Annals of Operations Research (2020) 290:95–113
https://doi.org/10.1007/s10479-018-2968-y

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Joint pricing and inventory decisions with carbon emission considerations, partial backordering and planned discounts

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Published online: 18 July 2018

Abstract
Typical economic order quantity models of inventory feature demand rate as a constant parameter and do not allow for backordering. Furthermore, the purchasing cost of the ordered materials is considered constant. In reality, the demand rate is related to the unit purchasing cost and other factors, such as time and availability of products in the market. A quantity discount is regularly applied to encourage ordering more products by decreasing the price. In some situations, carbon dioxide emissions are carefully scrutinized and a program to handle these. Greenhouse gases are put in place. Hence, for this research, the rate of demand in the model was assumed proportional to the unit purchasing cost and partial backordering was allowed as a fixed parameter. Because plants emit greenhouse gases (carbon dioxide), we considered mitigation efforts. A mathematical model and computational procedures are shown with the solution algorithms that demonstrate the capability of the model. An example problem was solved with the model and sensitivity analysis was conducted to inform the managerial insights offered.

Keywords All-units quantity discounts · Carbon emissions · Variable and price-dependent demand · Green supply chain

1 Introduction

In many inventory models, the demand rate has a fixed value and the price of the ordered products is constant regardless of the order level. However, in real inventory models, the demand rate depends on price. In this study, the annual demand rate was assumed to be related to the selling cost for a unit of goods, and because of an all-unit quantity discount for the customer, the unit purchasing cost depended on the order quantity.

Furthermore, because it has recently emerged as an environmental concern, carbon emission is addressed in this study. Plants sold from a greenhouse emit carbon dioxide, a

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greenhouse gas, and to keep the carbon emissions in the warehouse at the lowest possible level by quickly selling plants, we suggested that the seller offer all-units quantity discounts. Finally, to make it more realistic, the presented model allowed for partial backordering.

When it is a variable, the demand rate can be a function of price, stock level, or both. Therefore, typical models of inventory feature demand that is based on varying rates according to the availability of an item or level of stock. Sana and Chaudhuri (2004) built a model of economic order quantity (EOQ) for which the demand varied with the availability of the item and related costs. Their model takes into account a budget that included storage-capacity costs and optimized profit. Min and Zhou (2009) presented a model of inventory for perishable items for which demand depends on stock, backordering is partially allowed, and the maximum inventory level is limited. Yang et al. (2010) proposed another inventory model for perishable items for which there is a stock-dependent demand that allows for a few backorders and also accounts for the impact of inflation. Lee and Dye (2012) presented a model of EOQ for which few backorders are admissible, the demand varies with stock, and the decline rate of the product can be controlled; this model optimizes the profit and specifies the optimum ordering and maintenance policies. Many models are characterized by greater demand and lower price. For example, Mondal et al. (2003) proposed a model of inventory for perishable items according to a demand rate that had a linear relationship to the selling price, and the rate of putrefaction had a declining relationship with the period of time in storage. In the same way, Mukhopadhyay et al. (2005) extended an inventory control model for perishable items for which the rate of demand depends on selling price and the rate of putrefaction depends on storage time.

For products with demand rates that depend on price, Teng et al. (2005) proposed an EOQ model for specifying an order quantity and a selling price that benefit retailers. Transchel and Minner (2008) presented algorithms of optimized solutions for an EOQ model for which the quantity rebates and the demand vary with price, and they featured a particular case in which the price function is linear. Various strategies, based on centralization, through which the selling price and lot size are combined, and non-centralized conditions as well as dynamic pricing, have been offered. Zhengping (2010) studied an inventory model through which the demand varies with the price and little information about demand exists in a supply chain consisting of a retailer and a supplier. Some models were based on the supposition that the rate of demand varies with stock level and selling price. Hou and Lin (2006) developed an inventory model for perishable items in which the demand rate varies with the combined selling price and stock level, and their model accounts for the time factors related to the fees that optimize the net present value of the profit. You and Hsieh (2007) examined an inventory system with a demand rate related to selling price and for which the amount of the item is available for a specific time. Panda et al. (2010) studied an EOQ model characterized by a seller and numerous retailers of a product for which the demand rate has a linear relationship with stock level and price.

Many models were based on the assumption that the inventory holding cost varies. Ferguson et al. (2007) proposed an inventory model with inventory holding costs that depend on storage moments in a linear manner. Under this model, if the item value declines in a non-linear manner with the storage moment, then surcharges are used for infrequent order deliveries and the price declines for perishable products. Ghasemi and Afshar Nadjafi (2013) presented two EOQ models for which the inventory holding costs vary, and they looked at one situation that includes backorders and another that does not. In their models, the inventory holding cost remains unchanged up to a specific time period, after which the costs are determined by a function that increases with the length of the ordering cycle. San-José et al. (2015) examined an EOQ model in which minimal backordering is allowed and the inventory holding cost function has two elements: a fixed cost and a cost that is variable and grows with the storage moment.

Inventory models with a procurement cost depend on order size rely on all-units or incremental quantity discounts. In the all-units discount, a lower price is placed on all items in the order. Incremental, sequentially priced discounts decline for unit subsets on the basis of specific, pre-determined limits. Weng (1995) presented an inventory model to specify policies for discounting optimized quantities according to price-dependent demand. Weng's model optimizes profits with the use of all-units and incremental quantity discounts, but no shortages are allowed and the inventory holding cost is held constant. Burwell et al. (1997) proposed an inventory model to specify the optimal orders and selling prices under a pride-dependent demand rate, an inventory carrying cost that is related to the unit cost, and offers all-units discounts. The algorithm that Burwell et al. (1997) used in this model was updated by Chang (2013) to optimize the profit and specify the accurate optimum amounts of the lot size and selling price. Also, some related work can be found in the work of Taleizadeh et al. (2008, 2009, 2015, 2016 and 2018), Taleizadeh and Noori-daryan (2015, 2016) and Taleizadeh and Pentico (2014) and Taleizadeh (2014). A brief summary of the literature review is presented in Table 1.

The rest of this paper organized as follows. A description of the problem is given in Sect. 2. The model formulation and solution algorithm are provided in Sect. 3. To check the validity of the model, computational experiments are offered in Sect. 4. Finally, the conclusion and some ideas for future research are given in Sect. 5.

2 Problem description

As with all EOQ inventory models, the presented model includes regular cost factors, including purchasing, ordering, holding, and partial backordering costs, and accounts for carbon emission issues. In this study, partial backordering has a fixed constant value and is defined by the greenhouse keeper, who is the decision maker. The carbon emission comprises four parts:

1.1. The frequency of delivery (set up, order processes, and transportation)
1.2. Storage amount (production and related activities)
1.3. Environmental impact (inventory holding, material handling, and warehouse activities)
1.4. Carbon emissions from obsolete materials.

The assumptions of the model, which is the first of this type in the literature, can be applied to several real situations. For example, in the plant-shop industry, the number of plants in inventory must sufficiently meet demand, and any greenhouse gases t emitted must not hurt the greenhouse keepers. In addition, we took into account the carbon emissions from the means for and frequency of delivering the plants, and from obstacles in the greenhouse.

Moreover, because plants are non-specialty products that can be sold almost anywhere, the demand for them at the greenhouse tends to depend on price. As a result of the consumer paying attention to price, the greenhouse should offer quantity discounts to motivate consumers to buy more plants and to reduce the long-term inventory in the greenhouse.

A greenhouse is featured in the case study to illustrate the presented model because plants release carbon dioxide in the afternoon and overnight, and this makes the warehouse environment weather unpleasant for the plant keepers. Also, the emissions of this greenhouse gas are released into the atmosphere. Annual demand rate is affected by the selling price of a unit of ordered plants, which depends on the order level. The greenhouse keeper is highly

Table 1 A summarized literature review on research most related to this study

References	Backordering	Partial backordering	Price-dependent demand	Quantity discount	Carbon emission consideration	Solution method
Arslan and Turkay (2013)					✓	CF[a]
Hovelaque and Bironneau (2015)					✓	CF
Battini et al. (2014)					✓	CF
Wee (1999)		✓	✓	✓		NCF[b]
Ghosh et al. (2011)		✓	✓			NCF
Abad (2003)		✓	✓			NCF
Roy (2008)			✓			NCF
Maihami and Abadi (2012)		✓	✓			NCF
Lin and Ho (2011)			✓	✓		CF
You and Hsieh (2007)			✓			CF
Wee (1999)		✓	✓	✓		NCF
San-José and García-Laguna (2009)	✓			✓		NCF
Zhengping (2010)		✓				NCF
Min and Zhou (2009)	✓					CF
Kumar et al. (2013)	✓					CF
Pando et al. (2013)			✓	✓		NCF
Shi et al. (2012)				✓		CF
You and Hsieh (2007)				✓		NCF
Burwell et al. (1997)			✓			NCF
Panda et al. (2010)		✓				NCF
This paper		✓	✓	✓	✓	CF

[a] *CF* closed-form
[b] *NCF* non-closed-form

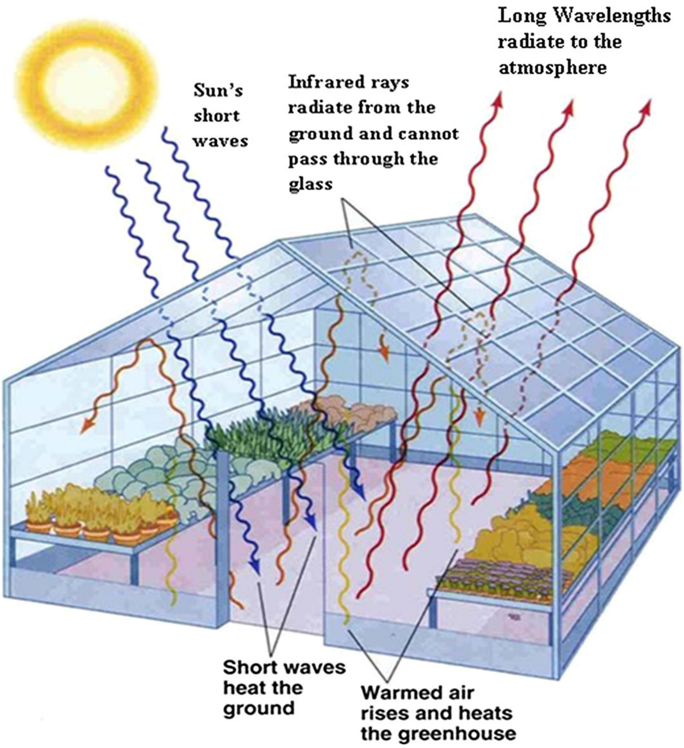

Fig. 1 Greenhouse gases emitted through a typical greenhouse

interested in selling the plants as soon as possible to make room for fresh plants. Because the plants are subject to decay over time, the seller offers quantity discounts such that the unit purchasing cost is decreased as the demand decreases. This discount selling method improves the emitted carbon dioxide levels for the warehouse keeper, and more plants are sold to increase profit (See Fig. 1).

This study was aimed at maximizing the total annual profit as improved by the EOQ model presented. The main characteristics of the model are as follows:

The linear demand function decreases if the selling price increases.

1. Carbon emission is considered a negative feature for inventory in the warehouse, as is the carbon that is emitted from obsolete materials.
2. The purchasing cost has a stage function that decreases as the order size increases, and it is calculated for all-units discounts.
3. Partial backordering is allowed in the model.

3 Mathematical model and solution algorithm

Figure 2 shows a schematic view of the relationships between different variables and parameters in the model.

Notations used in the model are as follows:

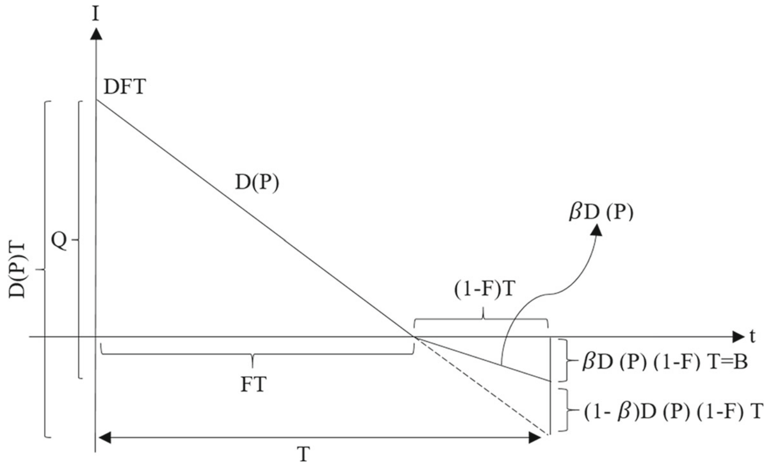

Fig. 2 Common EOQ inventory model with partial backordering

Indices	
j	Index for cost range ($j = 1, 2, \ldots, J$)
Parameters	
A	Fixed ordering cost for placing and receiving one order
c_j	Purchase cost for each unit for order size Q in range j
β	Constant fraction of backordering
π	Unit backordering cost for each period
π'	Unit lost sales cost
g_1	Unit goodwill loss
i	Constant coefficient for the holding cost
h	Unit holding cost per unit time
a	Potential market demand (greater than zero)
b	Coefficient of selling price in the demand rate function
F	Percentage of demand that will be filled from stock

Carbon emission parameters		
e	Carbon emission unit associated with order initiated (replenishment)	
g	Carbon emission unit in the warehouse that is related to stock level per unit of time	
k	Amounts of carbon emission associated per unit purchased or produced	
t	Carbon tax per unit (€/ton) (money units for each unit of carbon emitted as tax)	
P'	Unitary scrap price (€/unit)	
$b1$	Annual rate of average inventory obsolescence (%)	
$a1$	Weight of an obsolete unit in the warehouse [ton/unit]	
c_{eo}	Average cost of carbon emission coefficient of inventory waste for emissions during collection and disposal €/ton	
Decision variables		
Q^*		Maximum order level for purchasing cost range j (order quantity)
$C(Q)$		Unit purchasing cost that depends on order size Q
DFT		Maximum level of inventory
T		Cycle time
P		Unit selling price
$D(P)$		Annual demand rate
B		Backordered quantity
L		Lost sales quantity
$ATC(T, P)$		Annual total cost
$ATP(T, P)$		Annual total profit

The model is based on the following assumed characteristics:

1. The inventory carrying cost includes a fixed component, i, and a variable component, w, that is linearly increased with the unit purchasing cost. Therefore, unit carrying cost is proportional to the unit purchase cost, c_j; this assumption is expressed as follows:

$$h = i + wc_j \tag{1}$$

2. The unit purchasing cost is determined for the all-units quantity discount as

$$C(Q) = c_j,\ \text{if}\, q_{j-1} < Q \leq q_j,\, c_1 > c_2 > \cdots > c_j \tag{2}$$

3. As mentioned before, demand is a linear function of the selling price, which is a more realistic representation of the real world than is the assumption that the demand is a fixed parameter. It is proportional to a constant parameter, a, and a coefficient of the selling price, b. The linear demand function is considered to be

$$D(P) = a - bP \tag{3}$$

4. To make sure that the objective function is profitable and feasible, meaning that the demand is positive, the demand must be greater than 0 ($D(P) > 0$) and the selling price must be greater than the purchasing cost ($c_j < P$). Considering these two assumptions simultaneously, the following equation is satisfied:

$$c_j < P < a/b \tag{4}$$

5. Partial backordering is allowed.
6. All ordered items are identical.
7. Deterioration is not considered.
8. Carbon emission related issues considered in the model are:

   a. Frequency of delivery (setup, order processes, and transportation)
   b. Storage amounts (production and related activities)
   c. Environmental impact (inventory holding, material handling, and warehouse activities)
   d. Carbon emissions from obsolete materials.

The total profit for one year is calculated as

Annual total profit=Annual revenue − Annual total cost

Annual total cost comprises several parts, including purchasing, ordering, inventory holding, and backordering costs, as well as mitigation or fees related to carbon emissions. Therefore, the annual total cost is calculated as follows:

Annual total cost=Fixed ordering cost+ Purchasing cost+Holding cost+Lost sales cost+Backordering cost+Carbon emission cost+Waste cost

First, we define several variables in the diagram in the following:

$$Q^* = \text{Consumed quantity or order level } = D(P)FT + (1 - F)\beta D(P)T \tag{5}$$

$$DFT = \text{ Maximum level of inventory} = FTD(P) \tag{6}$$

$$D(P) = \text{Demand rate} = a - bP \tag{7}$$

$$L = \text{ Lost sales level} = (1 - \beta)D(P)(1 - F)T \tag{8}$$

$$B = \text{ Backorder level} = \beta(1 - F)D(P)T \tag{9}$$

$$\text{Average inventory level} = {D(P)F^2T}/{2} \tag{10}$$

Different parts of the objective function are detailed as described in the remainder of this section.

Average inventory level in 1 year is calculated in the following

$$\text{Average inventory level} = \int_0^{FT} (-D(P)(t - FT)dt) \Big/ T$$

$$= \int_0^{FT} (-(a - bP)(t - FT)dt) \Big/ T = D(P)F^2T \big/ 2$$

3.1 Total revenue

The final equation for total revenue gained from the inventory is gained from selling the quantity of order that is sold in the correct time (DFT) and the backorder level ($B = \beta(1 - F)D(P)T$), which is

$$\frac{P[DFT + \beta(1 - F)D(P)T]}{T} = P[F + \beta(1 - F)]D(P) = P(a - bP)(F + \beta - \beta F) \tag{11}$$

This final formulation is obtained from selling the ordered quantity based on the selling price selected in the algorithms that take into account quantity discounts. We should note that all of the model parts are divided by the cycle time T to calculate the annual costs and revenue.

3.2 Ordering cost

The annual fixed ordering cost is calculated as

$$\frac{A}{T} \tag{12}$$

3.3 Purchasing cost

According to Fig. 2, the purchasing cost is calculated for the order quantity per year $\frac{[DFT+\beta(1-F)D(P)T]}{T}$ as

$$c_j\left(\frac{[DFT + \beta(1 - F)D(P)T]}{T}\right) = c_j[F + \beta(1 - F)]D(P) = c_j(a - bP)(F + \beta - \beta F) \tag{13}$$

3.4 Holding cost

Holding cost is given by Eq. (14), and is based on the average inventory level in stock ($D(P)F^2T \big/ 2$). Because $i + wc_j$ is the cost of holding an item in each year, the following is obtained:

$$h \times the\, average\, of\, inventory\, level = h\left(D(P)F^2T \big/ 2\right) = \left(i + wc_j\right)\frac{(a - bP)}{2}F^2T \tag{14}$$

3.5 Backordering cost

Backordering cost is calculated with parameter π and takes into account the average number of backorders within a fixed percentage in each year (Fig. 2) and is given as

$$\begin{aligned} \pi \times \text{the average number of back orders} &= \pi \times \frac{B \times (1-F)T}{2T} \\ &= \pi \times \frac{(1-F)^2 T^2 \beta D(P)}{2T} \\ &= \pi \times \frac{(1-F)^2 T \beta D(P)}{2} \end{aligned} \tag{15}$$

where the average number of backorders within a fixed percentage in each year is calculated as

$$\int_{FT}^{(1-F)T} -\beta D(P)(t-T)dt \Big/ T = \beta(1-F)D(P)T \times (1-F)T \big/ 2T = B \times (1-F)T \big/ 2T$$

3.6 Lost sales cost

Lost sales cost is calculated with the parameter g_1 and is based on the number of orders that are lost, within a fixed percentage $(1-\beta)$, in each year (Fig. 2). The parameter g_1 accounts for both lost profit and goodwill loss $(P - c_j + \pi')$ as

$$g_1 \times \frac{L}{T} = g_1 \frac{(1-\beta)D(P)(1-F)T}{T} = g_1(1-\beta)(1-F)(a-bP) \tag{16}$$

where $g_1 = P - c_j + \pi'$

3.7 Waste (obsolescence) cost

There is a cost for collecting and disposing of products in storage that are wasted or are obsolete $\left(P - P'\right)$. The total annual cost to eliminate obsolete products is

$$\left(P-P'\right)b'\frac{DFT}{2T} = \frac{\left(P-P'\right)b'FD(P)}{2} = \frac{\left(P-P'\right)b'F(a-bP)}{2} \tag{17}$$

where the amount of order wasted or is obsolete in each year is proportional to the amount of inventory level.

3.8 Carbon emission cost

The carbon emission cost has three components. The first component is related to replenishment, the second component is related to the quantity of inventory in stock, and the third component considers the average amount of inventory kept in stock. These components are summated and briefly represented by

$$\frac{e}{T} + K(a-bP)(F+\beta-\beta F) + (g + b_1 a_1 c_{eo})\left(\frac{(a-bP)}{2}F^2 T\right) \tag{18}$$

where

a. Frequency of delivery $= e \big/ T$

b. Storage amounts $= g \times the\,average\,of\,inventory\,level = g \times \left(D(P)F^2T/_2\right)$
c. Environmental impact $= K \times order\,quantity\,per\,year = K\left(DFT + \beta(1-F)D(P)T/_T\right) = K(a - bP)(F + \beta - \beta F)$
d. Carbon emissions from obsolete materials $= b_1 a_1 c_{eo} \times the\,average\,of\,inventory\,level = b_1 a_1 c_{eo} \times \left(D(P)F^2T/_2\right)$

To obtain the closed form of the optimum values for P and T, the objective function is rewritten as

$$ATP(P,T) = P\psi_1 + P^2\psi_2 + \frac{1}{T}\psi_3 + T\psi_4 + TP\psi_5 + \psi_6 \tag{19}$$

where

$$\begin{aligned}
\psi_1 &= a(F + \beta - \beta F) + b\left[c_j + K(t+s)\right](F + \beta - \beta F) \\
&\quad - a(1-F) + a(1-F)\beta - b(1-F)c_j + b(1-F)\pi' \\
&\quad + b\beta(1-F)c_j - b\beta(1-F)\pi' - \frac{F}{2}b'a - \frac{F}{2}b'bP' \\
\psi_2 &= -b(F + \beta - \beta F) + b(1-F) - b(1-F)\beta + \frac{F}{2}b'b \\
\psi_3 &= -[A + e(t+s)] \\
\psi_4 &= -\frac{aF^2}{2}\left[g(t+s) + b_1 a_1 c_{eo}(t+s) + i + wc_j\right] \\
\psi_5 &= \frac{F^2 b}{2}\left[g(t+s) + b_1 a_1 c_{eo}(t+s) + i + wc_j\right] + \frac{b\beta\pi}{2}(1-F)^2 \\
\psi_6 &= -a\left[c_j + K(t+s)\right](F + \beta - \beta F) + a(1-F)c_j - a(1-F)\pi' - a(1-F)\beta c_j \\
&\quad + a(1-F)\beta\pi' + \frac{F}{2}b'aP'
\end{aligned}$$

In the next section, we explain that the objective function $ATP(P, T)$ is concave such that we can obtain the optimal values for variables P and T. We set the first partial derivations for the two variables, $ATP(P, T)$ equal to zero, and then simultaneously solved the equations. Specifically, to obtain the optimum value for P, the derivative of objective function at P is calculated and set equal to zero as follows:

$$\frac{\partial ATP(P, T)}{\partial P} = \psi_1 + 2\psi_2 P + \psi_5 T = 0$$

Therefore, the optimum value for P is

$$P^* = \frac{-\psi_5 T - \psi_1}{2\psi_2} \tag{20}$$

Then, the optimum value for T is calculated as

$$\frac{\partial ATP(P, T)}{\partial T} = \frac{-\psi_3}{T^2} + \psi_4 + \psi_5 P = 0$$

Replacing Eq. (20) which is the optimum value for P in the above formulation gives the following formulation for T:

$$\left(\frac{-\psi_5}{2\psi_2}\right)T^3 + \left(\frac{\psi_1 - 2\psi_2\psi_4}{2\psi_2}\right)T^2 + \psi_3 = 0$$

Now the optimum value for T is extracted from this third-degree polynomial equation. There is an analogous formula for polynomials of degree three (see "Appendix"). Then the optimum value for T is:

$$T^* = \sqrt[3]{\left(x^3+y\right)\left(1+\sqrt{1+\frac{x^6}{x^3+y}}\right)} + \sqrt[3]{\left(x^3+y\right)\left(1-\sqrt{1+\frac{x^6}{x^3+y}}\right)} + x \qquad (21)$$

where

$$x = \frac{-\psi_1 + 2\psi_2\psi_4}{3\psi_5}$$

and

$$y = \frac{-\psi_3\psi_2}{2\psi_5}$$

3.9 Concavity of the objective function

To insure that the profit function is concave and has an optimal value, we need to prove the following: First, the Hessian matrix $\begin{bmatrix} \frac{\partial ATP(P,T)}{\partial T^2} & \frac{\partial ATP(P,T)}{PT} \\ \frac{\partial ATP(P,T)}{\partial TP} & \frac{\partial ATP(P,T)}{\partial P^2} \end{bmatrix}$ is determined to check for a total annual profit that is concave. The first and the second principal minors of the Hessian matrix should be calculated and checked to determine if they are negative when the values of the parameters in the proposed model are used. The first leading principal minor of $ATP(P, T)$ is

$$H_1 = |H_{11}| = \frac{\partial ATP(P, T)}{\partial T^2} = \frac{2\psi_3}{T^3} = \frac{2[-[A+e(t+s)]]}{T^3} < 0$$

Next, the second leading principal minor of $ATP(P, T)$ is calculated as

$$H_2 = \begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} = \begin{vmatrix} \frac{\partial ATP(P,T)}{\partial T^2} & \frac{\partial ATP(P,T)}{PT} \\ \frac{\partial ATP(P,T)}{\partial TP} & \frac{\partial ATP(P,T)}{\partial P^2} \end{vmatrix} = 2\psi_2.\left(\frac{2\psi_3}{T^3}\right) - \psi_5^2 < 0$$

For all values for the parameters, the term $\frac{2(-(A+e[t+s]))}{T^3}$ is always negative. Thus,

$$2\left[-b(F+\beta-\beta F) - b(1-F)\beta + b(1-F) + \frac{F}{2}bb'\right]\left[\frac{2(-(A+e(t+s)))}{T^3}\right] - \left[\frac{F^2 b}{2}\left(g_3(t+s) + b_1 a_1 c_{eo}(t+s) + i + wc_j\right) + \frac{b\beta\pi}{2}(1-F)^2\right]^2 < 0$$

Because $\psi_2 > 0$,

$$-b(F+\beta-\beta F) - b(1-F)\beta + b(1-F) + \frac{F}{2}b'b > 0$$

To ensure that the second principal minor of the matrix is always negative, the following must be checked

$$-b(F+\beta-\beta F) > 0$$

Finally, if the following equation is satisfied, the total profit is always concave.

Check that $(1-2F)(1-\beta) + \frac{F}{2}b' > \beta$

Therefore, the Hessian matrix for the objective function is negative definite.

The percentage of a period in which the orders are filled from stock is assumed to be fixed. The algorithm, by which the optimum value for the profit function is obtained, is described as follows.

Step 0 Initialization. Set $j = J$ and $ATP(P, T)_{max} = 0$

Step 1 Substitute c_j and other parameters into Eqs. (20) and (21) and solve for P and Q (replace the equal value for Q)

- If Q is in the correct range for the purchasing cost ($q_{j-1} < Q \leq q_j$), then these values are feasible. Substitute the optimal values of T and P into Eq. (17) to calculate $ATP(P, T)$. If $ATP_j(P, T) > ATP(P, T)_{max}$, then set $ATP_j(P, T) = ATP(P, T)_{max}$. Go to Step 4.
- If Q is not in the correct range for the purchasing cost, then these values are infeasible. Go to Step 2.

Step 2 Substitute c_j, $Q = q_{j-1}$, and the other parameters into Eq. (20) and solve for P. Plug the value $Q = q_{j-1}$ and the related value of P into Eq. (19) to calculate $ATP_j(P, T)$. If $ATP_j(P, T) > ATP(P, T)_{max}$, then set $ATP(P, T)_{max} = ATP_j(P, T)$. Go to Step 3

Step 3 If $j \geq 2$, set $j = j - 1$ and go to Step 1.
If $j = 1$, then Go to Step 4

Step 4 The final resulting values are the feasible values related to $ATP(P, T)_{max}$. Optimal values of Q; P; T; $ATP_j(P, T)$; and $ATC_j(P, T)$ are defined, and then the algorithm is finished

3.10 Computational and practical results

For the case of a greenhouse, many plants are grown to be sold in a supply chain. The greenhouse in this study receives orders from retailers, florists, and farmers. The government sets taxes for the level of carbon dioxide emitted from the greenhouse to lower the greenhouse effect on the atmosphere and prevent global warming. This tax situation creates some costs for the greenhouse keeper. The plants emit carbon dioxide; therefore, to handle the costs, the amount of carbon dioxide emitted per each plant is calculated, and a special sales tax is applied to the price that retailers pay for the plants. The parameters are set as $A=100$, $a=200$, $b=1.5$, $K=2$, $t=0.01$, $s=0.01$, $F=0.8$, $\beta=0.6$, $e=1$, $g=1$, $b_1=0.1$, $a_1=0.002$, $c_{eo}=13$, $i=0.2$, $w=0.2$, $\pi=1$, $\pi'=2$, $b'=0.1$, $P'=5$, $c_1=25$, $c_2=25$, $c_3=20$, $q_1=0$, $q_2=30$, and $q_3=50$. The solution algorithm procedure is done in the following stages:

Stage 0 Start.
Set $j = 3$and$ATP(P, T)_{max} = 0$

Iteration1: $j = 3$

Stage 1 For $c_3 = 20(Q \geq 50)$, start with the minimum unit purchasing cost, $c_3 = 20$; plugging the given values into Eqs. (18) and (19) and solving for Q and P results in $P = 83.01$, $Q = 46.71$, and $ATP_3(P, T) = 2835.61$. Because $Q < 50$, these values are not feasible. Therefore, go to Stage 2

Stage 2 For $c_3 = 20(Q = 50)$;
that is, for the least cost, $c_3 = 20$ and when applying the minimum ordering level for this cost, $Q = 50$, plug other parameters into Eq. (19) and solve for P. As a result, P=80.13, $Q = 50$, and $ATP_3(P, T) = 2147.11$. Because $2147.11 > 0$, set $ATP_{max}(P, T) = 2147.11$. Go to Stage 3

Stage 3 Set $j = 2$ and Go to Stage 1
Iteration2: $j = 2$
Stage 1 For $c_2 = 25$, $(30 < Q < 50)$,
the unit purchase cost $c_2 = 25$ should be substituted with the other parameters into Eqs. (18) and (19) such that computing them results in: P = 80.33, Q = 40.01, and $ATP_3(P, T) = 3183.97$. Because these values are feasible, the solution is optimal. Go to Stage 4
Stage 4 Algorithm is finished

The relationship between parameters and the effects of them on the relative variables were extracted by sensitivity analysis. The values of parameters are in relative stages of 20% (− 40%, − 20%, + 20%, and + 40%), and were changed systematically to evaluate the effects on the total profit value and other main variables. Therefore, each function was calculated four times to account for the changes in parameters. The results are presented in Table 2.

Table 2 shows that the order level is based on the decreasing values of A and a, and if the purchase cost is lowered, the optimum inventory level decreases.

The price of plants is dependent on the values of A and a, and it increases if they increase. It also depends on the values of b and i in a negative way. The selling price is not related to the parameter g. We found that if the value of b increases, then the annual total cost per unit time $ATC_j(P, T)$ decreases, and if it increases, then all other parameters increase. In addition, if a increases, then the annual total profit per unit time $ATP_j(P, T)$ increases, and if all other parameters increase, then $ATP_j(P, T)$ also increases. It is also clear that parameters for calculating annual demand, primarily a and secondarily b, have the greatest impact on the profit function $ATP_j(P, T)$ and on the variables P, T, and $ATP_j(P, T)$. It is suggested that the greenhouse decision maker focus on increasing demand instead of reducing costs. The purchasing cost c_j has the most important role in the profit function, and the second most important parameter is the ordering cost A. Increasing the selling price does not always result in higher profit. If the ordering or purchase costs (A or c_j) are decreased, the profit with a lower unit-selling price P increases (Figs. 3 and 4).

4 Conclusions

Greenhouses offer a real example for which a program should be used to handle carbon emissions. In addition, a greenhouse should use inventory management to replenish and sell plants for an optimal profit. In the situation under study, carbon dioxide emissions were carefully considered. In reality, the annual demand rate is related to the unit purchasing cost and other factors, such as product time and availability in the market. By lowering the price, a quantity discount is regularly applied to encourage buyers to order more products. Therefore, in this paper, the rate of demand was assumed proportional to the unit purchasing cost, and partial backordering was allowed as a fixed parameter. A greenhouse was considered for the case study because plants emit greenhouse gases (carbon dioxide).

A mathematical model was proposed and computational procedures were completed along with solution algorithms to illustrate the capability of the model. An example was solved, sensitivity analysis was completed, and managerial insights were presented. According to the numerical results shown, we suggested that the greenhouse keeper apply marketing strategies to earn additional profit. In another way to increase profit, related costs, such as ordering cost, are reduced. The suggested extensions to this study, to make it more realistic, include assuming that the annual demand rate is a nonlinear function of price, inventory level, and

Table 2 Computational results

Parameter	First value	Changed values	T	P	Q	$ATC_j(T, P)$	$ATP_j(T, P)$
A	100	60	0.82	81.33	40.01	3691.58	318,397
		80	0.81	83.33	40.21	2731.58	318,397
		120	0.80	85.33	40.23	2756.58	388,397
		140	0.77	83.33	40.23	2811.58	316,397
F	0.8	0.32	0.81	80.33	40.01	2891.58	318,397
		0.56	0.81	84.33	40.21	3681.58	318,397
		0.98	0.82	88.33	48.23	2891.58	318,397
		1	0.83	80.33	40.23	2691.58	386,397
a	200	160	0.94	80.33	48.01	2691.58	318,397
		180	0.89	84.33	40.21	2691.68	358,394
		220	0.88	80.33	40.23	3261.58	348,397
		240	0.87	80.33	45.23	2651.58	318,399
b	1.5	1	0.8	80.33	40.01	2659.68	418,397
		1.25	0.9	86.33	40.21	2691.58	318,384
		1.75	0.8	80.33	40.23	2951.58	316,397
		1.90	0.5	80.33	49.23	2851.59	358,395
i	0.2	0.1	0.9	85.33	40.01	2651.58	418,397
		0.15	0.8	80.33	40.21	2661.58	318,399
		0.25	0.6	80.35	49.23	3651.56	318,397
		0.30	0.8	80.33	40.23	2661.58	358,395
w	0.2	0.1	0.5	80.35	45.01	2699.59	345,397
		0.15	0.8	86.33	40.21	2691.55	318,397
		0.25	0.5	80.33	49.23	3591.59	316,395
		0.30	0.8	80.33	40.23	2581.58	418,397
c_1, c_2, c_3	30, 25, 20	27	0.8	80.33	45.01	2659.58	318,397
		22	0.8	80.33	40.21	2691.68	358,394
		17	0.9	80.33	40.23	3699.58	318,397
		31	0.8	80.33	46.23	2891.55	418,399
		27	0.9	80.33	40.01	2651.58	318,394
		21	0.8	89.33	40.21	2669.58	318,397
		33	0.8	80.33	40.23	3691.58	318,397
		28	0.9	80.36	40.26	2891.58	358,397
		23	0.6	89.33	49.01	2691.68	418,394
		35	0.8	88.35	40.21	3891.55	318,397
		30	0.9	85.33	45.23	2651.58	315,394
		25	0.8	89.33	40.23	3691.55	418,395

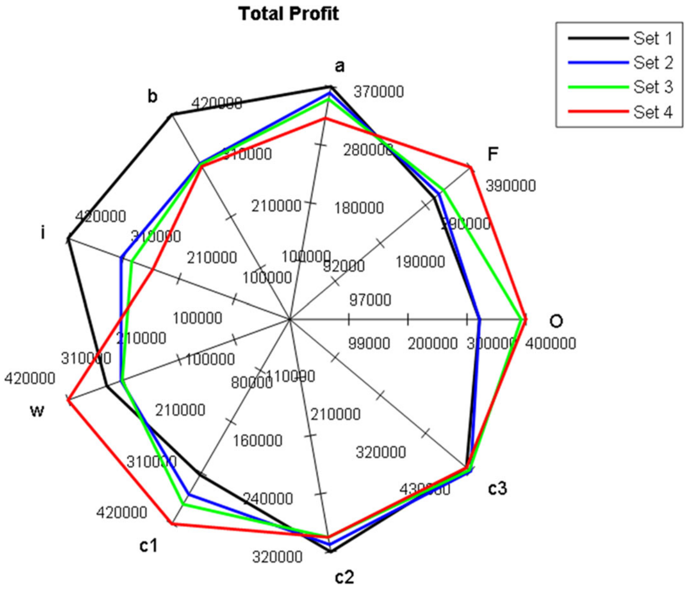

Fig. 3 The effects of parameters on total profit

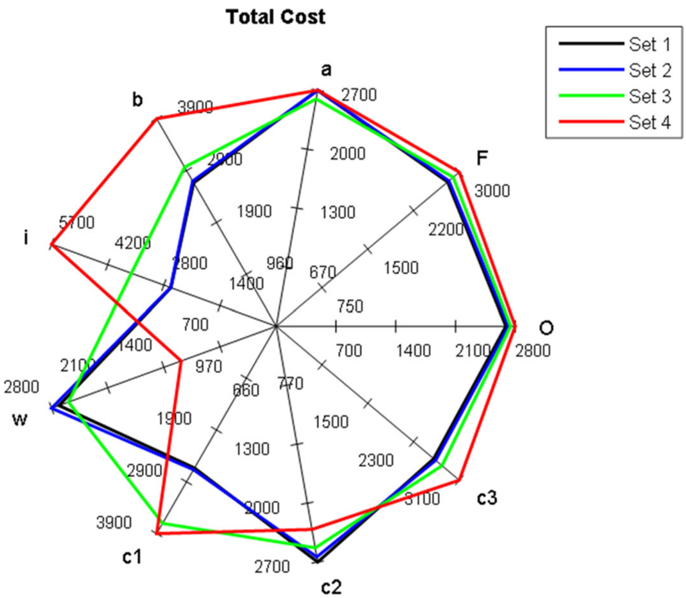

Fig. 4 The effects of parameters on total cost

time. Also, instead of using a fixed percentage for partial backordering such that the parameter F is a constant, F can be considered a variable value, which creates opportunities for the decision maker. Deterioration and variable backordering, which was not taken into account in this study, might also be considered in future research.

Acknowledgements The first author would like to thank the financial support of University of Tehran for this research under grant number 30015-1-04. This work was also supported by the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning [Grant No. 2017R1A2B2007812].

Appendix

There is an analogous formula for polynomials of degree three: The solution of

$$ax^3 + bx^2 + cx + d = 0$$

That is

$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a}\right)^3}}$$
$$+ \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a}\right)^3}} - \frac{b}{3a}$$

This can be briefly written as

$$x = \sqrt[3]{q + \sqrt{q^2 + \left(r - p^2\right)^3}} + \sqrt[3]{q - \sqrt{q^2 + \left(r - p^2\right)^3}} + p$$

where

$$p = \frac{-b}{3a}$$
$$q = p^3 + \frac{bc - 3ad}{6a^2}$$
$$r = \frac{c}{3a}$$

This formulation is used to calculate the optimal value for T, which is a polynomials of degree three.

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