributions. Nevertheless, start-ups are highly willing to participate in such programs and VCs benefit from their existence because of their "launch pad" role. This role means that acceptance into such programs produces a positive, highly informative signal for VCs about the success of the participants.
5) Institutional reforms targeting mentoring or entrepreneurial communities are unexpectedly detrimental to the efficacy of accelerators, while they may enhance the start-ups' equilibrium willingness to participate. They encourage accelerators to reduce their attention to their most vital services, their screening services, and pay more attention to their lower priority services, their mentorship services. Instead, given the priority of screening services, only institutional reforms that target screening services will enhance the efficacy of accelerators.

This article is not without limitations, and we have some suggestions for future research. We assumed that the accelerator focuses only on one industry. In the real world, some accelerators' portfolios are composed of start-ups in different industries. In this case, the productivity of the mentorship services depends on the industry of the start-ups. Therefore, an analytical or empirical model that deals with the selection of start-ups from different industries is relevant. We did not investigate the heterogeneity of accelerators in their resources. Therefore, it is relevant for future research to answer this question: how do different degrees of resource constraints of accelerators affect their performances? This article only revealed the adverse effects of mentorship services. However, accelerators may be counter-productive or ineffective because of the diseconomies of their short duration or adverse effects of their networking capabilities [71]. Therefore, it would be valuable to deeply investigate the duration of accelerators and the total effects of their mentorship and networking capabilities. It would also be valuable to investigate whether or not accelerators include other adverse effects.

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