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ORIGINAL ARTICLE

Complexity and relaxation methods for minimising total average cycle stock subject to practical constraints

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ABSTRACT

This paper considers the problem of minimising total average cycle stock that is subject to practical constraints, as first studied by Silver and Moon and later by Hsieh, and Billionnet. For the problem, reorder intervals of a population of items are restricted to a given set, and the total number of replenishments allowed per unit time is limited. Previous researchers proposed different mathematical programming formulations and relaxation methods without identifying the computational complexity of the problem. In this study, we investigate the computational complexity of the problem and analyse the proposed relaxation methods. We identify NP-hard and polynomial time solvable cases of the problem and compare three different relaxations in terms of the lower bounds provided by each relaxation method. We also show that the relaxation with the strongest bound can be solved using a linear time greedy algorithm instead of a general-purpose linear programming algorithm.

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Inventory; complexity; greedy algorithm

1. Introduction

Silver and Moon (1999) addressed the problem of setting reorder intervals for items within a population such that the total average stock is minimised. In their research, the stock was subjected to a restricted set of intervals, and a limit was placed on the total number of replenishments allowed per unit time. As Silver (2008) pointed out, a substantial gap exists between the theory and practice of inventory management, and researchers should strive to narrow it. The reorder interval problem provides a good example for showing the narrowing of the gap between the theory and practice of inventory management because, in practice, the reorder intervals are usually selected from a set such as {1 day, 2 days, … , 1 week, 2 weeks, 1 month, … }. The many grocers who invest in automated ordering must set the reorder intervals to coincide with the intervals of the delivery truck, which are quite regular and determined in advance (Strother, 2018; Wells, 2017). The second largest convenience store chain in Korea recently developed a smart ordering system in which items are sorted according to their reorder intervals, which are 1 day, 2 days, 3 days, … , 1 week, 2 weeks, etc. (Cho, 2016). Therefore, the inventory ordering problem with a restricted set of intervals has become of practical concern since automated ordering has been implemented steadily.

The mathematical programming formulation introduced by Silver and Moon clearly represented the problem. The following notations were used in this original work:

n = number of items,
m = number of possible discrete reorder intervals,
N = maximum total number of replenishments per unit time,
D_i = demand rate of item i, in units/unit time, $i = 1, 2, \ldots, n$,
v_i = unit variable cost of item i, in monetary units/unit time, $i = 1, 2, \ldots, n$,
$w_i \equiv \frac{D_i v_i}{2}$, $i = 1, 2, \ldots, n$,
t_i = decision variable representing the reorder interval of item i, $i = 1, 2, \ldots, n$,
T_j = jth possible reorder interval, $j = 1, 2, \ldots, m$,
$T = \{T_1, \ldots, T_m\}$.

The problem was described as follows:

$$\min \sum_{i=1}^{n} w_i t_i$$

$$\text{subject to } \sum_{i=1}^{n} \frac{1}{t_i} \le N$$

$$t_i \in \{T_1, \ldots, T_m\}, \quad i = 1, 2, \ldots, n$$

By setting $y_i = \frac{1}{t_i}$ for each i and using a suitably selected set of integer values $Y = \{Y_1, \ldots, Y_m\}$, the formulation was transformed into the equivalent problem, (P1).

$$(\text{P1}) \quad \min \sum_{i=1}^{n} w_i \frac{1}{y_i} \tag{1}$$

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$$\text{subject to } \sum_{i=1}^{n} y_i \le N \tag{2}$$

$$y_i \in \{Y_1, \ldots, Y_m\}, \quad i = 1, 2, \ldots, n \tag{3}$$

To handle the (P1) problem, Silver and Moon (1999) developed a dynamic programming algorithm that finds an optimal solution in pseudo-polynomial time. They also presented a heuristic. Hsieh (2001) showed that (P1) can be presented as a multiple-choice knapsack problem and developed a heuristic that was based on the linear programming (LP) relaxation of the multiple-choice knapsack model. Using the formulation of Hsieh, Billionnet (2003) proposed a way of using mixed integer programming software for solving a large-scale problem. All the previous studies were based on the conjecture that the problem is NP-hard, mainly because the problem featured a knapsack-type constraint. However, because the objective function is closely related to the constraint, the knapsack problem cannot be reduced to (P1), and hence, the NP-hardness of the knapsack problem cannot be used to verify the NP-hardness of (P1).

The previously cited studies also presented relaxation methods to obtain lower bounds of (P1). Silver and Moon (1999) used the Lagrangian relaxation of (P1) without Constraint (3), and Hsieh (2001) and Billionnet (2003) developed two slightly different LP relaxations for Hsieh's multiple-choice knapsack model. Silver and Moon's (P1) relaxation can be solved by an analytical method, while both LP relaxations are solved using general-purpose LP algorithms. Billionnet (2003) indicated that the LP relaxation used in the multiple-choice knapsack problem provides tighter lower bounds than that of Silver and Moon but did not compare them with those obtained through the LP relaxation of Hsieh.

The first contribution of our study stems from the investigation into the computational complexity of (P1). In the next section, we prove that (P1) is NP-hard and that it remains NP-hard even for cases in which every item has the same demand rate and unit variable cost. We also show that a polynomial time algorithm solves (P1) for a special case in which the possible number of orders per unit time, $Y_1, \ldots, Y_m$, is an arithmetic sequence; i.e., $Y_j = Y_1 + (j-1)c$ for each $j = 2, \ldots, m$. For example, when possible numbers of orders are integer multiples of a certain basic number, $Y_1, \ldots, Y_m$ becomes an arithmetic sequence. Analysis of the computational complexity of a problem is interesting and valuable research from both theoretical and practical points of view, and the same type of approach used for some inventory problems can be found in Akbalik, Hadj-Alouane, Sauer, and Ghribi (2017) and Li, Chen, and Tang (2017).

The second contribution of our study comes from the comparison of the three relaxations proposed by previous researchers. We show that Billionnet's LP relaxation provides lower bounds that are as good as or more constrained than those of Hsieh's LP relaxation method, and we show the development of a linear time-greedy algorithm for solving Billionnet's LP relaxation problem, which had originally been solved with a general-purpose LP algorithm.

2. Problem complexity

In this section, we will show that (P1) is NP-hard. For this purpose, we introduce the following decision version of the unbounded subset sum problem (USSP) that is NP-complete.

USSP INSTANCE: A set of given integer numbers is $a_1 < a_2 < \cdots < a_k$ and b.

QUESTION: Is there a set of integer numbers $x_1, x_2, \ldots, x_k$ that satisfies $\sum_{j=1}^{k} a_j x_j = b$?

Theorem 1. (P1) is NP-hard.

Proof. We show that USSP is reducible to the decision version of (P1). Given an instance of USSP, an instance of (P1) is associated with it by the following procedure. Set $n = \left\lceil \frac{b}{a_1} \right\rceil$ and select $w_1 \le w_2 \le \cdots \le w_n$ such that $\frac{ba_1}{(b-1)a_k} < \frac{w_n}{w_1} < \frac{b}{b-1}$ (C1). Set $m = k+1$ and select Y_1 such that $Y_1 > \frac{w_n(b-1)a_k - w_1 b a_1}{w_1 b - w_n(b-1)}$ (C2). It is noteworthy that the numerator and denominator are positive by C1; therefore, $Y_1 > 0$. Set $N = nY_1 + b$ and $Y_j = Y_1 + a_{j-1}$ for each $j = 2, \ldots, m$. The answer to the question about the instance USSP instance is *yes* if and only if the associated (P1) has a feasible solution with $\sum_{i=1}^{n} w_i \frac{1}{y_i} \le \sum_{i=1}^{n} w_i \frac{1}{Y_1} - \frac{w_1 b}{Y_1 Y_m}$.

For a solution vector $\{x_j\}$ of USSP with $\sum_{j=1}^{k} a_j x_j = b$, we constructed a solution vector, $\{y_i\}$, using $\{x_j\}$ such that the number of y_i variables having a Y_j value equals x_{j-1} for $j = 2, \ldots, m$. Because $n = \left\lceil \frac{b}{a_1} \right\rceil$, $n > \sum_{j=1}^{k} x_j$. We set the remaining $n - \sum_{j=1}^{k} x_j$ variables equal to Y_1. Also, we assigned the largest Y_j value to the y_i with the largest index as follows:

$$y_i = \begin{cases} Y_1, & \text{if } 1 \le i \le n - \sum_{s=1}^{k} x_s \\ Y_m, & \text{if } i > n - x_k \\ Y_j, & \text{if } n - \sum_{s=j-1}^{k} x_s \le i \le n - \sum_{s=j}^{k} x_s \end{cases}$$

We define a 0-1 indicator variable, δ_{ij}, for each i and j such that if y_i is assigned to Y_j, then δ_{ij} is 1,

and it is 0, otherwise. Thus, $y_i = \sum_{j=1}^{m} Y_j \delta_{ij}$, $\sum_{i=1}^{n} \delta_{i1} = n - \sum_{s=1}^{k} x_s$, and $\sum_{i=1}^{n} \delta_{ij} = x_{j-1}$ for $j = 2, \ldots, m$. Clearly, $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \sum_{j=1}^{m} Y_j \delta_{ij} = \sum_{i=1}^{n} \{Y_1 \delta_{i1} + \sum_{j=2}^{m} (Y_1 + a_{j-1}) \delta_{ij}\} = nY_1 + b$ and

$$\begin{aligned}
\sum_{i=1}^{n} w_i \frac{1}{y_i} &= \sum_{i=1}^{n} w_i \sum_{j=1}^{m} \frac{1}{Y_j} \delta_{ij} \\
&= \sum_{i=1}^{n} w_i \left[\frac{1}{Y_1} \delta_{i1} + \sum_{j=2}^{m} \left(\frac{1}{Y_1} - \frac{1}{Y_1} + \frac{1}{Y_j} \right) \delta_{ij} \right] \\
&= \sum_{i=1}^{n} w_i \left[\frac{1}{Y_1} - \sum_{j=2}^{m} \left(\frac{1}{Y_1} - \frac{1}{Y_j} \right) \delta_{ij} \right] \\
&= \sum_{i=1}^{n} w_i \left(\frac{1}{Y_1} - \sum_{j=2}^{m} \frac{a_{j-1}}{Y_1 Y_j} \delta_{ij} \right) \\
&= \sum_{i=1}^{n} \mathrm{w}_i \frac{1}{Y_1} - \sum_{i=1}^{n} w_i \sum_{j=2}^{m} \frac{a_{j-1}}{Y_1 Y_j} \delta_{ij} \\
&\le \sum_{i=1}^{n} w_i \frac{1}{Y_1} - w_1 \sum_{i=1}^{n} \sum_{j=2}^{m} \frac{a_{j-1}}{Y_1 Y_m} \delta_{ij} \\
&= \sum_{i=1}^{n} w_i \frac{1}{Y_1} - w_1 \frac{1}{Y_1 Y_m} \sum_{s=1}^{k} a_s x_s \\
&= \sum_{i=1}^{n} w_i \frac{1}{Y_1} - w_1 \frac{b}{Y_1 Y_m}
\end{aligned}$$

To show the conditional part of the claim, we use a given feasible solution, $\{y_i\}$, of (P1) with $\sum_{i=1}^{n} w_i \frac{1}{y_i} \le \sum_{i=1}^{n} w_i \frac{1}{Y_1} - \frac{w_1 b}{Y_1 Y_m}$. If δ_{ij} is defined with the same 0-1 indicator variables as we have described, then

$$\begin{aligned}
\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \sum_{j=1}^{m} Y_j \delta_{ij} &= \sum_{i=1}^{n} \{Y_1 \delta_{i1} + \sum_{j=2}^{m} (Y_1 + a_{j-1}) \delta_{ij}\} \\
&= nY_1 + \sum_{i=1}^{n} \sum_{j=2}^{m} a_{j-1} \delta_{ij} \le nY_1 + b.
\end{aligned}$$

Therefore, either $\sum_{i=1}^{n} \sum_{j=2}^{m} a_{j-1} \delta_{ij} = b$ or $\sum_{i=1}^{n} \sum_{j=2}^{m} a_{j-1} \delta_{ij} \le b - 1$. We assume that the latter case is true. Therefore, we write

$$\begin{aligned}
\sum_{i=1}^{n} w_i \frac{1}{y_i} &= \sum_{i=1}^{n} w_i \sum_{j=1}^{m} \frac{1}{Y_j} \delta_{ij} \\
&= \sum_{i=1}^{n} w_i \frac{1}{Y_1} - \sum_{i=1}^{n} w_i \sum_{j=2}^{m} \frac{a_{j-1}}{Y_1 Y_j} \delta_{ij} \\
&\ge \sum_{i=1}^{n} w_i \frac{1}{Y_1} - w_n \sum_{i=1}^{n} \sum_{j=2}^{m} \frac{a_{j-1}}{Y_1 Y_2} \delta_{ij} \\
&\ge \sum_{i=1}^{n} w_i \frac{1}{Y_1} - w_n \frac{b-1}{Y_1 Y_2} \\
&> \sum_{i=1}^{n} w_i \frac{1}{Y_1} - w_1 \frac{b}{Y_1 Y_m}
\end{aligned}$$

The last inequality stems from $w_1 \frac{b}{Y_1 Y_m} - w_n \frac{b-1}{Y_1 Y_2} = \frac{w_1 b (Y_1 + a_1) - w_n (b-1)(Y_1 + a_k)}{Y_1 Y_2 Y_m} > 0$ (by C2). Therefore, $\sum_{i=1}^{n} \sum_{j=2}^{m} a_{j-1} \delta_{ij} = b$. If we construct a solution vector $\{x_j\}$ of USSP such that $x_{j-1} = \sum_{i=1}^{n} \delta_{ij}$ for $j = 2, \ldots, m$, then, $\sum_{j=1}^{k} a_j x_j = b$. □

C1 can be satisfied when $w_1 = w_n$. In other words, equal values of w_i can be used to satisfy C1. Therefore, even when every item has the same demand rate and unit variable cost, the problem is still NP-hard.

Corollary 1. (P1) remains NP-hard even if $w_1 = w_2 = \cdots = w_n$.

3. Polynomial time solvable case

In this section, we consider a special case in which the number of possible orders per unit time, $Y_1, \ldots, Y_m$, is an arithmetic sequence. We assume that $Y_j = Y_1 + (j-1)c$ for each $j = 2, \ldots, m$. We show that the following algorithm optimally solves this case:

Algorithm Exact

(Step 1) Set $y_i = Y_1$ for $i = 1, \ldots, n$; $z = \sum_{i=1}^{n} w_i \frac{1}{Y_1}$; $\overline{N} = N - nY_1$.

(Step 2) Order the indices (i, j) for $i = 1, \ldots, n$ and $j = 2, \ldots, m$ by the decreasing values of $\frac{w_i}{Y_{j-1} Y_j}$, and when ties occur, place the index with smaller i first.

(Step 3) For each index (i, j) in a given order,

if $\overline{N} - c \ge 0$, set $y_i = Y_j$; $\mathrm{z} = \mathrm{z} - \frac{w_i}{Y_{j-1} Y_j}$; $\overline{N} = \overline{N} - c$ else STOP.

Theorem 2. The algorithm Exact optimally solves (P1) when $Y_j = Y_1 + (j-1)c$ for each $j = 2, \ldots, m$.

Proof. Solution $\{y_i^*\}$ is obtained from Exact but it is not an optimal solution of (P1). We let $\{\hat{y}_i\}$ be an optimal solution of (P1). Then, we conclude that $1 \le i1 \le n$ with $y_{i1}^* < \hat{y}_{i1}$. As $N - \sum_{i=1}^{n} y_i^* < c$ by Step 3 of Exact, we conclude that $1 \le i2 \le n$ with $y_{i2}^* > \hat{y}_{i2}$. By ordering of indices (i, j) in Step 2, we obtain $\frac{w_{i1}}{y_{i1}^* (y_{i1}^* + c)} \le \frac{w_{i2}}{(y_{i2}^* - c) y_{i2}^*}$. Without loss of generality, we assume that $\frac{w_{i1}}{y_{i1}^* (y_{i1}^* + c)} < \frac{w_{i2}}{(y_{i2}^* - c) y_{i2}^*}$ because we can find the pair $i1$ and $i2$ by modifying $\{y_i^*\}$ through the following process: If the two ratios are equal, we simultaneously increase y_{i1}^* and decrease y_{i2}^* by c, which does not change the objective value for the modified $\{y_i^*\}$. Then, the following relationship holds:

$$\frac{w_{i1}}{(\hat{y}_{i1} - c)\hat{y}_{i1}} \le \frac{w_{i1}}{y_{i1}^* (y_{i1}^* + c)} < \frac{w_{i2}}{(y_{i2}^* - c) y_{i2}^*} \le \frac{w_{i2}}{\hat{y}_{i2} (\hat{y}_{i2} + c)}$$

We construct a new solution $\{y_i'\}$ such that $y_{i1}' = \hat{y}_{i1} - c$, $y_{i2}' = \hat{y}_{i2} + c$ and $y_i' = \hat{y}_i$ for all i other than $i1$ and $i2$. Then,

$$\sum_{i=1}^{n} w_i \frac{1}{y'_i} = \sum_{i=1}^{n} w_i \frac{1}{\hat{y}_i} - w_{i1}\frac{1}{\hat{y}_{i1}} + w_{i1}\frac{1}{y'_{i1}} - w_{i2}\frac{1}{\hat{y}_{i2}} + w_{i2}\frac{1}{y'_{i2}}$$

$$= \sum_{i=1}^{n} w_i \frac{1}{\hat{y}_i} + \frac{w_{i1}c}{(\hat{y}_{i1}-c)\hat{y}_{i1}} - \frac{w_{i2}c}{\hat{y}_{i2}(\hat{y}_{i2}+c)} < \sum_{i=1}^{n} w_i \frac{1}{\hat{y}_i}$$

contradicting the optimality of $\{\hat{y}_i\}$. □

4. Relaxation methods

Silver and Moon (1999), Hsieh (2001), and Billionnet (2003) proposed relaxation methods that provide lower bounds of (P1). We compare their relaxation methods and develop a linear time greedy algorithm to solve the relaxation that provides the strongest bound. For expositional convenience herein, we use the following notation: For an optimisation problem, P, z(P) and F(P) denote the optimal objective value and the feasible region of P, respectively. Silver and Moon (1999) relaxed the problem by discarding Constraint (3). Let $(\overline{P1})$ be the problem (P1) without Constraint (3). Hsieh (2001) and Billionnet (2003) proposed LP relaxations that were based on the following multiple choice knapsack model:

$$(P2)\ \min = \sum_{i=1}^{n}\sum_{j=1}^{m} w_i \frac{1}{Y_j} x_{ij} \tag{4}$$

$$\text{subject to} \sum_{i=1}^{n}\sum_{j=1}^{m} Y_j x_{ij} \le N \tag{5}$$

$$\sum_{j=1}^{m} x_{ij} = 1,\ \ i = 1, 2, ..., n \tag{6}$$

$$x_{ij} \in \{0, 1\},\ \ i = 1, 2, ..., n,\ \ j = 1, 2, ..., m \tag{7}$$

where x_{ij} =1, if where $y_i = Y_j$ and 0, otherwise.

Let $(\overline{P2})$ be the LP relaxation of (P2) in which Constraint (7) is replaced by non-negativity constraints. Billionnet (2003) showed that $F(\overline{P2}) \subseteq F(\overline{P1})$ and that a data instance exists for which $z(\overline{P1}) < z(\overline{P2})$. To obtain a lower bound of (P2), Billionnet (2003) solved $(\overline{P2})$ using a general-purpose LP algorithm. Hsieh (2001) introduced a more strengthened relaxation using the idea of Silver and Moon (1999) that the optimal solution of (P2) always satisfies $y_1 \le y_2 \le \cdots \le y_n$. Hsieh added the following constraints to (P2)

$$\sum_{j=1}^{m} Y_j x_{ij} \le \sum_{j=1}^{m} Y_j x_{i+1,j}\ \ i = 1, 2, ..., n-1 \tag{8}$$

With Constraint (8), $(\overline{P2})$becomes$(\overline{P3})$. Clearly, $F(\overline{P3}) \subseteq F(\overline{P2})$; therefore, $z(\overline{P2}) \le z(\overline{P3})$. To obtain an improved lower bound, Hsieh (2001) solved $(\overline{P3})$ using a general-purpose LP algorithm that was based on the interior point algorithm. Contrary to expectation, we show that $z(\overline{P2}) = z(\overline{P3})$; in other words, the optimal solution of $(\overline{P2})$ always satisfies Constraint (8). We describe the relationships among the three relaxations, including a proof, for $z(\overline{P2}) = z(\overline{P3})$, and we offer a proof for $F(\overline{P2}) \subseteq F(\overline{P1})$ that is simpler than the one that appeared in Billionnet (2003).

Theorem 3 (i) $F(\overline{P3}) \subseteq F(\overline{P2}) \subseteq F(\overline{P1})$
(ii) $z(\overline{P1}) \le z(\overline{P2}) = z(\overline{P3})$

Proof. (i) $F(\overline{P3}) \subseteq F(\overline{P2})$ is straightforward. To show $F(\overline{P2}) \subseteq F(\overline{P1})$, for an arbitrary feasible solution, $\{x_{ij}\}$, of $(\overline{P2})$, we set $y_i = \sum_{j=1}^{m} Y_j x_{ij}$ for $i = 1, \ldots, n$. Then, we obtain $y_i \in F(\overline{P1})$ because $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n}\sum_{j=1}^{m} Y_j x_{ij} \le N$.

(ii) Suppose that $\{x_{ij}\}$ is an optimal solution of $(\overline{P2})$ but $\sum_{j=1}^{m} Y_j x_{ij} > \sum_{j=1}^{m} Y_j x_{i+1,j}$ for some i, $1 \le i \le n-1$. If $\sum_{j=1}^{m} \frac{1}{Y_j} x_{ij} \le \sum_{j=1}^{m} \frac{1}{Y_j} x_{i+1,j}$, interchange the values of x_{ij} and $x_{i+1,j}$, and otherwise (i.e., $\sum_{j=1}^{m} \frac{1}{Y_j} x_{ij} > \sum_{j=1}^{m} \frac{1}{Y_j} x_{i+1,j}$) set the values of both x_{ij} and $x_{i+1,j}$ equal to the previous value of $x_{i+1,j}$. Then the resulting solution does not increase the objective value while satisfying the constraints of $(\overline{P3})$. □

Theorem 3 states that the more constrained LP relaxation, $(\overline{P3})$, cannot provide better lower bounds than $(\overline{P2})$. We also show that a greedy algorithm can be used to find an optimal solution of $(\overline{P2})$ because $(\overline{P2})$ is a specific version of the linear multiple-choice knapsack problem. We propose a greedy algorithm that is a simplified version of the algorithm by Sinha and Zoltners (1979) for the linear multiple-choice knapsack problem as a means of solving $(\overline{P2})$.

Algorithm Greedy

(Step 1) Set x_{i1} =1 for $i = 1, \ldots, n$; z = $\sum_{i=1}^{n} w_i \frac{1}{Y_1}$; $\overline{N} = N - nY_1$.

(Step 2) Order the indices (i, j) for $i = 1, \ldots, n$ and $j = 2, \ldots, m$ by the decreasing values of $\frac{w_i}{Y_{j-1}Y_j}$, and when ties occur, place the index with smaller i first.

(Step 3) For each index (i, j) in a given order,

if $\overline{N} - Y_j + Y_{j-1} > 0$, set x_{ij} =1; $x_{i,j-1} = 0$; z = z $- \frac{w_i}{Y_{j-1}Y_j}$; and $\overline{N} = \overline{N} - Y_j + Y_{j-1}$**else** set $x_{ij} = \frac{\overline{N}}{Y_j}$; $x_{i,j-1} = 1 - x_{ij}$; z = z $- \frac{w_i}{Y_{j-1}Y_j} x_{ij}$; and STOP.

When the indices (i, j) are preordered, the running time of the algorithm Greedy is $O(mn)$. However, without preordering, the algorithm can be carried in $O(mn)$ using the algorithm from Zemel (1984).

5. Conclusions

In this paper, we addressed the problem of setting reorder intervals for a population of items, as

originally addressed by Silver and Moon (1999), Hsieh (2001), and Billionnet (2003). In the three previously published studies, the computational complexity of the problem was not clearly identified and different relaxations were used. In this paper, we showed that the problem is NP-hard and remains NP-hard even when every item has the same demand rate and unit variable cost. We also showed that when a restricted set of intervals satisfies a certain condition, the problem is polynomial time solvable. We compared the relaxations proposed in the three initial papers and found that the more constrained LP relaxation of Hsieh (2001) cannot provide better lower bounds than the relaxation proposed by Billionnet (2003). We also showed that the strongest relaxation by Billionnet (2003) can be solved by a greedy algorithm for the linear multiple-choice knapsack problem in linear time.

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