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Cooperative sales promotion with a point-sharing policy: Advantages and limitations[☆]

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ABSTRACT

Consumption-point programs have been commonly implemented in retail industries in efforts to promote sales and improve customer loyalty. In Korea, many retailers from different industries use a point-sharing policy to augment the conventional consumption-point program of each retailer. In a multi-retailer coalition under such a cooperative sales promotion policy, by purchasing from one coalition retailer, customers earn points that they can redeem points at other retailers in the coalition. On one hand, the introduction of this policy gives customers great flexibility for redeeming earned points, which can increase the demand at all retailers who promote the policy. On the other hand, the additional product costs associated with the points created by one retailer may spill over and be partly borne by other retailers, possibly distorting the coalition members' equilibrium decisions under decentralized control. Under the general assumptions about the demand functions, we developed a model consisting of two retailers with fixed retail prices and addressed the retailers' equilibrium decisions under a pure point-sharing policy. The findings suggest that the policy resulted in a cost spillover phenomenon. Then, we revealed that a pure point-sharing policy may fail to maximize the total profit of the coalition. Moreover, we showed that a pure point-sharing policy does not dominate the individual point scheme, which may explain the reason that point sharing is useful but not ubiquitously used in the real world. Our numerical examples also illustrate the way a pure point-sharing policy influences retailers' profits when retail prices are decision variables. To improve the overall profit under the point-sharing policy further, we propose a target rebate contract to coordinate a pair of retailers. This contract can maximize the total profit and arbitrarily split the profit between retailers.

1. Introduction

In modern retail industries, a consumption-point mechanism is commonly used for promoting sales and boosting customer loyalty. Under such a mechanism, customers can obtain and accumulate some points when they purchase products from a retailer. The amount of points generated from one purchase usually depends on a customer's total payment and the conversion ratio set by the retailer. Customers can redeem the accumulated points at the same retailer in a mechanism also recognized as a type of customer loyalty membership. Consumption-point mechanisms fall into several customer loyalty membership categories. In one type of point mechanism, customers receive a product from the retailer by redeeming the required number of points, instead of paying with money. For example, in China Starbucks encourages customers to apply for membership to obtain points whenever they make a purchase. Customers can accumulate points with a conversion ratio of approximately 10% and redeem the points to receive a product from Starbucks without incurring any charge. This practice is similar to the "buy-x-get-one-free" scheme (e.g., Gandomi and Zolfaghari [15]). In another popular point mechanism, customers can use the points as money at the same retailer to avoid paying full price for another product purchased in the future. In the final type addressed herein, customers can redeem points for gifts that the retailer purchased from a third-party supplier.

Customer loyalty memberships are used to increase customer demand and to cultivate customer loyalty in the long run. A trade-off between the increased demand and the cost of customers' point redemption characterize these programs. In the United States, the number of customer loyalty memberships, which enable customers to obtain rewards by redeeming their accumulated points,

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increased by 26.7% since 2010, exceeding the 2.65 billion memberships recorded for 2012 [1,11].

In recent years, many Korean retailers from different industries have been implementing cooperative sales promotions with a point-sharing (PS) policy that alters their own performances and those of an entire coalition. Under PS policies upon which two retailers agree, customers can earn and redeem points at either retailer. A Twosome Place (ATP), for example, is a famous dessert cafe in Korea that sells cake, coffee, and other drinks, and CGV is a popular cinema brand. These two companies implement a PS policy to promote their respective sales and performance. Customers can earn points at ATP or CGV and redeem them at either company. Because ATP and CGV have common target customers (e.g., young people and white-collar professionals), such a policy provides customers with great flexibility in redeeming their points. Moreover, these two retailers, which provide different products, do not compete with each other. Therefore, the demands at these two retailers are both increased under the PS policy.

When the advantages of PS policies were noticed by practitioners, some networks involving more retailers developed a third-party membership card company. CJONE, a membership card company in Korea, covers more than 20 retailers from different industries, including those in shopping, food, and entertainment, among others (http://www.cjone.com). Customers can accumulate consumption points generated from sales at member retailers featured on the CJONE card and also can redeem the points when purchasing products from any of the member retailers.

Shin and Cha [25] collected one-year survey data from 1000 people in Korea to analyze the point-card usage frequency and demand under PS programs. The results showed that the program appealed to many customers and increased the point-card usage frequency and point accumulation by encouraging cross buying. Moreover, the researchers estimated that the sales revenue of the famous cosmetic retailer in Korea, *Olive Young*, has been significant since it joined a PS program in 2008 [9].

Retailers that implement PS policies have specific characteristics. First, customers usually return and conduct repeat purchasing at the retailers, and thus, consumption-point programs play an important role in sales promotion. Second, the accumulated points can significantly reduce the customers' purchasing costs when buying at retail prices. However, a useful PS policy between a shipbuilding company and a café, for example, seems unlikely because customers would experience difficulty in accumulating sufficient points from the café to purchase a ship at or near retail price. Third, retailers have common target customers but different target markets. In this case, customers might be interested in redeeming points earned from one retailer at the other retailer. Moreover, competition between these types of retailers can be avoided. Hence, in this case, increasing demand at one retailer does not negatively affect the sales of the other retailer.

Although the advantages of PS policies are straightforward, retailers' individual decision making and the effect of those decisions on the total profit remain unexplored. To distinguish among the different coalition settings, we defined the PS policy without any coordinating contract as a pure PS policy. Each retailer attempts to maximize its own profit under the pure PS policy that is a type of decentralized control. On one hand, the demands at the retailers can be increased because customers enjoy the flexibility of redeeming their points at more than one retailer. On the other hand, customers can redeem their points at all member retailers such that the product cost associated with the points created at one retailer is passed to the retailer that honors the points, thus making the decision framework more complex. That is, when retailers promote their sales by allowing the flexibility of point redemption, the promotional point costs spill over and switch to coalition partners at the same time. Because it is difficult for one retailer to influence a coalition partner's decision on the point-conversion ratio and selling price, this cost spillover phenomenon may distort the equilibrium decisions of both retailers such that the maximum total profit may not be achieved under the pure PS policy. In some cases, the distortion even makes the total profit under the pure PS policy lower than it is under an individual point schemes. In other words, the pure PS policy puts coalition members in a situation in which seeking their own maximum profit may result in greater costs to other members than is optimal for all. In this study, we address the following three questions:

(i) How do retailers make their decisions under the pure PS policy?
(ii) Does the pure PS policy always outperform individual point schemes? If it does not, then what factors influence the performance of the pure PS policy?
(iii) How is a contract able to coordinate two retailers and arbitrarily divide the profit under a PS policy?

We examined a PS policy with two retailers from different industries. This study contributes to the literature in two ways. First, we analyzed retailers' equilibrium decisions under the pure PS policy and showed the bottleneck these decisions create in the pathway to achieving maximum total profit. We discussed two cases: One in which the retail prices of retailers are exogenous parameters and another in which retail prices are decision variables. We also uncovered some unique characteristics of cooperative sales promotion with a PS policy. The results shed light on the coalition partner performances and customer welfare under the pure PS policy. In addition, practitioners may obtain managerial insights into their decision making from this study. Second, we proposed a flexible contract for the coordination of two retailers under the PS policy. The idea of a popular target rebate (TR) contract was used to design our contract, and the optimal setting of the contract parameters is discussed. In addition, we showed the advantages of TR contracts in terms of ease of easy implementation.

The PS policy is different from the point-pooling (PP) policy, which involves multiple retailers and a third-party company. Under the PP policy, retailers purchase the consumption points from a third-party company and award their customers these points. Customers can redeem their points from the third-party company and obtain rewards from a pre-determined reward chart [5]. One retailer bears all of the cost resulting from consumer redemption of points generated by it, and its decision on the point-conversion ratio does not affect other retailers. Moreover, customers' point redemption is limited by the third-party company. The core operational problem is in the vertical channel; that is, the ways retailers and the third-party company make optimal decisions on point ordering and award purchasing create complexity. The third-party company plays an important role in the channel by setting prices for the points and creating the reward chart.

Under the PS policy, however, one retailer's decision on the point-conversion ratio affects the profits of other retailers because of point-switching redemption and cost spillover. Customers have relatively great flexibility in redeeming any number of their points from any participating retailer. In this case, the coalition should pay attention to the way retailers' decisions influence each other and coalition performance. Any third-party company under a PS policy provides only information management and card services.

The remaining parts of this paper are organized as follows. We summarize the literature in related areas and present it in Section 2. Section 3 offers a discussion on the decision framework and the optimal decisions of two retailers under the pure PS policy. Section 4 offers an explanation of the proposed TR contract under the PS policy and shows the performance of it in coordinating the two retailers. We provide managerial insights and concluding remarks, and suggest directions for future work in Section 5.

2. Literature review

To the best of our knowledge, coordination of retailers under the PS policy has rarely been investigated in previous studies. One stream of related research involves a coalition loyalty program (CLP), also known as multi-vendor loyalty program, which is an integrated customer loyalty program involving multiple companies. In addition to consumption-point mechanisms, other sales promotion and customer loyalty programs, such as those that include advertising, finance compensation, gifts, and membership privileges, are also used in a CLP. In this field, the literature has focused mostly on customer relationships with retailers from the marketing perspective, and only a few researchers have discussed the problem by considering the operational decisions on reward supply planning of the third-party company in CLPs. Lemon and Wangenheim [20] conducted a longitudinal analysis to show the effects of partner selection in CLPs and found that a strong fit between the products in CLPs could be beneficial. Dorotic et al. [10] summarized studies on loyalty programs and showed that the size of spillover effects in CLPs remained inadequately investigated. Considering that CLPs feature multiple participating companies and cross selling, Lee et al. [19] developed a behavior-scoring model to analyze customers' preferences in OKCashBag, which is the largest CLP in Korea. Schumann et al. [23] discussed the spillover effect of service failures in CLPs and found that one program partner's service failure negatively affected customer loyalty toward the CLP. So et al. [24] revealed one potential weakness of CLPs: Customers' associations with any program partner can be diluted. Vallacé-Molinero et al. [28] conducted an empirical analysis to determine the way a CLP influences customer behavioral loyalty. The results showed that companies need to select a CLP carefully before joining and should implement differentiated relationship management of customers. Chang and Wong [6] carried out an empirical analysis that focuses on the impacts of psychological reactance on the performance of a CLP.

In recent years, a few studies have investigated operational issues in sales promotion and CLPs. Cao et al. [3] developed a stochastic linear programing model for rewards-supply planning in CLPs, and a sampling-based stochastic optimization heuristic algorithm was proposed to obtain the optimal solution. Cao et al. [4] studied the rewards-supply planning problem with the cooperative sales effort of the third-party company. Cao et al. [5] designed option contracts for the rewards-supply planning problem of CLPs by considering the budget constraint of the third-party company. In the supply chain models described herein, a third-party company sells points to participating retailers of CLPs, and retailers reward their customers with these points. Then, customers can redeem points to obtain products from other suppliers.

The PS policy is a type of marketing mechanism that promotes sales at extra effort costs. Many studies in this field have focused on different types of sales promotions, such as advertising, coupon, and group buying [7,17,21,33,34]. When considering sales efforts in supply chains, researchers usually concentrate on the free-riding phenomenon of sales effort. Promotion cost sharing is a common approach to achieve coordination. Krishnan et al. [18] incorporated the promotion cost-sharing policy into a buy-back contract to coordinate supply chains with a single manufacturer and a single retailer. Tsao and Sheen [27] developed a promotion cost-sharing policy to coordinate supply chains with a single supplier and multiple retailers. Their study showed that, under decentralized control, each retailer is willing to make a smaller promotional effort than the channel-wide optimal effort required. Under this cost-sharing policy, the supplier undertakes a fraction of the promotion cost of each retailer to achieve supply chain coordination. Jin et al. [16] analyzed contract design for supply chains with a single manufacturer and a capital-constrained retailer. The supply chain performances under four business models, in which either the manufacturer or the retailer has the decision rights for sales promotion, were discussed.

The PS policy differs from the supply-chain stream of studies in the following two main respects. First, the sales efforts under multiple channels usually involve companies that offer the same or substitute products, which is not essential under the PS policy. Second, one company can enjoy the increased demand resulting from the sales efforts made by other companies in existing models. However, one coalition partner must undertake the point cost switching from the other partner under the PS policy. Moreover, the point-cost function could also be complex because of the point redemption. As a consequence, the PS policy has a different impact on the partners' profit structures than other types of sales efforts do.

In another related area, channel coordination with contracts is considered, and a large body of literature exists in this field. Use of channel contracts is a popular approach to achieve channel coordination. Different types of contracts, such as those of revenue sharing, buy back, and quantity flexibility, have been developed and implemented in the real world. For a more complex supply chain model, some composite contracts have been developed to achieve high total profit and great flexibility in profit split [13,29,32].

The TR contract (also known as *channel rebate contract*) is popular in several industries. Using it, the supplier pays the retailer a rebate for each unit sold beyond a certain target value. In particular, the channel rebate is a linear rebate when the target value is equal to zero. Taylor [26] studied a channel rebate contract in a two-stage supply chain by considering sales efforts. The results showed that a single TR contract could fail to coordinate the supply chain in a way that can be implemented. Wong et al. [30] found that TR contracts combined with a vendor-managed inventory policy could perfectly coordinate supply chains with multiple retailers. Chiu et al. [8] extended the TR contract for supply chains with risk-sensitive retailers. Xing and Liu [31] discussed the free-riding phenomenon of sales effort in supply chains with a single manufacturer and two retailers. In their study, selective rebate contracts with price match were developed to coordinate the supply chain to achieve higher total profit than could be earned under the traditional TR contract.

Despite the increasing practice of the PS policy in the real world, modeling and analyses of it, which would provide managerial insights, remain rare. As Table 1 shows, most existing studies that considered point sharing for CLPs were based on empirical analysis. Only a few studies have accounted for operational decisions within CLPs; however, these studies concentrated on obtaining the optimal award schedule and did not address the PS policy. Existing channel models of sales effort were devoted to cases with the free-riding phenomenon of sales effort, and contracts were developed to eliminate the negative effect of channel members' optimal decisions on sales efforts.

3. The pure point-sharing policy

A symmetric model consisting of two independent retailers (Retailers 1 and 2), each of which sells one type of product, respectively, was considered for this study. The product sold by Retailer 1 is not a substitute for the one sold by Retailer 2. The unit production cost for retailer i is c_i, and retailer i sells the product to customers at a retail price, p_i. We defined λ_i as the point-conversion ratio of retailer i, and customers can obtain $\lambda_i p_i$ points when they purchase one unit of product from retailer i. The model was developed based on three main assumptions.

Assumption 1. The demand at either retailer does not negatively affect demand at the other one.

Table 1
Comparison of some relevant studies and this study.

Relevant study	CLP mode	Methodology	Optimal operation decision	Coordinating mechanism
Lemon and Wangenheim [20]	G	E	/	/
Dorotic et al. [10]	G	Q	/	/
Lee et al. [19]	G	M	/	/
Schumann et al. [23]	G	E	/	/
Villacé-Molinero et al. [28]	G	E	/	/
Chang and Wong [6]	G	E	/	/
Cao et al. [3]	PP	M	√	/
Cao et al. [4]	PP	M	√	/
Cao et al. [5]	PP	M	√	/
This study	PS	M	√	√

Note: "G" represents general CLP; "E", "Q", and "M" represent empirical analysis, qualitative analysis, and mathematical model, respectively. "√" represents "covered" and "/" represents "not covered".

Assumption 2. When they have accumulated sufficient points, customers can redeem their points at one retailer by obtaining the corresponding number of products without paying any fee (this practice is common among many retailers in the real world) and there is no stockout.

Assumption 3. Each customer who redeems any positive X units of points at retailer i can purchase X/p_i units of products from that retailer. This assumption implies that the total accumulated points of one customer are not of lower magnitude than the retailer prices. This assumption also enables us to focus on the analysis of retailers' operational decisions.

Assumption 4. Customers are willing to redeem all of their points. This assumption does not influence the discussion or main conclusions.

In many real cases, prior to implementing the PS policy, many retailers had run their own consumption-point program for a time and have stable retail prices. In the real world, they may prefer adjusting the point-conversion ratio with implementation of the PS policy than changing the price. In the CJONE program, for example, *ATP* faced fierce competition in the café industry because each café fixed a retail price for a relatively long time. In some industries, suppliers, such as *Coca Cola* and film distributors, have the power to decide or influence the retail price. One real example is Olive Young, which did not change the retail prices of products after it joined the PS program. Hence, herein, we first present the case in which retail prices are exogenous parameters.

Let $D_i(\lambda_i)$ be the quantity of products sold by retailer i in one selling season. We defined $D_i(\lambda_i) = A_i(\lambda_i)+\Delta D_i(\lambda_i)$, where in $A_i(\lambda_i)$ is the demand under the individual point scheme and $\Delta D_i(\lambda_i)$ is the extra demand attracted by the PS policy. We assumed that $dA_i(\lambda_i)/d\lambda_i > 0$, $d\Delta D_i(\lambda_i)/d\lambda_i > 0$, $d^2A_i(\lambda_i)/d(\lambda_i)^2 \leq 0$ and $d^2\Delta D_i(\lambda_i)/d(\lambda_i)^2 \leq 0$, $i = 1, 2$. Let θ_i be the percentage of points of retailer i that is redeemed at retailer i and θ_i be limited within the range of [0, 1]. Then, $1-\theta_1$ of the points of Retailer 1 is redeemed at Retailer 2. In this case, the total quantity of products of Retailer 1 given through point redemption is $\theta_1\lambda_1 D_1(\lambda_1)+p_2\lambda_2(1-\theta_2)D_2(\lambda_2)/p_1$. The latter part tells the quantity of products related to the points generated by Retailer 2 but redeemed at Retailer 1. According to the practices described in this paper, we assumed that a redemption policy does not bring new points from retailers. Hence, we considered θ_i as an exogenous parameter independent of λ_i and λ_j. Different values of θ_i may be found for customers engaged with an individual point scheme than for new customers attracted by the PS policy. However, we used a uniform θ_i for all demands of retailer i for two reasons. First, individual point schemes are offered for customers who repeatedly conduct transactions at the same retailer, which means that a part of $\Delta D_i(\lambda_i)$ has the same customer source as $A_i(\lambda_i)$ does, such that managers would struggle to find the exact values for point-switching ratios of $A_i(\lambda_i)$ and $\Delta D_i(\lambda_i)$. Second, a uniform θ_i did not significantly affect the main conclusions of this study.

3.1. Comparison between the pure PS policy and the conventional individual point scheme

In this subsection, we discussed the fundamental profit source of the pure PS policy and illustrated its advantage/disadvantages compared with the individual point scheme. Retailer i can satisfy the demand by selling products to customers and awarding the customers with products through point redemption.

Denote $\Pi_i(\lambda_i)$ as the profit of retailer i, $i = 1, 2$. Under the pure PS policy, the retailers' objective functions are as follows.

$$\Pi_1(\lambda_1) = (p_1 - c_1)D_1(\lambda_1) - c_1\theta_1\lambda_1 D_1(\lambda_1) - \frac{p_2 c_1}{p_1}(1-\theta_2)\lambda_2 D_2(\lambda_2) \tag{1}$$

$$\Pi_2(\lambda_2) = (p_2 - c_2)D_2(\lambda_2) - c_2\theta_2\lambda_2 D_2(\lambda_2) - \frac{p_1 c_2}{p_2}(1-\theta_1)\lambda_1 D_1(\lambda_1) \tag{2}$$

In Eq. (1), the first term is the gross profit without considering costumer redemption, and the second term is the cost resulting from costumers' local redemption. The third term represents the cost of customers' switching redemption from Retailer 2. $\Pi_2(\lambda_2)$ shares the same profit structure with $\Pi_1(\lambda_1)$.[1]

Under the individual point scheme, the retailers' objective functions are as follows:

$$\Pi_1(\lambda_1) = (p_1 - c_1 - c_1\lambda_1)A_1(\lambda_1). \tag{3}$$

$$\Pi_2(\lambda_2) = (p_2 - c_2 - c_2\lambda_2)A_2(\lambda_2). \tag{4}$$

Let λ_i^o the equilibrium λ_i that maximize $\Pi_i(\lambda_i)$ under the pure PS policy and λ_i^I be the λ_i that maximize $\Pi_i(\lambda_i)$ under the individual point scheme, respectively, $i = 1, 2$.

Proposition 1. *Under the pure PS policy, (i) if there exists a $\{\lambda_1, \lambda_2\}$ that satisfies*

$$d\Pi_1(\lambda_1)/d\lambda_1 = d\Pi_2(\lambda_2)/d\lambda_2 = 0, \tag{5}$$

then retailers' equilibrium point-conversion ratios, $\{\lambda_1{}^\circ, \lambda_2{}^\circ\}$ under the pure PS policy, can be obtained from Eq. (5),

(ii) if the condition in (i) holds, then λ_i^o is decreasing with θ_i, i = 1, 2,

[1] When $\theta_i = 1$, there is no point-switching redemption. If we let $p_i' = p_i - c_i\lambda_i$, then we can transform the profit functions into new profit functions where in p_i' can be considered as retail price of retailer i and as a decision variable, $i = 1{,}2$.

Table 2
Functions of the marginal gross profit and point cost of retailers under three demand scenarios.

	Marginal gross profit (MGP)	Marginal point cost (MPC)
$\Delta D_i(\lambda_i) = 0$	$(p_i - c_i)b_i$	$a_ic_i + 2b_ic_i\lambda_i$
$\Delta D_i(\lambda_i) = e_i\lambda_i$	$(p_i - c_i)(b_i + e_i)$	$a_ic_i\theta_i + 2(b_i + e_i)c_i\theta_i\lambda_i$
$\Delta D_i(\lambda_i) = e_i\ln(\lambda_i + 1)$	$(p_i - c_i)[b_i + e_i/(\lambda_i + 1)]$	$a_ic_i\theta_i + 2b_ic_i\theta_i\lambda_i + e_ic_i\theta_i\ln(\lambda_i + 1) + e_ic_i\theta_i\lambda_i/(\lambda_i + 1)$
$\Delta D_i(\lambda_i) = e_i(-(\lambda_i)^2 + \lambda_i)$	$(p_i - c_i)(b_i + e_i - 2e_i\lambda_i)$	$a_ic_i\theta_i + 2(b_i + e_i)c_i\theta_i\lambda_i - 3e_ic_i\theta_i(\lambda_i)^2$

(iii) under linear demand functions of $A_i(\lambda_i) = a_i + b_i\lambda_i$ and $\Delta D_i(\lambda_i) = e_i\lambda_i$, retailer i is incentivized to select a higher λ_i under the pure PS policy than under the individual point scheme, i.e., $\lambda_i{}^o > \lambda_i$, with a_i, b_i, $e_i > 0$ and $i = 1, 2,^I$.

All proofs are shown in the appendix. The PS policy provides a new approach of improving the overall performance of the two retailers by allowing flexible point redemption. We defined the production cost for the products obtained from point redemption as point cost. A larger λ_i implies a greater flexibility of point redemption that can bring a larger demand, but it also leads to a higher point cost. The increased demand brought by a larger λ_i is totally enjoyed by retailer *i*. However, the point cost is partly undertaken by the other retailer, and the cost spillover effect is determined by θ_i. For retailer *i*, a larger θ_i means that more customers want to redeem the points from retailer *i* at retailer *i*. In this case, the total point cost determined by λ_i will be mainly undertaken by itself and retailer *i* may not have a high incentive to select a higher λ_i. In particular, $\lambda_i{}^o$ will decrease to the value under the individual point program when $\theta_i = 1$.

Motivated by the flexibility the PS policy offers, customers may be willing to purchase more products than they would under a retailer's individual point scheme. For this case, $\Delta D_i(\lambda_i)$ is described under the PS policy as extra demand for Retailer *i*. By fixing $A_i(\lambda_i) = a_i + b_i\lambda_i$, we investigated three cases to compare the pure PS policy and the individual point scheme for which $\Delta D_i(\lambda_i) = e_i\lambda_i$, $\Delta D_i(\lambda_i) = e_i\ln(\lambda_i + 1)$, and $\Delta D_i(\lambda_i) = e_i(-(\lambda_i)^2 + \lambda_i)$, $0 < \lambda_i < 1$. We set $(p_i - c_i)D_i(\lambda_i)$ to be the gross profit of retailer *i*. The point cost of retailer *i* is $c_i\theta_i\lambda_iD_i(\lambda_i) + p_j(1 - \theta_j)c_iD_j(\lambda_j)/p_i$. Then we obtained the marginal gross profit and the marginal point cost of retailer *i* for each of the three cases, as summarized in Table 2.

Under the three demand scenarios, the MGP values of retailer *i* were non increasing with λ_i. Moreover, the MGP and MPC values for retailer *i* under the pure PS policy were always higher than those from the individual point scheme because of the extra demand resulting from the flexibility of the pure PS policy. The following figures showed the equilibrium λ_1 under the pure PS policy when $p_1 = \$10$, $c_1 = \$6$, $b_1 = 300$, $a_1 = 10$, and $e_1 = 200$. A small θ_i means that a large portion of the customers redeem their points earned from retailer *i* at retailer *j*. As shown in Fig. 1, the value of $(\lambda_1{}^\circ - \lambda_1{}^I)$ is higher when θ_1 is decreased from 95% to 85%. This finding means that when more customers who purchase products at Retailer 1 want to redeem the points at Retailer 2, the MGP of Retailer 1 decreased and it is incentivized to set a relatively high λ_1 through a pure PS policy. In addition, Proposition 1 explains that when $\Delta D_i(\lambda_i) = e_i\lambda_i$, $\lambda_i{}^o$ is always greater than $\lambda_i{}^I$ regardless of the value of θ_i. If $\lambda_i{}^o > \lambda_i{}^I$, then each existing or new customer can enjoy a higher point-conversion ratio at retailer *i*. However, not every customer under the policy benefits from the pure PS policy. As Fig. 1(f) shows, if the customers of retailer *i* are sensitive to λ_i, then we can find cases wherein $\lambda_i{}^o < \lambda_i{}^I$. In this case, existing customers who are not interested in redeeming the points from the other retailer may lose some utility under the pure PS policy.

Remark 1. Customers might benefit from a PS policy with a greater flexibility of point redemption and higher point-conversion ratios. However, in some cases, equilibrium conversion ratios under the PS policy are lower than under the individual point scheme.

3.2. Comparison between the pure PS policy and the centralized control

In this subsection, we analyzed overall optimal $\{\lambda_1, \lambda_2\}$ of the two retailers and uncover the potential improvement under the pure PS policy.

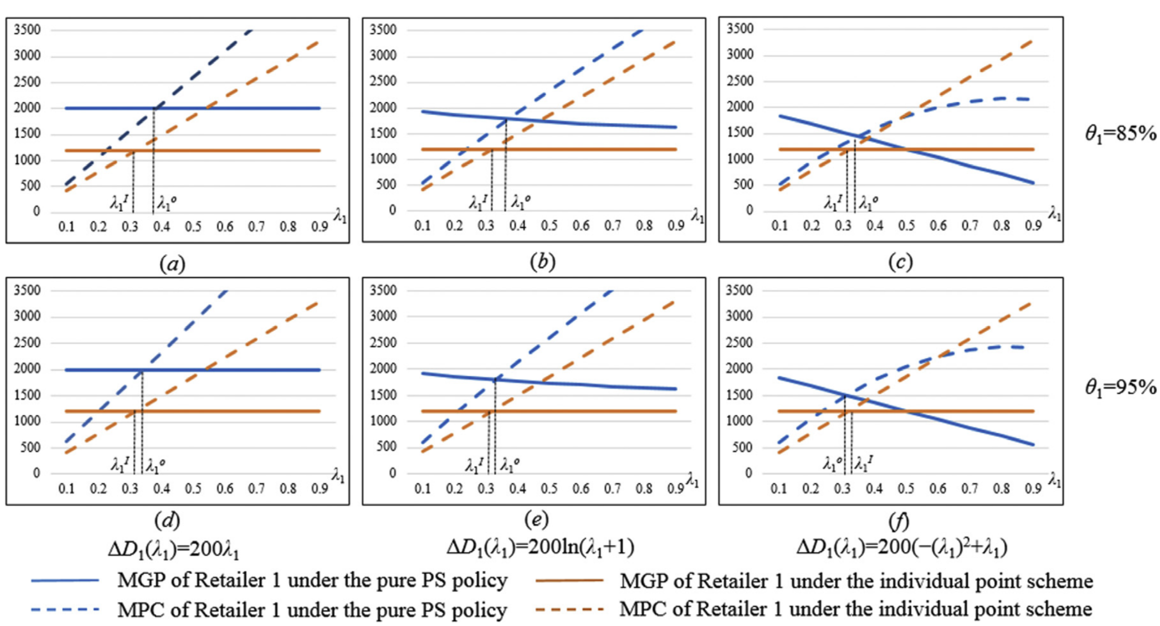

Fig. 1. $\lambda_1{}^\circ$ and $\lambda_1{}^I$ under three demand scenarios and two values of θ_1.

Table 3
Influence of Retailer 1's point-switching redemption when $\theta_2 = 85\%$.

θ_1 (%)	Pure PS policy					Overall optimal				
	$\lambda_1{}^\circ$	$\lambda_2{}^\circ$	$\Pi_1(\lambda_1{}^\circ)$	$\Pi_2(\lambda_2{}^\circ)$	$\Pi_{ch}(\lambda_1{}^\circ, \lambda_2{}^\circ)$	$\lambda_1{}^*$	$\lambda_2{}^*$	$\Pi_1(\lambda_1{}^*)$	$\Pi_2(\lambda_2{}^*)$	$\Pi_{ch}(\lambda_1{}^*, \lambda_2{}^*)$
55	0.396	0.138	303.04	218.44	521.48	0.101	0.081	325.17	268.19	593.36
65	0.297	0.138	316.83	241.49	558.32	0.102	0.081	320.51	272.99	593.50
75	0.224	0.138	319.87	256.19	576.06	0.103	0.081	315.80	277.85	593.65
85	0.169	0.138	318.10	265.97	584.07	0.104	0.081	310.96	282.83	593.79
95	0.124	0.138	314.00	273.12	587.12	0.105	0.081	306.02	287.92	593.94

Denote $\Pi_{ch}(\lambda_1, \lambda_2)$ as the total profit of the two retailers and let $\{\lambda_1{}^*, \lambda_2{}^*\}$ be the overall optimal $\{\lambda_1, \lambda_2\}$ that maximizes $\Pi_{ch}(\lambda_1, \lambda_2)$. We can obtain the following objective function.

$$\begin{aligned}\Pi_{ch}(\lambda_1, \lambda_2) = {} & (p_1 - c_1)D_1(\lambda_1) + (p_2 - c_2)D_2(\lambda_2) - c_1\theta_1\lambda_1 D_1(\lambda_1) \\ & - \frac{p_2 c_1}{p_1}(1-\theta_2)\lambda_2 D_2(\lambda_2) - c_2\theta_2\lambda_2 D_2(\lambda_2) \\ & - \frac{p_1 c_2}{p_2}(1-\theta_1)\lambda_1 D_1(\lambda_1) \qquad (6)\end{aligned}$$

Proposition 2. *Under the pure PS policy,*

(i) *If there exists a $\{\lambda_1, \lambda_2\}$ that satisfies*

$$\frac{\partial \Pi_{ch}(\lambda_1, \lambda_2)}{\partial \lambda_1} = \frac{\partial \Pi_{ch}(\lambda_1, \lambda_2)}{\partial \lambda_2} = 0, \qquad (7)$$

then retailers' overall optimal decision, $\{\lambda_1{}^*, \lambda_2{}^*\}$ can be obtained from Eq. (7).

(ii) *If the conditions in (i) holds, then $\lambda_i{}^o > \lambda_i{}^*$ $i = 1,2$.*

Under the pure PS policy, both retailers have increased demands because customers have high flexibility in terms of point redemption. However, the cost resulting from the point redemption spills over to the other member and each retailer's profit function contains part of the cost that is determined by its decision. Then, the marginal cost of λ_i at retailer i is decreased, thus distorting the retailer's decision and the overall optimal solution fails to be the Nash equilibrium under the pure PS policy. In particular, the two retailers can be too radical in setting point-conversion ratio. As Proposition 2 shows, retailer i is incentivized to select a $\lambda_i{}^o$ greater than $\lambda_i{}^*$. Therefore, the pure PS policy may not maximize the total profit. Although such a phenomenon can be beneficial to customers, it may have a negative effect on the incentive of retailers to implement the PS policy (Example 1 shows that cases exist in which the total profit under the pure PS policy can be smaller than the one under the individual point schemes). In addition, if one member of a large-scale generates many points through sales, then, a small partner faces the risk of undertaking a high point cost when the customer seeks to redeem the points from the smaller retailer. This is consistent with the real observation that a typical PS policy consists of members with similar transaction scales.

Proposition 3. *If the conditions in Proposition 2 hold, then (i) $\lambda_i{}^*$ is decreasing with θ_i when $p_i/c_i < p_j/c_j$, $i, j = 1,2$, and $i \neq j$. (ii) $\lambda_i{}^*$ is increasing with θ_i when $p_i/c_i > p_j/c_j$, $i, j = 1, 2$, and $i \neq j$.*

When we aim to maximize the total profit of the two retailers, the relationship between $\lambda_i{}^*$ and θ_i can be complicated because the two retailers usually have different cost performances. Retailer i has a higher cost performance than retailer j if $p_i/c_i > p_j/c_j$. Then, enabling customers to redeem their points from the retailer with a high value of p_i/c_i can be beneficial for the coalition of the two retailers. In this case, $\lambda_i{}^*$ increases with θ_i when $p_i/c_i > p_j/c_j$, $i, j = 1,2$, and $i \neq j$. Let $\Pi_{ch}{}^*$ be the maximum overall profit under the PS policy.

Proposition 4. *$\Pi_{ch}{}^*$ decreases with θ_i and increases with θ_j when $p_i/c_i < p_j/c_j$, $i, j = 1,2$, and $i \neq j$.*

In addition to increasing demand, the PS policy implies a new source of overall performance improvement. As Proposition 4 shows, the percentage of customer points switching between the two retailers affects the total profit. Therefore, point-switching ratio and cost performance should be considered in partner selection to maximize the total profit. Moreover, by encouraging customers to redeem more points from the retailer with a higher cost performance, the policy can reduce the total cost of point redemption of the two retailers.

Example 1. To obtain further managerial insights, we conducted three experiments to analyze retailers' equilibrium decisions and corresponding profits under the pure PS policy. Let $D_1(\lambda_1) = 100+150\lambda_1+40\ln(\lambda_1+1)$ and $D_2(\lambda_2) = 80+120\lambda_2+30\ln(\lambda_2+1)$, where in $\Delta D_1(\lambda_1) = 40\ln(\lambda_1+1)$ and $\Delta D_2(\lambda_2) = 30\ln(\lambda_2+1)$. Let the demands at Retailers 1 and 2 under the individual point scheme be $100+150\lambda_1$ and $80+120\lambda_2$, respectively. Let $c_1 = \$4$ and $c_2 = \$5$. In this experiment, we set the values of p_1 and p_2 to \$7 and \$8.5, respectively.

Table 3 shows the profit comparison of the pure PS policy and the overall optimal solution under different values of θ_1. According to Table 2, we can obtain that the retailer's optimal point-conversion ratios under the individual point scheme, $\{\lambda_1{}^I, \lambda_2{}^I\}$, is $\{0.042, 0.017\}$. Then the profits are $\Pi_1(\lambda_1{}^I) = \$301.04$, $\Pi_2(\lambda_2{}^I) = \$280.17$, and $\Pi_{ch}(\lambda_1{}^I, \lambda_2{}^I) = \$581.21$. Table 3 shows that the total profit cannot be maximized under a pure PS policy. Both Retailers 1 and 2 obtained higher profits under the pure PS policy than under an individual point scheme when $\theta_1 = 95\%$. Compared with the individual point scheme, the pure PS policy may fail to bring higher profits to the retailers if the cost spillover effect is significant. Moreover, $\{\lambda_1{}^*, \lambda_2{}^*\}$, which cannot guarantee a win-win profit split for both retailers, may fail to reach equilibrium. We can conjecture that the pure PS policy has difficulty in reaching $\{\lambda_1{}^*, \lambda_2{}^*\}$ if the contract determined by bargaining requires the retailers to set only $\{\lambda_1, \lambda_2\}$. Therefore, the pure PS policy may not be ubiquitous in the real world because of the limited total profit and profit split. When the pure PS policy is not acceptable to both retailers, customers cannot enjoy any flexibility of point redemption. What is worse, customers lose more benefits because the two retailers implement the individual point schemes under which λ_i is smaller than the one under the PS policies. Moreover, as $p_1/c_1 > p_2/c_2$, the maximum total profit increases with θ_1 when customers redeem more points for products with a higher cost performance. Due to the symmetry of Retailers 1 and 2, we omit the experiments on θ_2.

Table 4 shows the decisions and the corresponding profits under different values of a_1, respectively. From Eq. (2), we found that a_1 did not have an impact on λ_2. Because the value of a_1 indicates the scale of demand which is not influenced by the PS policy, a higher a_1 could only increase the MPC without affecting the marginal revenue (see Table 2). Therefore, the $\lambda_1{}^I$, $\lambda_1{}^\circ$, and $\lambda_1{}^*$ are decreased under the three scenarios when a_1 is increased. Moreover, the pure PS policy could not always guarantee a higher total profit than the individual point scheme. A small a_1 implies that the extra revenue (partly depending on e_1) generated from the PS

Table 4
Comparisons of $\{\lambda_1, \lambda_2\}$ and profits under different values of a_1.

a_1	Individual point scheme			Pure PS policy			Overall optimal		
	$\lambda_1{}^I$	$\lambda_2{}^I$	$\Pi_{ch}(\lambda_1{}^I, \lambda_2{}^I)$	$\lambda_1{}^\circ$	$\lambda_2{}^\circ$	$\Pi_{ch}(\lambda_1{}^\circ, \lambda_2{}^\circ)$	$\lambda_1{}^*$	$\lambda_2{}^*$	$\Pi_{ch}(\lambda_1{}^*, \lambda_2{}^*)$
60	0.175	0.017	478.54	0.342	0.145	485.35	0.214	0.085	500.56
80	0.108	0.017	527.21	0.289	0.145	530.20	0.162	0.085	545.40
100	0.042	0.017	581.21	0.237	0.145	579.29	0.109	0.085	594.49
120	0.000	0.017	640.17	0.184	0.145	632.61	0.056	0.085	647.77
140	0.000	0.017	700.17	0.132	0.145	690.18	0.004	0.085	705.34

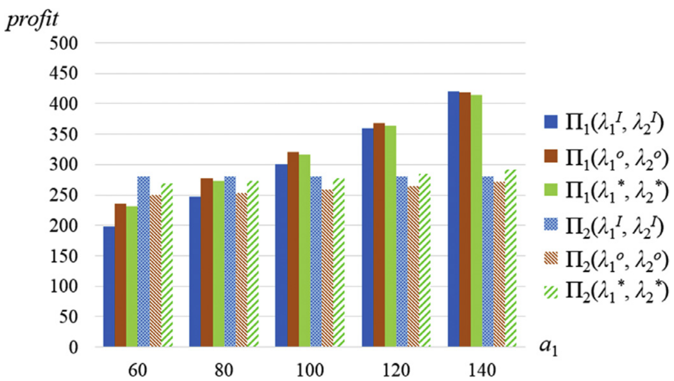

Fig. 2. Profits of Retailers 1 and 2 under different values of a_1.

policy may play a more important part in the total demand than other parameters. In this case, the point cost due to a high λ_1 value is not as significant as the extra revenue. When a_1 is increased, the demand under the individual point scheme is higher and it is dominant in the total demand under a pure PS policy. As a result, the higher point cost under the PS policy makes the total profit lower than the individual point scheme.

Based on Eq. (1) and $\{\lambda_1{}^\circ, \lambda_2{}^\circ\}$, shown in Table 4, the number of points generated by Retailer 1 is 294.6 and 270.1 when $a_1 = 60$ and $a_1 = 80$, respectively. This finding means that the total points generated by Retailer 1 are decreased because of a low λ_1, even when a_1 is increased. In this case, fewer points from Retailer 1 are redeemed at Retailer 2 and, Retailers 1 and 2 both benefit from a high a_1 under the pure PS policy because of the low point cost. Therefore, one retailer may be more likely to select a partner to join in a pure PS policy who has a relatively large number of customers not attracted by the PS policy.

Fig. 2 illustrates the profits of Retailers 1 and 2 under the three scenarios of point management. When a_1 is no greater than 120, only Retailer 1 can obtain a higher profit under the pure PS policy than under the individual point scheme. Hence, Retailer 1 may encourage Retailer 2 to join the pure PS policy by transferring compensation money if the overall profit of the coalition under the pure PS policy is higher than it is under the individual point scheme (see $a_1 = 60$ and $a_1 = 80$ in Table 4). When $a_1 = 100$ and $a_1 = 120$, Retailer 1 cannot offer compensation money to make both retailers benefit from joining the policy because $\Pi_{ch}(\lambda_1{}^\circ, \lambda_2{}^\circ) < \Pi_{ch}(\lambda_1{}^I, \lambda_2{}^I)$. When $a_1 = 140$, neither retailer can improve the profit by switching from the individual point scheme to the pure PS policy. In these cases, no retailer is willing to implement the pure PS policy, and customers cannot enjoy the flexibility of point redemption and higher point-conversion ratios.

Remark 2. The amount of inherent demand that is not influenced by the point-conversion ratio and PS policy has an impact on the performance of the pure PS policy. This demand should also be considered by managers during coalition partner selection.

Remark 3. In some cases, the performances of both retailers are poorer when switching from the individual point scheme to the pure PS policy.

3.3. Model when retail prices are decision variables

To make our model more general, we discussed the cases wherein retailers can decide their own retail prices. We extended $D_i(\lambda_i)$ as $D_i(p_i, \lambda_i)$ with $\partial D_i(p_i, \lambda_i)/\partial p_i < 0$. In this case, under the pure PS policy, the retailers' objective functions are as follows:

$$\begin{aligned}\Pi_1(p_1, \lambda_1) &= (p_1 - c_1)D_1(p_1, \lambda_1) - c_1\theta_1\lambda_1 D_1(p_1, \lambda_1)\\ &\quad - \frac{p_2 c_1}{p_1}(1-\theta_2)\lambda_2 D_2(p_2, \lambda_2)\end{aligned} \tag{8}$$

$$\begin{aligned}\Pi_2(p_2, \lambda_2) &= (p_2 - c_2)D_2(p_2, \lambda_2) - c_2\theta_2\lambda_2 D_2(p_2, \lambda_2)\\ &\quad - \frac{p_1 c_2}{p_2}(1-\theta_1)\lambda_1 D_1(p_1, \lambda_1)\end{aligned} \tag{9}$$

The total profit function satisfies $\Pi_{ch}(p_1, \lambda_1, p_2, \lambda_2) = \Pi_1(p_1, \lambda_1) + \Pi_2(p_2, \lambda_2)$.

We can obtain

$$\frac{\partial \Pi_1(p_1, \lambda_1)}{\partial \lambda_1} = (p_1 - c_1 - c_1\theta_1\lambda_1)\frac{\partial D_1(p_1, \lambda_1)}{\partial \lambda_1} - c_1\theta_1 D_1(p_1, \lambda_1) \tag{10}$$

$$\begin{aligned}\frac{\partial \Pi_1(p_1, \lambda_1)}{\partial p_1} &= D_1(p_1, \lambda_1) + (p_1 - c_1 - c_1\theta_1\lambda_1)\frac{\partial D_1(p_1, \lambda_1)}{\partial p_1}\\ &\quad + \frac{p_2}{p_1^2}c_1\lambda_2(1-\theta_2)D_2(p_2, \lambda_2)\end{aligned} \tag{11}$$

With the same approach, we can obtain $\partial \Pi_2(p_2, \lambda_2)/\partial p_2$ and $\partial \Pi_2(p_2, \lambda_2)/\partial \lambda_2$. If $\{p_1{}^*, p_2{}^*, \lambda_1{}^*, \lambda_2{}^*\}$ satisfies $\frac{\partial \Pi_{ch}(p_1^*, p_2^*, \lambda_1^*, \lambda_2^*)}{\partial p_1} = \frac{\partial \Pi_{ch}(p_1^*, p_2^*, \lambda_1^*, \lambda_2^*)}{\partial \lambda_2} = \frac{\partial \Pi_{ch}(p_1^*, p_2^*, \lambda_1^*, \lambda_2^*)}{\partial \lambda_2} = \frac{\partial \Pi_{ch}(p_1^*, p_2^*, \lambda_1^*, \lambda_2^*)}{\partial p_2} = 0$, then we have $(p_1^* - c_1 - c_1\theta_1\lambda_1^* - \frac{p_1^* c_2}{p_2^*}(1-\theta_1)\lambda_1^*)\frac{\partial D_1(p_1^*, \lambda_1^*)}{\partial \lambda_1^*} = (c_1\theta_1 + \frac{p_1^* c_2}{p_2^*}(1-\theta_1))D_1(p_1^*, \lambda_1^*)$.

Suppose that $p_1 = p_1{}^*$, $p_2 = p_2{}^*$, and $\lambda_2 = \lambda_2{}^*$. In this case, we have

$$\frac{\partial \Pi_1\left(p_1^*, \lambda_1\right)}{\partial \lambda_1} = (p_1^* - c_1 - c_1\theta_1\lambda_1)\frac{\partial D_1\left(p_1^*, \lambda_1\right)}{\partial \lambda_1} - c_1\theta_1 D_1(p_1^*, \lambda_1)$$

with

$$\begin{aligned}\frac{\partial^2 \Pi_1\left(p_1^*, \lambda_1\right)}{\partial (\lambda_1)^2} &= (p_1^* - c_1 - c_1\theta_1\lambda_1)\frac{\partial^2 D_1\left(p_1^*, \lambda_1\right)}{\partial (\lambda_1)^2}\\ &\quad - 2c_1\theta_1\frac{\partial D_1\left(p_1^*, \lambda_1\right)}{\partial \lambda_1}.\end{aligned}$$

It is easy to show that there exist λ_1 that satisfies $\partial \Pi_1(p_1{}^*, \lambda_1)/\partial \lambda_1 = 0$ and $\partial^2 \Pi_1(p_1{}^*, \lambda_1)/\partial(\lambda_1)^2 < 0$. Then, the optimal λ_1 satisfies $\partial \Pi_1(p_1{}^*, \lambda_1)/\partial \lambda_1 = 0$ and it is different from $\lambda_1{}^*$. It means that, when Retailer 2 selects $\{p_2{}^*, \lambda_2{}^*\}$, Retailer 1 can obtain a higher profit by selecting $\{p_1, \lambda_1\}$ different from $\{p_1{}^*, \lambda_1{}^*\}$. Therefore, if there exists a $\{p_1{}^\circ, p_2{}^\circ, \lambda_1{}^\circ, \lambda_2{}^\circ\}$ that is a pure-strategy Nash

Table 5
Influence of Retailer 1′s point-switching redemption under the decentralized control when $\theta_2 = 90\%$.

θ_1 (%)	$p_1{}^{\circ}$	$p_2{}^{\circ}$	$\lambda_1{}^{\circ}$	$\lambda_2{}^{\circ}$	$\Pi_1(p_1{}^{\circ}, \lambda_1{}^{\circ})$	$\Pi_2(p_2{}^{\circ}, \lambda_2{}^{\circ})$	$\Pi_{ch}(p_1{}^{\circ}, p_2{}^{\circ}, \lambda_1{}^{\circ}, \lambda_2{}^{\circ})$
0.71	99.4	101.69	1.02	0.32	10,125.69	21,233.04	31,358.73
0.72	104.05	106.08	1.15	0.42	9977.13	21,308.28	31,285.41
0.73	109.05	110.82	1.30	0.53	9775.02	21,405.05	31,180.06
0.74	114.51	115.94	1.45	0.65	9516.67	21,514.62	31,031.29
0.75	120.37	121.55	1.62	0.79	9185.45	21,669.58	30,855.03

Table 6
Influence of Retailer 1′s point-switching redemption under the centralized control when $\theta_2 = 90\%$.

θ_1 (%)	$p_1{}^*$	$p_2{}^*$	$\lambda_1{}^*$	$\lambda_2{}^*$	$\Pi_1(p_1{}^*, \lambda_1{}^*)$	$\Pi_2(p_2{}^*, \lambda_2{}^*)$	$\Pi_{ch}(p_1{}^*, p_2{}^*,\lambda_1{}^*, \lambda_2{}^*)$
0.71	85.49	129.59	0.41	0.81	7554.45	26,503.18	34,057.63
0.72	84.92	128.97	0.40	0.80	7511.74	26,529.85	34,041.59
0.73	84.35	128.35	0.38	0.78	7472.13	26,554.09	34,026.22
0.74	83.77	127.75	0.36	0.77	7435.60	26,575.90	34,011.49
0.75	83.19	127.16	0.34	0.76	7402.15	26,595.28	33,997.43

Table 7
Demands and transformed prices under the pure PS policy and decentralized control when $\theta_2 = 90\%$.

θ_1 (%)	$D_1(p_1{}^{\circ}, \lambda_1{}^{\circ})$	$D_1(p_1{}^*, \lambda_1{}^*)$	$D_2(p_2{}^{\circ}, \lambda_2{}^{\circ})$	$D_2(p_2{}^*, \lambda_2{}^*)$	$p_1{}^{to}$	$p_1{}^{t*}$	$p_2{}^{to}$	$p_2{}^{t*}$
0.71	353.72	290.19	503.66	454.01	56.18	57.51	68.33	72.79
0.72	351.24	289.83	504.91	454.09	55.02	57.57	66.43	73.13
0.73	348.32	289.50	506.35	454.16	53.86	57.64	64.67	73.47
0.74	344.93	289.20	508.00	454.21	52.70	57.71	63.02	73.80
0.75	340.98	288.92	509.87	454.26	51.53	57.80	61.50	74.13

equilibrium under the pure PS policy, then it differs from $\{p_1{}^*, p_2{}^*, \lambda_1{}^*, \lambda_2{}^*\}$. This situation implies that when retail prices are decision variables, the maximum total profit cannot be achieved under the pure PS policy.

Example 2: We conducted the following experiments to analyze the cases wherein retail prices are decision variables. Let $D_1(p_1, \lambda_1) = 1000{+}300\lambda_1{-}10p_1{+}60\ln(\lambda_1{+}1)$ and $D_2(p_2, \lambda_2) = 1200{+}300\lambda_2{-}8p_1{+}80\ln(\lambda_2{+}1)$. We obtained the equilibrium decisions under the pure PS policy and the optimal decisions under the centralized control with PS policy from the mathematic software, *Matlab*. Hessian Matrices have been checked to guarantee that the solutions are optimal to the decision frameworks. Table 5 summarizes retailers' equilibrium decisions under the pure PS policy and different values of θ_1 when $\theta_2 = 90\%$. When θ_1 is increased, less customers who obtain points from Retailer 1 redeem the points at Retailer 2. Therefore, for a $\{p_1, p_2, \lambda_1, \lambda_2\}$, the point cost of Retailer 1 is increased while the one of Retailer 2 is decreased. As Table 5 shows, $\Pi_1(p_1{}^{\circ}, \lambda_1{}^{\circ})$ is decreased and $\Pi_2(p_2{}^{\circ}, \lambda_2{}^{\circ})$ is increased with θ_1.

The equilibrium decisions also change when θ_1 varies. According to Eq. (8), we can see that Retailer 1′s gross profit is $(p_1{-}c_1)D_1(p_1, \lambda_1)$, and the point cost consists of two parts, i.e., $c_1\theta_1\lambda_1D_1(p_1, \lambda_1)$ and $p_2(1{-}\theta_2)c_1D_2(p_2, \lambda_2)/p_1$. Given a $\{p_1, p_2, \lambda_1, \lambda_2\}$, a higher θ_1 means that a smaller percentage of points generated by Retailer 1 is redeemed at Retailer 2, which can increase the point cost of Retailer 1 while can decrease the point cost of Retailer 2 (see Eqs. (8) and 9). In this case, we can conjecture that Retailers 1 and 2 would be incentivized to decrease and increase the demands, respectively, to meet the new situation of point cost. Then, Retailer 1 would prefer a higher $p_1{}^{\circ}$ and higher $\lambda_1{}^{\circ}$ under a higher θ_1, considering that it can bring higher gross profit and decrease a part of the point cost, as Table 5 shows. Note that $\lambda_1{}^{\circ}$ in Table 5 are higher than 1 which can bring more free products to customers than the "buy-one-get-one-free" scheme. The symmetry of $\Pi_1(p_1, \lambda_1)$ and $\Pi_2(p_2, \lambda_2)$ implies that Retailer 2 also prefers increasing p_2 and λ_2 simultaneously when aiming to increase the demand.

Table 6 illustrates the optimal decision and corresponding profits under the centralized control. According to Tables 5 and 6, we can obtain $D_1(p_1{}^{\circ}, \lambda_1{}^{\circ})$, $D_2(p_2{}^{\circ}, \lambda_2{}^{\circ})$, $D_1(p_1{}^*, \lambda_1{}^*)$, and $D_1(p_1{}^*, \lambda_1{}^*)$ under the pure PS policy and the centralized control which are summarized in Table 7. We can see that $D_1(p_1{}^{\circ}, \lambda_1{}^{\circ}) > D_1(p_1{}^*, \lambda_1{}^*)$ and $D_2(p_2{}^{\circ}, \lambda_2{}^{\circ}) > D_1(p_1{}^*, \lambda_1{}^*)$ under different values of θ_1. Moreover, $D_1(p_1{}^{\circ}, \lambda_1{}^{\circ})$ is decreasing while $D_2(p_2{}^{\circ}, \lambda_2{}^{\circ})$ is increasing with θ_1. Because of the point-switching redemption under the pure PS policy, Retailer 2 undertakes a part of point cost of the points generated by Retailer 1. However, Retailers 1 and 2 need to consider that part of point cost, i.e., $\frac{p_1c_2}{p_2}(1-\theta_1)\lambda_1D_1(p_1, \lambda_1)$ when they achieve an agreement of a coordinating contract and are motivated to maximize the overall profit. The demands shown in Table 7 imply that the consideration of this cost may make the coalition of Retailers 1 and 2 select a lower demand at Retailer 1 than under the pure PS policy.

Considering the point redemption, customers may get some free products that makes the average price of the received products at each retailer complicated. On the other hand, retailers can influence the prices by adjusting retail prices and point-conversion ratios so that the customers' welfare is not straightforward. We defined $p_i{}^{to}$ and $p_i{}^{t*}$ as the average prices of products at retailer i under the pure PS policy and the centralized control with the following equations.

$$p_i^{to} {=} \frac{p_i^o D_i\left(p_i^o, \lambda_i^o\right)}{D_i\left(p_i^o, \lambda_i^o\right) + \theta_i\lambda_i^o D_i\left(p_i^o, \lambda_i^o\right) + \frac{p_j^o(1-\theta_j)\lambda_j^o D_j\left(p_j^o, \lambda_j^o\right)}{p_i^o}} i, j{=}1, 2, i \neq j \tag{12}$$

$$p_i^{t*} {=} \frac{p_i^* D_i\left(p_i^*, \lambda_i^*\right)}{D_i\left(p_i^*, \lambda_i^*\right) + \theta_i\lambda_i^* D_i\left(p_i^*, \lambda_i^*\right) + \frac{p_j^*(1-\theta_j)\lambda_j^* D_j\left(p_j^*, \lambda_j^*\right)}{p_i^*}} i, j{=}1, 2, i \neq j \tag{13}$$

In Eq. (12), the numerator is the customers' monetary payment to retailer i, and the denominator contains three part, (i) the number of products sold to customers by retailer i, (ii) the number of products related to local redemption at retailer i, and (iii) the number of products related to point-switching redemption from retailer j to retailer i. Hence, p_i^{to} represents the average price of the products that customers obtained from retailer i by purchasing or point redemption under the pure PS policy. With the similar discussion, p_i^{t*} represents the average price under the centralized control. As Table 7 shows, customers can enjoy lower average prices at Retailers 1 and 2 under the pure PS policy than under the centralized control. Because of the point-switching redemption, each retailer's point cost is partially undertaken by the coalition partner. In this case, retailers would achieve equilibrium decisions which lead to lower average prices than those under the centralized control, respectively.

Remark 4. Customers would prefer incompact coalitions of retailers without a coordinating contract. On the other hand, the retailers may achieve higher profits with such a contract that may negatively influence the customers' welfare.

4. The target rebate contract

In this section we proposed a TR contract to coordinate the two retailers under which the total profit can be arbitrarily split. The TR contract achieves coordination by making retailers' profits be linear functions of the total profit, which is a common approach of contract design [12,22]. Our TR contract is developed based on the idea of the channel rebate contract which has been used to coordinate many supply chains in the world [26]. Under the channel rebate contract, the supplier sets a target level, and the rebate is paid from the supplier to the retailer for each unit sold beyond that specified target level. By using this idea, we developed the TR contract to coordinate the point setting of the retailers. Because different channels or retailers may have different ranges of absolute value of points, we used the percentage of the amount of points switching from Retailer 1 to Retailer 2 in the total amount of points switching between the two retailers. A high percentage implies that many points generated by Retailer 1 are redeemed at Retailer 2, when Retailer 2 may undertake a high point cost.

Let $S_1(p_1, \lambda_1)$ be the amount of points switching from Retailer 1 to Retailer 2 in one selling season. Let $S_2(p_2, \lambda_2)$ be the amount of points switching from Retailer 2 to Retailer 1. Then, we have

$$S_1(p_1, \lambda_1) = p_1\lambda_1(1-\vartheta_1)D_1(p_1, \lambda_1). \tag{14}$$

$$S_2(p_2, \lambda_2) = p_2\lambda_2(1-\vartheta_2)D_2(p_2, \lambda_2). \tag{15}$$

Under the TR contract, retailers decide on a target percentage of $S_1(p_1, \lambda_1)$ or $S_2(p_2, \lambda_2)$ in $S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)$. Without loss of generality, the target value of Retailer 1 is denoted as a and $0<a<1$. Then, the target value of Retailer 2 is $1-a$. Retailer 1 pays γ to Retailer 2 for each percent unit that exceeds the target a, and Retailer 2 pays γ, as rebates, to Retailer 1 for each percent unit that exceeds $1-a$. $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a$ and $S_2(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-(1-a)$ cannot be greater (or smaller) than zero simultaneously. Therefore, only one retailer needs to pay this type of rebate to the other one, and the retailers need to decide only the value of a. If $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]$ is greater than a, then Retailer 1 pays Retailer 2 $[S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a]\gamma$; otherwise, Retailer 2 pays Retailer 1 $[a-S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]]\gamma$. Mathematically speaking, this statement is equivalent to "*Retailer 1 pays Retailer 2* $[S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a]\gamma$ *and a negative payment means a fund flow from Retailer 2 to Retailer 1*".

The TR contract has two contract parameters: a and γ. The former determines the target percentage and the latter is the payment from Retailer 1 to Retailer 2 for a percent unit that exceeds a, and the payment from Retailer 2 to Retailer 1 is for a percent unit that exceeds $1-a$. We set Φ as the split scenario of the total profit, which is limited to the range of [0, 1]. The timing of events under a TR contract is as follows:

- Before a selling season, two retailers negotiate and decide the value of Φ to fix the profit allocation.
- The retailers calculate a and γ using Eqs. (16) and (17) based on Φ.
- Retailer i decides p_i and λ_i, $i = 1, 2$.
- At the end of the selling season, Retailer 1 pays Retailer 2 $[S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a]\gamma$ if $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]>a$; otherwise, Retailer 2 pays Retailer 1 $[S_2(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-(1-a)]\gamma$.

Theorem 1 shows that for any Φ, there exist contract parameters under which coordination between both retailers can be achieved.

Theorem 1. *For any $\Phi\in[0, 1]$, consider the TS contract with*

$$a = \frac{t_1}{t_1+t_2} \tag{16}$$

$$\gamma = \frac{(t_1+t_2)(S_1(p_1,\lambda_1)+S_2(p_2,\lambda_2))}{p_1(1-\theta_1)\lambda_1 p_2(1-\theta_2)\lambda_2}, \tag{17}$$

where in

$$t_1 = p_1\lambda_1(1-\theta_1)\left(\Phi(p_2-c_2-c_2\lambda_2\theta_2) + (1-\Phi)\frac{p_2c_1\lambda_2}{p_1}(1-\theta_2)\right)$$

and

$$t_2 = p_2\lambda_2(1-\theta_2)\left((1-\Phi)(p_1-c_1-c_1\lambda_1\theta_1) + \Phi\frac{p_1c_2\lambda_1}{p_2}(1-\theta_1)\right).$$

Retailer 1 pays Retailer 2 $[S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a]\gamma$ if $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]>a$; otherwise, Retailer 2 pays Retailer 1 $[S_2(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-(1-a)]\gamma$. Under this TR contract, the profit function of Retailer 1 is $\Phi\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$, and both retailers are incentivized to choose $\{p_1^*, p_2^*, \lambda_1^*, \lambda_2^*\}$.

Theorem 1 explains that, for any profit-split scenario, the way to set a and γ to incentive the retailers to accept the overall optimal decisions on $\{p_1, p_2, \lambda_1, \lambda_2\}$ that maximizes the total profit of Retailers 1 and 2. The value of Φ was obtained from the negotiation between the two retailers because it can influence the final profit split. Moreover, both a and γ are influenced by Φ.

Lemma 1. *The overall optimal solution, $\{p_1^*, p_2^*, \lambda_1^*, \lambda_2^*\}$, satisfies $p_1^* - c_1 - c_1\theta_1\lambda_1^* - \frac{p_1^*c_2\lambda_1^*}{p_2^*}(1-\theta_1) > 0$ and $p_2^* - c_2 - c_2\theta_2\lambda_2^* - \frac{p_2^*c_1\lambda_2^*}{p_1^*}(1-\theta_2) > 0$.*

Consider that Φ increases by $\Delta\Phi>0$. A unique $a+\Delta a$ exists that satisfies $a+\Delta a = t_1(\Phi+\Delta\Phi)/[t_1(\Phi+\Delta\Phi)+t_2(\Phi+\Delta\Phi)]$. As t_1 increases with Φ and t_2 decreases with Φ when $p_1 - c_1 - c_1\theta_1\lambda_1 - \frac{p_1c_2\lambda_1}{p_2}(1-\theta_1) > 0$ and $p_2 - c_2 - c_2\theta_2\lambda_2 - \frac{p_2c_1\lambda_2}{p_1}(1-\theta_2) > 0$, $t_2(\Phi+\Delta\Phi)/t_1(\Phi+\Delta\Phi)<t_2(\Phi)/t_1(\Phi)$. Then, $\Delta a>0$. As $\{p_1^*, p_2^*, \lambda_1^*, \lambda_2^*\}$ can always be achieved under the TR contract. According to Lemma 1, we can infer that for $\{p_1, p_2, \lambda_1, \lambda_2\}$, the overall optimal value of a increases with Φ.

The TR contract can coordinate both retailers and arbitrarily split the total profit. By designing a as the threshold of the point-switching ratio rather than the threshold of the absolute value of switching points, we can make our TR contract easier to implement in different cases. Inevitably, administrative cost is involved in the implementation of contracts between retailers; this involvement of administrative cost is a limitation of the wide adoption of contracts. Under the revenue-sharing contract, for example, one source of administrative cost is that the supplier must monitor the retailer's sales revenue [2,13]. The TR contract has two advantages in this aspect. First, information flow is simple and the cost of monitoring point switching is small. In modern consumption point programs, customer information that includes amount of points is usually saved in membership cards that can be easily read and updated by the retailer's information systems. The amount of switching points can be available to both retailers. Second, at most, one fund flow between retailers is involved under the TR contract because retailers cannot exceed their target values of the switching-point ratio simultaneously. Moreover, in contrast to the PP policy, retailers are not needed to purchase points from the third-party point company, thus making the decision framework and fund flows of the two retailers simple.

When p_1 and p_2 are exogenous parameters, we developed a new approach of setting contract parameters that only depend on retail prices and production costs. The new TS contract can coordinate the two retailers and lead to a Pareto-optimal split of the total profit that is relatively fair to both retailers.

Theorem 2. *When p_1 and p_2 are exogenous parameters, the TS contract with the following are considered:*

$$a=\frac{c_1 p_2}{c_1 p_2+c_2 p_1}. \tag{18}$$

$$\gamma=\frac{(c_1 p_2+c_2 p_1)(S_1(p_1,\lambda_1)+S_2(p_2,\lambda_2))}{p_1 p_2}. \tag{19}$$

The payment rule between Retailers 1 and 2 is the same as that in Theorem 1. Under this TR contract, both members are incentivized to choose $\{\lambda_1^*, \lambda_2^*\}$ and the profit split is fixed.

The simplified TS contract coordinates the two retailers by making the profit of Retailer i dependent on all revenues and costs related to λ_i. In this case, the cost spillover effect under the PS policy is eliminated, while the advantage of PS policy is still available. Moreover, the contract parameter a only depends on retail prices and production costs, which are certain values.

One weakness of this simplified TS contract is the limited range of profit allocation between the two retailers. As Theorem 2 shows, this contract can achieve a certain profit allocation, which may negatively affect the flexibility of the contract. However, such a profit allocation can be acceptable in many real cases. First, it is a Pareto-optimal solution for both retailers. A solution is Pareto-optimal if any other solution cannot make one retailer better off without making the other one worse off [14]. Second, the fixed profit split is relatively fair between two retailers. Under such a contract, one retailer's profit only depends on the demand and cost resulting from its own decision. The retailer needs to take care of all the costs, including production cost and consumption points, it generates while also enjoying the incremental demand due to the PS policy.

5. Managerial insights and conclusions

5.1. Managerial insights

Our results showed that the point-switching ratio influences the maximum total profit. In the case where $c_i/p_i>c_j/p_j$, the maximum total profit was increased when θ_i was decreased. or θ_j was decreased when the demand functions satisfy the condition in Proposition 1. Therefore, encouraging more customers to redeem their points from the retailer with the higher cost performance may benefit retailers.

When the demand at one retailer increases under the PS policy, new customers are usually interested in redeeming their points from the other retailer. Therefore, the switching ratio can be positively associated with increased demand in the real world. Because the increased demand is enjoyed by the retailer, and the cost spills over and is partially undertaken by the other retailer, the retailer with the increased high demand can benefit much from the pure PS policy. The retailer that fails to obtain a high increased demand bears three disadvantages: low increased demand, low proportion of points switching to the coalition partner, and a high percentage of point switching in from the partner. Therefore, finding appropriate partners for the pure PS policy is important to ensure that both retailers can obtain higher profits than they could under individual point schemes.

We found two weak points of the pure PS policy, and they could explain the reasons a pure PS policy is not ubiquitous in the real world. First, the total profit cannot be maximized under the pure PS policy because the cost spillover distorts the retailers' optimal decision on $\{\lambda_1, \lambda_2\}$. Because administrative expense is involved in the pure PS policy, limited profit improvement can negatively affect the managers' incentive to implement such a policy. This limitation can be more critical when the total profit under the pure PS policy is smaller than the one under an individual point scheme. Second, the total profit split is fixed under the policy. In some cases, one retailer may obtain a smaller profit under the policy than under the individual point scheme such that the retailer cannot agree on the PS policy despite the improved total profit.

Both retailers may experience higher profits in cases based on $\{\lambda_1^*, \lambda_2^*\}$ than those based on $\{\lambda_1^\circ, \lambda_2^\circ\}$. On one hand, $\{\lambda_1^*, \lambda_2^*\}$ cannot create an equilibrium under the pure PS policy because either retailer (e.g., Retailer 1) can always benefit from selecting the λ_1 value that is higher than the λ_1^* value when the other retailer is bound by the overall optimal point-conversion ratio. On the other hand, for those retailers engaged in a simple and effective bargaining process, cases based on $\{\lambda_1^*, \lambda_2^*\}$ can leave space for retailers to achieve a win outcome. Hence, managers might increase the total and individual profits by equipping the pure PS policy with such a bargaining mechanism rather than implementing a TR contract.

5.2. Concluding remarks

In this study, we analyzed the PS policy that articulates a common practice in the real world. We discussed the cost spillover between retailers and the ways it influences the overall performance of the two retailers under the pure PS policy. We found that when the retail prices are fixed, retailers prefer high point-conversion ratios to the overall optimal ones. Therefore, total profit cannot be maximized under the pure PS policy. When θ_i is sufficiently small, retailer i benefits from setting a λ_i^o value higher than the λ_i^I value. Although customers benefited by obtaining more points, the total profit under the pure PS policy was smaller than under the individual point scheme. Moreover, the pure PS policy could not guarantee a win profit split to retailers when the individual point scheme was used as a benchmark. When the point-switching ratio is small, some cases showed $\lambda_i^o<\lambda_i^I$ such that customers lose some benefit under the pure PS policy because of a low point-conversion ratio. When retail prices are decision variables, the overall optimal decisions are different from the retailers' Nash equilibrium under the pure PS policy.

Our numerical experiments showed that each retailer can be incentivized to generate a higher demand when some of the point cost is undertaken by a partner. Moreover, the average price of the products operated by each retailer is also lower under the pure PS policy than under centralized control. This finding means that, although the retailers' overall profit may not be maximized under the pure PS policy, customers may experience greater welfare than when facing two retailers under centralized control or with coordinating contracts.

TR contracts are developed to coordinate the two retailers and arbitrarily split the total profit when coupled with the PS policy.

We showed that the TR contract studied had advantages in terms of low administrative cost. For cases with fixed retail prices, we developed a simplified TR contract to coordinate the two retailers and achieve a fixed profit allocation. Under such a contract, each retailer's profit consists and only consists of the terms that are influenced by its own decision. In this case, retailers face higher demands and undertake all their own production and point costs. Then, the profit allocation is Pareto optimal and fair to both retailers.

We developed our model with a general form of demand function. By considering some specific assumptions about the form of the functions of demand and the point-switching ratio, more analytic results could be obtained for cases in which retail prices are decision variables. Then, managers can determine the ways the retail pricing and point-conversion ratio jointly influence their own profits and the overall performance of both retailers under the PS policy.

Moreover, analyzing the advantages of PS policies in more complicated models can be fruitful directions of future work. First, the sales promotion by one retailer can be more remarkable when more partners are joined by a PS policy because customers can take advantage of the greater flexibility of point redemption. Moreover, their point accumulation effectiveness can also be improved under this increased number of point sources. In this case, designing a contract to coordinate such retailers and arbitrarily split the profit among retailers can be a challenging endeavor. Second, whether PS polices can be implemented to improve the performance of competing retailers or not is an interesting direction to extend the theoretical and practical insights of PS policies. Although PS policies may make the competition fiercer, the total demand can be increased meaningfully because the retailers would share many common customers.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.omega.2019.02.007.

Appendix

Proof. of Proposition 1.

(i) From Eq. (1), we **can** obtain

$$\frac{d\Pi_1(\lambda_1)}{d\lambda_1} = (p_1 - c_1)\frac{dD_1(\lambda_1)}{d\lambda_1} - c_1\theta_1 D_1(\lambda_1) - c_1\theta_1\lambda_1\frac{dD_1(\lambda_1)}{d\lambda_1}, \tag{A1}$$

and

$$\frac{d^2\Pi_1(\lambda_1)}{d(\lambda_1)^2} = (p_1 - c_1 - c_1\theta_1\lambda_1)\frac{d^2D_1(\lambda_1)}{d(\lambda_1)^2} - 2c_1\theta_1\frac{dD_1(\lambda_1)}{d\lambda_1}. \tag{A2}$$

Suppose that $\lambda_1{}^\circ$ satisfies $d\Pi_1(\lambda_1{}^\circ)/d\lambda_1 = 0$, then we have

$$(p_1 - c_1 - c_1\vartheta_1\lambda_1{}^o)dD_1(\lambda_1{}^o)/d\lambda_1 = c_1\vartheta_1 D_1(\lambda_1{}^o). \tag{A3}$$

As $D_1(\lambda_1)>0$ and $dD_1(\lambda_1)/d\lambda_1>0$, we have $(p_1-c_1-c_1\theta_1\lambda_1{}^\circ)>0$. As $d^2D_1(\lambda_1)/d\lambda_1{}^2 <0$, we obtain that $\lambda_1{}^\circ$ satisfies $d^2\Pi_1(\lambda_1)/d\lambda_1{}^2 <0$. Therefore, $\lambda_1{}^\circ$ is the equilibrium decision of Retailer 1 under the pure PS policy. With the same approach, we can prove that $\lambda_2{}^\circ$ is the equilibrium decision of Retailer 2.

(ii) The above proof tells that $\lambda_1{}^\circ$ satisfies Eq. (A3) and $(p_1-c_1-c_1\theta_1\lambda_1{}^\circ)>0$. Define the function $f(\lambda_1|\theta_1) = (p_1-c_1-c_1\theta_1\lambda_1)dD_1(\lambda_1)/d\lambda_1-c_1\theta_1 D_1(\lambda_1)$. Suppose that θ_1 is increased by a positive $\Delta\theta_1$, and then a new equilibrium λ_1 of Retailer 1 under the pure PS policy can be determined by $f(\lambda_1|\theta_1+\Delta\theta_1) = 0$ according to the above proof. Consider an x^o satisfies $f(x^o|\theta_1+\Delta\theta_1) = 0$; therefore the value of $\lambda_1{}^\circ$ is equal to x^o under $\theta_1+\Delta\theta_1$. Because $f(\lambda_1|\theta_1+\Delta\theta_1)<f(\lambda_1|\theta_1)$ for any certain pair of $\{\lambda_1, \theta_1, \Delta\theta_1\}$, we can obtain that $f(x^o|\theta_1)>0$. Considering that $f(\lambda_1|\theta_1)$ is decreasing with λ_1 for any θ_1, we can infer that the $\lambda_1{}^\circ$ satisfying $f(\lambda_1{}^\circ|\theta_1) = 0$ should be greater than x^o. Therefore, the value of $\lambda_1{}^\circ$ decreases when θ_1 is increased by $\Delta\theta_1$.

(iii) From Eqs. (3) and (4), we can obtain that $\Pi_i(\lambda_i) = (p_i-c_i-c_i\lambda_i)(a_i+b_i\lambda_i)$ under the individual point scheme. It is easy to show that $\lambda_i{}^I$ should satisfy

$$\lambda_i^I = \frac{(p_i - c_i)b_i - a_i c_i}{2b_i c_i} \tag{A4}$$

Under the pure PS policy, we have $\Pi_i(\lambda_i) = (p_i-c_i-c_i\theta_i\lambda_i)(a_i+b_i\lambda_i+e_i\lambda_i)-p_jc_iD_j(\lambda_j)/p_i$. It can be shown that $\lambda_i{}^o$ should satisfy

$$\lambda_i^o = \frac{(p_i - c_i)(b_i + e_i) - a_i c_i\theta_i}{2(b_i + e_i)c_i\theta_i} \tag{A5}$$

Then, we have

$$\begin{aligned}\lambda_i^o - \lambda_i^I &= \frac{(p_i - c_i)(b_i + e_i) - a_i c_i\theta_i}{2(b_i + e_i)c_i\theta_i} - \frac{(p_i - c_i)b_i - a_i c_i}{2b_i c_i} \\ &= \frac{[(p_i - c_i)(b_i+e_i)-a_ic_i\theta_i]b_i-[(p_i - c_i)b_i-a_ic_i](b_i + e_i)\theta_i}{2(b_i + e_i)b_i c_i\theta_i} \\ &= \frac{(1-\theta_i)(p_i - c_i)(b_i + e_i)b_i + a_i c_i e_i\theta_i}{2(b_i + e_i)b_i c_i\theta_i} > 0\end{aligned} \tag{A6}$$

Therefore, we can obtain that $\lambda_i{}^o > \lambda_i{}^I$ if the conditions in (i) and (ii) hold. With the same approach, we can prove that $\lambda_2{}^\circ > \lambda_2{}^I$. □

Proof. of Proposition 2.

(i) From Eq. (3), we can obtain

$$\begin{aligned}\frac{\partial\Pi_{ch}(\lambda_1,\lambda_2)}{\partial\lambda_1} &= (p_1 - c_1)\frac{dD_1(\lambda_1)}{d\lambda_1} - c_1\theta_1 D_1(\lambda_1) \\ &\quad - c_1\theta_1\lambda_1\frac{dD_1(\lambda_1)}{d\lambda_1} \\ &\quad - \frac{p_1c_2}{p_2}(1-\theta_1)\left[D_1(\lambda_1)+\lambda_1\frac{dD_1(\lambda_1)}{d\lambda_1}\right]\end{aligned} \tag{A7}$$

and

$$\begin{aligned}\frac{\partial\Pi_{ch}^2(\lambda_1,\lambda_2)}{\partial(\lambda_1)^2} &= \left(p_1 - c_1 - c_1\theta_1\lambda_1 - \frac{p_1c_2\lambda_1}{p_2}(1-\theta_1)\right)\frac{d^2D_1(\lambda_1)}{d(\lambda_1)^2} \\ &\quad - \left(2c_1\theta_1 + \frac{2p_1c_2}{p_2}(1-\theta_1)\right)\frac{dD_1(\lambda_1)}{d\lambda_1}\end{aligned} \tag{A8}$$

Suppose that $\lambda_1{}^*$ satisfies $\partial\Pi_{ch}(\lambda_1, \lambda_2)/\partial(\lambda_1) = 0$, we have

$$\begin{aligned}&\left(p_1 - c_1 - c_1\theta_1\lambda_1^* - \frac{p_1c_2\lambda_1^*}{p_2}(1-\theta_1)\right)\frac{dD_1\left(\lambda_1^*\right)}{d\lambda_1} \\ &= \left(c_1\theta_1 + \frac{p_1c_2}{p_2}(1-\theta_1)\right)D_1(\lambda_1^*).\end{aligned} \tag{A9}$$

Given that $D_1(\lambda_1)>0$ and $dD_1(\lambda_1)/d\lambda_1>0$, we have $p_1-c_1-c_1\theta_1\lambda_1{}^*-p_1c_2\lambda_1{}^*(1-\theta_1)/p_2>0$. As $d^2D_1(\lambda_1)/d\lambda_1{}^2 < 0$, we obtain that $\lambda_1{}^*$ satisfies $\partial^2\Pi_{ch}(\lambda_1, \lambda_2)/\partial\lambda_1{}^2 < 0$. Moreover, we can show that $\partial^2\Pi_{ch}(\lambda_1, \lambda_2)/\partial(\lambda_1)\partial(\lambda_2) = 0$. Therefore, $\lambda_1{}^*$ is the overall optimal λ_1. With the same approach, we can prove that $\lambda_2{}^*$ is the overall optimal λ_2.

(ii) **For** any λ_1, we have $(p_1-c_1-c_1\theta_1\lambda_1)>(p_1-c_1-c_1\theta_1\lambda_1-p_1c_2\lambda_1(1-\theta_1)/p_2)$ and $(c_1\theta_1+p_1c_2(1-\theta_1)/p_2)>c_1\theta_1$. As $d^2D_1(\lambda_1)/d\lambda_1{}^2<0$, then $(p_1-c_1-c_1\theta_1\lambda_1-\frac{p_1c_2\lambda_1}{p_2}(1-\theta_1))\frac{dD_1(\lambda_1)}{d\lambda_1}$ is decreasing with λ_1 when $(p_1-c_1-c_1\theta_1\lambda_1-p_1c_2\lambda_1(1-\theta_1)/p_2)>0$. Moreover, $(c_1\theta_1+\frac{p_1c_2}{p_2}(1-\theta_1))D_1(\lambda_1)$ is increasing with λ_1. Therefore, we can obtain that $\lambda_1{}^\circ>\lambda_1{}^*$ if the conditions in (i) and (ii) hold. With the same approach, we can prove that $\lambda_2{}^\circ>\lambda_2{}^*$. □

Proof. of Proposition 3.
(i) From the proof of Proposition 2, $\lambda_i{}^*$ satisfies Eq. (A9) and $(p_i-c_i-c_i\theta_i\lambda_i{}^*-p_ic_j\lambda_i{}^*(1-\theta_i)/p_j)>0$. Suppose that θ_i increases by $\Delta\theta_i$ and $\lambda_i{}^*$ can be determined by

$$\left(p_i-c_i-c_i(\theta_i+\Delta\theta_i)\lambda_i^*-\frac{p_ic_j\lambda_i^*}{p_j}(1-(\theta_i+\Delta\theta_i))\right)\frac{dD_i\left(\lambda_i^*\right)}{d\lambda_i}$$
$$=\left(c_i(\theta_i+\Delta\theta_i)+\frac{p_ic_j}{p_j}(1-(\theta_i+\Delta\theta_i))\right)D_i\left(\lambda_i^*\right) \quad \text{(A10)}$$

When $p_i/c_i<p_j/c_j$, we have $(c_i\theta_i+p_ic_j(1-\theta_i)/p_j)<(c_i(\theta_i+\Delta\theta_i)+p_ic_j(1-(\theta_i+\Delta\theta_i))/p_j)$ and $(p_i-c_i-c_i\theta_i\lambda_i-p_ic_j\lambda_i(1-\theta_i)/p_j)>(p_i-c_i-c_i(\theta_i+\Delta\theta_i)\lambda_i-p_ic_j\lambda_i(1-(\theta_i+\Delta\theta_i))/p_j)$. Using the similar approach in (i), we can obtain that $\lambda_i{}^*$ decreases when θ_i increases by $\Delta\theta_i$ and $p_i/c_i<p_j/c_j$.
(ii) Using the similar approach of the above proof, we can find that $\lambda_i{}^*$ decreases when θ_i increases by $\Delta\theta_i$ and $p_i/c_i>p_j/c_j$. □

Proof. of Proposition 4. (i) Without loss of generality, suppose that θ_1 increases by $\Delta\theta_1$. Then, the function of the total profit is

$$\Pi_{ch}(\lambda_1,\lambda_2|\theta_1+\Delta\theta_1)$$
$$=(p_1-c_1-c_1(\theta_1+\Delta\theta_1)\lambda_1)D_1(\lambda_1)+(p_2-c_2-c_2\theta_2\lambda_2)D_2(\lambda_2)$$
$$-\frac{p_2c_1}{p_1}(1-\theta_2)\lambda_2D_2(\lambda_2)-\frac{p_1c_2}{p_2}(1-(\theta_1+\Delta\theta_1))\lambda_1D_1(\lambda_1) \quad \text{(A11)}$$

and we have

$$\Pi_{ch}(\lambda_1,\lambda_2|\theta_1+\Delta\theta_1)-\Pi_{ch}(\lambda_1,\lambda_2|\theta_1)$$
$$=\left(\frac{p_1c_2}{p_2}-c_1\right)\Delta\theta_1\lambda_1D_1(\lambda_1). \quad \text{(A12)}$$

Therefore, for any $\{\lambda_1, \lambda_2\}$, $\Pi_{ch}(\lambda_1, \lambda_2|\theta_1+\Delta\theta_1)$ is smaller than $\Pi_{ch}(\lambda_1, \lambda_2|\theta_1)$ when $p_1/c_1<p_2/c_2$ and is greater than $\Pi_{ch}(\lambda_1, \lambda_2|\theta_1)$ when $p_1/c_1>p_2/c_2$. We can obtain that $\Pi_{ch}{}^*$ is decreasing with θ_i and increasing with θ_j when $p_i/c_i<p_j/c_j$. □

Proof. of Theorem 1. Under the TR contract, the profit functions of retailers are

$$\Pi_1(p_1,\lambda_1,a,\gamma)=(p_1-c_1)D_1(p_1,\lambda_1)-\frac{p_1c_1}{p_1}\theta_1\lambda_1D_1(p_1,\lambda_1)$$
$$-\frac{p_2c_1}{p_1}(1-\theta_2)\lambda_2D_2(p_2,\lambda_2)-\left(\frac{S_1}{S_1+S_2}-a\right)\gamma \quad \text{(A13)}$$

$$\Pi_2(p_2,\lambda_2)=(p_2-c_2)D_2(p_2,\lambda_2)-c_2\theta_2\lambda_2D_2(p_2,\lambda_2)$$
$$-\frac{p_1c_2}{p_2}(1-\theta_1)\lambda_1D_1(p_1,\lambda_1)-\left(\frac{S_2}{S_1+S_2}-(1-a)\right)\gamma \quad \text{(A14)}$$

Suppose that $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]>a$. In this case, Retailer 1 pays Retailer 2 $[S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a]\gamma$, and Retailer 1′s profit is

$$\Pi_1(p_1,\lambda_1,a,\gamma)$$
$$=(p_1-c_1)D_1(p_1,\lambda_1)-\frac{p_1c_1}{p_1}\theta_1\lambda_1D_1(p_1,\lambda_1)$$
$$-\frac{p_2c_1}{p_1}(1-\theta_2)\lambda_2D_2(p_2,\lambda_2)$$
$$-\frac{S_1(t_1+t_2)}{p_1(1-\theta_1)\lambda_1p_2(1-\theta_2)\lambda_2}+\frac{t_1(S_1+S_2)}{p_1(1-\theta_1)\lambda_1p_2(1-\theta_2)\lambda_2} \quad \text{(A15)}$$

By inserting t_1, t_2, S_1, and S_2 into the profit function, we can obtain $\Pi_1(p_1, \lambda_1, a, \gamma)=\Phi\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$ and $\Pi_2(p_2, \lambda_2, a, \gamma)=(1-\Phi)\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$.
With the same approach, we can show that $\Pi_1(p_1, \lambda_1, a, \gamma)=\Phi\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$ and $\Pi_2(p_2, \lambda_2, a, \gamma)=(1-\Phi)\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$ under the TR contract if $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]\le a$. Therefore, for any $\Phi\in[0, 1]$, the retailers' profits can be maximized if and only if the maximum total profit is achieved and they are incentivized to choose $\{p_1{}^*, p_2{}^*, \lambda_1{}^*, \lambda_2{}^*\}$. □

Proof. of Lemma 1. When $p_1{}^*\le c_1$ and $p_2{}^*\le c_2$, the two retailers cannot obtain a positive total profit. By setting $p_1>c_1$, $p_2>c_2$, $\lambda_1=0$, and $\lambda_2=0$, the total profit becomes greater than 0. Therefore, the optimal retail prices should satisfy $p_1{}^*>c_1$ and $p_2{}^*>c_2$. From Eqs. (8) and (9), $\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$ can be obtained as follows.

$$\Pi_{ch}(p_1,p_2,\lambda_1,\lambda_2)$$
$$=\left[p_1-c_1-c_1\theta_1\lambda_1-\frac{p_1c_2}{p_2}(1-\theta_1)\lambda_1\right]D_1(p_1,\lambda_1)$$
$$+\left[p_2-c_2-c_2\theta_2\lambda_2-\frac{p_2c_1}{p_1}(1-\theta_2)\lambda_2\right]D_2(p_2,\lambda_2) \quad \text{(A16)}$$

Suppose that a solution $\{p_1{}^t, \lambda_1{}^t\}$ satisfies $p_1{}^t$-c_1-$c_1\theta_1\lambda_1{}^t$-$p_1{}^tc_2\lambda_1{}^t(1$-$\theta_1)/p_2\le 0$. Then, we have

$$\left[p_1^t-c_1-c_1\theta_1\lambda_1^t-\frac{p_1^tc_2\lambda_1^t}{p_2}(1-\theta_1)\right]D_1\left(p_1^t,\lambda_1^t\right)\le 0. \quad \text{(A17)}$$

We can always find a solution, $\{p_1{}^t, \lambda_1{}^t-\Delta\lambda_1{}^t\}$, with $\Delta\lambda_1{}^t>0$ that satisfies $p_1{}^t-c_1-c_1\theta_1(\lambda_1{}^t-\Delta\lambda_1{}^t)-p_1{}^tc_2(\lambda_1{}^t-\Delta\lambda_1{}^t)(1-\theta_1)/p_2>0$ when $p_1{}^t>c_1$. As other elements in $\Pi_{ch}(p_1, p_2, \lambda_1, \lambda_2)$ are independent of λ_1, we can obtain $\Pi_{ch}(p_1{}^t, p_2, \lambda_1{}^t, \lambda_2)<\Pi_{ch}(p_1{}^t, p_2, \lambda_1{}^t-\Delta\lambda_1{}^t, \lambda_2)$. Therefore, for any such a $\{p_1{}^t, \lambda_1{}^t\}$, the two retailers can always obtain a higher profit than $\{p_1{}^t, \lambda_1{}^t\}$ by setting $p_1-c_1-c_1\theta_1\lambda_1-p_1c_2\lambda_1(1-\theta_1)/p_2>0$.
With the same approach, we can show that $\{p_2, \lambda_2\}$ is not the optimal decision of Retailer 2 under $\{p_1, \lambda_1\}$ when $p_2-c_2-c_2\theta_2\lambda_2-p_2c_1\lambda_2(1-\theta_2)/p_1<0$. □

Proof. of Theorem 2. Under the TR contract, the profit functions of retailers are

$$\Pi_1(\lambda_1,a,\gamma)=(p_1-c_1)D_1(\lambda_1)-\frac{p_1c_1}{p_1}\theta_1\lambda_1D_1(\lambda_1)$$
$$-\frac{p_2c_1}{p_1}(1-\theta_2)\lambda_2D_2(\lambda_2)-\left(\frac{S_1}{S_1+S_2}-a\right)\gamma \quad \text{(A18)}$$

$$\Pi_2(\lambda_2,a,\gamma)=(p_2-c_2)D_2(\lambda_2)-c_2\theta_2\lambda_2D_2(\lambda_2)$$
$$-\frac{p_1c_2}{p_2}(1-\theta_1)\lambda_1D_1(\lambda_1)-\left(\frac{S_2}{S_1+S_2}-(1-a)\right)\gamma \quad \text{(A19)}$$

Suppose that $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]>a$. In this case, Retailer 1 pays Retailer 2 $[S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]-a]\gamma$, and Retailer 1′s profit is

$$\Pi_1(\lambda_1,a,\gamma)$$
$$=(p_1-c_1)D_1(\lambda_1)-\frac{p_1c_1}{p_1}\theta_1\lambda_1D_1(\lambda_1)-\frac{p_2c_1}{p_1}(1-\theta_2)\lambda_2D_2(\lambda_2)$$
$$-\frac{S_1(c_1p_2+c_2p_1)}{p_1p_2}+\frac{c_1(S_1+S_2)}{p_1}. \quad \text{(A20)}$$

By inserting S_1 and S_2 into the profit function, we can obtain

$$\Pi_1(\lambda_1, a, \gamma) = (p_1 - c_1)D_1(\lambda_1) - c_1\theta_1\lambda_1 D_1(\lambda_1) - \frac{p_1 c_2}{p_2}(1-\theta_1)\lambda_1 D_1(\lambda_1). \tag{A21}$$

$$\Pi_2(\lambda_2, a, \gamma) = (p_2 - c_2)D_2(\lambda_2) - c_2\theta_2\lambda_2 D_2(\lambda_2) - \frac{p_2 c_1}{p_1}(1-\theta_2)\lambda_2 D_2(\lambda_2). \tag{A22}$$

With the same approach, we can show that Equations (A21) and (A22) still hold under the TR contract when $S_1(p_1, \lambda_1)/[S_1(p_1, \lambda_1)+S_2(p_2, \lambda_2)]\leq a$. Therefore, for any $\Phi\in[0, 1]$, retailers' profits can be maximized if and only if the maximum total profit is achieved and retailers are incentivized to choose $\{\lambda_1{}^*, \lambda_2{}^*\}$. □

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