

ANALYZING THE EFFECTS OF USING BOTH FOLDABLE AND STANDARD CONTAINERS IN OCEAN TRANSPORTATION

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We analyze the effects of foldable containers using a newly developed multi-port and multi-period container planning model. The proposed model is a large-scale optimization problem, for which we develop an efficient heuristic algorithm to get near-optimal solutions within a reasonable time. Our model improves existing models by including practical assumptions on the supply of empty containers at each port in each period. Through intensive computational experiments, we analyze the effect of the imbalance in demand and the decreases in the purchasing cost and the handling cost of foldable containers on the potential economic benefits of deploying foldable containers in ocean transportation.

Keywords: empty container repositioning; foldable container; ocean transportation

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1. INTRODUCTION

Allocating containers to meet demand is one of the most challenging jobs of a shipping company. The allocation decision is complicated by an imbalance in the demand and supply of containers at each port during each period. At deficit ports, a shipping company may use empty containers stored in inventory and/or purchase new ones. A shipping company can also reposition empty containers from surplus ports to deficit ports. These strategies must be completed while minimizing the total costs for purchasing, repositioning, and storage. The use of foldable containers reduces inventory and repositioning costs by saving on storage space, but it involves folding and unfolding costs. Therefore, whether foldable containers can save on the total cost of container allocation is an interesting issue for study.

Numerous studies on empty container repositioning have been published. In early research, Crainic *et al.* (1993) proposed dynamic deterministic formulations for empty container allocation problems inland transportation. Then, they proposed a two-stage stochastic programming model to handle the uncertainty of demand and supply data. Cheung and Chen (1998) proposed a two-stage stochastic network model for the empty container repositioning problem and developed a stochastic quasi-gradient method and a stochastic hybrid approximation algorithm to solve the problem. Recently, Moon *et al.* (2010) proposed a mathematical model for empty container repositioning that takes into account leasing and purchasing of containers. They developed a hybrid genetic algorithm to reduce the computation time. Brouer *et al.* (2011) considered loaded and empty containers as decision variables to be determined simultaneously. They formulated the problem as a multi-commodity flow problem that they solved with a delayed column generation algorithm. Song and Dong (2013) proposed a long-haul liner service route design problem that included route structure design, ship deployment, and empty container repositioning. They included empty container repositioning in the route design. Zhang *et al.* (2014) considered stochastic demand and lost sales for empty container repositioning. They formulated the single-port case as an inventory problem and proposed the optimal policy with a pair of critical points. They also developed a polynomial-time algorithm to determine critical points. Then, they formulated the multi-port problem and developed a polynomial algorithm to obtain an approximation policy. Chen *et al.* (2016) studied a shipping market with carriers while considering empty container repositioning. They proposed the optimal pricing strategy for a monopoly and a duopoly model of the shipping market. Song and Dong (2015) discussed the various details about empty container repositioning with numerous statistics, causes, literature citations, and solutions.

Despite the practical significance of such studies, very few researchers have investigated the effects of using foldable containers. Konings and Thijs (2001) and Konings (2005) analyzed the potential cost savings of foldable containers. Shintani *et al.* (2010) also considered cost savings of foldable containers in the hinterland using a single-

period model. Recently, Moon *et al.* (2013) developed three different multi-port and multi-period container planning models for ocean transportation with different usages of standard and foldable containers. Two models assume that either of the two types of containers is used, and the last one assumes that both types of containers are used. The last model of Moon *et al.* (2013), which we call the mixed usage model, is the first model dealing with the case where both standard and foldable containers are used simultaneously. Moon and Hong (2016) developed a mathematical model for empty container repositioning considering both standard and foldable containers. Due to the highly complex nature of the model, they developed linear programming-based and hybrid genetic algorithms to obtain approximate solutions. Goh and Lee (2016) studied the commercial viability of foldable containers by conducting cost-benefit and sensitivity analyses for operating foldable containers. They showed the realistic cost savings of using foldable containers. In fact, although many container planning models have been presented in the literature, most of the models were concerned with only standard containers (Crainic *et al.* (1993), Shen and Khoong (1995), Cheang and Lim (2005), Li *et al.* (2007), Shintani *et al.* (2007), Dong and Song (2009), Song and Carter (2009), Moon *et al.* (2010), Song and Dong (2010), Meng and Wang (2011), Song and Dong (2011), Song and Zhang (2010))

In the real world, shipping companies would use both standard and foldable containers. Shintani *et al.* (2012) developed an integer programming model to obtain an optimal fleet mix of foldable and standard containers in liner shipping networks and showed that the mixed usage of both standard and foldable containers could potentially save the costs of container fleet management. Recently, Zhang *et al.* (2017) studied the empty container repositioning problem in intermodal transport networks using both standard and foldable containers. A mixed-integer linear program model was formulated, and an artificial bee colony algorithm was developed to find near-optimal solutions quickly. The mixed-usage model of Moon *et al.* (2013) justified using foldable containers in empty container repositioning. However, we found a practical shortcoming in this model. Moon *et al.* (2013) developed three models in a unified framework and assumed that the number of empty containers supplied from consignees was a given parameter. This assumption does not affect the practicality of models in which one type of containers is used, but it critically damages the mixed-usage model. When the supply of empty containers is assumed as a specific parameter, the purchase of one type of containers in earlier periods does not relate with the supply of the same type needed in later periods. In an interesting finding of the numerical experiments of Moon *et al.* (2013), no foldable containers were purchased in the mixed-usage model. We think that the result is due to the unrealistic assumption about the supply of containers.

In this paper, we analyze the effects of the foldable container when both standard and foldable containers are used, which is a more realistic assumption about the supply of containers than the assumption used in the single-container models. For this purpose, we developed a new multi-port and multi-period model. In our new model, the supply of empty containers at each port in each period is not given as a parameter; rather, empty containers are assumed to be supplied after devanning (i.e., the process at the destination port in which containers are delivered to customers, unpacked, and then returned to the port) is finished. We also assume that the time needed for devanning is not deterministic. Using this more realistic model, we carried out the same numerical experiments as done in Moon *et al.* (2013) and show the extent to which foldable containers can replace standard ones.

The remainder of the paper is organized as follows. In Section 2, we describe the new model and compare it with the model of Moon *et al.* (2013). In Section 3, we present an algorithm to solve the new model, and in Section 4, we report the numerical experiments. Some concluding remarks are presented in Section 5.

2. MODEL FORMULATION

In this section, we present two container planning models in which both standard and foldable containers are used simultaneously. One is the mixed-usage model of Moon *et al.* (2013), and the other is a new model that we developed to remedy the unrealistic assumptions of Moon *et al.* (2013). In both container planning models, the demand from a departure port to a destination port in each period is assumed known. To satisfy the demand, a shipping company can use containers available in different ways: stocked in inventory; transported to a port fully loaded and available after the freight is unloaded and returned to the port, and repositioned empty and transported to a port for repositioning. The available containers that exceed the demand can be stocked or repositioned.

Either a standard or a foldable container can be used, but when a foldable container is provided in the folded state, it must undergo an unfolding operation. A stocked and repositioned foldable container is delivered in the folded state. A foldable container supplied after the freight is unloaded may be delivered either in the unfolded or folded state, as Shintani *et al.* (2010) discussed. In the model of Moon *et al.* (2013), all the foldable containers are assumed to be delivered in the unfolded state, but in our new model, we allow the supplied foldable containers to be delivered in both states. Both container planning models are used to decide the number of containers for each type to purchase and reposition to minimize the sum of the costs for purchasing, repositioning, holding inventory, and unfolding/folding the containers.

2.1 Notations

We use the following notations to describe the parameters:

- P : set of ports, $P = \{1, 2, \dots, n_p\}$
 T : set of periods, $T = \{1, 2, \dots, n_t\}$
 D_{ijt} : demand for empty containers from port i to port j in period t
 H_i^S : unit storage cost of a standard container at port i in a period
 H_i^F : unit storage cost of a foldable container at port i in a period
 A_i^S : amortized unit purchasing cost of a standard container at port i
 A_i^F : amortized unit purchasing cost of a foldable container at port i
 C_{ij}^S : unit repositioning cost of a standard container from port i to port j
 C_{ij}^F : unit repositioning cost of a foldable container from port i to port j
 L_i^F : unit folding cost of a foldable container at port i
 L_i^U : unit unfolding cost of a foldable container at port i
 τ_{ij} : transportation time (in terms of time periods) from port i to port j .

The following notations are used as decision variables:

- x_{ijt}^S : number of standard containers to be used to satisfy the demand from port i to port j in period t
 x_{ijt}^F : number of foldable containers to be used to satisfy the demand from port i to port j in period t
 f_{ijt}^S : number of standard containers to be transported (for repositioning) from port i to port j in period t
 f_{ijt}^F : number of foldable containers to be transported (for repositioning) from port i to port j in period t
 y_{it}^S : number of standard containers to be purchased at port i in period t
 y_{it}^F : number of foldable containers to be purchased at port i in period t
 I_{it}^S : inventory level of standard containers at port i in period t
 I_{it}^F : inventory level of foldable containers at port i in period t
 l_{it} : number of foldable containers unfolded at port i in period t
 k_{it} : number of foldable containers folded at port i in period t .

In addition, we use the following notations to present an aggregated term:

$$D_{it} = \sum_{j \in P - \{i\}} D_{ijt}; \quad x_{it}^S = \sum_{\substack{j \in P - \{i\} \\ \tau_{ij} + t \leq n_t}} x_{ijt}^S; \quad x_{it}^F = \sum_{\substack{j \in P - \{i\} \\ \tau_{ij} + t \leq n_t}} x_{ijt}^F.$$

We also need the following notations to represent the number of standard and foldable containers supplied at port i in period t after devanning is finished:

- W_{it}^S : number of standard containers to be supplied to port i in period t
 W_{it}^{FU} : number of foldable containers to be supplied in the unfolded state to port i in period t
 W_{it}^{FF} : number of foldable containers to be supplied in the folded state to port i in period t .

The notation terms were assumed to be given in Moon *et al.* (2013), but they are decision variables in our new model.

2.2 Total cost function and constraints

In both models, the total cost consists of purchasing, repositioning, storage, unfolding, and folding expenses and can be described as follows:

$$\begin{aligned}
 TC = & \sum_{i \in P} \sum_{j \in P - \{i\}} \sum_{t \in T} (C_{ij}^S f_{ijt}^S + C_{ij}^F f_{ijt}^F) + \sum_{i \in P} \sum_{t \in T} (H_i^S I_{it}^S + H_i^F I_{it}^F) \\
 & + \sum_{i \in P} \sum_{t \in T} (n_t - t + 1) (A_i^S y_{it}^S + A_i^F y_{it}^F) + \sum_{i \in P} \sum_{t \in T} \{L_i^U l_{it} + L_i^F k_{it}\}
 \end{aligned}$$

The constraints used in our study were used in both the Moon *et al.* (2013) model and in our new model. We must satisfy all of the following demands:

$$x_{ijt}^S + x_{ijt}^F = D_{ijt}, \quad \forall i, j \in P, t \in T. \quad (1)$$

They can be expressed in the following aggregated form:

$$x_{it}^S + x_{it}^F = D_{it}, \quad \forall i \in P, t \in T \quad (1')$$

Constraints (2) and (3) are the inventory balance constraints at each port in each period for the standard and the foldable containers, respectively.

$$I_{it-1}^S + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} < t}} f_{jit-\tau_{ji}}^S + y_{it}^S + W_{it}^S = x_{it}^S + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ij} + t \leq n_t}} f_{ijt}^S + I_{it}^S \quad \forall i \in P, t \in T \quad (2)$$

$$I_{it-1}^F + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} < t}} f_{jit-\tau_{ji}}^F + y_{it}^F + W_{it}^{FU} + W_{it}^{FF} = x_{it}^F + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ij} + t \leq n_t}} f_{ijt}^F + I_{it}^F \quad \forall i \in P, t \in T \quad (3)$$

We also need a set of constraints to define the variables l_{it} and k_{it} . A number of foldable containers need to be unfolded and folded at port i in period t . The foldable containers supplied in the unfolded state must be used first because the folding and unfolding operation generates cost. Therefore, if $x_{it}^F \geq W_{it}^{FU}$, we would unfold $x_{it}^F - W_{it}^{FU}$ containers to satisfy a portion of demand and in addition, $W_{it}^{FU} - x_{it}^F$ containers are folded to keep for inventory or repositioning. In other words, $l_{it} = \max[x_{it}^F - W_{it}^{FU}, 0]$ and $k_{it} = \max[W_{it}^{FU} - x_{it}^F, 0]$ for all $i \in P$ and $t \in T$. These relations can be described as the following constraints:

$$x_{it}^F - W_{it}^{FU} = l_{it} - k_{it}, \quad \forall i \in P, t \in T. \quad (4)$$

2.3 Mixed usage model of Moon *et al.* (2013)

We first introduce the model developed by Moon *et al.* (2013). Their model assumes that W_{it}^S , W_{it}^{FU} , and W_{it}^{FF} are given and $W_{it}^{FF} = 0$. Their model can be expressed as follows:

$$(P1) \quad \text{minimize } TC \\ \text{subject to. } (1'), (2), (3), (4),$$

where all variables are nonnegative integers.

The original mixed-usage model in Moon *et al.* (2013) has an additional repositioning capacity constraint, but we present the simplified version that appeared in Myung and Moon (2014). When a single type of containers is used, no decision variables exist in (1) or (1'), and thus the number of empty containers supplied can be estimated from demand. However, in the mixed-usage model, W_{it}^S , W_{it}^{FU} , and W_{it}^{FF} in period t are closely related with x_{it}^S and x_{it}^F from earlier periods. Therefore, it is impractical to assume that the number of empty containers supplied is a given parameter.

2.4 The new model

In this paper, we develop a realistic model in which containers are supplied to each port after being delivered to customers and unpacked, and then returned to the port. As discussed in Shintani *et al.* (2010), a foldable container may be returned to a port either in the unfolded or folded state. The portion of foldable containers returned in either state depends on the cost structure in hinterland transportation. Our model deals only with ocean transportation and assumes that the portion of foldable containers returned in each state to each port is given as a parameter. We also assume that the time needed for devanning is not deterministic; instead, a shipping company can estimate the devanning time of each loaded container. Therefore, we define the following parameters to reflect the new realistic assumption:

v_i : maximum devanning time (in terms of time periods) at port i

α_{ik} : portion of empty containers returned to port i , k periods after transported to the port

β_i : portion of foldable containers returned in the unfolded state at port i .

In our new model, W_{it}^S , W_{it}^{FU} , and W_{it}^{FF} are decision variables and satisfy the following constraints:

$$W_{it}^S = \sum_{k=0}^{v_i} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-k}^S, \quad \forall i \in P, t \in T \quad (5)$$

$$W_{it}^{FU} = \beta_i \sum_{k=0}^{v_i} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-k}^F, \quad \forall i \in P, t \in T \quad (6)$$

$$W_{it}^{FF} = (1 - \beta_i) \sum_{k=0}^{v_i} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-k}^F, \quad \forall i \in P, t \in T \quad (7)$$

If we want the variables W_{it}^S , W_{it}^{FU} , and W_{it}^{FF} to have integer values, we need to modify Constraints (5), (6), and (7). For this purpose, we assume that among the total number of the loaded containers transported to port i in period t , i.e., $\sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} < t}} x_{jit-\tau_{ji}}^{S(or F)}$, the number of empty containers returned k periods after transported to the port is

$\left[\alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} < t}} x_{jit-\tau_{ji}}^{S(or F)} \right]$ when $k = 0, 1, \dots, v_i - 1$ and the remaining amounts are returned v_i periods after. We also

assume that the number of foldable containers returned in the unfolded state at port i is $\lfloor \beta_i W_{it}^F \rfloor$ where W_{it}^F is the number of foldable containers to be supplied to port i in period t . Under these assumptions, (5), (6), and (7) are modified as follows:

$$W_{it}^S = \left[\sum_{k=0}^{v_i-1} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-k}^S \right] + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + v_i < t}} x_{jit-\tau_{ji}-v_i}^S - \left[\sum_{k=0}^{v_i-1} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-v_i}^S \right], \quad \forall i \in P, t \in T \quad (5')$$

$$W_{it}^{FU} = \lfloor \beta_i W_{it}^F \rfloor, \quad \forall i \in P, t \in T \quad (6')$$

$$W_{it}^{FF} = W_{it}^F - W_{it}^{FU}, \quad \forall i \in P, t \in T \quad (7')$$

where,

$$W_{it}^F = \left[\sum_{k=0}^{v_i-1} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-k}^F \right] + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + v_i < t}} x_{jit-\tau_{ji}-v_i}^F - \left[\sum_{k=0}^{v_i-1} \alpha_{ik} \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} + k < t}} x_{jit-\tau_{ji}-v_i}^F \right].$$

Our new model is shown as follows:

$$(P2) \quad \text{minimize } TC \\ \text{subject to. } (1), (2), (3), (4), (5'), (6'), (7'),$$

where all variables are nonnegative integers.

3. HEURISTIC ALGORITHM

(P2) is a pure integer programming problem and has complicated sets of Constraints (5'), (6'), and (7'). Moreover, (P2) for a real-world problem would be very large (for example, (P2) has about 150,000 constraints and 570,000 variables when the number of ports and periods are 30 and 156, respectively), and it would take a great deal of computing time to obtain an exact solution. Thus, we developed a heuristic to get near-optimal solutions to (P2) based on optimal solutions to the following linear programming problem.

$$(LP) \quad \text{minimize } TC \\ \text{subject to. } (1), (2), (3), (4), (5), (6), \text{ and } (7),$$

where all variables are nonnegative values.

Our algorithm first solves (LP) and rounds any fractional solution to get a feasible integer solution of (P2). However, it is not easy to round fractional values in a way that the resulting solution satisfies all the constraints of (P2). Therefore, we first rounded the values of the variables, x_{ijt}^S and x_{ijt}^F while satisfying (1), which is easy because of the simple structure of Constraints (1). Then, we used the fixed integer values for variables, x_{ijt}^S and x_{ijt}^F . We determined the variables l_{it} , k_{it} , W_{it}^S , W_{it}^{FU} , and W_{it}^{FF} through Constraints (4), (5'), (6'), and (7'). The remaining variables can be found using the following subproblem (SP) of (P2).

$$\begin{aligned}
(\text{SP}) \quad & \text{Min} \sum_{i \in P} \sum_{j \in P - \{i\}} \sum_{t \in T} (C_{ij}^S f_{ijt}^S + C_{ij}^F f_{ijt}^F) + \sum_{i \in P} \sum_{t \in T} (H_i^S I_{it}^S + H_i^F I_{it}^F) + \sum_{i \in P} \sum_{t \in T} (n_t - t + 1)(A_i^S y_{it}^S + A_i^F y_{it}^F) \\
& \text{s. t.} \quad I_{it-1}^S + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} < t}} f_{jit-\tau_{ji}}^S + y_{it}^S + R_{it}^S = \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ij} + t \leq n_t}} f_{ijt}^S + I_{it}^S \quad \forall i \in P, t \in T \quad (2') \\
& I_{it-1}^F + \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ji} < t}} f_{jit-\tau_{ji}}^F + y_{it}^F + R_{it}^F = \sum_{\substack{j \in P \setminus \{i\} \\ \tau_{ij} + t \leq n_t}} f_{ijt}^F + I_{it}^F \quad \forall i \in P, t \in T \quad (3')
\end{aligned}$$

where $R_{it}^S = W_{it}^S - x_{it}^S$ and $R_{it}^F = W_{it}^{FU} + W_{it}^{FF} - x_{it}^F$ for each $i \in P$ and $t \in T$, respectively, and all variables are nonnegative integers.

When all the variables, x_{ijt}^S and x_{ijt}^F are fixed, R_{it}^S and R_{it}^F are also fixed. Therefore, R_{it}^S and R_{it}^F are constants in (SP). Our two-stage approach is based on the observation that (SP) can be viewed as a single-commodity minimum-cost flow problem and thus we can obtain an optimal solution of (SP) using an efficient minimum-cost network flow algorithm. The following lemma verifies our assertion.

Lemma 1. A directed network exists such that a feasible flow on the network has a one-to-one correspondence to a feasible solution of (SP).

Proof. We show that the system of Equations (2') and (3') can be modified to represent the flow conservation constraints of a network flow model on a directed network. We add all equations in the form of (2') and (3') and obtain the following equation:

$$\sum_{i \in P} \sum_{t \in T} (y_{it}^S + y_{it}^F) + \sum_{i \in P} \sum_{t \in T} (R_{it}^S + R_{it}^F) = \sum_{i \in P} (I_{in_t}^S + I_{in_t}^F) \quad (0)$$

We claim that the coefficients of the system of Equations (0), (2'), and (3') constitute a node-arc incidence matrix. Each variable appears in exactly two equations, and the coefficient of each variable is 1 in one equation and -1 in the other (assuming that we rearrange the variables). The corresponding network is shown in Figure 1, in which Node 0 represents Equation (0) and Nodes $i-t-S$ and $i-t-F$ represent Equations (2') and (3') for each $i \in P$ and $t \in T$, respectively. Numbers associated with nodes indicate the required net flow of the node. The net flow of a node is defined as inflows subtracted from the outflows of the node. In the network, we associate each arc with a variable such that the flow of the arc corresponds to the value of the variable labeled on the arc.

□

We test the performance of our two-stage rounding heuristic on a fairly large number of instances and show that in most of the cases, our heuristic finds near-optimal solutions within a reasonable amount of time.

4. EXPERIMENTS USING THE NEW MODEL

In this section, we describe the numerical experiments using our new model. For intensive tests, we generated many data instances. We first tested the performance of our two-stage rounding heuristic and then conducted a scenario analysis to determine the extent that foldable containers should replace standard containers under different market situations. We generated various data sets of different parameter values for the demand pattern, the numbers of ports and periods, the maximum devanning time, the purchasing costs of foldable containers, and the folding and unfolding costs.

4.1 Generation of test instances of (P2)

To prepare the test instances of (P2), we first generated the basic instances and then generated the test instances based on them. We generated the basic instances with various numbers of ports (3, 10, and 30) and periods (52, 104, and 156). Here, we assume that a period corresponds to one week. For each of the 9 combinations of n_p and n_t , we generated 10 basic instances. For each of these instances, demand between two ports for each period (D_{ijt}) was randomly selected between 100 and 200 containers such that $D_{ijt} = D_{jit}$. So, demand data for each period of the basic instances is symmetric. The other parameters of the basic instances are as follows: (1) Based on the market prices of standard (\$4,500) and foldable (\$10,000) containers, 15 years of the economic lifetime, and 15% of yearly capital cost, the amortized unit

purchasing cost of a foldable container per period was assumed to be about twice as much as that of a standard container (\$15 for a standard container, \$33 for a foldable container); (2) The repositioning cost was estimated as the weekly sailing cost (\$140/standard container) multiplied by τ_{ij} periods and the estimated storage cost was \$8/standard container in a one-week period; (3) The repositioning and storage costs for a foldable container was 1/4 of those for a standard container; (4) The folding and unfolding costs are the same (\$50); (5) The transportation time, τ_{ij} is selected randomly between 1 and 5 periods; (6) The maximum devanning period of each port, v_i is randomly selected in the range of [1, 4]; (7) There is no on-hand inventory of standard and foldable containers.

Our test instances consist of those basic instances and the additional instances obtained by modifying the basic instances to reflect an imbalance in the demand and the supply of containers at each port. For each basic instance, we generated two more instances with different types of demand patterns. We selected a subset of ports and made outgoing demand from a port in the set higher than incoming demand to that port. For example, when $n_p = 3$, we selected $S = \{1\}$ and set $D_{ijt} = k \times D_{jit}$ for $i \in S, j \notin S, t \in T$ where $k = 2, 3$. Note that demand data of each basic instance corresponds to the case where $k = 1$. For the cases of 10 and 30 ports, we set $S = \{1, 2, 3\}$ and $S = \{1, 2, \dots, 10\}$, respectively. To sum up, we generated 10 test instances for each of the 27 combinations of n_p, n_t , and k .

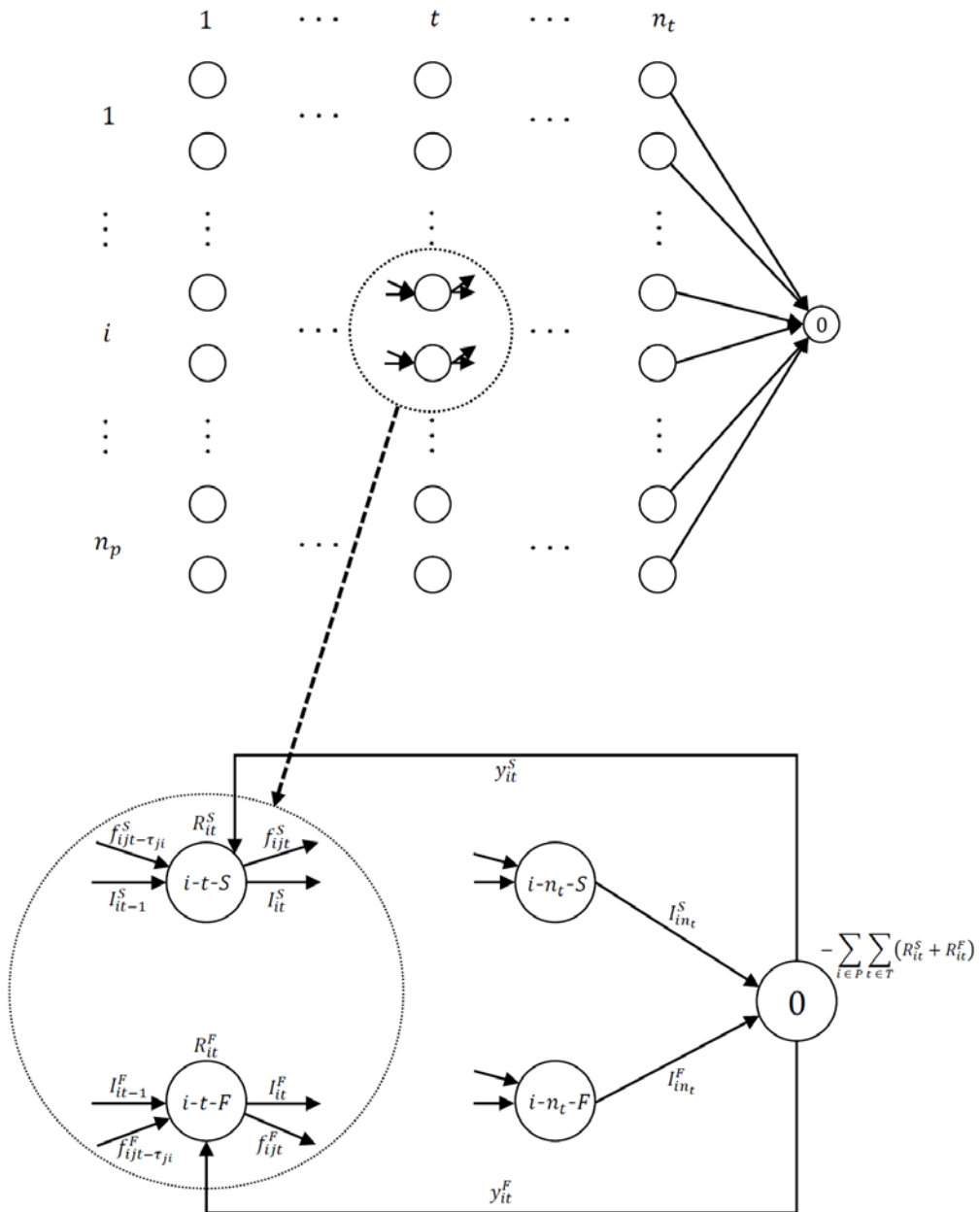


Figure 1. Decision variables corresponding to arcs

4.2 Performance of the heuristic

Our heuristic sequentially solves two linear programming problems, (LP) and (SP). As shown in Section 3, (SP) is a minimum-cost network flow problem and the linear programming solution of (SP) has an integral property. To evaluate the quality of our heuristic solutions, we need to find an optimal solution of (P2) but that is a difficult job, especially due to Constraints (5'), (6'), and (7'). So, we indirectly test the performance of the heuristic based on the observation that for the specific values of α_{ik} and β_i , the objective value of (LP) is a lower bound for the optimal objective value of (P2). One such case is that $\alpha_{ik} = 1$, for $k = v_i$ and 0, otherwise and $\beta_i = 0$. For the instances with such α_{ik} and β_i , we evaluate the performance of our algorithm by comparing the objective value of our heuristic ($z(H)$) with the objective value obtained from (LP) ($z(LP)$). We conducted computational tests on the test instances generated as described in Section 4.1. The tests was conducted on a PC (2.9 GHz CPU, 16 G RAM), and we used a commercial optimization software library (Xpress 8.0 (2016)) to implement our heuristic algorithm. The comparison between the two objective values is shown in Table 1. The figures in the table represent the averages for the 10 instances for each combination of the number of ports ('# Ports'), the number of periods ('# Periods'), and the demand imbalance parameter k . The table also shows the average computation times for our heuristic.

Table 1. Quality of our heuristic and computing time

# Ports	# Periods	$z(LP)/z(H)$ (%)			Computation time (seconds)		
		$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
3	52	100.00	100.00	100.00	0.052	0.040	0.048
	104	100.00	99.98	99.99	0.102	0.123	0.103
	156	100.00	99.98	99.99	0.165	0.191	0.194
10	52	100.00	100.00	100.00	0.476	0.492	0.469
	104	100.00	100.00	100.00	1.422	1.525	1.414
	156	100.00	100.00	100.00	2.754	2.888	2.790
30	52	100.00	100.00	100.00	6.648	6.838	6.885
	104	100.00	100.00	100.00	23.021	23.140	22.796
	156	100.00	100.00	100.00	44.689	41.841	42.913

As shown in Table 1, the results revealed that our heuristic finds near-optimal solutions for even fairly large problems within a reasonable amount of time. The average ratio of $z(LP)$ to $z(H)$ is 100% for most of the 27 combinations, which means the solutions obtained by our heuristic are almost optimal solutions to (P2). Also, note that the average computation times of our heuristic to solve test instances of (P2) for the case of 30 ports and 156 periods, which are very large-scale integer programs as mentioned in Section 3, is less than 45 seconds on a PC.

4.3 Scenario analysis

As mentioned in Section 1, Moon *et al.* (2013) conducted a scenario analysis through numerical experiments using their mixed-usage model. Their results showed that no foldable containers were purchased unless the purchase cost of a foldable container was less than the price of a standard container. The reason, we think, is due to their assumption that the number of empty containers supplied is given as a parameter. Here, we carry out similar numerical experiments to determine the ratio of foldable containers to all the containers used in our new mixed-usage model.

Our first set of data instances were generated as described in Section 4.1 and had three different demand patterns (with $k = 1, 2$, and 3) and nine different combinations of n_p and n_t . For the scenario analysis, we set $\alpha_{ik} = 1$ only if $k = v_i$ and set it to 0, otherwise. Since some portion of empty containers can be returned to the port earlier than the maximum devanning time in practice, so our analysis considers the worst case with respect to the devanning time and the results remain valid regardless of the actual distribution of the portions of empty containers returned earlier than the maximum devanning time. We also set $\beta_i = 0$. The results of the experiments are described in Table 2 and all figures in the table represent the average for the 10 data instances. The cost reduction columns in the table show that the ratio of the reduced cost to the total cost incurred when only standard containers were used.

Unlike the results in Moon *et al.* (2013), foldable containers were purchased in our model. The result supports our conjecture that the unrealistic assumption on supply parameters led to no foldable containers purchased in the numerical

experiments in Moon *et al.* (2013). However, in our experiments, the foldable to standard container ratio was also lower than we expected. The reason, we think, is that the purchase of a foldable container is beneficial only when the cost savings in repositioning and stocking foldable containers reach a certain level. Our reasoning is supported by the results that showed that the portion of foldable containers increased as k increased (i.e., the imbalance in the demand and the supply of containers rises). In addition, the increase in the number of ports suppresses the use of foldable containers, which are purchased over relatively long planning periods. This finding may be attributed to the mitigation of the imbalance in the demand and the supply of containers when more ports are used and when repositioning and stocking are undertaken over a long period.

Next, we tested the effects of the time needed for devanning by changing the range of maximum devanning periods. We tested two more ranges of devanning periods, [1, 3] and [1, 2]. The results are summarized in Table 3, and each figure in the table represents the average for the 10 data instances. In comparison to the results given in Table 2 for which the range of maximum devanning periods was [1, 4], we can see that the decreases in the time needed for devanning increases the economic benefit of deploying foldable containers. These results are intuitively clear in that longer devanning time causes lower utilization of foldable containers.

Table 2. Effects of demand imbalance on the portion of foldable containers and the total cost reduction

k	# Ports	# Periods	Max Devanning Periods : 1 ~ 4					Cost Reduction (%)
			Portion of foldable containers (%)					
			Demand	Reposition	Storage	Purchase		
1	3	52	0.00%	1.15%	0.02%	0.01%	0.00%	
		104	0.02%	7.74%	0.07%	0.03%	0.01%	
		156	0.00%	0.58%	0.01%	0.00%	0.00%	
	10	52	0.00%	0.00%	0.00%	0.00%	0.00%	
		104	0.02%	4.15%	0.07%	0.03%	0.00%	
		156	0.05%	12.04%	0.11%	0.07%	0.01%	
	30	52	0.00%	0.00%	0.00%	0.00%	0.00%	
		104	0.00%	1.24%	0.01%	0.00%	0.00%	
		156	0.01%	1.72%	0.02%	0.01%	0.00%	
2	3	52	4.69%	24.20%	1.95%	3.55%	0.39%	
		104	15.30%	64.01%	15.45%	17.65%	4.28%	
		156	14.74%	61.73%	10.00%	17.27%	4.06%	
	10	52	2.84%	17.55%	0.41%	2.69%	0.33%	
		104	5.58%	32.86%	3.36%	6.63%	1.48%	
		156	7.20%	39.03%	3.36%	8.56%	2.56%	
	30	52	0.12%	0.77%	0.02%	0.11%	0.01%	
		104	0.75%	4.76%	0.07%	0.67%	0.07%	
		156	0.95%	5.90%	0.11%	0.90%	0.13%	
3	3	52	7.77%	24.71%	2.63%	4.81%	0.52%	
		104	24.60%	64.41%	21.74%	24.01%	5.24%	
		156	23.52%	61.83%	14.73%	24.00%	5.13%	
	10	52	4.28%	15.56%	0.52%	3.46%	0.39%	
		104	9.14%	32.09%	3.71%	9.46%	1.83%	
		156	11.94%	38.09%	4.49%	12.66%	3.22%	
	30	52	0.15%	0.55%	0.01%	0.13%	0.01%	
		104	1.01%	3.71%	0.07%	0.83%	0.08%	
		156	1.35%	4.96%	0.11%	1.17%	0.15%	

We also tested the extent to which the portion of foldable containers increases when both the purchase price of a foldable container and the folding and unfolding costs decrease. The results are shown in Tables 4 and 5, and each figure in the table represents the average for the 10 data instances. Naturally, the decreases in folding and unfolding costs and the purchase price of a foldable container increase the usage of foldable containers. The results help to estimate the conditions that affect the time and extent that standard containers are replaced by foldable containers.

Table 3. Effects of maximum devanning periods on the portion of foldable containers and the total cost reduction

k	# Ports	# Periods	Max Devanning Periods : 1 ~ 3					Max Devanning Periods : 1 ~ 2				
			Portion of foldable containers (%)				Cost Reduction (%)	Portion of foldable containers (%)				Cost Reduction (%)
			Demand	Reposition	Storage	Purchase		Demand	Reposition	Storage	Purchase	
1	3	52	0.00%	1.93%	0.02%	0.01%	0.00%	0.00%	2.18%	0.02%	0.01%	0.00%
		104	0.01%	3.93%	0.03%	0.02%	0.01%	0.01%	4.19%	0.03%	0.02%	0.01%
		156	0.02%	6.08%	0.07%	0.03%	0.00%	0.02%	6.40%	0.07%	0.04%	0.00%
	10	52	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.16%	0.04%	0.01%	0.00%
		104	0.03%	5.73%	0.05%	0.04%	0.00%	0.04%	7.97%	0.08%	0.05%	0.01%
		156	0.07%	16.54%	0.14%	0.09%	0.02%	0.08%	20.34%	0.20%	0.11%	0.02%
	30	52	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.19%	0.00%	0.00%	0.00%
		104	0.01%	2.50%	0.02%	0.01%	0.00%	0.01%	4.34%	0.03%	0.02%	0.00%
		156	0.01%	3.55%	0.03%	0.01%	0.00%	0.02%	6.11%	0.04%	0.02%	0.00%
2	3	52	6.70%	32.60%	4.80%	6.05%	0.54%	8.38%	39.74%	7.50%	8.25%	0.73%
		104	17.62%	73.41%	18.23%	20.63%	5.20%	20.34%	84.07%	20.36%	23.69%	6.11%
		156	14.75%	61.90%	9.43%	17.61%	5.16%	16.94%	70.53%	11.94%	20.21%	6.39%
	10	52	3.25%	19.66%	0.38%	3.20%	0.41%	4.16%	24.57%	0.51%	4.21%	0.61%
		104	6.62%	37.90%	3.70%	8.12%	1.87%	7.14%	40.25%	3.71%	8.94%	2.42%
		156	7.98%	43.22%	4.13%	9.76%	3.08%	9.20%	48.82%	4.95%	11.13%	3.59%
	30	52	0.20%	1.30%	0.03%	0.19%	0.02%	0.36%	2.30%	0.05%	0.37%	0.03%
		104	1.01%	6.27%	0.13%	0.95%	0.10%	1.49%	8.92%	0.17%	1.46%	0.14%
		156	1.06%	6.63%	0.12%	1.06%	0.16%	1.74%	10.52%	0.13%	1.81%	0.28%
3	3	52	10.69%	32.52%	6.30%	7.99%	0.70%	13.91%	41.19%	9.92%	11.33%	0.95%
		104	27.88%	73.13%	25.59%	27.35%	6.26%	32.12%	83.96%	28.58%	31.16%	7.29%
		156	23.51%	62.42%	13.85%	24.17%	6.42%	27.07%	70.69%	17.32%	27.50%	7.82%
	10	52	5.01%	17.93%	0.29%	4.18%	0.48%	6.48%	22.79%	0.60%	5.58%	0.71%
		104	11.07%	37.03%	4.66%	11.77%	2.31%	12.02%	39.73%	5.22%	13.06%	2.97%
		156	13.28%	42.07%	6.04%	14.30%	3.84%	15.20%	47.59%	7.06%	16.30%	4.45%
	30	52	0.27%	1.10%	0.03%	0.24%	0.02%	0.49%	1.91%	0.04%	0.45%	0.04%
		104	1.43%	5.30%	0.13%	1.23%	0.12%	2.01%	7.22%	0.16%	1.79%	0.16%
		156	1.54%	5.72%	0.13%	1.43%	0.18%	2.47%	8.96%	0.14%	2.33%	0.31%

Table 4. Effects of reduction in purchase cost of foldable containers on the portion of foldable containers and the total cost reduction

k	# Ports	# Periods	Purchase cost reduction (%) : As-Is (0%)					Purchase cost reduction (%) : 15%					Purchase cost reduction (%) : 30%				
			Portion of foldable containers (%)				Cost Reduction (%)	Portion of foldable containers (%)				Cost Reduction (%)	Portion of foldable containers (%)				Cost Reduction (%)
			Demand	Reposition	Storage	Purchase		Demand	Reposition	Storage	Purchase		Demand	Reposition	Storage	Purchase	
1	3	52	0.00%	1.15%	0.02%	0.01%	0.00%	0.02%	6.23%	0.19%	0.05%	0.01%	0.19%	30.29%	0.78%	0.29%	0.05%
		104	0.02%	7.74%	0.07%	0.03%	0.01%	0.03%	11.96%	0.12%	0.05%	0.02%	0.10%	32.93%	1.04%	0.24%	0.06%
		156	0.00%	0.58%	0.01%	0.00%	0.00%	0.04%	11.77%	0.15%	0.06%	0.01%	0.14%	38.75%	1.33%	0.29%	0.05%
	10	52	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	7.49%	0.11%	0.05%	0.00%	0.16%	29.62%	0.67%	0.22%	0.04%
		104	0.02%	4.15%	0.07%	0.03%	0.00%	0.09%	18.13%	0.28%	0.12%	0.02%	0.23%	39.58%	1.47%	0.32%	0.09%
		156	0.05%	12.04%	0.11%	0.07%	0.01%	0.14%	30.60%	0.44%	0.18%	0.05%	0.26%	50.88%	1.92%	0.37%	0.13%
	30	52	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	1.97%	0.01%	0.01%	0.00%	0.06%	16.25%	0.13%	0.07%	0.01%
		104	0.00%	1.24%	0.01%	0.00%	0.00%	0.04%	12.30%	0.09%	0.05%	0.01%	0.12%	30.07%	0.49%	0.14%	0.03%
		156	0.01%	1.72%	0.02%	0.01%	0.00%	0.05%	12.93%	0.09%	0.05%	0.01%	0.11%	28.29%	0.50%	0.13%	0.03%
2	3	52	4.69%	24.20%	1.95%	3.55%	0.39%	14.18%	62.96%	13.79%	14.24%	2.75%	19.57%	80.88%	26.93%	22.01%	7.14%
		104	15.30%	64.01%	15.45%	17.65%	4.28%	20.60%	84.90%	19.49%	23.12%	8.67%	23.13%	94.62%	23.34%	26.05%	13.80%
		156	14.74%	61.73%	10.00%	17.27%	4.06%	17.20%	71.13%	11.56%	19.55%	8.03%	18.37%	75.77%	13.65%	20.94%	12.40%
	10	52	2.84%	17.55%	0.41%	2.69%	0.33%	6.68%	40.42%	2.57%	6.79%	1.52%	11.52%	63.06%	8.22%	12.63%	3.85%
		104	5.58%	32.86%	3.36%	6.63%	1.48%	8.70%	50.05%	6.10%	10.22%	3.94%	12.17%	66.94%	11.86%	14.99%	6.61%
		156	7.20%	39.03%	3.36%	8.56%	2.56%	11.25%	60.96%	5.96%	13.04%	5.01%	13.45%	72.62%	9.88%	15.97%	8.43%
	30	52	0.12%	0.77%	0.02%	0.11%	0.01%	1.36%	8.66%	0.16%	1.23%	0.14%	4.68%	28.11%	0.44%	4.70%	0.87%
		104	0.75%	4.76%	0.07%	0.67%	0.07%	3.48%	20.87%	0.19%	3.37%	0.46%	6.33%	36.80%	0.67%	6.79%	1.78%
		156	0.95%	5.90%	0.11%	0.90%	0.13%	3.33%	20.87%	0.25%	3.35%	0.62%	5.46%	32.82%	1.19%	6.00%	1.91%
3	3	52	7.77%	24.71%	2.63%	4.81%	0.52%	22.73%	63.68%	18.97%	19.53%	3.49%	30.65%	80.97%	39.05%	30.43%	9.03%
		104	24.60%	64.41%	21.74%	24.01%	5.24%	32.53%	84.87%	28.70%	31.17%	10.57%	36.40%	94.83%	35.93%	35.32%	16.80%
		156	23.52%	61.83%	14.73%	24.00%	5.13%	27.45%	71.46%	17.58%	27.31%	10.06%	29.27%	75.88%	20.59%	29.22%	15.50%
	10	52	4.28%	15.56%	0.52%	3.46%	0.39%	10.79%	38.83%	2.87%	9.63%	1.91%	19.41%	62.26%	10.37%	19.31%	5.07%
		104	9.14%	32.09%	3.71%	9.46%	1.83%	14.98%	50.43%	7.65%	15.80%	4.62%	20.53%	67.39%	15.94%	22.88%	8.69%
		156	11.94%	38.09%	4.49%	12.66%	3.22%	18.76%	60.50%	8.52%	19.76%	6.47%	22.69%	72.56%	14.39%	24.59%	11.03%
	30	52	0.15%	0.55%	0.01%	0.13%	0.01%	1.96%	7.51%	0.15%	1.63%	0.17%	7.64%	27.29%	0.47%	7.22%	1.13%
		104	1.01%	3.71%	0.07%	0.83%	0.08%	4.97%	17.85%	0.27%	4.45%	0.55%	10.74%	35.71%	0.42%	10.73%	2.28%
		156	1.35%	4.96%	0.11%	1.17%	0.15%	5.05%	18.78%	0.25%	4.72%	0.75%	9.42%	32.95%	1.10%	9.76%	2.50%

Table 5. Effects of reduction in folding/unfolding cost on the portion of foldable containers and the total cost reduction

k	# Ports	# Periods	Folding/Unfolding cost reduction, As-Is (0%) : \$50					Folding/Unfolding cost reduction, 50% : \$25					Folding/Unfolding cost reduction, 100% : \$0				
			Portion of foldable containers (%)				Cost Reduction (%)	Portion of foldable containers (%)				Cost Reduction (%)	Portion of foldable containers (%)				Cost Reduction (%)
			Demand	Reposition	Storage	Purchase		Demand	Reposition	Storage	Purchase		Demand	Reposition	Storage	Purchase	
1	3	52	0.00%	1.15%	0.02%	0.01%	0.00%	0.02%	5.85%	0.05%	0.03%	0.00%	0.09%	22.59%	0.31%	0.15%	0.03%
		104	0.02%	7.74%	0.07%	0.03%	0.01%	0.03%	12.98%	0.10%	0.05%	0.02%	0.03%	18.19%	0.13%	0.06%	0.03%
		156	0.00%	0.58%	0.01%	0.00%	0.00%	0.04%	13.19%	0.06%	0.04%	0.01%	0.08%	26.83%	0.22%	0.11%	0.03%
	10	52	0.00%	0.00%	0.00%	0.00%	0.00%	0.07%	17.07%	0.15%	0.09%	0.01%	0.33%	60.40%	1.06%	0.36%	0.11%
		104	0.02%	4.15%	0.07%	0.03%	0.00%	0.15%	30.39%	0.28%	0.17%	0.04%	0.36%	62.10%	1.08%	0.39%	0.17%
		156	0.05%	12.04%	0.11%	0.07%	0.01%	0.17%	39.23%	0.41%	0.21%	0.06%	0.31%	63.73%	1.20%	0.36%	0.19%
	30	52	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	11.36%	0.06%	0.04%	0.00%	0.25%	59.85%	0.79%	0.24%	0.08%
		104	0.00%	1.24%	0.01%	0.00%	0.00%	0.09%	24.66%	0.19%	0.09%	0.02%	0.30%	68.20%	0.88%	0.29%	0.12%
		156	0.01%	1.72%	0.02%	0.01%	0.00%	0.10%	25.77%	0.10%	0.09%	0.02%	0.31%	68.84%	0.75%	0.28%	0.13%
2	3	52	4.69%	24.20%	1.95%	3.55%	0.39%	14.12%	68.43%	13.38%	13.94%	3.40%	21.83%	96.96%	30.48%	25.91%	8.71%
		104	15.30%	64.01%	15.45%	17.65%	4.28%	20.45%	86.65%	21.91%	24.20%	8.95%	22.97%	96.30%	21.56%	29.14%	14.67%
		156	14.74%	61.73%	10.00%	17.27%	4.06%	17.95%	75.86%	11.46%	20.79%	8.21%	20.32%	85.18%	12.19%	23.66%	13.11%
	10	52	2.84%	17.55%	0.41%	2.69%	0.33%	7.94%	51.15%	6.75%	8.19%	2.12%	14.10%	84.75%	23.26%	15.81%	6.25%
		104	5.58%	32.86%	3.36%	6.63%	1.48%	9.76%	56.95%	9.52%	11.79%	4.18%	16.11%	89.84%	22.05%	18.92%	9.00%
		156	7.20%	39.03%	3.36%	8.56%	2.56%	11.95%	66.52%	10.15%	14.30%	5.76%	17.07%	93.45%	16.99%	19.73%	10.84%
	30	52	0.12%	0.77%	0.02%	0.11%	0.01%	2.84%	19.54%	0.75%	2.53%	0.44%	9.26%	58.35%	14.19%	8.64%	2.92%
		104	0.75%	4.76%	0.07%	0.67%	0.07%	4.31%	26.45%	1.53%	4.27%	0.98%	12.38%	71.47%	17.05%	12.05%	4.29%
		156	0.95%	5.90%	0.11%	0.90%	0.13%	4.54%	29.14%	1.59%	4.78%	1.09%	11.72%	67.18%	13.50%	11.21%	4.32%
3	3	52	7.77%	24.71%	2.63%	4.81%	0.52%	22.71%	67.93%	15.89%	18.99%	4.33%	35.97%	97.82%	40.32%	38.32%	11.03%
		104	24.60%	64.41%	21.74%	24.01%	5.24%	32.75%	86.44%	29.03%	33.37%	10.91%	37.05%	96.27%	29.48%	41.76%	17.87%
		156	23.52%	61.83%	14.73%	24.00%	5.13%	28.65%	75.43%	16.22%	29.39%	10.29%	32.91%	85.46%	16.66%	34.91%	16.40%
	10	52	4.28%	15.56%	0.52%	3.46%	0.39%	12.46%	47.06%	6.11%	11.27%	2.65%	23.48%	82.69%	26.94%	24.67%	8.12%
		104	9.14%	32.09%	3.71%	9.46%	1.83%	16.37%	56.46%	11.81%	17.88%	5.37%	26.11%	86.69%	28.25%	29.15%	11.56%
		156	11.94%	38.09%	4.49%	12.66%	3.22%	20.00%	65.83%	13.39%	21.88%	7.38%	28.04%	90.94%	22.42%	31.08%	13.92%
	30	52	0.15%	0.55%	0.01%	0.13%	0.01%	4.18%	16.68%	0.21%	3.39%	0.53%	14.23%	52.97%	14.58%	12.74%	3.72%
		104	1.01%	3.71%	0.07%	0.83%	0.08%	6.39%	23.32%	1.03%	5.85%	1.48%	19.35%	66.25%	19.05%	18.49%	5.46%
		156	1.35%	4.96%	0.11%	1.17%	0.15%	7.18%	27.12%	1.02%	7.08%	1.33%	18.21%	62.26%	15.45%	17.29%	5.43%

5. CONCLUSIONS

In this paper, we have considered a multi-port and multi-period container planning problem where both standard and foldable containers can be used simultaneously. We developed a mathematical programming model for deciding the type of containers to use while satisfying demand at the minimum total cost. Our model improves the practicality of operations by setting the supply of empty containers at each port in each period to be equal to the fully loaded containers that arrive at the same port. In the Moon *et al.* (2013) model, the supply of empty containers at each port in each period was given as a parameter and not linked to decisions made in earlier periods. Using the new model, we conducted intensive numerical experiments and analyzed the effect of introducing foldable containers. Through all the computational experiments, we found that a shipping company can save on costs by replacing a certain portion of standard containers with foldable ones. The portion of foldable containers increased as the purchasing price for a foldable container and the folding and unfolding costs decreased.

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