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Wartime logistics model for multi-support unit location–allocation problem with frontline changes

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Abstract

To reflect a realistic, changing front line, wartime logistics are illustrated by a dynamic location–allocation model. In this paper, a mixed integer programming (MIP) model is developed for use in deciding the timing of unit relocation for continuous resupply, safe locations for support units, and delivery amounts that minimize total risk to the logistics service. Total risk in wartime logistics is represented by unsatisfied demand, hazard at the support site, and the number of relocations. The proposed MIP model reflects realistic factors in battle situations, such as maximum distance, vehicle capacity, basic load carried by combat units, and limited supplies during unit relocation. Furthermore, special operators for crossover and mutation are developed to maintain feasibility of possible solutions, and an efficient hybrid genetic algorithm is proposed to find optimal and near-optimal solutions.

Keywords: dynamic location–allocation problem; hybrid genetic algorithm; mixed integer programming; wartime logistics system

1. Introduction

To secure line of communications (LOCs) and continuous supplies are recognized as important factors in wartime success. The supply chain of the Republic of Korea (ROK) Army features a multilevel structure that reserves inventory for emergencies, such as during isolation or urgent deployment. Military commodities are classified into nine categories such as food, ammunition, maintenance item, and so forth. One type of commodity with similar attributes, which is defined as a *class*, has a different priority for transport compared to other types of commodities. The ROK Army logistics structure is composed of a hierarchical organization consisting of supply

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and transportation, maintenance, and ammunition. Materials in one type of class may be handled by more than two support units, and a support unit may transport materials of more than one class. Support units analyze previous logistics requirements, estimate current demand, and deliver materials. Combat units in engagement require resupply and relocate their bases to the site of relative advantage over the enemy for taking the initiative as situations evolve.

The provision of supplies for initiatives at the right time, in the right place, and with appropriate quantity is an integral part of success in wartime logistics. The main decisions involve the timing of relocations to block risks to increasingly long LOCs, determination of a new position to relocate that reduces risk from enemy threats, the amount of supplies that satisfy daily demand. Support units need flexibility to maintain successive supply operations by keeping reserves and adjusting daily supplies as dictated by uncertainties in the battlefield. Frontline changes resulted from the engagement and damage to friendly forces are the main causes of uncertainties in wartime logistics and can lead to problems in establishing future operations. Uncertainties in the battlefield result from difficulties in predicting an enemy attack. The location of combat units and the demand for supplies change with levels of enemy hostility. Hence, to establish continuous supply operations, commanders of support units must consider many factors when deciding when and where to move.

To evaluate the performance of a military logistics services, one must simultaneously analyze the geographical advantages of possible locations of support units as well as service levels necessary to meet demand. Therefore, the focus of this paper is on a location–allocation problem in which the optimal location and delivery schedule are determined at the same time. The proposed model, which includes evaluation factors, is practical for deciding the time of unit relocation, the location that minimizes risk from enemy threats, and the delivery amounts that maximize service levels of a wartime logistics support system. The optimal supply system suggested in this paper might serve as a scientific decision tool that commanders use to make determinations quickly.

Although the location problem is widely studied in the private sector, to adapt it to the army logistics is difficult because of the special characteristics of wartime logistics. Sim et al. (2013) summarized the differences between military and commercial supply chains in terms of goals, objectives, key performance indicators, procurement criteria, demand characteristics, supply chain networks, and processes for product acquisition. Although the military logistics system aligns with an integrated logistics system, the frequently changing environment makes adaptation a challenge. Sim et al. (2013) suggested a mathematical model that minimizes the total cost of the supply chain to determine the optimal location and number of facilities to open. As Sim et al. (2013) pointed out, the goal of the military supply chain is to minimize inventory shortages rather than to minimize inventory holding costs. Also, the demand during wartime is both unstable and unforecastable, supply points are changeable, and material has priority over other products. In these ways, military logistics systems feature several characteristics that differ from those of a private-sector supply chain.

First, for military logistics, support for a successful operational plan is much more important than total cost reduction. To the contrary, improvements to the private-sector supply chain are aimed at maximizing the profit of the enterprise or minimizing the cost. Hence, the purpose of the private-sector supply chain is fundamentally different from that of wartime logistics. Therefore, many researchers have shown that, for the military logistics support system, maximizing the total effectiveness to guarantee a successful operation is more appropriate than the commonly taken approach of reducing total distribution costs. The information from the second-best solution, which is typically more expensive than the best one, could be chosen for a future military operation. Continuous resupply, at any cost, is of the utmost importance to a successful war effort; therefore, securing uninterrupted supply delivery is more important than saving money.

After Wesolowsky (1973) suggested that the relocation cost can be used to solve a dynamic location problem, Levin and Friedman (1982) proposed use of maximizing total effectiveness to establish the deployment plan of support units. They assumed that three main factors influence the effectiveness of wartime logistics services: the proximity of support units to combat units, the safety of the support unit location, and the effort required to occupy the current location. With these factors, Kim (2004) suggested three types of cost related to the cost penalty and unit relocation, and the safety to solve the uncapacitated, multiple support unit, location problem with dynamic programming, and a genetic algorithm (GA). In other words, the total cost not only consisted that calculated for unsatisfied demands but also those for improving the total effectiveness by transferring other factors to the cost calculations. Likewise, Gue (2003) proposed a location and material flow model for sea-based logistics (SBL) with the objective of minimizing the inventory of moving units. Moon (2017) suggested SBL models with a mathematical formulation that can be used to find the minimum number of aircraft liftoffs.

Second, in a rapidly changing battlefield, where units are frequently moved and demands fluctuate according to the damage to combat units, estimating demands of the combat units and the appropriate amount of materials to deliver proves difficult. The dynamic location problem is appropriate to consider uncertainties of the real world for long-term and strategic decisions, such as those associated with changing market demands and facility relocation; however, unlike in private-sector situations, in wartime, as the frontline changes with combat unit movements, support units must change location to secure LOCs while minimizing the enemy threat.

Melo et al. (2009) reviewed numerous research findings on location problems and supply chain management, and they emphasized six classification factors to consider for a location problem: capacity, inventory, procurement, production, routing, and transportation mode. A few researchers have studied the problems in which the capacity and inventory problems are addressed simultaneously (Gue, 2003). Although an important factor for a distribution network (Perl and Daskin, 1985; Lee et al., 2010), routing for a military logistics system can be considered when only a few vehicles are available.

Third, a wartime logistics system requires special scientific tools for commanders to use in making quick decisions while addressing changing situations. Therefore, near optimal alternatives for solving problems in the shortest possible time, rather than solving the most optimal problems over a relatively long time, constitutes military strategy. However, the location problem must be addressed at the same time as the inventory problem, which makes wartime logistics NP-hard problems such that a heuristic approach is required to solve them as quickly and accurately as possible.

Hormozi and Khumawala (1996) used mixed integer programming (MIP) with dynamic programming to generate a guaranteed optimal solution. Dynamic programming that combines each optimal solution does not guarantee an optimal solution over all periods; therefore, many researchers have developed heuristic approaches, including those that integrate an MIP with a dynamic programming and a GA. The application of the GA to solve various types of large-scale NP-hard optimization problems was presented by Chan and Lee (2005), who argued that meta-heuristics are needed to obtain near optimal solutions for problems of modern, complex, and large supply chains. Jaramillo et al. (2002) compared the performance of a GA for different types of location problems. They show that a GA for the capacitated facility location problem yielded an optimal solution but required a long computation time. Kim (2004) proved that the performance by a GA to find solutions, which was based on crossover, was better than dynamic programming for a wartime logistics system. To solve the integrated inventory distribution problem, Abdelmaguid and Dessouky (2006) suggested horizontal and vertical breakdown, to retain the feasibility for crossover, and backward delivery through mutation. Deb (2000) shows that a GA is a good method for addressing a combinatorial problem by summarizing five ways it can be used to handle constraints related to feasibility preservation and a penalty function by distinguishing between feasibility and infeasibility.

For the study presented herein, the time line differs from studies on private-sector logistics. In many logistics problems for business, the decisions about the number and location of warehouses are based on a long-term need. However, on battlefields, decisions are made about the number and location of support units, which must be moved frequently, regardless of the cost incurred; that is, military support units make up a special location–allocation problem. Many studies from the private sector are too limited to be adopted for situations in the military sector, which have distinct characteristics. Although the dynamic facility location problem deals with the changing market, changes in wartime happen very quickly, and the fixed cost to relocate military units is low compared to that of plant relocation. Hence, a specialized location model is required to design a wartime logistics system. The study presented herein can be classified as a dynamic capacitated facility location–allocation problem. The proposed model is used to determine simultaneous timing for relocating support units, determining the sites with fewest enemy threats, and delivering appropriate amounts as needed in the operational environment.

The purpose of this paper is to establish a wartime logistics model compatible with practical considerations. Basic load, which is defined as the quantity a moving unit must accommodate, is considered so that maximum vehicle capacity and maximum distance are used to illustrate the real environment of wartime. In this paper, the sequential locations of support units and the delivery schedule in each period are determined. Unsatisfied demand, resulting from insufficient vehicle capacity, is defined as a shortage of commodities to satisfy demand at the end of each period. The distance from the front line to support units includes hazards imposed by enemy threats. Thus, location of support units affects all logistics services.

In this paper, the MIP model contains realistic constraints, for which feasible solutions are difficult to find with the procedures used in a GA. Therefore, understanding the entire structure of the GA and development of special genetic operators are critical to obtaining the best solutions. Constraints in the MIP model are related to capacity, distance, and inventory. A multi-period problem is complex because genetic operators, such as crossover and mutation, are applied in each period. As a result, understanding the structure of all constraints is the most important starting point to ensure feasibility.

In the next section, a description of the military logistics service system is explained and the MIP formulation is established. A GA combined with an efficient heuristic approach, to guarantee higher performance with an optimal and a near-optimal solution, is presented in Section 3. The computational results of the branch and cut method and the hybrid GA (HGA) are compared and the performance of the HGA is explained in Section 4. In the last section, conclusions about the experiments and suggestions for further research in military logistics are offered.

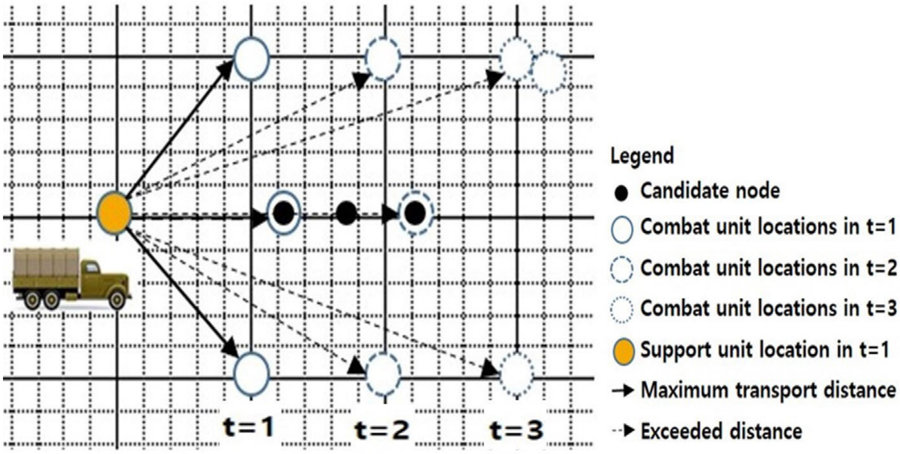

Fig. 1. Wartime logistics system.

2. Mathematical model

2.1. Problem description

This paper presents the determination of simultaneous sequential locations of support units and the schedules used to deliver materials; therefore, the study is classified as a multi-period, multi-commodities, multi-support unit, location–allocation problem. To solve this problem, Fig. 1 is presented to promote understanding about wartime logistics system in which support units transport materials on a daily basis even as combat units move to new locations. The support unit transports materials within the maximum transportable distance, and if the distance between the support and combat units becomes too large, the support unit must relocate to a position closer to the combat unit. Herein, the *maximum transport distance* is the length of LOCs. In wartime logistics, if the LOCs is too long, enemy threats to supply replenishment increase and materials cannot be delivered promptly. Therefore, the lengths of LOCs should be maintained appropriately with support unit relocation as necessary. For this study, support candidate nodes were pre-determined by the analysis of the operational area. Every location, which is a node, was marked as a (x, y) coordinate, and the distance between two points, $a(x_1, y_1)$ and $b(x_2, y_2)$, was calculated by $d_{ab} = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}$.

In this situation, the timing and location of the support unit relocation and the amount that the support unit transports to meet demands are important. Although support units could move whenever combat units move, the high relocation costs and delays in transportation may affect the ability of the support units to meet combat unit demands. Therefore, to maintain a proper distance so that the supply line is neither too long nor too short, the commander must decide the timing of any relocation, find the safest site, and ensure the proper delivery amount to maximize the effectiveness of the logistics system (see Fig. 1).

To solve the problem, there are several realistic considerations for the wartime logistics system. Each support unit transports commodities to every combat unit. A support unit relocates to another site according to distance restrictions. With the assumption that locations and demands of combat units are known, the timing of relocation, the safe location of support units, and the delivery amount for each period are decided. Although the assumption might be unrealistic, operations could be conducted on the basis of the analysis of operational areas and enemy threats by intelligence staffs. The assumption, by which demands and locations of combat units in the short period can be predicted, could be complemented by updates on current situations in a rolling horizon so that commanders assess wartime situations quickly and make prompt decisions. The maximum distance, maximum vehicle capacity, and basic loads of combatants are practical constraints. The objective is to minimize total costs related to penalty, relocation, and hazard. To define relationships among penalty, relocation, and hazard, the cost minimization approach is used; however, the total costs are used to define weights for each type of restriction as determined through experiments.

2.2. Assumptions

The battlefield is described with a graph that consists of combat units, support units, and routes. Units are located at nodes, and vehicles travel on arcs. Only one unit can be located at one node. Candidate sites for support units must have many attributes, but distance from the support base to the combat units was the only factor taken into account for this study. Other assumptions are as follows:

1. Initial locations of combat and support units were already determined as presumptions of the study. As the battlefield situation changes, the front line is changed according to the movement of the combat units. Support units are relocated to new sites as necessary to deliver materials continuously. Commanders of the support units quickly decide the location and amount for material delivery according to the limited battlefield information available.
2. The estimate of the daily demand depends on the degree of damage to the combat unit, and supply priority is changed according to the damage rate. To decide the amount and target locations for material delivery, commanders decide the supply priority for combat units according to current situations; therefore, locations and demands of combat units with different supply priorities could be predicted in the short term.
3. Military commodities are classified into m categories and include food, ammunition, and maintenance items. Each support unit manages several items of similar attributes within the same group of commodities. Two supports units can manage the same item.
4. Each support unit must have sufficient quantities to satisfy daily demand and transport items with vehicles in a *forward supply system*. Combat units cannot carry much material because they need to remain mobile. The minimum amount required for survival is determined as the basic load that combat units carry.
5. Relocation involves fixed costs related to the characteristics of the support units. For example, support units that manage class 3, such as oil, exert relatively great effort to move the garrison; however, those dealing with class 6 goods such as repair parts, need to exert relatively little effort to relocate. In addition, because troops use support unit vehicles while relocating a base, the "*limited supply rule*" is adopted during relocation; that is, the capacity to transport goods is reduced during the move.

6. Unsatisfied demand occurs in two cases in this study: when the capacity of the vehicle to make a one-time transport is less than the demand and when the number of vehicles to transport materials to combat units is insufficient because of relocation (limited supply rule).
7. All parts in the operational area could be attacked by enemies; however, the risk of attack increases when units move close to the enemy. The hazard to support candidate sites linearly decreases with increased distance from the front line.

The time horizon consists of three phases. In the beginning of each period, combat unit deployment is completed so troops are ready to fight. The support unit checks the distance from the base to each combat unit location to decide whether to relocate the current base or to transport supplies without relocating. Combat units receive supplies within each period, then check the unsatisfied demand, and move to another location to resume operations on the next day.

2.3. Mathematical model

We develop an MIP model for a dynamic capacitated multi-support unit location–allocation problem using the following notations:

i, j	index of arbitrary nodes ($i, j = 1, 2, \ldots, N_t$)
s	index of support units ($s = 1, 2, \ldots, S$)
c	index of combat units ($c = 1, 2, \ldots, C$)
k	index of commodities ($k = 1, 2, \ldots, K_s$)
t	index of periods ($t = 1, 2, \ldots, T$)
N_t	number of support candidate nodes for support units in period t
S	number of support units
C	number of combat units
K_s	number of commodities for each support unit s
T	time periods
P_{cnt}	0–1 matrix to represent location of combat unit c for all nodes n over all periods
D_{ckt}	demand of commodity k for combat unit c in period t
d_{ij}	distance from node i to node j
$\max D$	maximum transportable distance for support unit s per period
$\max I_{ck}$	maximum inventory of commodity k for combat unit c
C_{sk}	maximum capacity of support unit s to transport commodity k
E_h	weight for hazard cost
E_c	weight for priority to supply because of damage to combat unit c
E_r	weight for relocation cost of support unit s
∂_k	weight of commodity k
b	required capacity during relocation
x_{sckt}	amount of commodity k transported from support unit s to combat unit c in period t
Z_{sit}	1, if support unit s decides to be located at node i in period t 0, otherwise
U_{ckt}	unsatisfied demand of commodity k for combat unit c in period t

R_{st} 1, if support unit s is stationed at the same node from period $t-1$ to t
0, otherwise

I_{ckt} inventory of commodity k for combat unit c at the end of period t (initial inventory, I_{ck0} is given)

The objective function consists of the penalty, relocation, and hazard costs. The penalty cost results from the unsatisfied demand of each period. In wartime logistics, the priority of transporting materials depends on the damage to combat units. Without immediate replenishment for combat units that take serious damage, the entire operation could fail. In this model, the penalty cost is higher when unsatisfied demand takes place at high priority goods or locations. The relocation cost is a fixed expense for different types of support units moving bases in period $t-1$ to another site in period t. Support units have large inventories, so moving a location requires quite a bit of effort. Because a limited number of support unit vehicles must be used to move a location, the support unit cannot transport materials at the same time it is relocating. Therefore, by including the relocation cost in the objective function, the model can be implemented more realistically. The hazard cost is used to find a less hazardous site far from the front line. A support unit can supply materials while minimizing relocation by moving as close as possible to the front lines that are moving; however, this approach increases the risk of enemy attack on the support unit. Therefore, the hazard cost prevents the support unit from moving too close to the front lines, and it is located at the safe sites while maintaining an appropriate LOCs distance.

Minimize

$$\sum_t \sum_s \sum_c \sum_k E_c \times U_{sckt} + \sum_t \sum_s E_r \times (1 - R_{st}) + \sum_t \sum_s \sum_c \sum_i \sum_j E_h \times Z_{sit} \times \frac{P_{cjt}}{d_{ij}}.$$

Subject to

$$I_{ckt} = I_{ckt-1} + \sum_{s \in S} x_{sckt} - D_{ckt} + U_{ckt} - U_{ck,t-1} \quad \forall c, \forall k, \forall t, \tag{1}$$

$$\sum_c x_{sckt} \le C_{sk} \quad \forall s, \forall k, \forall t, \tag{2}$$

$$\sum_{i \in N_t} Z_{sit} = 1 \quad \forall s, \forall t, \tag{3}$$

$$\sum_{s \in S} Z_{sit} \le 1 \quad \forall i \in N_t, \forall t, \tag{4}$$

$$\sum_{s \in S} Z_{sit} + \sum_{c \in C} P_{cit} \le 1 \quad \forall i \in N_t, \forall t \tag{5}$$

$$\sum_{i \in N_t} \sum_{j \in N_t} \left(Z_{sit} \times P_{cjt} \times d_{ij} \right) \le \max D \quad \forall s, \forall c, \forall t, \tag{6}$$

$$Z_{sit} - Z_{si,t-1} \leq 1 - R_{st} \quad \forall s, \forall i, \forall t, \tag{7}$$

$$\sum_{c \in C} x_{sckt} \leq C_{sk} \times \left(R_{st} + b\right) \quad \forall s, \forall k, \forall t, \tag{8}$$

$$\partial_k \times I_{ckt} \leq \max I_{ck} \quad \forall c, \forall k, \forall t, \tag{9}$$

$$x_{sckt}, U_{ckt}, I_{ckt} \geq 0 \quad \forall s, \forall c, \forall k, \forall t \text{ integers}, \tag{10}$$

$$Z_{sit}, R_{st} \in \{0, 1\} \quad \forall s, \forall i \in N_t, \forall t. \tag{11}$$

Constraint (1) refers to inventory balance equations. Constraint (2) indicates that the number of commodities transported should be less than the maximum vehicle capacity. Constraints (3)–(5) restrict the unit such that only one can be located at one node and all the support units are on the graph over all periods. A support unit locates at a node within the boundary of maximum distance as indicated by Constraint (6). Constraint (7) restricts the number of relocations. The objective function includes the relocation cost, and this model finds the location of support units such that unnecessary relocations are reduced. Constraint (8) limits the vehicle capacity for transport during a relocation. Constraint (9) suggests that the basic load of combatants restricts the amount of commodities held for combat units.

To solve the problem, support units move to new locations when the distance between them and the combat units exceeds the maximum transportable distance. The information about the support unit location includes the timing of relocation such that the restriction of the amount of delivery is of b percentage. The amount of delivery is determined by the inventory balance equation, and the current amount of inventory and unsatisfied demand are defined. The objective function is to minimize the total cost by finding the optimal solution that minimizes the unsatisfied demand and the number of relocations for a delivery schedule and that determines serial support unit locations that are not too close to the front line.

3. Hybrid GA

Although the real-world problem can be described by a mathematical formulation, increasing the problem size affects the computation time. When it comes to solving the problem in Section 2 using Xpress, computation time increases considerably as the number of support units and the types of commodity increases. The wartime situation of operational area expansion can be illustrated in the model by the number of support candidate nodes such that as the number of support candidate nodes increases, the complexity of the problem increases. For the NP-hard optimization problem, such as the wartime logistics system for which urgent decisions for successful operations are required, heuristic approaches should be considered (Chan and Lee, 2005). A GA is based on the survival of the fittest as characterized in nature; that is, the chromosome that evolves in a way that maximizes the ability of the organism to endure the environment survives through that organism to successive generations. Starting from an initial chromosome of randomly determined characteristics, each generation is produced to create the desired population size, and the fitness of each chromosome is evaluated. The fittest chromosomes are chosen through selection and become parents for the next generation. Crossover is a means by which possible characteristics among available chromosomes in the neighborhood are searched by the GA so that the best characteristics of parents can be captured and passed to their offspring. Mutation is conducted to introduce new features in a parent by replacing selected information points in a single parent chromosome with different information.

Although the performance of GA to search solutions is great, the procedure does not always guarantee possible solutions. The characteristics of the proposed algorithm, which include practical constraints, make feasibility of solutions difficult to maintain. In this study, five types of decision variables were deemed important to the outcome: delivery amount, location of support units, unsatisfied demand, timing of relocation, and inventory of combat units. Delivery amount and location of support units were decided, and the number of relocations was minimized by the objective. Constraints considered while proceeding with the GA included maximum distance, vehicle capacity, and basic load. To solve this dynamic location–allocation problem with constraints, heuristic approaches were used to find feasible solutions. A heuristic was used to generate initial solutions and develop genetic operators.

On the basis of the technique of Abdelmaguid and Dessouky (2006), who suggested two approaches to handle capacity constraint violations, procedures to adjust delivery amount were adopted. The technique of Kim (2004) for representing the information of unit location was used in the GA procedure. Because the proposed algorithm includes different types of information in a chromosome, the combination of techniques to handle constraints and find sequential unit location was required.

3.1. Genetic representation

Each chromosome has multiple components that include information on the delivery schedule, support unit location, timing of relocation, unsatisfied demand, and holding inventory of combat units. All components in the chromosome are initiated by the GA because of the importance of them. The support unit location and the delivery schedule are decisive components, and the other types of information comprise dependent components. Finding serial support unit locations requires information about the timing for support unit movement. In other words, the timing of relocation, R_{st}, can be calculated by the information of Z_{sit}. Finding the optimal amount of delivery allows for calculations on the deficit in demand and the amount of inventory combat units possess. In each chromosome, decisive components, x_{sckt} and Z_{sit}, are determined mainly through crossover and mutation; then, the dependent components, R_{st}, U_{ckt}, and I_{ckt}, are calculated. For each generation, the genetic information of the parents, including dependent components, critically influences the offspring to determine the decisive components.

Chromosome length is determined by the number of variables it contains. Figure 2 shows an example of a chromosome. The solution is encoded into a genetic form and includes a delivery schedule, support unit location, unsatisfied demand, timing of relocation, and holding inventory. Each component is transformed into special forms for the crossover and mutation procedures. Then, all components are connected in series to represent the general form of a chromosome.

To describe a specific representation of each component in a chromosome, a sample is used. Table 1 shows each type of information included in a chromosome. In this case, the delivery

x_{sckt}						Z_{sit}												U_{ckt}						R_{st}			I_{ckt}					
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
4	96	100	0	60	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	69	0	1	1	0	4	96	0	41	0	0

Fig. 2. Genetic representation of a chromosome.

Table 1
Genetic representation of each information type for the sample case

Type of information in the chromosome		Time period											
		t_1		t_2			t_3						
Delivery amount from the support unit to each combat unit	$s_1\ k_1\ c_1$	4			100					60			
	$s_1\ k_1\ c_2$	96			0					0			
Sequential locations of the support unit	N_t	1	2	1	2	3	1	2	3	4	5	6	7
	Z_{sit}	1	0	1	0	0	0	0	0	0	0	1	0
Unsatisfied demands of combat unit	$k_1\ c_1$	0			0					69			
	$k_1\ c_2$	0			0					0			
Timing of the support unit relocation	s_1	1			1					0			
Inventory of each combat unit	$k_1\ c_1$	4			0					0			
	$k_1\ c_2$	96			41					0			

schedule is two-dimensional with two rows and three columns. Each cell in the matrix represents the delivery amount of commodity k from support unit s to combat unit c in each period. The support unit location is represented by cells arrayed in two rows and 12 columns. The column shows the support candidate node in each period. The first row represents the set of support candidate nodes in each period, and the number of cells represents the total number of support candidate nodes over the periods. The second row shows the location of the support unit in each period. For the case of multi-support units, an integer value is used to represent the location of each support unit. For unsatisfied demand and inventory, the row shows the number of combat units and commodities, and the columns represent the planning horizon. The timing of relocation is reflected in the information on the support unit in the single row.

3.2. Initialization

The procedure to generate an initial solution for the GA is featured in this subsection. Figures 3 and 4 are flowcharts that represent the process used to produce an initial random solution based on a greedy algorithm. The procedure is started with determination of the distance between units, then it proceeds to either transporting materials with full capacity or relocating the support unit position. Figure 3 shows the flowchart of the main problem for initialization.

Figure 4 shows the sub-problem (SUB), which is used when the distance between the support unit and all combat units exceeds the maximum distance. The SUB is used to find an alternative location and appropriate delivery amounts.

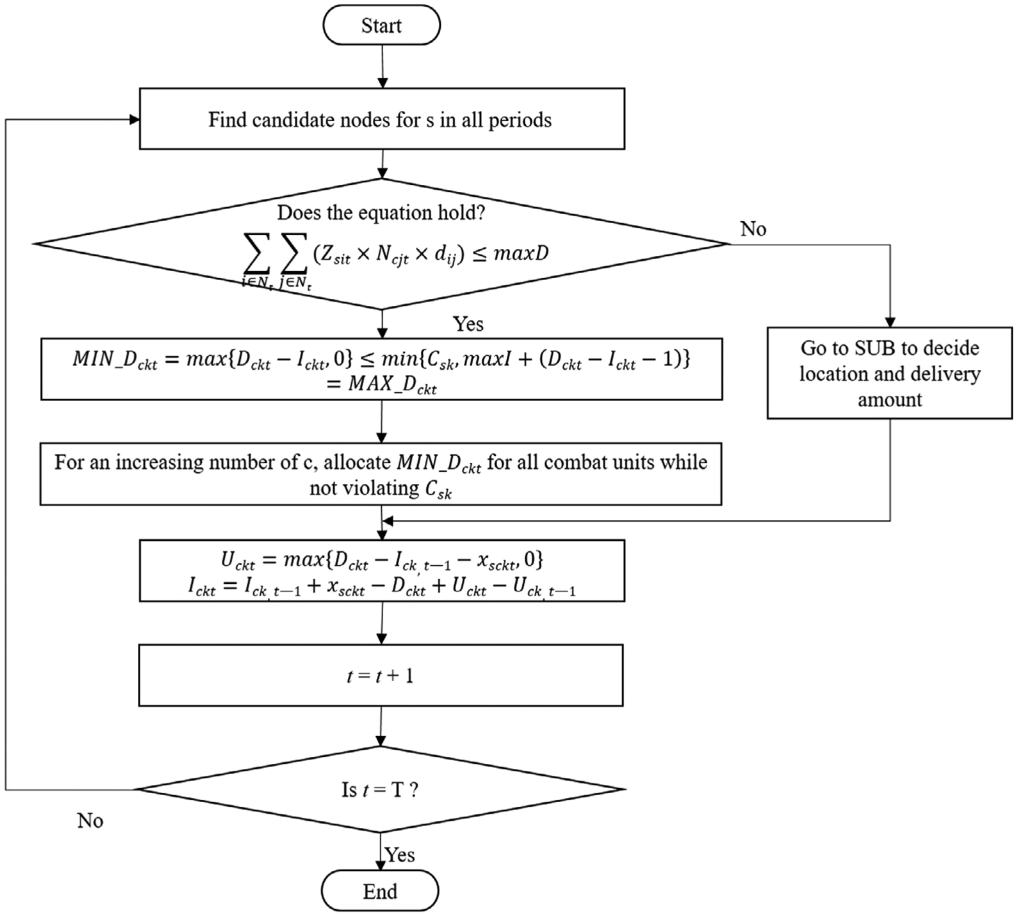

Fig. 3. Flowchart of the main problem for genetic algorithm initialization.

A chromosome is selected in a random process through a roulette wheel. The selector determines and evaluates the fitness of each chromosome in the beginning of each generation and chooses chromosomes through a probability that is based on fitness. The following fitness equation is based on the quality of the proportionality method. It returns the fitness of the best chromosome that is k times higher than the fitness value of the worst chromosome. Generally, k is an integer between 2 and 4, which controls the selective pressure. At a higher value of k, the gap between the probability of choosing a superior chromosome and that of choosing an inferior chromosome increases. The fitness function is as follows:

$$fit_i = C_w - C_i + \frac{(C_w - C_b)}{(k-1)},$$

where C_w is the worst total cost in population, C_b is the best total cost in population, and C_i is the total cost of ith population.

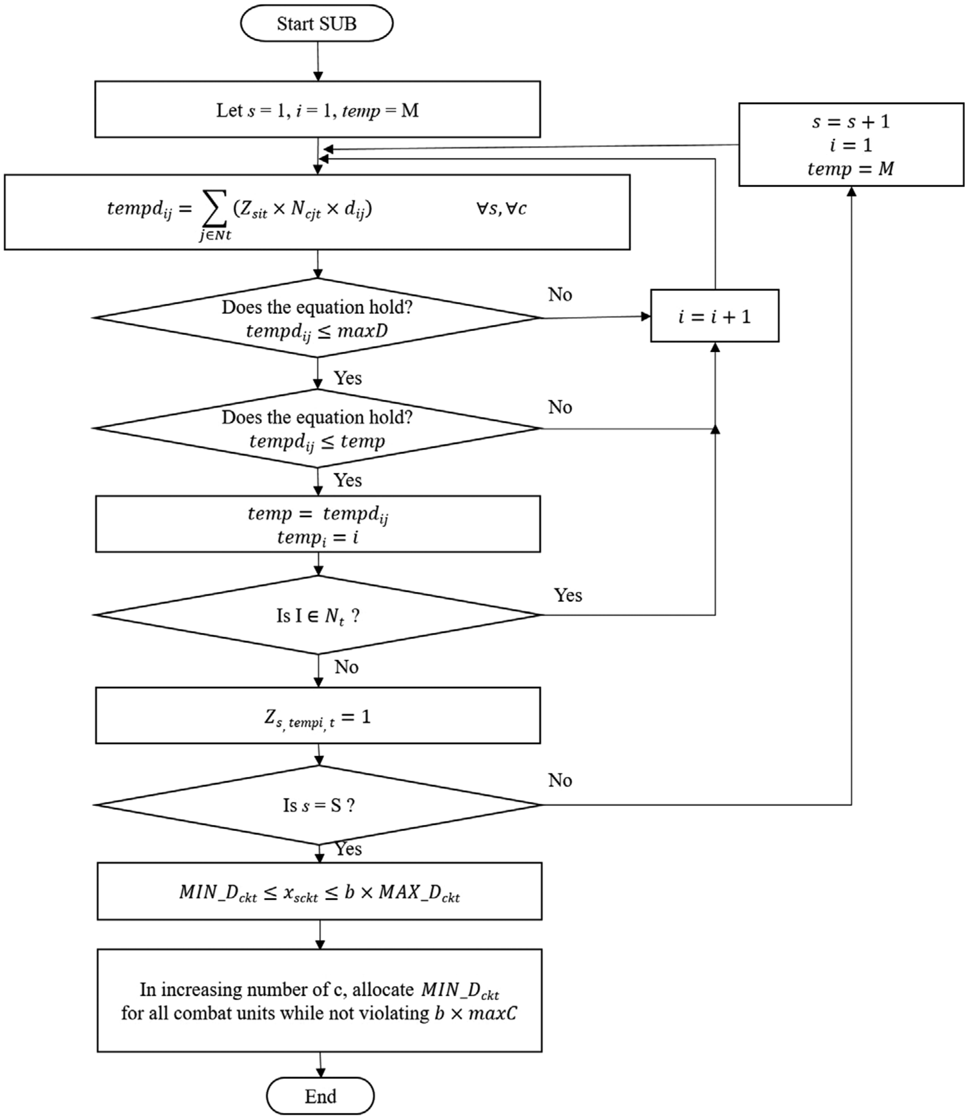

Fig. 4. Flowchart of the sub-problem used for initialization of the genetic algorithm selection operator.

3.3. Crossover operator

3.3.1. Crossover for the delivery schedule

Abdelmaguid and Dessouky (2006) proposed an appropriate GA for the integrated inventory-distribution problem. They designed a crossover operator rule to maintain feasibility. Vertical breakdown arranges the delivery schedule of each period. Horizontal breakdown adjusts the delivery schedule of the selected combat unit over periods. By curtailing unsatisfied demand and adjusting the delivery amount, the horizontal breakdown was adopted to reduce unsatisfied demand.

The delivery schedule for each combat unit was exchanged, which could cause the vehicle capacity to be violated. Maximum transport capacity changes according to the relocation information. The remaining capacity was kept through crossover procedures to maintain feasibility. Then, a vertical breakdown was used for specific periods in which the capacity constraint had been violated. This procedure reduces the chances of violating the capacity constraint and combines fitter features of parents to produce relatively evolved offspring. The steps to maintain feasibility are as follows:

Step 1: Exchange a randomly selected row and check the remaining capacity.
Step 2: Find vehicle capacity constraint violations, and conduct horizontal breakdown for the selected row.
  Step 2.1: Adjust the current delivery to satisfy the maximum vehicle capacity and check the unsatisfied demand for each period.
  Step 2.2: Adjust the delivery amount in the previous periods to minimize unsatisfied demand.
Step 3: If the remaining capacity is less than 0, then conduct a vertical breakdown.
  Step 3.1: Adjust the delivery amount for other combat units in this period.
Step 4: Adjust unsatisfied demand and inventory according to the decisive component.

Figure 5 shows the crossover procedure for the delivery schedule for the sample case. The sample case is too small to show all crossover steps included in the vertical breakdown. However, the proposed procedure outlined in Step 3 can be adopted for large problems.

3.3.2. Crossover for location

For support unit location, an order crossover, which Davis (1985) designed for the permutation type of chromosome, was used. Procedures of order crossover are illustrated in Fig. 6 and outlined as follows:

Step 1: Select two points in each period at random.
Step 2: Generate offspring by copying the element of parent between two points and use it in the same position for the offspring.
Step 3: Fill the element of the other parent from the second point into the temporary offspring in the order of support candidate node. Delete the elements that are already in the offspring.
Step 4: Place the elements from the next point of the second point according to the order of support candidate node.

3.4. Mutation operator

Mutation is conducted according to the types of information in the chromosome. The main factor in deciding the quality of the chromosome is the location of support units because the transportable capacity changes on the basis of information related to relocation. Thus, the mutation is directed at the location of the information portion of the chromosome to remove unnecessary relocations.

First, two points were chosen in each period. One point contained the node occupied by a support unit, and the other point represented an unallocated support candidate node. Then, the

Step 1

Parent 1	t_1	t_2	t_3
x_{111}	16	50	60
x_{112}	24	50	0
Remaining vehicle capacity	60	0	0

Parent 2	t_1	t_2	t_3
x_{111}	16	73	60
x_{112}	24	27	0
Remaining vehicle capacity	60	0	0

Step 2

Offspring 1	t_1	t_2	t_3
x_{111}	16	50	60
x_{112}	24	27	0
Remaining vehicle capacity	60	23	0

Offspring 2	t_1	t_2	t_3
x_{111}	16	73	60
x_{112}	24	50	0
Remaining vehicle capacity	60	-23	0

Step 2-1

Offspring 1	t_1	t_2	t_3
x_{111}	16	50	60
x_{112}	24	U	0
U_{21}	0	36	75
Remaining vehicle capacity	60	0	0

Offspring 2	t_1	t_2	t_3
x_{111}	16	73	60
x_{112}	24	27	0
U_{21}	0	59	75
Remaining vehicle capacity	60	0	0

Step 2-2

Offspring 1	t_1	t_2	t_3
x_{111}	16	50	60
x_{112}	84	50	0
U_{21}	0	36	15
Remaining vehicle capacity	0	0	0

Offspring 2	t_1	t_2	t_3
x_{111}	16	73	60
x_{112}	84	27	0
U_{21}	0	0	74
Remaining vehicle capacity	0	0	0

Fig. 5. Crossover for the delivery component when dealing with vehicle capacity violation.

element at each point was swapped with the selected point in each period so that the number of relocations was reduced and fewer hazardous locations were found. The more possible solutions created when a support candidate node is changed is associated with a greater possibility that a nonhazardous location is identified. Then, the timing of relocation was adjusted. Second, the transportable and remaining vehicle capacity for transport was calculated. Changes to transportable capacity may generate vehicle capacity violations. In cases of vehicle capacity violations, procedures were adopted to maintain feasibility through adjustments of current delivery, delivery amounts in previous periods, and delivery amounts to other combat units in the current period (these are further described in Section 3.3.1). For example, if 40% of total vehicles were assumed to be used during relocation, only 60% of total maximum capacity of materials can be transported in the relocation period. In this situation, vertical and horizontal breakdowns are used to satisfy the limited supply rule while minimizing unsatisfied demand. Last, unsatisfied demands and inventory information were adjusted. Figure 7 shows the procedure for introducing mutations in the sample case.

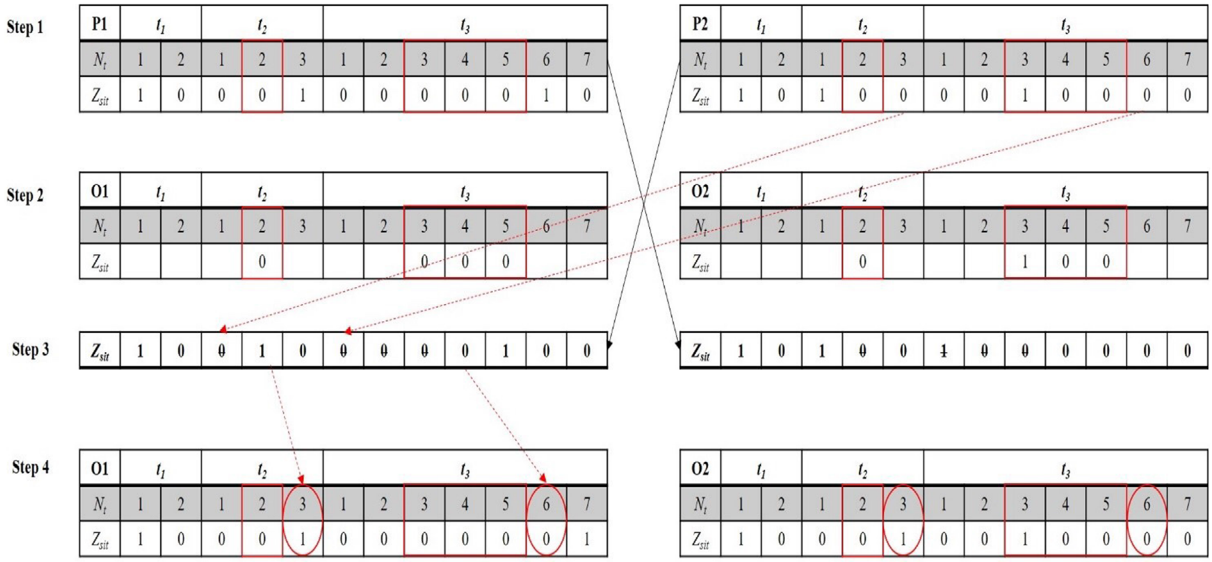

Fig. 6. Order crossover for the location component.

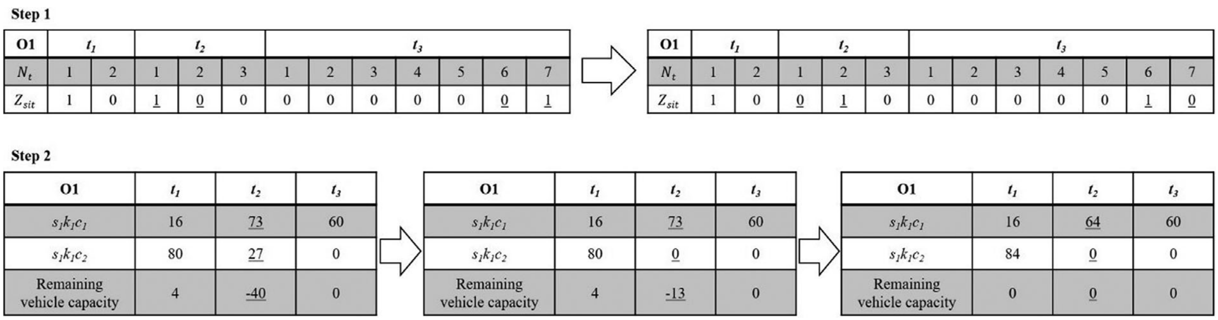

Fig. 7. Mutation procedure to maintain feasibility for the sample case.

4. Computational experiments

Two experiments were conducted to evaluate the performance of the MIP model and the HGA. Experiment 1 defined the credible parameters with restrictions on predelivery to determine the relocation timing when demand was relatively small. Experiment 2 informed predelivery, which encouraged holding inventory of the basic load for combat units. The MIP models presented in Section 3 were coded using XPRESS 7.7 on a PC with an Intel® Core™ i5-3470 CPU of 3.20 GHz and 8 GB RAM. The system featured 32 MB of RAM and Intel Pentium. The proposed HGA was programmed using MATLAB R2014b.

The experiment was based on the following settings: the number of support and combat units, types of materials each support unit handles, and vehicles each support unit possesses; the maximum capacity of each type of vehicle and the timing the support unit needs for replenishment. In addition to this, the following values were necessarily determined in advance: initial and serial locations of

Table 2
Daily demands for combat units

Combat unit	Class 1 Food	Class 1 Water	Class 3 Fuel	Class 5 Ammunition
Infantry company	806	7.644	230	842
Armor platoon	154	1470	3430	2243
Tank platoon	205	1974	14,280	3259

combat units, information about the support candidate nodes, and daily demands over the period. All the possible sites of the units within the operational areas were selected as candidate nodes, and the initial locations of the support and combat units were designated. According to the operation plan, the serial locations of combat units were predicted. Because the support units must be located behind the combat units, the number of support candidate nodes was determined according to the information of the serial combat unit location by each period.

To enhance the reliability of the experiments, input data for demand were created using the daily requirement featured in Beddoes (1997), who studied SBL, which has been useful for comparing different characteristics of combat units.

The experimental values are more meaningful when they account for the different characteristics of combat units. Daily demand varied on the basis of the characteristics of each combat unit, which consisted of infantry, armor, artillery, and so forth. For example, armored units required fewer supplies for personnel but more parts to repair mobile equipment. According to the characteristics of each combat unit featured in Table 2, the daily demand for combat units was generated within a deviation of $\pm 5\%$.

This study addressed a multi-commodity situation, which means that one support unit handled several types of material class consisting of various items. Many researchers studying multi-commodities in military logistics assumed that each support unit handled one type of commodity and that the number of support units was identical to the number of classes. For the real-world troops on which this study was based, however, nine different types of classes are grouped and managed together, and each class comprises various goods. For example, class 1 featured rice, rations, and water. Although fuel was classified as a class 3 commodity, classes 1 and 3 were managed by the same support unit. Class 5 was used to describe all types of ammunition used by the army. In emergencies, ammunition delivery was given a higher priority than goods in other classes. In other words, each support unit handled several different classes and each class was prioritized uniquely.

Unit movements and resupply were performed within the operational area. Combat units occupied new locations to seize, retain, and exploit the initiative for success over the enemy, and locations of the combat units defined the front line in each period. Support unit commanders made decisions for continuous resupply on the basis of frontline changes. For example, a commander of the support unit must decide the timing of the relocation, safest location from enemy attack, and delivery amounts for each period. Support units did not proceed to the front line ahead of combat units. Support units were assumed to have a number of vehicles corresponding to the number of combat units they service. The operation rate of vehicles remained at 85%, because some were assumed in operation while undergoing maintenance. The type of commodities determined the type of vehicles needed; for example, an oil tanker was needed for class 3 item delivery, and a truck delivered class 5 goods. Engaged combat units could be greatly damaged and required urgent deliveries of supplies to make repairs. Some key commodities, such as ammunition, must be supplied immediately. Combat units held limited inventory, defined as basic load, to maintain mobility.

4.1. Experiment 1

A battlefield consists of 40 nodes, two support units, three types of commodities (food, oil, and ammunitions), and five combat units (three infantry, one armor, and one artillery). The duration of operations is 30 days. Distances between nodes were calculated using Euclidean distance. Demands were generated randomly and sequential locations of combat units over the periods were also generated randomly. Each combat unit moves at the beginning of each period, and each support unit decides whether to deliver or relocate. One support unit manages food and oil, and another support unit handles ammunition. In Experiment 1, the relationship between the coefficient for the relocation cost, E_r, and the coefficient for the hazard, E_h, is tested. Support units have sufficient vehicle capacity to deliver materials to meet all demands, except in emergency cases such as when urgent resupply for combat units is not a mission that the support unit is capable of fulfilling. Therefore, the mathematical model finds the optimal delivery amount by transporting materials in advance to deal with emergency cases. Predelivery for an emergency is restricted so that correlation values of the coefficients can be compared. Table 3 shows the results of Experiment 1 as completed with Xpress using the heuristic option. Xpress searches solutions using branch and cut algorithm and cuts. The number of combat units and the maximum distance are fixed.

Cases 1, 2, and 3 show that increasing the number of nodes encourages support units to find locations with fewer enemy threats, which results in a decreased total cost. In Case 3, for example, the penalty and hazard costs were decreased. Two approaches were used to decide the timing of a relocation with restriction of predeliveries. Table 4 shows the mathematical model for the optimal timing in situations of relatively low demand so that the total logistics system incurs a minimum penalty cost.

The MIP model yields the optimal timing of relocation because of the demand levels of each period. The type of commodity with a shortage is type ②, which is managed by s_1. Vehicle capacity for type ② is 29,750, and it is 17,850 during relocation. The optimal sequential location suggests that the best timing for relocation of s_1 is t_{20}, but distances between units exceed the maximum distance in t_{22}. Demand for type ② in t_{20} is 25,910, it is 26,249 for t_{21}, and it is 27,604 for t_{22}. Table 5 shows how this model decides the timing of relocation.

A conservative solution for relocation timing was found when the distances between units exceeded the maximum distance. In this problem, a location at t_{22} made up the conservative solution, but it corresponded to a relatively high unsatisfied demand. The optimal solution, relocating in advance, minimized the unsatisfied demand. The model found the timing of relocation with low demand to minimize penalty cost.

Case 5 in Table 3 shows that support unit 3, which handles type ②, reduced unsatisfied demands. Support unit 3 can be considered an additional supplier in a high echelon. Cases 1, 6, and 7 show that changes in coefficients E_r and E_h affected the number of relocations. Table 6 shows a sensitivity analysis for parameters E_r and E_h.

Table 3
Comparison of results from Experiment 1

Case	s	c	K s_1	K s_2	K s_3	N	E_r s_1	E_r s_2	E_r s_3	E_h	Sum of total costs	Relocation cost	Hazard cost	Penalty cost	Number of relocations
1	2	5	①②	③	–	40	500	1000	–	10^3	12,429	3000	6694	2735	4
2	2	5	①②	③	–	80	500	1000	–	10^3	12,245	3000	6510	2735	4
3	2	5	①②	③	–	120	500	1000	–	10^3	11,864	3000	6283	2580	4
4	3	5	①	②	③	40	500	500	1000	10^3	16,526	4000	9970	2555	6
5	3	5	①②	③	②	40	500	500	1000	10^3	13,970	4000	9970	0	6
6	2	5	①②	③	–	40	500	1000	–	4×10^3	32,233	4000	24,949	3284	5
7	2	5	①②	③	–	40	500	1000	–	9×10^3	63,353	4500	54,668	4185	6

Table 4
Two approaches to find the timing of relocation

Support unit	Timing of relocation						Optimal sequential location		
	Conservative			Optimal					
s_1	1	9	22	1	9	20	$1(t_1)$	$7(t_9)$	$18(t_{20})$
s_2	1	13	26	1	13	26	$2(t_1)$	$8(t_{13})$	$19(t_{26})$

Table 5
Demand and delivery amount in each period

Combat unit	Demand			Delivery amount		
	t_{20}	t_{21}	t_{22}	t_{20}	t_{21}	t_{22}
c_1	6948	8156	7923	6948	8156	7923
c_2	7814	6895	8049	7814	6895	8049
c_3	7544	7575	8039	0	2799	1878
c_4	1457	1474	1554	1457	0	
c_5	2147	2149	2039	1631	0	0
Sum	25,910	26,249	27,604	17,850	17,850	17,850
$U_{c,2,t}$	0	0	0	8060	8300	9745

Table 6
Sensitivity analysis for parameters

E_r	E_h $(\times 10^3)$										
	2	4	6	8	10	12	14	16	17	18	20
1000	4	5	5	5	5	5	5	5	6	6	6
1500	4	4	4	5	5	5	5	5	5	6	6
3000	4	4	4	4	5	5	5	5	5	5	5

Figure 8 illustrates the result of the sensitivity analysis. The larger the ratio of E_r to E_h, the more frequently support units relocated. Because the objective function was used to minimize total cost of the logistics service, the model found safe locations, but relocations should be done such that E_h was four times bigger than E_r. The number of relocations and the limited supply rule affected future operations, and the parameters in Cases 1–5 resulted in fewer relocations being adopted for the Experiment 2.

4.2. *Experiment 2*

For the practical experiment, delivery in advance was allowed. E_r and E_h from Cases 1 to 5 of Experiment 1 were adopted to minimize the number of relocation. The following experiments were conducted by increasing the number of support units, nodes, and overlapping suppliers. Table 7 shows the computation results as completed with Xpress using the branch and cut approach.

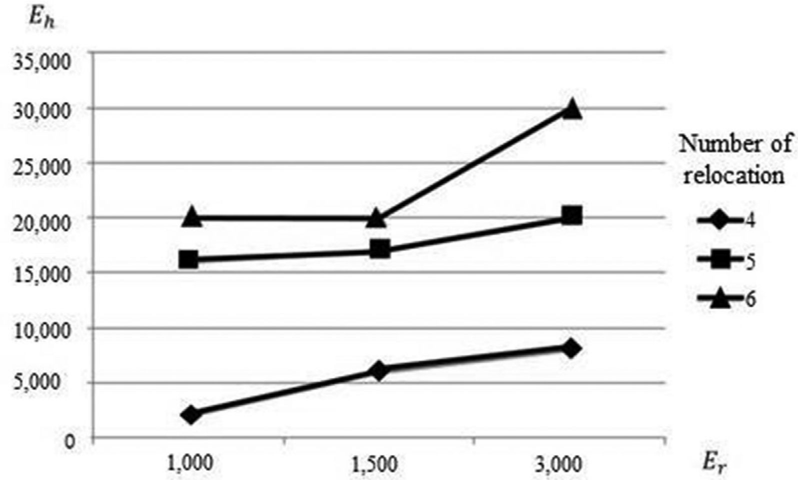

Fig. 8. Relationship of coefficients for the number of relocations.

Table 7
Computation results from Experiment 2 using the branch and cut approach

Case	Support unit	Combat unit	Type	Node	Sum of total costs	Relocation cost	Hazard cost	Penalty cost	Computation time (seconds)
1	2	5	3(1:1:1)	40	9538.26	3000	6538.26	0	10.9
2	2	5	3(1:1:1)	80	9353.92	3000	6353.92	0	75.3
3	2	5	3(1:1:1)	120	9251.87	3000	6251.87	0	155.3
4	3	5	3(1:1:1)	40	13,970.43	4000	9970.43	0	18.8
5	3	5	3(1:1:1)	80	13,784.79	4000	9784.79	0	419.9
6	3	5	3(1:1:1)	120	13,694.88	4000	9894.88	0	67,435.0
7	3	5	3(1:2:1)	40	13,970.43	4000	9970.43	0	14.1
8	3	5	3(1:2:1)	80	13,784.79	4000	9784.79	0	2771.4
9	3	5	3(1:2:1)	120	13,677.72	4000	9677.72	0	7464.0

The branch and cut approach tended to find an optimal solution that minimized total cost. It reduced penalty costs by allowing for delivery of materials in advance, hazard costs by locating support units at safe nodes, and relocation costs by adjusting the timing of relocation. For the size of Experiment 2 with allowance of predelivery, the optimization model found a delivery schedule with no unsatisfied demand. Because the delivery schedule was adjusted by holding inventory for combat units, the branch and cut approach took the conservative approach, which guaranteed a low hazard cost by keeping the current location of support units as long as possible. Table 7 shows that the number of nodes and support units increased the complexity of the problem because of the increased combinations of locations; however, the number of supplier types and the rate of overlapping suppliers did not affect the computation time because the support unit was assumed to have sufficient capacity to transport different commodities to combat units. For example, in Case 6, the branch and cut approach found feasible solutions in an 18-hour computation time, and the

Table 8
Comparison of results from the branch and cut algorithm and the HGA

			K				Branch and cut			HGA		
Case	s	c	s_1	s_2	s_3	Node	Sum of total costs		Computation time (seconds)	Sum of total costs	Computation time (seconds)	Gap
1	2	5	①②	③	–	40	9538.26	Optimal	10.9	9538.26	25.84	Optimal
2	2	5	①②	③	–	80	9353.92	Optimal	75.3	9353.92	24.50	Optimal
3	2	5	①②	③	–	120	9251.87	Optimal	155.3	9251.87	29.82	Optimal
4	3	5	①	②	③	40	13,970.43	Optimal	18.8	13,970.43	29.05	Optimal
5	3	5	①	②	③	80	13,784.79	Optimal	419.9	13,784.79	30.28	Optimal
6	3	5	①	②	③	120	13,694.88	Feasible	67,435.0	13,684.03	31.75	–
7	3	5	①②	③	②	40	13,970.43	Optimal	14.1	13,970.43	25.89	Optimal
8	3	5	①②	③	②	80	13,784.79	Optimal	2771.4	13,784.79	28.19	Optimal
9	3	5	①②	③	②	120	13,677.72	Optimal	7464.0	13,684.00	28.95	0.99

solution is worse than the best solution with the HGA. Most of the results from the branch and cut model reflect optimal solutions, but the computation time for such a large problem is long.

Although the MIP model depicts wartime logistics with practical constraints and the branch and cut approach finds optimal solutions, wartime logistics call for urgent decisions and credible solutions, which must be found as soon as possible so that commanders can make important decisions quickly. Battlefield situations change frequently, so belated decisions are useless. Therefore, an efficient heuristic algorithm is required for wartime logistics of a practical size. The HGA finds the optimal and near-optimal solutions in a relatively short time by searching neighborhoods for the best characteristics to combine in a solution. The proposed HGA was developed by accounting for the characteristics of the decision variables, and special genetic operators were proposed to find feasible solutions in every evolutionary situation.

Table 8 shows the comparison results from the Xpress, which conducted the branch and cut approach to find solutions, and the HGA. In most cases, the HGA found the optimal solution in the shortest time. Optimal solutions of the HGA were verified by visualizing the locations of support units and comparing delivery amounts with those features determined by the branch and cut approach. As the size of a problem increased, the computation time of the branch and cut approach also increased, but the HGA found optimal and near-optimal solutions within 40 seconds. In the HGA, the population size was 100, and termination was complete when either the optimal solution was found or the near-optimal solution (within a 1% gap) was determined, or the generation size was 100. The convergence of the HGA was rapid from the beginning, and a small gap emerged by the 10th generation.

For Case 6, the MIP model could not find the optimal solution, and the HGA found a better solution in less computation time. The HGA found solutions quickly by using an efficient heuristic in every generation. For small problems, initial solutions were generated on the basis of the greedy algorithm, and a 2% or smaller gap from the optimal solution was found. For large problems, the fitness of the initial solution was poor. By initiating genetic operators, the performance of the HGA improved. By searching the set of support candidate nodes and using crossover, the HGA moved support units to the next location and thus guaranteed fewer hazards. Crossover for the delivery amount was conducted by choosing a row that demonstrates unsatisfied demand such that feasible solutions that did not violate the vehicle capacity restriction survive to the next generation. The proposed HGA allocated delivery amount in each period by adjusting current delivery amount, delivery amount in the previous period, and delivery amount for multiple combat units in the current period. Mutations for locations were conducted to find more safe locations for support units. In these procedures, the HGA found solutions at a higher level of performance than the MIP model did.

5. Conclusions

A multi-support unit location–allocation model in wartime was addressed in this study. Realistic constraints, such as maximum distance, maximum vehicle capacity, limited supply rule effected during relocation, and basic load for combatants were considered. The MIP model was proposed to minimize the total cost related to unsatisfied demand, relocation, and hazard. Logistics model in tactical level was conducted with the assumption that demands and locations of combat units in future operations are predictable. The results show that wartime logistics problem were easily solved by a mathematical model and led to optimal solutions for the supply plan.

Furthermore, an HGA for wartime logistics was developed for commanders to estimate situations and make decisions quickly. The GA was combined with an effective heuristic algorithm to find feasible solutions quickly. In less computation time than taken by branch and cut procedures using Xpress, the proposed HGA for wartime logistics suggested the optimal and near-optimal solutions for the timing of relocation, delivery amount, and safe locations.

Although input data were not based on real training data, by adopting daily requirements from previous research (Beddoes, 1997), experiments increased the credibility of the findings. The objective function had three different types of cost and we additionally scaled the cost values so that they were the same. Therefore, further research on multi-objective optimization with penalty, hazard, and relocation costs can yield meaningful findings. A sample case, as modified from previous research (Kim, 2004), was tested to verify the optimal solution generated by the mathematical model. The locations and demands of combat units were assumed known, but in reality, the battlefield is characterized by uncertainties created by enemy attack. Therefore, expanding this study to a stochastic allocation problem would be an interesting research area; in addition, the probability that enemy threat from support areas might be higher than reckoned in the assumption of the hazard rate for candidate sites. Thus, a simulation model of wartime logistics with stochastic demands and enemy threats might offer a visualized decision tool that commanders could use to deal with uncertainties. As the heuristic algorithm using an initialization procedure in GA finds good initial solutions for small problems, improving the heuristic algorithm without adopting GA procedures might yield better performances in finding solutions for the proposed model. With various practical combat scenarios, the proposed model could be used in supply operations.

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