Effects of variable setup cost, reliability, and production costs under controlled carbon emissions in a reliable production system

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Abstract: Although important for production industries to reach fully sustainable manufacturing processes, those implementing production systems face challenges in reaching this reliability goal. In this direction, a production system is modelled through a basic economic-production paradigm under carbon emissions with a storage constraint and demand-dependent unit production cost. More reliable production houses produce fewer defective products than the unreliable production system. As the model contains a power-function, a geometric programming procedure is employed to obtain a quasi-closed form of the optimal solution. A numerical example based on data from the literature and a case study based on industry data, are provided to demonstrate geometric programming as a valuable analytical tool to resolve this type of problem for a production system under carbon emissions. Finally, a sensitivity analysis and graphical illustration are provided to illustrate the model. Numerical results show that the production system becomes completely reliable when the recommended model is used. [Submitted: 18 May 2019; Accepted: 8 March 2021]

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1 Introduction

A reliable production system means that the system should always move with perfect products without any defects or deterioration. There are several reasons to create reliable production systems, including saving the environment, increasing the output of perfect products, and making the best possible products. If a production system is sustainable, then it must follow three pillars of sustainability – economic, environmental, and social benefit. If a production system is reliable, then the number of defective products tends to be zero. This indicates that the production system produces less waste, which itself is an indicator of the environmental benefit of producing less waste and pollution. This also indicates the economic benefit as the number of reworking/remanufacturing/refurbishing units is reduced. Additionally, a social benefit is shown, as the maintenance of any reliable system needs more skilled workers, which creates more jobs than the traditional production system. Rasay et al. (2019) studied a two-stage maintenance model, where the quality of the second stage is dependent upon the first stage. A maintenance planning is considered for general deterioration. The process control follows a continuous distribution. They developed a stand-alone model for statistical process control. Recently, Tiwari et al. (2018) discussed the effect of carbon emissions for managing inventory. They found that considering carbon emissions is very much essential to hold inventory to save the environment and make any reliable production system. In this direction, Laforest et al. (2013) discussed the optional production system with cleaning and better way within a multi-decision criteria system. Jawad et al. (2015) extended the economic order quantity model with energy counting criteria to make the system reliable. Cárdenas-Barrón et al. (2015) developed a production system to determine the optimum lot size and the best transportation way by reworking defective items. Ozturk et al. (2016) extended the textile industry problem with the production system by maintaining the eco-friendly environment. Tayyab and Sarkar (2016) obtained the optimal production batch size within a multi-stage production process by considering random defective production rate. Using variable backorder rates in an imperfect production, Kim and Sarkar (2017) developed a production system without accounting for the direction of reliability of the production system. Several authors, such as Sarkar et al. (2011), Sarkar (2012), Saha et al. (2017) and Sett et al. (2017) acknowledged the role of a reliable production system, but none of them took into account the reliability of the systems they described. Instead, most of the cited works were based on shortages and backorders that were caused by defective products.

The main research gap found in the literature involves the reliability of the production system because reliability is one of the most credible measures of a production system. This study explains that a system is reliable when the number of defective units is reduced in the production process. The research gap is fulfilled by this paper, which is an extension of Leung's (2007) model within a production environment with more realistic assumptions based on carbon emissions during reliable production setup, during continuous production, and holding those products in warehouse. Rest of this study is designed as follows: the development of mathematical model is explained in Section 3. In Section 4, a solution procedure is provided to the mathematical model thorough geometric programming. In Section 5, an illustrative numerical example and sensitivity analysis are given to explain the model. Finally, conclusions are given in Section 6.

2 Literature review

The literature review is being made based on the following specific research areas of the production-inventory model.

2.1 Production-inventory models

The basic production-inventory model is based on two important assumptions regarding fixed setup cost and production of a perfect-quality product. In a short-term manufacturing situation, the setup cost may be fixed or constant, but for long-term manufacturing operations, it may not always be fixed. It tends to decrease due to investment in new technologies and machines, and the flexibility of the manufacturing processes largely depends upon system reliability. Every firm, regardless of size or incurred costs, would like to produce perfect-quality items within a reliable production system. Is such a goal achievable in either the short or long-run? Generally, it is impossible to get a perfect product in every production run, but quality is directly related to reliability within a production system. Thus, one can expect to obtain continuously good quality products if the system is reliable. However, to make a system reliable, a large initial investment must be allocated to the production system. As the system is continuing for a long-time, carbon emissions will be more during production setup and production as well as for holding those products. Carbon emissions during productions or holding products become a global issue nowadays. Every industry is taken care of this matter as the first priority-based because of recent government regulation. Many countries started to follow Euro-6 rules to save the environment. Recently, Ahmed and Sarkar (2018) developed the priority-based optimisation in sustainable production system, where they proved the cost is minimum when the total cost is being calculated subject to carbon emissions constraint.

Wang et al. (2018) proved that the decision of manufacturing or remanufacturing depends on carbon emissions and the low carbon emissions can control the economy easily. Sinha and Chaturvedi (2018) extended this field of production with reduction of carbon emissions and minimising energy consumption. They proved that the flexibility production schedule can reduce carbon emissions during production time. Sarkar et al. (2018) developed the effect of variable production rate on quality of products in a supply chain management without having any concept of reliability of production system. Majumder et al. (2017) investigated the effect of quality improvement under the controllable lead time scenario. They found the effect of setup cost reduction in a two-echelon supply chain management. Moon et al. (2018) studied a supply chain management for online selling. The manufacturer has the opportunity to sell products both in centralised and decentralised way. They observed that the manufacturer is beneficial for the decentralised policy but the entire supply chain earns more profit by the centralised policy. Kang et al. (2018a, 2018b) discussed the effect of human quality based inspection, safety stock, and planned backorder in two smart production models without any concept of carbon emissions or reliability. Lin et al. (2019) introduced the production system's capacity extension with coordinated production planning. They utilised a flexible production capacity to solve the model due to the linear problem. Sahin et al. (2020) increased the production system's performance by a system analysis, but they concentrated only on the quality of products without thinking about obtaining those perfect qualitative products. Recently, Nahas (2020) discussed a serial production model with an unreliable production system. Nevertheless, the concept of making the production system reliable was not considered within it, though the consideration of equipment of the production system with the buffer was considered.

2.2 Reliable production-inventory models

Silver (1992) suggested that 'changing the givens' plays an important role in manufacturing systems, and qualitative models offer the most important tool for making managerial decisions. Silver (1992) described the changes in constant parameters such as setup cost, setup time, demand, and production rate. The literature on inventory modelling that explains the effect of 'changing the givens' in manufacturing decisions is rapidly growing. Porteus (1985) discussed an inventory model with a reduced system cost and later explained that the quality of a product can be controlled by investment levels (Porteus, 1986). Van Beek and Putten (1987) discussed issues related to improved flexibility in a production and inventory management. They defined flexibility as an ability to change a situation with the minimal effort in a short time, explaining, "the flexibility possesses three dimensions: the ranges of states a system can adopt, the cost of moving from one state to the other, and the time which is necessary to move from one state to other."

Cheng (1989a) characterised a reliable production process as consistently generating acceptable quality products, which are defined as those demonstrating acceptable performance under a certain condition. If the reliability increases, the costs of scrap items and wasted materials as well as expenses associated with reworking substandard products decrease. To produce highly reliable products, investments in new technologies and machines as well as prioritised setup cost reductions are required. Based on this concept, Cheng (1989b) developed an economic production quantity model considering manufacturing process flexibility and reliability. Due to scale of economies, the unit production cost is assumed as less to satisfy more demand. An annual fractional cost of the capital investment function and price elasticity parameter are introduced within the unit production cost to increase the process reliability due to investment in technology. Based on this idea, Leung (2007) extended Cheng's (1989b) model by adding a reliability constraint and obtained a closed-form solution for the model when the degree of difficulty is positive.

If the entire production system cannot be transformed into a reliable system, then machines might breakdown. To control total system costs, machine breakdowns must be prevented through reliable production. Although Sarkar and Saren (2016) and Sett et al. (2017) proved that a stable system reduces costs, the production systems they described were not completely reliable. As a result, these researchers used defective costs and warranty costs as proxies of reliability. If any defective product is sold to a customer, the company may suffer from a poor brand image within the industry. To promote a positive brand image, the production system must be fully reliable such that no defective items will be manufactured. Sarkar et al. (2011) and Sarkar (2012) developed two new production models using the number of failures and working hours as indicators of system reliability. If the ratio of failure percentages to total work hours is decreased, the system becomes more reliable such that a fewer defective items are produced. These researchers initiated a direction of study toward reliability, but they did not address the ways the production system can become reliable. They used a control theory optimisation to solve for a closed-form solution. Asim et al. (2019) introduced a reliability cost within an uncertain integrated production-inventory model. Although they used the concept of reliability in a production system, they considered multi-objective optimisation instead of geometric programming. Thus, they could not reach a fully reliable production system.

In all cases, researchers have been unable to extend the model of a power function or power demand pattern. The models of Sarkar et al. (2011) or Sarkar (2012), based on a control theory approach, cannot solve models that contain a power function. Therefore, past research leaves a major research gap leading to the question: how can a closed-form or quasi-closed-form solution be obtained when system reliability is taken into account for reliable production? This research gap is fulfilled by this study considering geometric programming procedure.

2.3 Reliable production-inventory models with constraints

Generally, adding more constraints in the research model makes the model more realistic, as in reality there are several constraints in every industry sector. The major constraints in almost every industry sector is a budget constraint. Sometimes, space constrains play an important role. Roy and Maiti (1998) developed a multi-objective inventory model by considering budget and space constraints in fuzzy environment. They solved the model with a fuzzy technique. Yi and Sarker (2013) discussed an optimum policy for an integrated inventory system under consignment stock policy with space limitation. Sarkar and Moon (2014) developed the quality improvement of products manufactured in an imperfect process, but they did not address corrections for the production system. Paul et al. (2015) extended their own production model by considering several disruption strategies and several constraints from single-stage disruption to multi-stage disruption, even though they can easily consider system reliability within the production system to reduce more disruptions. Recently, Saha et al. (2017) developed a model of optimal investment for the retailer and preservation of deteriorated products through green operations to make a system clean. However, they did not take into account the reliability of the entire system under carbon emissions. They only considered the quality and improved quality of products. An inventory management for perishable products was discussed by Balugani et al. (2019), where the demand follows a Bernoulli demand pattern. The management of those products was really challenging because of the intermittent consumption, limited lifespan, and expiration. They found that the solution of this type of problem was driven by the demand size, not by intermittent and expiration. These all researchers developed models to improve product quality, but they did not consider the reliability of the machinery systems on which the items were produced. The reliability of any production system is considered as an important measurement of industrial and management design. A series or parallel reliable system is more preferable, but not economical. Thus, many industries do not prefer the redundancy system to control continuous production of perfect products. Gago et al. (2013) developed an exact cost minimisation of a series-parallel reliable system with multiple component choices using an algebraic method for a nonlinear program. Das et al. (2021) solved a production inventory problem with system reliability in which there was no concept of a geometric program solution for a variable demand and production setup. They only considered system reliability within the confines of a production system with perfect products exclusively. They found that if the machinery system was very reliable, then defective items were not produced within such a reliable production system. This study fulfills the research gap that exists in the recent literature. If the machinery system is the most reliable, then defective items are not produced within the reliable production system.

2.4 Necessity of geometric programming as a solution methodology

To accommodate different changes in inventory models, the solution procedure becomes more difficult to implement. Generally, differential calculus provides the strongest analytical tool to obtain the optimal solution for continuous optimisation problems, but the power function in the model makes obtaining a closed-form solution more difficult. Many industry problems require the optimal solution of the nonlinear program of complex nature. Among them, generalised reduced gradient (GRG) is one of the best method which was proved by Duffuaa et al. (1993). Choi and Bricker (1996) proved that geometric programming is an equivalent techniques for solving the same problem for which GRG is the best. They obtained the same results for both the procedure. They proved the effectiveness of geometric programming is more than others for optimisation. Since then, geometric programming is an important tool, with advantages over other optimisation methods, to solve a nonlinear program. Duffin et al. (1967) explained the basic tenets of geometric programming: in geometric programming problems, the degree of difficulty (i.e., the dimension of the dual problem) is defined as the number of variables (minus one) subtracted from the number of terms. According to geometric programming terminology, if the degree of difficulty is negative, then geometric programming is not directly applied; if it is zero, then only one unique solution to the problem can be found; if it is positive, then an infinite number of solutions is possible. Cárdenas-Barrón (2001) extended a production model with an algebraic approach to obtain the closed-form solution. He proved that this algebraic method can be utilised for the research problems without using optimisation through calculus. Yang and Wee (2002) extended the application of the algebraic method in an economic lot size model through the vendor-buyer relation. They also proved that using algebraic approach, the optimum results can be obtained faster than the calculus-method. Wee et al. (2003) wrote a note on the inventory model through the algebraic procedure to obtain the optimum results. Chung and Wee (2007) utilised the algebraic method in the supply chain model and proved that they obtained the optimum results without the complex calculation of calculus method. Cárdenas-Barrón (2008) proved the arithmetic-geometric mean can solve more faster way than the basic algebraic method. Thus, he recommended to utilise arithmetic-geometric method for optimisation instead of calculus. Sarkar (2013) proved that for the supply chain problem, the algebraic method is quite useful to obtain the closed-form solution. The important benefit of this method is that the Hessian matrix calculation and other complex calculation is not needed for obtaining the optimum solution.

Liu (2007) discussed how geometric programming can be used for profit maximisation problem. He extends the geometric programming approach which gives the global optimum solution as well as information to obtain the relationship between profit maximisation and returning scale for the problem solution. Liu (2008) developed an extended interval-exponent posynomial problem with coefficients. The resulting solution procedure is based on the duality theorem and the separation-of-variables method for the exponents present in the objective function. Liu (2009) derived a profit maximisation model with interval coefficients and a quantity discount by using signomial geometric programming to derive the interval profit value. Receiving both bounds for the range, Liu (2009) employed two-level mathematical programming and utilised the duality theorem and the separation-of-variables technique to transform the two-level geometric programming into a single-level geometric programming. Mahapatra and Mondal (2012) developed a posynomial parametric geometric programming setup with an interval valued coefficient in an imprecise environment. They optimised the objective function without changing the equivalent transformed model. Xu (2013) developed an iterative strategy to address the steady-state optimisation of biochemical systems by considering nonlinear kinetic models known as generalised mass action (GMA) models. Samadi et al. (2013) introduced a geometric approach for the fuzzy inventory model. They proved that variable pricing made a significant effect on the total profit. Ojha and Biswal (2014) introduced an ϵ-constraint method to solve the multi-objective geometric programming problem, but they obtained a non-inferior solution. They finally used the duality theory and found a Pareto optimal solution. However, an iterative procedure can be used to calculate an optimal solution by solving a series of geometric programming. Some notable books and research articles have been written by Sun et al. (2015).

2.5 Novelty of this research

After a long survey of literature, one can find a literature gap that no authors yet considered the reliable production system to reduce the waste for making a reliable production system with good quality products. There are two aspects for novelty for this model:

2.5.1 Theoretical aspect

Nowadays, many in the industry maintain green manufacturing systems to save the environment and address other issues. Controlling carbon emissions within the whole production system is one of the way to maintain a good environment. For example, under controlled carbon emissions, reduction in the number of defective items minimises waste, which contributes to a good environment and is an indicator of a reliable production system. Furthermore, reduction in the number of defective items during production means that the system is reliable, and this reliability translates to relatively few failures per working hour. Thus, a reliable green production system is ultimately a reliable production system-as determined by the output of high quality products and reduced waste-that causes less damage to the environment than an inefficient production system. However, these important outcomes have not been considered by any other researcher to date.

2.5.2 Methodological aspect

A geometric programming procedure is typically employed for solving algebraic nonlinear problems for which the main goal is optimisation. The most remarkable property of geometric programming is that it can solve an optimisation problem with highly nonlinear constraints because of the powerful duality. Generally, two types of geometric programming problems are solved: posynomial and signomial. In a posynomial problem, all coefficients are positive and in an signomial problem, at least one coefficient is negative. Posynomial and signomial problems each contain a power function. For the proposed model, a posynomial problem is formulated. The model has been solved with geometric programming to obtain a closed-form solution even though the model solved in the usual way obtained a quasi-closed-form solution. The finding is based on a high degree of difficulty, and a unique solution is obtained. This is a major contribution from the methodological perspective.

The proposed model establishes general results for a reliable production system under the effect of carbon emissions by using arithmetic-geometric inequalities and geometric programming under storage space and reliability constraints. A quasi-closed-form solution is found analytically for each of the decision variables of demand, lot size, setup cost, and reliability. Geometric programming is a powerful tool to solve the posynomial, and the purpose of this study is to reduce the total system cost by obtaining a quasi-closed-form solution with the help of geometric programming and an arithmetic-geometric inequality for an economic reliable production quantity problem. The contribution of the paper, compared with existing studies, is summarised in Table 1.

3 Mathematical model

This section contains the basic concept of reliable production with the problem definition addressed by the model, notation, basic assumptions, and a full description of a reliable production system characterised by reliability.

3.1 Basic concept of a reliable production system

A production system is said to be a *reliable production system* if it controls or reduces the amount of waste generated throughout the process in a way that maintains the quality of the products and reduces any negative effects on the environment. The novelty of the production model presented is that it represents a reliable production system under carbon emissions that minimises the amount of defective items. Controlling waste while maintaining the quality of manufactured products is the key to create a reliable production system under carbon emissions. See Figure 1 to gain a better understanding of a reliable production.

Figure 1 Basic concept of a reliable production system (see online version for colours)

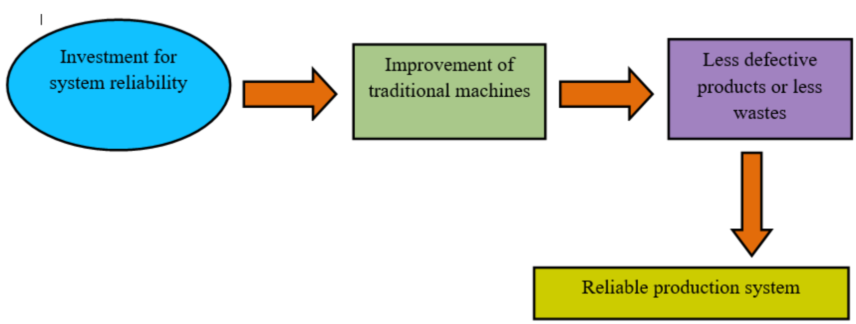


Table 1 Comparison between contributions of different authors

Previous works	*Geometric programming*	*Closed/quasi-closed-form solution*	*Variable reliability cost*	*Variable production*	*Variable setup cost*	*Variable lot size*	*Reliability constraint*	*Space constraint*
Cheng (1989b)	√	√	√					
Cárdenas-Barrón (2001)		√				√		
Leung (2007)	√	√		√	√	√	√	
Chung and Wee (2007)		√				√		
Sarkar (2012)		√		√		√		
Cárdenas-Barrón et al. (2015)		√				√		
Paul et al. (2015)		√				√		
Ozturk et al. (2016)		√				√		
Tayyab and Sarkar (2016)		√				√		
This study	√	√	√	√	√	√	√	√

A basic economic production system is the basis for making a reliable production system model that featured reliability as a decision variable under carbon missions. The aim of the model is to reduce costs and improve existing production systems by introducing reliability as a measured characteristic under carbon emissions. To create a reliable production system model, the following basic assumption is adopted: More reliable production systems create fewer defective products. The fundamental goal is to make a completely reliable carbon-controlled production system such that the entire system is defect free, or totally clean under controlled carbon emissions, which is an indicator of a good environment for green manufacturing. An improved methodology of a geometric programming is employed to obtain closed-form or quasi-closed form solutions. Using a geometric programming procedure with a positive degree of difficulty, it is very difficult to obtain a closed-form solution. For this model, using an analytical geometric programming procedure, one can obtain a closed-form or quasi-closed-form of the optimal solution.

The nomenclature of the model is given in Table 2.

Table 2 Notation used in to the model

Decision	*Variables*
S	Setup cost per setup under controlled carbon emissions cost ($/setup)
D	Demand rate (units/year)
R	Reliability of the reliable production process
$Q(t)$	Production quantity at time t per batch (units/batch)
Dependent	*Variables*
p	Demand dependent unit production cost ($/item)
Parameters	*Variables*
$\Psi(S,R)$	Total cost of depreciation and interest per production cycle ($/year)
$C(S,D,R,Q)$	Total average cost of the reliable production system ($/year)
β	Price elasticity ($\beta > 1$)
α	Annual fractional cost of the capital investment including carbon emissions ($\alpha > 0$)
h	Holding cost per unit per unit time under controlled carbon emissions ($/unit/year)
ω_1	Storage space area per unit (square feet/unit)
A	Total storage space area (square feet)

The following assumptions are considered to develop the model.

1 A reliable production process handles a single type of items. All items are inspected and all defective items are discarded to maintain the reliable production system. All costs are considered under controlled carbon emissions. The whole production is being made such that the amount of carbon emissions is less for saving the environment.

2 Generally many models consider a fixed-unit production costs. However, for large manufacturing systems, the unit production cost will be less because it is based on the product's demand. This model assumes that the unit production cost under

carbon emissions is dependent on demand as $p = \alpha D^{-\beta}$, where α is the partial fractional cost of the total investment annually and β is the price elasticity parameter $(\alpha > 0, \beta > 1)$.

3 The total depreciation and interest cost under carbon emissions per production cycle is directly proportional to the reliability and inversely proportional to the setup cost: $\Psi(S, R) = aS^{-b}R^{c}$, where $a, b, c > 0$ are three constants (real numbers) as the shape parameters.

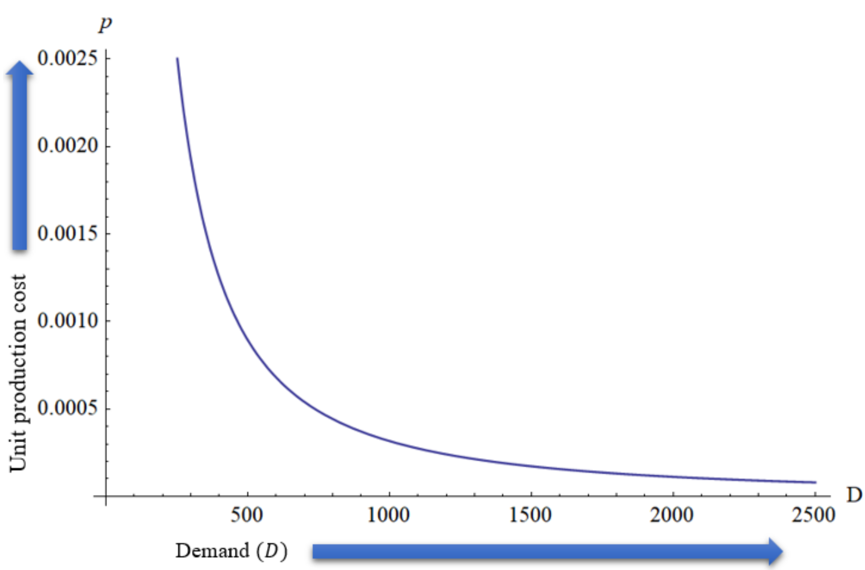
4 The time horizon is infinite and production rate is instantaneous.

The unit production cost under carbon emissions depends on demand and high demand of reliable products translates to reduced unit production; that is, if demand increases, then production must increase to prevent shortages. Hence, without considering shortages, increasing the value of production rate translates to a decreasing value of unit production cost. Therefore, the unit production rate must follow an inverse relation with the demand rate. The graphical representation of the relationship is given in Figure 2. The unit production cost under carbon emissions is considered as

$$p = \alpha D^{-\beta}, \text{ where } \alpha \text{ is the annual fractional cost of the capital investment and } \beta \text{ is the price elasticity parameter } (\alpha > 0,\ \beta > 1). \tag{1}$$

Products can be made more efficiently through use of modern technology. Small batches require less holding costs, but managers may produce in large batches when demand for the reliable product is high. Thus, a storage space constraint is considered. As no product is perfect reliable, another constraint on reliability is used.

Figure 2 Relationship between demand and unit production cost (see online version for colours)



It is examined for the cost in terms of unreliability as $1 - R$:

$$\Psi(S,R) = aS^{-b}(1-R)^{-c_1}$$

where $a, b, c_1 > 0$ are constant (real numbers) as the shape parameters. (2)

If $R \to 1$, then $\Psi(S,R) \to \infty$; that is, no product is fully reliable, and if this unreliable equation is used, then the closed-form solution cannot be found from this model. However, equation (2) fits the data better than equation (3), thus a geometric programming is employed rather than differential calculus.

The production system depends on the reliability R of the product, which indicates that R% of the items with acceptable quality can meet the market demand. Hence, the cycle length is equal to RQ/D, and D/RQ indicates the average number of cycles per year.

Now the total inventory related cost under controlled carbon emissions per production cycle becomes

Setup cost under controlled carbon emissions
+production cost under controlled carbon emissions
+inventory holding cost under controlled carbon emissions
+interest and depreciation cost under controlled carbon emissions.

Thus, the total average cost is

$$C(S,Q,R,D) = DSQ^{-1}R^{-1} + DpR^{-1} + \frac{HQR}{2} + DaS^{-b}Q^{-1}R^{c-1} \tag{3}$$

and the required storage area is $\omega_1 RQ$. One can consider

$$P = \alpha D^{-\beta}$$

along with the two constraints, $0 \le R \le 1$ and $RQ\omega_1 \le A$.

Thus, it can be rewritten as

$$\begin{aligned}
&\text{Min } C(S,Q,R,D) = DSQ^{-1}R^{-1} + \alpha D^{1-\beta}R^{-1} \\
&+\frac{HQR}{2} + DaS^{-b}Q^{-1}R^{c-1} \\
&\text{subject to } 0 \le R \le 1 \\
&\qquad RQ\omega_1 \le A \\
&\qquad D, S, Q, R > 0.
\end{aligned} \tag{4}$$

In the basic economic production quantity model, the product is considered as perfect, neither inspection nor rework costs are considered. However, production of fully reliable products is very difficult to achieve. Therefore, to make an accurate assumption, the total interest and depreciation costs under carbon emissions per production cycle of the modern flexible system are considered as: both of these costs under carbon emissions vary with reliability levels and setup costs. Van Beek and Putten (1987) explained a similar type of expression to model the production of a reliable product in a basic economic production quantity model without carbon emissions.

$$\Psi(S,R) = aS^{-b}R^{c}$$

where $a, b, c > 0$ are three constants as of the shape parameters. (5)

4 Geometric programming solution to the mathematical model

By using geometric programming, a solution is obtained for the mathematical model by considering the following equations:

$$u_1 = DSQ^{-1}R^{-1} \tag{6}$$
$$u_2 = \alpha D^{1-\beta}R^{-1} \tag{7}$$
$$u_3 = \frac{HRQ}{2} \tag{8}$$
$$u_4 = DaS^{-b}Q^{-1}R^{c-1} \tag{9}$$
$$u_5 = R \tag{10}$$
$$u_6 = \frac{RQ\omega_1}{A}. \tag{11}$$

From the weighted arithmetic-geometric inequality for the positive integers, one can write the cost equation under controlled carbon emissions as

$$\begin{aligned} C(S,Q,R,D) &= u_1 + u_2 + u_3 + u_4 \\ &\geq \left(\frac{u_1}{y_1}\right)^{y_1}\left(\frac{u_2}{y_2}\right)^{y_2}\left(\frac{u_3}{y_3}\right)^{y_3}\left(\frac{u_4}{y_4}\right)^{y_4} \end{aligned} \tag{12}$$

where $y_1 + y_2 + y_3 + y_4 = 1$ and $y_1, y_2, y_3, y_4 > 0$ and one can consider the constraints in the following way:

$$1 \geq R = u_5 \geq u_5^{\Delta_5} \tag{13}$$
$$1 \geq \frac{RQ\omega_1}{A} = u_6 \geq u_6^{\Delta_6} \tag{14}$$

where

$$\Delta_5 \in \begin{Bmatrix} (1,\infty) & \text{if } u_5 < 1 \\ (-\infty,\infty) & \text{if } u_5 = 1 \end{Bmatrix} \tag{15}$$

and

$$\Delta_6 \in \begin{Bmatrix} (1,\infty) & \text{if } u_6 < 1 \\ (-\infty,\infty) & \text{if } u_6 = 1 \end{Bmatrix}. \tag{16}$$

Multiplying the geometric inequality (12) by the two extreme sides of constraint inequalities (13) and (14), one can obtain

$$\begin{aligned} C(S,Q,R,D) &= u_1 + u_2 + u_3 + u_4 \\ &\geq \left(\frac{u_1}{y_1}\right)^{y_1}\left(\frac{u_2}{y_2}\right)^{y_2}\left(\frac{u_3}{y_3}\right)^{y_3}\left(\frac{u_4}{y_4}\right)^{y_4} u_5^{\Delta_5}u_6^{\Delta_6} \\ &\equiv \phi(\overline{y}, \Delta_5, \Delta_6, S, Q, R, D). \end{aligned} \tag{17}$$

From the geometric programming theory, the left-hand side and right-hand side of inequality (17) is known as the *primal function* and *pre-dual function*, respectively. The column vector $\overline{y}$ $(= y_1, y_2, y_3, y_4)$ consists of normalised dual variables. Δ_5 and Δ_6 are the non-normalised dual variables.

Using the values of u_i's, $i = (1, 2, ..., 6)$, one can find

$$\phi(\overline{y}, \Delta_5, \Delta_6, S, Q, R, D) = \Big(\frac{DSQ^{-1}R^{-1}}{y_1}\Big)^{y_1}\Big(\frac{\alpha D^{1-\beta}R^{-1}}{y_2}\Big)^{y_2}\Big(\frac{HQR}{2y_3}\Big)^{y_3}$$
$$\Big(\frac{DaS^{-b}Q^{-1}R^{c-1}}{y_4}\Big)^{y_4} = \Big(\frac{1}{y_1}\Big)^{y_1}\Big(\frac{\alpha}{y_2}\Big)^{y_2}\Big(\frac{H}{2y_3}\Big)^{y_3}\Big(\frac{a}{y_4}\Big)^{y_4}\Big(\frac{\omega_1}{A}\Big)^{\Delta_6}$$

$$D^{y_1+y_2(1-\beta)+y_4}S^{y_1-by_4} \tag{18}$$
$$Q^{-y_1+y_3-y_4+\Delta_6}R^{-y_1-y_2+y_3+y_4(c-1)+\Delta_5+\Delta_6}. \tag{19}$$

Because $\overline{y}$, Δ_5, and Δ_6 are arbitrary, the exponent of the four decision variables are set to zero. Therefore, equation (18) is of the following form:

$$\phi(y_1, y_2, y_3, y_4) = \Big(\frac{1}{y_1}\Big)^{y_1}\Big(\frac{\alpha}{y_2}\Big)^{y_2}\Big(\frac{H}{2y_3}\Big)^{y_3}\Big(\frac{a}{y_4}\Big)^{y_4}\Big(\frac{\omega_1}{A}\Big)^{\Delta_6}. \tag{20}$$

Equation (19) is accepted when the orthogonality and normality conditions hold:

$$y_1 + y_2(1-\beta) + y_4 = 0 \tag{21}$$
$$y_1 - by_4 = 0 \tag{22}$$
$$-y_1 + y_3 - y_4 + \Delta_6 = 0 \tag{23}$$
$$-y_1 - y_2 + y_3 + y_4(c-1) + \Delta_5 + \Delta_6 = 0 \tag{24}$$
$$y_1 + y_2 + y_3 + y_4 = 1. \tag{25}$$

Equations (20) to (23) refer to the orthogonality conditions, and equation (24) shows the normality condition.

The degree of difficulty = number of terms in equation (17)
−number of variables − 1 = 6 − 4 − 1 = 1.

an infinite number of solutions is possible. Taking $Z = (1 + b) - c(\beta - 1)$, one can obtain solutions of dual variables in terms of Δ_5.

$$y_1 = \frac{\Delta_5 b(\beta - 1)}{Z} \tag{26}$$
$$y_2 = \frac{\Delta_5(1 + b)}{Z} \tag{27}$$
$$y_3 = 1 - \frac{\Delta_5\beta(1 + b)}{Z} \tag{28}$$
$$y_4 = \frac{\Delta_5(\beta - 1)}{Z} \tag{29}$$
$$\Delta_6 = \frac{\Delta_5(2\beta - 1)(1 + b)}{Z} - 1. \tag{30}$$

These values are substituted into equation (19) to obtain

$$\phi(\Delta_5^*) = \left[\frac{Z}{\Delta_5^* b(\beta-1)}\right]^{\frac{\Delta_5^* b(\beta-1)}{Z}} \left[\frac{\alpha Z}{\Delta_5^*(1+b)}\right]^{\frac{\Delta_5^*(1+b)}{Z}} \left[\frac{H}{2\left(1-\frac{\Delta_5^*\beta(1+b)}{Z}\right)}\right]^{1-\frac{\Delta_5^*\beta(1+b)}{Z}} \left[\frac{aZ}{\Delta_5^*(\beta-1)}\right]^{\frac{\Delta_5^*(\beta-1)}{Z}} \left(\frac{\omega_1}{A}\right)^{\frac{\Delta_5^*(2\beta-1)(1+b)}{Z}-1}. \tag{31}$$

The dual of the above function for Δ_5 is considered as

$$\phi(\Delta_5) = \left[\frac{Z}{\Delta_5 b(\beta-1)}\right]^{\frac{\Delta_5 b(\beta-1)}{Z}} \left[\frac{\alpha Z}{\Delta_5(1+b)}\right]^{\frac{\Delta_5(1+b)}{Z}} \left[\frac{H}{2\left(1-\frac{\Delta_5\beta(1+b)}{Z}\right)}\right]^{1-\frac{\Delta_5\beta(1+b)}{Z}} \left[\frac{aZ}{\Delta_5(\beta-1)}\right]^{\frac{\Delta_5(\beta-1)}{Z}} \left(\frac{\omega_1}{A}\right)^{\frac{\Delta_5(2\beta-1)(1+b)}{Z}-1}. \tag{32}$$

According to Duffin et al. (1967), the substituted dual function $\phi(\Delta_5)$ can be maximised by the optimal weight Δ_5^*. To maximise this expression, a logarithm of the above function is taken.

$$X(\Delta_5) = \ln\phi(\Delta_5) = \frac{\Delta_5 b(\beta-1)}{Z}\ln\frac{Z}{\Delta_5 b(\beta-1)} + \frac{\Delta_5(1+b)}{Z}\ln\frac{\alpha Z}{\Delta_5(1+b)} + \left\{1-\frac{\Delta_5\beta(1+b)}{Z}\right\} \ln\frac{H}{2\left(1-\frac{\Delta_5\beta(1+b)}{Z}\right)} + \frac{\Delta_5(\beta-1)}{Z}\ln\frac{\alpha Z}{\Delta_5(\beta-1)} + \left\{\frac{\Delta_5(2\beta-1)(1+b)}{Z}-1\right\}\ln\left(\frac{\omega_1}{A}\right). \tag{33}$$

Differentiating and equating with zero, one can find that the value of Δ_5 satisfies the equation:

$$\left(\frac{Z}{\Delta_5 b(\beta-1)}\right)^{\frac{b(\beta-1)}{Z}} \left(\frac{\alpha Z}{\Delta_5(1+b)}\right)^{\frac{1+b}{Z}} \left(\frac{aZ}{\Delta_5(\beta-1)}\right)^{\frac{\beta-1}{Z}} \left(\frac{\omega_1}{A}\right)^{\frac{(2\beta-1)(1+b)}{Z}} = \left(\frac{H}{2\left(1-\frac{\Delta_5\beta(1+b)}{Z}\right)}\right)^{\frac{\beta(1+b)}{Z}} \tag{34}$$

and the value of Δ_6 satisfies the equation

$$\Delta_6 = \frac{\Delta_5(2\beta-1)(1+b)}{Z} - 1.$$

Therefore, the optimal weights Δ_5^* and Δ_6^* must satisfy equations (29) and (33). With these values, other optimal weights can be determined from equations (25), (26), (27), and (28). Equation (19) gives

$$\begin{aligned} &C(S,Q,r,D) \geq \phi^*(y_1^*, y_2^*, y_3^*, y_4^*) \equiv \phi(\Delta_5^*, \Delta_6^*) \\ &= \phi(y_1^*, y_2^*, y_3^*, y_4^*) \\ &= \left(\frac{1}{y_1}\right)^{y_1} \left(\frac{\alpha}{y_2}\right)^{y_2} \left(\frac{H}{2y_3}\right)^{y_3} \left(\frac{a}{y_4}\right)^{y_4} \left(\frac{\omega_1}{A}\right)^{\frac{\Delta_5(2\beta-1)(1+b)}{Z}-1}. \end{aligned} \tag{35}$$

Considering the equality of equation (34), it is found as

$$\text{Min } C(S,Q,r,D) = \text{Max } \phi^*(y_1^*, y_2^*, y_3^*, y_4^*) = \phi^*(y_1^*, y_2^*, y_3^*, y_4^*). \tag{36}$$

To obtain the global optimal solution of the optimisation problem with constraints, the necessary steps are as follows:

The equality case in equation (17) is assumed, which is possible when inequalities (12), (13), and (14) are considered as equal, which gives

$$\frac{u_1^*}{y_1^*} = \frac{u_2^*}{y_2^*} = \frac{u_3^*}{y_3^*} = \frac{u_4^*}{y_4^*}. \tag{37}$$

It is assumed

$$\frac{u_1^*}{y_1^*} = \frac{u_2^*}{y_2^*} = \frac{u_3^*}{y_3^*} = \frac{u_4^*}{y_4^*} = \theta,$$
$$u_5^{\Delta_5^*} = 1,$$

and

$$u_6^{\Delta_6^*} = 1.$$

After substituting these values into equation (17), one can obtain

$$\begin{aligned} &\theta^{y_1^*+y_2^*+y_3^*+y_4^*} = \theta = \phi^* \\ &\text{i.e., } u_i^* = y_i^*\phi^*,\ i = 1,2,3,4. \end{aligned} \tag{38}$$

Equation (37) gives

$$\begin{aligned} &u_1^* = y_1^*\phi^* \\ &\text{which implies } Q^* = \frac{D^*S^*Z}{\Delta_5 b\phi^* R^*(\beta-1)} \end{aligned} \tag{39}$$

and

$$\begin{aligned} &u_2^* = y_2^*\phi^* \\ &\text{which implies } R^* = \frac{\alpha Z D^{*(1-\beta)}}{\Delta_5^*(1+b)\phi^*}. \end{aligned} \tag{40}$$

Using equations (38) and (39) in

$$u_3^* = y_3^* \phi^*$$

$$\text{which gives } D^* = \frac{2\phi^{*2}\Delta_5^* b(\beta-1)}{S^* Z H}\left(1 - \frac{\Delta_5{}^* \beta(1+b)}{Z}\right). \tag{41}$$

Substituting equation (40) into equation (38), one can find

$$Q^* = \frac{D^{*\beta} S^*(1+b)}{\alpha b(\beta-1)}. \tag{42}$$

Finally, using equations (39), (40), and (41) in

$$u_4^* = y_4^* \phi^*$$

$$\text{which gives } S^* = \left[ab\left(\frac{Z}{\Delta_5^*}\right)^{\beta c}\left(\frac{2b(\beta-1)\left(1-\frac{\Delta_5\beta(1+b)}{Z}\right)}{H}\right)^{(1-\beta)c}\right.$$

$$\left.\left(\frac{\alpha}{1+b}\right)^c \phi^{(1-2\beta)c}\right]^{\frac{1}{b+c+1-\beta c}}. \tag{43}$$

The optimal solutions of the model are given by equations (39), (40), (41), and (42). According to geometric programming theory, the optimal dual variables are obtained first and using these, the optimal cost and finally optimal decision variables are found.

5 Numerical example, sensitivity analysis, and managerial insights

This section contains two numerical examples, sensitivity of both examples, and major insights.

5.1 Numerical examples

Two numerical examples are presented to illustrate the model. Example 1 was conducted with data available in the literature, and Example 2 was conducted with data available through a newly launched company, in West Bengal, India.

Example 1: For illustration of the geometric programming procedure developed in Section 3, under the effects of the space constraint and the demand-dependent unit production cost under controlled carbon emissions, an EPQ model with the data used in Leung (2007) is assumed as: a = 1, b = 1, c = 1, H = \$5 per item per year. For variable unit production cost under carbon emissions and a space constraint, it is assumed β = 1.4, α = 1,000, ω_1 = 10 square feet per item, and A = 1,500 square feet. Then, the cost of interest and depreciation under controlled carbon emissions due to production is

$$\Psi(S, R) = S^{-1} R$$

and the corresponding cost under controlled carbon emissions becomes

$$\text{Min } C(S,Q,R,D) = DSQ^{-1}R^{-1} + 1{,}000D^{1-1.4}R^{-1} + \frac{5QR}{2} + DS^{-1}Q^{-1}$$
$$\text{subject to } 0 \leq R \leq 1$$
$$10\ RQ \leq 1{,}500$$
$$D, S, Q, R > 0.$$

By using the geometric programming procedure, one can obtain the following values of the dual variables: $\Delta_5^* = 0.0937651$, $\Delta_6^* = -0.789028$, $y_1^* = 0.0234413$, $y_2^* = 0.117206$, $y_3^* = 0.835911$ and $y_4^* = 0.0234413$.

The values of the optimal dual variables satisfy the normality condition: $y_1^* + y_2^* + y_3^* + y_4^* = 1$.

The corresponding minimum total cost of the system under controlled carbon emissions is $C^* = \$448.61$ per year, which is less than the minimum cost obtained by Leung (2007). Therefore, this model has saved \$98.65 per year. The values of the decision variables are $D^* = 1{,}577$ items per year, $R^* = 1$, $S^* = \$1$ per batch, and $Q^* = 150$ items per batch.

Example 2: A real medium-sized company seeks to obtain fully perfect quality products for the system to maintain a positive brand image of the company. They produce packets made by jute. It is totally green products without having less or almost zero-carbon emissions. Therefore, management at the company appreciates the reliable production system with a space constraint under controlled carbon emissions. The numerical data is taken from the above mentioned industry as follows: $a = 10$, $b = 1$, $c = 1$, $H = \$0.5$/unit/year, $\beta = 1.4$, $\alpha = 2{,}200$, $\omega_1 = 10$ square per item, and $A = 1{,}000$ square feet.

The optimum result of the industry is obtained as total cost $C_{Int}^* = 226.701$ and decision variables $D^* = 911.19$ units, $R^* = 0.99$, $S^* = \$3.16$ per batch, $Q^* = 100$ items per batch.

5.2 Sensitivity analysis

A sensitivity analysis has been performed to prove the effect of key parameters on the optimal cost for both Examples 1 and 2. The results are summarised in Tables 3 and 4.

From the sensitivity analysis of Example 1, one can observe the following results:

1. The increasing value of a indicates the increasing value of the total system cost; that is, a is proportional with the total system cost. Table 3 shows that a is the least sensitive parameter. Under controlled carbon emissions, the value of scaling parameter increases, total cost increases, which is an indicator of sustainable production.
2. Increased holding cost under controlled carbon emissions increases the total system cost. If the available space is fixed, the holding cost is also directly proportional to the total system cost. Therefore, the extent of positive and negative changes in holding cost is similar to that of total cost. However, the signs for the

resulting two cost equations are opposite. i.e., the holding cost under carbon emissions maintain an equilibrium position.

3 An increasing value of α, which is related to the production cost under controlled carbon emissions of the system, increases the total system cost. ω_1 is inversely proportional to the total system cost because it is related to the storage space per item. Therefore, if ω_1 increases, the total system cost decreases. Results indicate that ω_1 is the most sensitive parameter. The increasing value of the total storage space A indicates an increased total system cost; that is, these costs are directly proportional.

Almost all analyses are similar, only the least and the most sensitive parameters are changed and all changes are in similar directions in both examples.

Table 3 Sensitivity analysis for key parameters in Example 1

Parameters	*Changes*	C^*	*Parameters*	*Changes*	C^*
a	–50%	–1.55	H	–50%	–41.80
	–25%	–0.66		–25%	–20.90
	+25%	+0.53		+25%	+20.90
	+50%	+0.98		+50%	+41.80
α	–50%	–6.41	ω_1	–50%	+80.64
	–25%	–3.05		–25%	+26.57
	+25%	+2.84		+25%	–15.64
	+50%	+5.51		+50%	–25.85
A	–50%	–38.20			
	–25%	–19.49			
	+25%	+19.88			
	+50%	+40.01			

Table 4 Sensitivity analysis for key parameters in Example 2

Parameters	*Changes*	C^*	*Parameters*	*Changes*	C^*
a	–50%	–8.39	H	–50%	–5.51
	–25%	–3.58		–25%	–2.76
	+25%	+2.88		+25%	+2.76
	+50%	+5.30		+50%	+5.51
α	–50%	–34.74	ω_1	–50%	+13.97
	–25%	–16.53		–25%	+4.86
	+25%	+15.37		+25%	–2.74
	+50%	+29.89		+50%	–4.22
f	–50%	–4.96			
	–25%	–3.34			
	+25%	+3.65			
	+50%	+7.25			

From the sensitivity analysis of Example 2, one can observe the following results:

1 Similar as in Table 3, the increasing value of a indicates the increasing value of the total system cost; that is a is proportional with the total system cost. But a is not the least sensitive parameter among all other cost parameters here.

2 Similar as in Example 1, for Example 2, the increased holding cost increases the total system cost and if the available space is fixed, the holding cost is also directly proportional to the total system cost. Therefore, the extent of positive and negative change in holding cost is similar to that of total cost, but the signs for the resulting two cost equations are opposite. Similar analysis is obtained as on last case with the holding cost as the least sensitive parameter among all other cost parameters.

3 An increasing value of α, which is related to the production cost of the system, increases the total system cost and this is the most sensitive parameter among all other cost parameters. If the storage space per item ω_1 increases, the total system cost decreases. It is also similar as in Example 1. As total storage space A is directly proportional to the total cost, its increasing value indicates an increased total system cost.

4 Figure 3 explains the sensitiveness of each parameter with respect to the changes from –50% to +50%. It is found that α is the most sensitive in case of negative change (–50%), whereas ω_1 shows the reverse pattern with α. It indicates if the annual fractional cost for the capital investment is reduced, then the total cost is reduced in a huge amount. This is the effectiveness of the continuous investment. In each cycle, the total cost will be continuously reducing. But if the discrete investment is utilised, then the total cost only reduces in the investment cycle only, then it may be increased again. Thus, the continuous improvement makes the major benefit for the cost reduction strategy. But the storage space per unit area behaves totally the reverse direction of the annual fractional cost.

Figure 3 Sensitivity of different key parameters (see online version for colours)

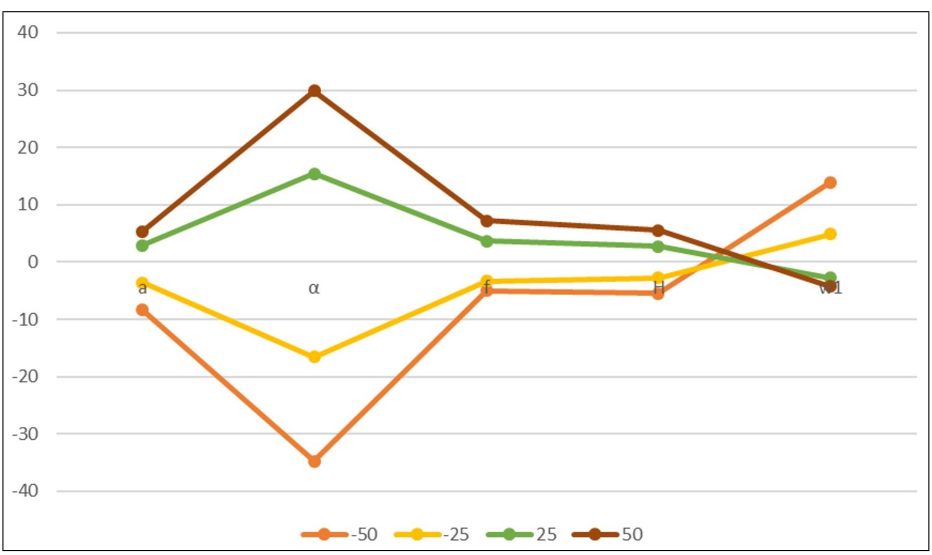
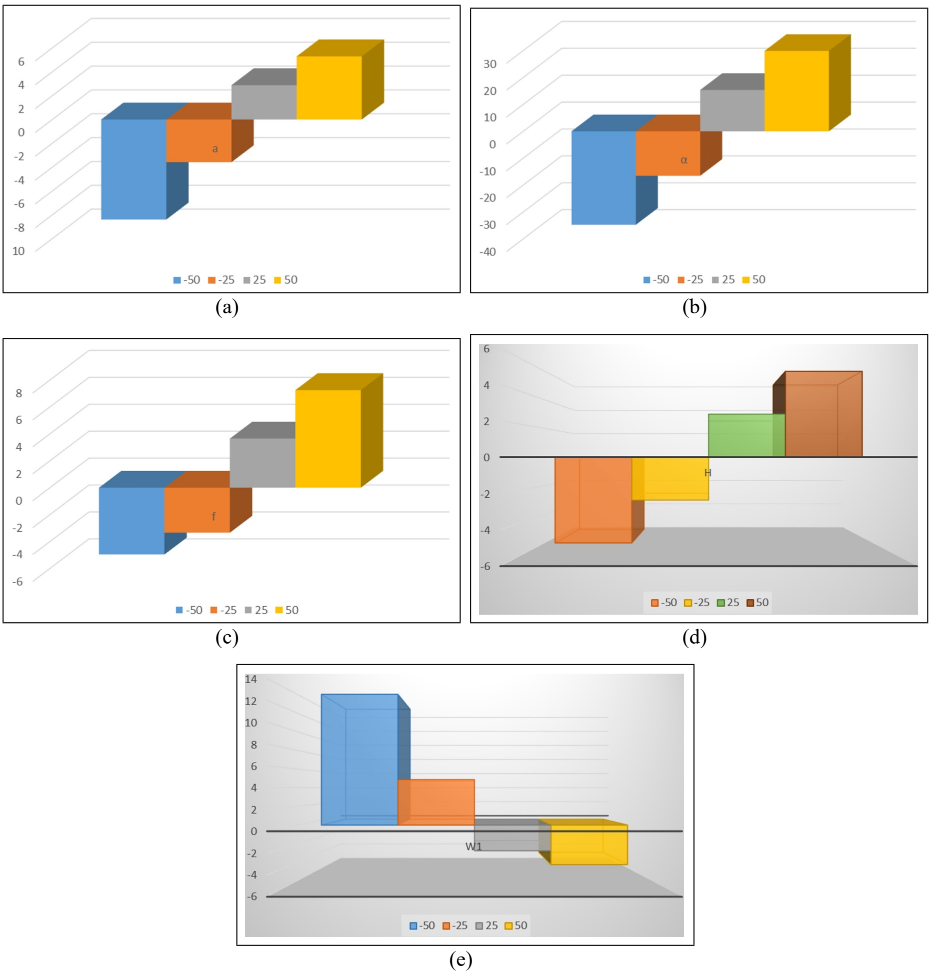


Figure 4 Changes of total cost with changes of different parameters separately, (a) changes of total cost with changes of parameter a (b) changes of total cost with changes of parameter α (c) changes of total cost with changes of parameter f (d) changes of total cost with changes of parameter H (e) changes of total cost with changes of parameter ω_1 (see online version for colours)



5.3 *Managerial insights*

1 From the five figures in Figure 4, it can be found that within five parameters, only the storage space per unit area follows the reverse direction with the other sensitive parameters of the total cost. The industry manager must be careful about the rented space and the amount of the total available rented space, because these parameters prove to be the most sensitive in the reverse direction of the total cost reduction.

2 Among the other four parameters, the industry manager may not concentrate on the holding cost, as it is the least sensitive, but it does hold an equilibrium change

in both positive and negative sides. As the demand and production rate are related, that is the main reason that the holding cost is the least sensitive among all parameters.

6 Conclusions

The model of Leung (2007) was extended by using a space constraint and a demand-dependent unit production cost for a reliable production system under controlled carbon emissions. In reality, it is very difficult to make a reliable production system under controlled carbon emissions, as all production systems have to control emissions. Therefore, for each section the cost is increased. This model, however, proved that using controlled carbon emissions costs within each cost itemised saved more than other models researched in the existing literature (Leung, 2007). The same data were used in this study and found more savings than in other studies. The geometric programming procedure was employed for obtaining a closed-form global optimum solution of this model, because this type of solution is difficult to find by applying calculus to a posynomial function. As the degree of difficulty is positive, a unique result is impossible to obtain. However, the same solution procedure as was used by Leung (2007) was used in this study and obtained a quasi-closed-form solution. This model proved that a reliable production system can produce more perfect products and can save the environment by controlling carbon emissions. This model can be extended to a model that includes a budget constraint. To make the system reliable, the energy cost and water resource cost can be included within a reliable production system. Thus, considering these factors, the research would point to interesting findings. A data analysis could help to make the system more reliable by reducing the ripple effect of supply chain management (Dubey et al., 2019). This model considered a single-stage production system, which could be extended to a multi-stage production system with many products or assembled products. A new design of two or three industries could result in the outcome of them working together to reduce carbon emissions to solve other unreliability issues in order to make the system more reliable. To make a production system fully reliable, a large amount of initial investment is needed, and as a result, the initial setup costs might increase. The setup costs of a reliable production system might be reduced by investment (Sarkar et al., 2014; Moon et al., 2014). This study could be extended by considering a trade-credit policy, and a constraint optimisation of geometric programming could be utilised. Furthermore, the production flexibility with learning effect could be considered together with cost savings for further extension of a multi-item reliable production system.

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