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Online banner advertisement scheduling for advertising effectiveness

Gwang Kim[a], Ilkyeong Moon[a,b,*]

[a] Department of Industrial Engineering, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul, Republic of Korea
[b] Institute for Industrial Systems Innovation, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul, Republic of Korea

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ABSTRACT

Online banner advertisement, which is the most common form of web advertising, has been catching the eye of online users. It is important for online advertisement publishers to schedule advertisements to maximize advertising effectiveness, which might be a predicted value of click-through-rate. In this paper, a model for online banner advertisement scheduling is presented with a new objective function that includes four factors that influence advertising effectiveness. This paper provides not only the integer programming model with a nonlinear objective function but also two heuristic and meta-heuristic algorithms as solution methodologies. Most importantly, the heuristic approach finds good lower and upper bounds by using the properties of the model. Both randomly generated and standard data sets are used to test the problem. The results indicate the effectiveness and efficiency of these algorithms tested in small and large data sets. In standard data sets, the results show the advertisement specification for which tighter bounds are obtained by using the heuristic approach.

1. Introduction

Advertising is used as one of the critical methods for companies to promote products or services. There are various platforms to present advertisements, such as broadcast, print, or the Internet. With a drastic increase in online communities, many companies have been paying attention to web advertising. Besides, web advertising has more traceability, cost effectiveness, reach, and interactivity than other platforms, so the popularity of web advertising can be expected to continue (Manik, Gupta, Jha, & Govindan, 2016). According to an Internet Advertising Bureau (IAB) report, Internet advertising revenue in the United States was $107.5 billion in 2018, an increase of 22% over that of 2017 (IAB, 2019).

With the advent of web advertising, studies related to web advertising have also been initiated. Yager (1997) provided a framework for the competitive selection of advertisements on websites. Novak and Hoffman (1997) described terminologies for web advertising measurements and proposed exposure and interactivity metrics. McCandless (1998) explained the growth of web advertising and proposed a web advertising model for effectiveness.

Banner advertisement is the most common form of web advertising. A banner is a rectangular advertisement positioned on either side, the top, or the bottom of a web page, such as an Internet article or an online shopping mall. An image in the banner advertisement is clickable and linked to a target web page (McCandless, 1998). By being displayed on a web page, the banner attracts the attention of online users who are interested in the associated products or services.

Online advertisement publishers must construct layout partitioning and allocate the appropriate banners at the respective time (Gopal, Li, & Sankaranarayanan, 2011; Marszałkowski & Drozdowski, 2013). A well-constructed advertisement scheduling increases advertising effectiveness. As the advertising effectiveness increases, advertisers' requests to display their advertisements on the website increase. As the requests increase, online advertisement publishers can ultimately generate more revenue. Hence, online banner advertisement scheduling is important to online advertisement publishers. Online banner advertisement scheduling for online advertisement publishers is addressed in this paper.

Many researchers have focused on online banner advertisement scheduling from an optimization perspective and provided mathematical models. Adler, Gibbons, and Matias (2002) presented a MAXSPACE problem of banner advertisement scheduling; it was designed to find a banner advertisement scheduling solution in which space utilization of various-sized advertisements is maximized for specific time slots. Fig. 1 shows an example solution to the problem (left) and a webpage for the second slot (right) in the planning period. In this example, there are six advertisements (A to F) and ten slots in the planning period. The advertisements each have different lengths, and they can be assigned to each slot. Each slot can be represented as a period of five, ten, thirty minutes, etc. A banner is positioned on the left side of

* Corresponding author.
E-mail addresses: gwangkim91@snu.ac.kr (G. Kim), ikmoon@snu.ac.kr (I. Moon).

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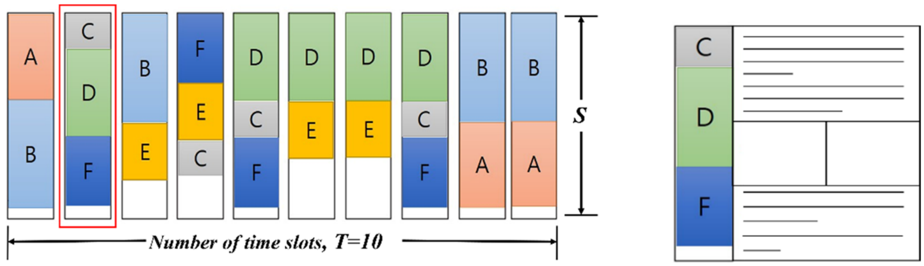

Fig. 1. Example of MAXSPACE problem (left) and webpage for the second slot (right).

the webpage in this figure.

Recent research on online banner advertisement scheduling has focused on maximizing advertising effectiveness, not space utilization by introducing new objective functions (Ahmed & Kwon, 2014; Deane & Agarwal, 2012a; Gamzu & Koutsopoulos, 2018; Ji et al., 2015). They pointed out that maximizing the space utilization of time slots does not directly translate to maximizing online advertisement publishers' revenue. To measure advertising effectiveness, they primarily took into account the timing when an advertisement is displayed and the number of advertisement exposures. However, they have not extensively investigated the effects of other advertisements displayed in the same time slot. Vakratsas and Ambler (1999), Henningsen, Heuke, and Clement (2011), and Agyei (2018) explicitly demonstrated that the effectiveness of any given advertisement might be diminished by the presence of competing advertisements sorted and served together with it in a time slot. Such interference could play an important role, in particular, when the companies display advertisements of their own products or services on the website. Therefore, this study develops an online banner advertisement scheduling model that includes critical factors, such as competitive advertising interference, influencing advertising effectiveness.

In this paper, there are four distinct features for the online banner advertisement scheduling considering advertising effectiveness. First, to reflect the advertising effectiveness, this model considers factors that influence the tendency for online users to click on the advertisement. The tendency is defined as the *click-through-rate* (CTR) (Gatarski, 2002; Lin & Chen, 2009). This study proposes four factors that influence the CTR: exposure, involvement, size, and competition. In addition to these factors, the contents of an advertisement—features such as message, colour, and animation—also influence the CTR. However, because online advertisement publishers cannot control content features, we do not consider them in this study. That is, this study assumes that the contents of each advertisement are fixed before the banner advertisement scheduling is implemented. Second, this study is the first one in which competition (competitive advertising interference) is introduced to online banner advertisement scheduling. We call the interference 'competition'. Competition is defined as the extent of interference when consumers are exposed to advertisements for competing products, and negatively affects advertising effectiveness (Henningsen et al., 2011; Vakratsas & Ambler, 1999). When more advertisements are displayed together in a time slot, each advertisement is adversely affected by the high competition within the slot. So, it may not be optimal to display as many advertisements as possible in a time slot.

Third, this study presents an expected CTR function as an objective function of the model. The function is based on the consideration of the four factors mentioned above. This study uses the demand function developed by Corstjens and Doyle (1981) to devise the expected CTR function. Each component of the demand function can be interpreted as one of the factors of the expected CTR function. Fourth, variable display frequency bounds proposed by Deane and Agarwal (2012b) are used in our model. The constraint not only reflects a realistic situation but also offers more flexibility for online advertisement publishers to display advertisements in the model compared to fixed display frequency.

With a given set of advertisements, online advertisement publishers have to design a well-constructed advertisement scheduling model to maximize advertising effectiveness. It leads to an increase in advertisers' requests to display their advertisements on the website and ultimately generates more revenue for online advertisement publishers. The model that considers the four features, as mentioned above, can be represented as an integer programming model with the non-linear objective function. This study not only develops an online banner advertisement scheduling model to maximize advertising effectiveness but also provides a solution methodology to obtain good solutions effectively and efficiently: a heuristic approach to finding good lower and upper bounds of the model. To compare the result from large data sets, we also present a hybrid tabu search.

1.1. Related work

There exists various literature on online banner advertising scheduling and mathematical models. Aggarwal, Wolf, and Philip (1998) proposed a banner assignment problem by using the formulation of the minimum cost flow problem. Adler et al. (2002) presented a MAXSPACE problem of banner advertisement scheduling. They also showed that problem is NP-hard. A heuristic algorithm called SUBSET-LSLF was developed for obtaining good advertisement scheduling solutions for web pages. Dawande, Kumar, and Sriskandarajah (2003) and Kumar, Jacob, and Sriskandarajah (2006) presented better heuristic algorithms to solve the MAXSPACE problem than those offered by to Adler et al. (2002). Freund and Naor (2004) presented a $(3 + \epsilon)$-approximation algorithm for a profit maximization problem (equal to the MAXSPACE problem).

Furthermore, some studies extended the original MAXSPACE problem by considering realistic situations. Amiri and Menon (2006) presented the MAXSPACE problem regarding multiple display frequencies in which the customer is allowed to specify a set of acceptable frequencies. Deane and Agarwal (2012b) presented the MAXSPACE problem that incorporated variable display frequencies. Online advertisement companies apply acceptable advertisement frequency ranges to offer more flexibility to the publishers and advertisers. Purnamawati, Nababan, Tsani, Taqyuddin, and Rahmat (2018) proposed an advertisement scheduling problem with different advertisement frequencies between prime and regular times. Deane and Pathak (2009) stated the importance of online advertisement targeting, and Deane (2012) extended the problem of Deane and Agarwal (2012b) by considering advertisement targeting. Some studies proposed placement models and formulations in which two-dimensional display time slots are considered (Boskamp, Knoops, Frasincar, & Gabor, 2011; Anshu Gupta, Aggarwal, Kaul, & Jha, 2016; A Gupta, Manik, Aggarwal, & Jha, 2011; Kaul et al., 2018; Knoops, Boskamp, Wojciechowski, & Frasincar, 2009; Manik et al., 2016). da Silva, Schouery, and Pedrosa (2019) considered release dates and deadlines in the MAXSPACE problem and developed a polynomial-time approximation scheme for the problem.

Meanwhile, some studies provided new pricing models that reflect applicability in industry. The studies emphasized maximizing advertising effectiveness rather than space utilization. Kumar, Dawande, and

Table 1
Four factors that influence the click-through-rate.

Factors	Descriptions	References
Exposure	The extent to how often the advertisement is exposed over a planning horizon	Rethans, Swasy, and Marks (1986), Wu, Wei, and Chen (2008)
Involvement	The number of website users who have high-affinity with the advertisement or the degree of relationship between the website and the advertisement	Bhat et al. (2002), Wu et al. (2008), Rosenkrans (2010)
Size	The relative or absolute size of the advertisement	Li and Bukovac (1999), Rosenkrans (2010)
Competition	The extent of interference when consumers are exposed to advertisements for competition products	Vakratsas and Ambler (1999), Henningsen et al. (2011)

Mookerjee (2007) solved the online banner advertisement scheduling problem with a hybrid pricing model. A hybrid pricing model is one where pricing is based on a combination of the number of advertisement exposures (the cost-per-thousand impressions model) and the number of clicks on the advertisement (click-through model). Palade and Banerjee (2011), Einziger, Chiasserini, and Malandrino (2018), Jea, Wang, and Hsu (2019), and Czerniachowska (2019) discussed the importance of the position and timing when managers assign advertisements to the slots in advertisement channels, such as online web, mobile device, book, and TV. Deane and Agarwal (2012a) extended Deane and Agarwal (2012b)'s model and presented a nonlinear pricing model which reflects the quantity discount pricing strategy. Also, some studies addressed new pricing models combined with the characteristic of online users. Ahmed and Kwon (2014) discussed a contract problem for online advertisements with pay-per-view pricing scheme. Deza, Huang, and Metel (2015)'s model is to accomplish that each advertisement is displayed as proportionally as possible to all targeted viewers types. Ji et al. (2015) and Gamzu and Koutsopoulos (2018) considered online users' preference and fatigue regarding online advertising scheduling. Lejeune and Turner (2019) presented an online display advertising planning problem to maximizes the spreading of impressions across targeted audience segments, while limiting demand shortfalls.

Compared to the previous literature, this study presents a new approach to measuring advertising effectiveness. The approach considers four factors that influence the tendency for online users to click on the advertisement and presents an expected CTR function to reflect these factors. To the best of our knowledge, this study first presents the expected CTR function in this problem. Among the factors, the degree to competition has not yet been discussed extensively in the literature on maximizing advertising effectiveness. The remainder of the paper is organized as follows. Section 2 presents descriptions of the assumptions and the formulation of the online banner advertisement scheduling for advertising effectiveness. Section 3 presents the solution methodologies used to obtain good lower and upper bounds of the problem efficiently. In Section 4, experimental results for small and large problems are provided. The results using the standard data sets provided by the IAB are also presented. Section 5 offers conclusions of the research and suggestion for future studies.

2. Mathematical model

In this section, the development of the integer programming model with a non-linear objective function for the online banner advertisement scheduling problem is explained. The objective of the model is to select a set of advertisements for each time slot so that the advertising effectiveness is maximized. A time slot is a space for displaying advertisements in a unit time interval. There are three types of constraints in our model. First, the selected advertisements for a time slot should fit in the available space. Second, each advertisement can be displayed at most one time in a time slot. Third, if an advertisement is selected, its display frequency in all time slots must be between lower and upper bounds.

2.1. Objective function

Unlike prior models of online banner advertisement scheduling, the model presented herein is based on considerations of advertising effectiveness (Deane & Agarwal, 2012b; Kaul et al., 2018; Manik et al., 2016). Accordingly, the advertising effectiveness is reflected in the objective function of the model. This study proposes an expected CTR function as an objective function of the model. CTR is typically defined as the number of clicks divided by the number of times the advertisement has been exposed to time slots; thus, it represents the effectiveness of an advertisement (Bhat, Bevans, & Sengupta, 2002; Gatarski, 2002). The expected CTR function also deals with exposures of advertisements based on the cost-per-thousand impressions model.

The expected CTR function used in this study consists of four main factors: exposure, involvement, size, and competition. The four factors are not only closely related to CTR but also those that online advertisement publishers can control through changing a subset of advertisements displayed when they display advertisements on websites. Because online advertisement publishers cannot control the contents of an advertisement itself, factors such as message, colour, and animation are not considered. In other words, it is assumed that the contents of each advertisement are fixed. Table 1 describes the four factors that compose the expected CTR function and presents previous research which found statistically significant relationships between the four factors and CTR.

This paper presents an expected CTR function based on the consideration of the four factors listed in Table 1. The function was made by referring to the demand function developed by Corstjens and Doyle (1981). The demand function of Corstjens and Doyle (1981) is defined as follows:

$$D_{ik}\,(expected\ \ demand\ \ for\ \ product\ 'i'\ \ displayed\ \ on\ \ shelf\ \ 'k') = \alpha_i (s_{ik} x_{ik})^{\beta_{ik}} \prod_{\forall j \neq i} \sum_{m=1}^{K} (s_{jm} x_{jk})^{\delta_{ij}} \tag{1}$$

The demand function was used to solve the shelf-space allocation problem. (K is the number of shelves.) The demand function was postulated and proved through broad empirical findings (Corstjens & Doyle, 1981; Curhan, 1973; Lee, 1961). An empirical estimation with cross-sectional data, a meta-analysis, and the method of least square have been used to estimate the parameters of the demand function (Desmet & Renaudin, 1998; Dreze, Hoch, & Purk, 1994; Eisend, 2014). The shelf-space allocation problem is used to optimize the retailer's allocation of shelf space for a set of alternative products. The shelf-space allocation and banner advertisement scheduling are structurally similar. First, the manager chooses a set of items (products or advertisements) and displays them in limited spaces. Second, the objective is to maximize the manager's revenue. In addition, the total demand is dependent on the allocation of selected products in the shelf space allocation problem. Likewise, in the online banner scheduling problem, the total advertising effectiveness is dependent on a set of displayed advertisements. Because of the similarities between the two problems, for this study, the expected CTR function was developed using the components of the demand function. Each component of the demand

Table 2
Comparisons between the demand function developed by Corstjens and Doyle (1981) and the expected CTR function.

	Demand function developed by Corstjens and Doyle (1981)	CTR function developed by this paper
α_i	Space scale parameter for product i	Scale parameter for contents of ad i
$s_{ik}(s_i)$	Quantity of product i displayed on shelf k	*Size* of ad i
x_{ik}	Product i displayed on shelf k or not	Ad i displayed in slot k or not (*exposure*)
β_{ik}	Space elasticity of product i displayed on shelf k	Degree to *involvement* of users for ad i in slot k
δ_{ij}	Cross space elasticity between product i and j	Degree to *competition* between ad i and j

function can be interpreted as one of the factors of the expected CTR function. In particular, exposure (positive) and competition (negative) effects are clearly reflected in the demand function.

The expected CTR function is presented as follows:

$$\pi_{ik}(expected\ \ CTR\ \ for\ \ ad\ \ 'i'\ \ in\ \ time\ \ slot\ \ 'k') = \alpha_i(s_i x_{ik})^{\beta_{ik}} \prod_{\forall j\neq i, x_{jk}=1} (s_j x_{jk})^{\delta_{ij}} \quad (2)$$

The expected CTR function includes the four factors that influence CTR. Table 2 shows the comparisons between the demand function developed by Corstjens and Doyle (1981) and the expected CTR function. The components of Corstjens and Doyle (1981)'s demand function can be translated into the factors this paper proposes. The components of Table 2, except x_{ik}, are all parameters. The parameters are assumed to be empirically estimated by historical data for advertising effectiveness. This study uses the expected CTR function as the objective function.

In shelf-space allocation problem, α_i is the space scale parameter for product i. The value of α_i is translated into the quality of the contents of advertisement i in the CTR function. The space elasticity is defined as the ratio of changes in sales to changes in space. The value of β_{ik} depends on characteristics of product i and shelf k. Similarly, in the CTR function, β_{ik} represents the degree to involvement of users for advertisement i in slot k. In other words, β_{ik} emphasizes the importance of timing when advertisement i is displayed, while α_i means the effect of advertisement $i$itself regardless of the timing. The cross-space elasticity δ_{ij} is defined as the ratio of changes in sales of product i to changes in product j. The value of δ_{ij} depends on whether products i and j are complementary or substitute products. Similarly, in the CTR function, δ_{ij} represents the degree to competition between advertisements i and j.

2.2. Notations and formulation

The definitions of the parameters and decision variables used in the integer programming model with a non-linear objective function are presented as follows:

N	Number of advertisement types
T	Number of time slots
H	Height of banner in all time slots
s_i	Height of advertisement i
L_i	Lower bound on display frequency of advertisement i if displayed
α_i	Scaling parameter for ad i
β_{it}	Degree to involvement and tendency of users of ad i for time slot$t$
δ_{ij}	Degree to competition between ad i and j
x_{it}	Binary decision variable, whose value is 1 if advertisement i is assigned to time slot t, 0 otherwise
y_i	Binary decision variable, whose value is 1 if advertisement i is assigned, 0 otherwise

It is assumed that the height of the banner is the same in every time slot and that each advertisement can be displayed in each time slot no more than once. These assumptions are the basic ones for the online banner advertisement scheduling problem (Boskamp et al., 2011; Deane & Agarwal, 2012a, 2012b; Gupta et al., 2016). In addition, to consider the effect of competition based on the combination of advertisements in a slot, we assume that each advertisement has a height less than or equal to one-half the height of the banner ($\forall\ s_i \le \frac{H}{2}$). In accordance with the definition of the *competition* factor, all the values of the degree to competition are assumed to be less than or equal to 0 ($\forall\ \delta_{ij} \le 0$).

The integer programming formulation with a non-linear objective function for online banner advertisement scheduling is

$$\text{Max} \sum_{i=1}^{N}\sum_{t=1}^{T} \pi_{it} x_{it}$$

$$\pi_{it} = \alpha_i(s_i)^{\beta_{it}} \prod_{\forall j\neq i} (1 - x_{jt} + s_j x_{jt})^{\delta_{ij}} \qquad \forall\ i,\ t \quad (3)$$

$$\sum_{i=1}^{N} s_i x_{it} \le H, \qquad \forall\ t \quad (4)$$

$$L_i y_i \le \sum_{t=1}^{T} x_{it} \le T y_i, \qquad \forall\ i \quad (5)$$

$$x_{it} \in \{0,\ 1\}, \qquad \forall\ i,\ t \quad (6)$$

$$y_i \in \{0,\ 1\}, \qquad \forall\ i \quad (7)$$

The objective of the formulation is to find the optimal banner advertisement scheduling that maximizes advertising effectiveness. The objective function means the total expected CTR value that indicates advertising effectiveness. Eq. (3) defines π_{it} that is an expected CTR value for advertisement i in time slot$t$. Constraint (4) guarantees that the sum of the heights for advertisements displayed in each time slot cannot exceed the banner height. Constraint (5) enforces frequency bounds for each advertisement. For any displayed advertisement, the number of times that the advertisement is shown should be between the lower bound, L_i, and the number of time slots on the time horizon. Constraints (6) and (7) define x_{it} and y_i as binary variables, respectively.

3. Solution methodologies

The optimization model presented in Section 2 featured a non-convex objective function, which reflects the effectiveness of advertising, with linear constraints. Therefore, in a large data set, whose optimal solution can be intractable to compute, the problem might not be solved directly using optimization solvers within a reasonable time. This paper explains the alternate algorithm proposed to solve the problem efficiently and effectively. The alternate algorithm finds good lower and upper bounds through the optimization model. Because optimization solvers might not provide good bounds or solutions within a reasonable time, a hybrid tabu search, which is a meta-heuristic approach, is also presented as a means to compare the results from large data sets.

3.1. Heuristic approach to finding lower and upper bounds using an integer programming model with a non-linear objective function

The heuristic approach focuses on finding valid lower and upper bounds by using the properties of the optimization model as presented in the previous section. The way to find lower and upper bounds is presented as follows:

[Upper bound]

The objective function of the optimization model can be divided into two parts. The two parts are called the *involvement part* ($\alpha_i(s_i)^{\beta_{it}}x_{it}$) and the *competition part* ($\prod_{\forall j\neq i}\left(1-x_{jt}+s_j x_{jt}\right)^{\delta_{ij}}$). As the number of advertisements displayed in a time slot increases, the advertising effectiveness, as caused by the involvement part, positively increases; the advertising effectiveness caused by the competition part adversely increases. In other words, for the latter, the value of $\prod_{\forall j\neq i}\left(1-x_{jt}+s_j x_{jt}\right)^{\delta_{ij}}$ is always in the range of (0, 1], and the value decreases as the competition effect increases. In addition to this fact, the following proposition could be made:

Proposition 1. *For a specific time slot $t\in\{1,2,\cdots,T\}$ and a set of advertisements A ($A=\{1,2,\cdots,N\}$), let A_1 be subsets of set$A$such that$n(A_1)=k$ assigned to the time slot t, and A_2 be subsets of set A such that$n(A_2)=k+1$ assigned to the time slot t ($k=1,2,\cdots N-1$). Let V_{MAX} be the maximum value of $\sum_{i=1}^{N}\pi_{it}x_{it}$ among the entire subsets A_1, and V_{MIN} be the minimum value of $\sum_{i=1}^{N}\pi_{it}x_{it}$ among the entire subsets A_2. If $V_{MAX}<V_{MIN}$, then any solution for which $k$advertisements are displayed in the time slot$t$is not an optimal solution.*

Proof. To get a contradiction, assume that let X^* ($X^*=(x_{11}^*,x_{12}^*,\cdots,x_{NT}^*)$) be an optimal solution such that k advertisements are displayed in the time slot t. Let $\widehat{X}$ be a feasible solution identical to X^* except for the time slot$t$ in which$k+1$ advertisements are displayed. Let $\Pi(X^*)$ and $\Pi(\widehat{X})$, respectively, be the objective values of the problem using X^* and $\widehat{X}$. It follows that $\Pi(X^*)\leq V_{MAX}+C$ (C is equal to $\Pi(X^*)-\sum_{i=1}^{N}\pi_{it}x_{it}^*$). Because of $\Pi(\widehat{X})\geq V_{MIN}+C$ and $V_{MAX}<V_{MIN}$, the outcome is $\Pi(\widehat{X})\geq\Pi(X^*)$. This contradicts the fact that X^* is the optimal solution. ■

In summary, as the number of advertisements displayed in a banner increases, the competition effect negatively increases. Therefore, banners with as many advertisements as possible may not always lead to desirable outcomes. If it is guaranteed that one of the solutions with more than k advertisements displayed in a particular time slot is optimal, at least the competition effect in the time slot of the optimal solution is greater than the minimum competition effect when $k+1$ advertisements are displayed. In the following lemma, an upper bound of the optimization model is obtained. In Lemma 2, the value of P is used to determine the range of w. If the N advertisements are sorted in descending order of size, then we define $\bar{S}_i$ as the size of the i^{th} advertisement. The value of P is now defined as the maximum integer value satisfying $\sum_{i=1}^{P}\bar{S}_i\leq H$.

Lemma 2. *For a time slot $t\in\{1,2,\cdots,T\}$ and an advertisement $i\in\{1,2,\cdots,N\}$, two values (L_{it}^w,R_{it}^w) are defined as follows ($w\in\{2,3,\cdots,P\}$):*

$$L_{it}^w=\begin{cases}\alpha_i(s_i)^{\beta_{it}} & w=2\\ \alpha_i(s_i)^{\beta_{it}}*Q_1^{max}+(Q_2^{max}*Q_3^{max})*(w-2) & w\neq 2\end{cases}\tag{8}$$

$$R_{it}^w=\alpha_i(s_i)^{\beta_{it}}*Q_1^{min}+(Q_2^{min}*Q_3^{min})*(w-1)\tag{9}$$

where

$$Q_1^{max}=\max(s_{j_1})^{\delta_{ij_1}}\cdots(s_{j_{w-2}})^{\delta_{ij_{w-2}}},\ Q_1^{min}=\min(s_{j_1})^{\delta_{ij_1}}\cdots(s_{j_{w-1}})^{\delta_{ij_{w-1}}},\ \forall j_1,j_2\cdots j_{w-1}\in\{1,2,\cdots,N\},j_1\neq j_2\neq\cdots\neq j_{w-1},$$

(Q_1^{max} is defined as the maximum value of the competition part for advertisement i when any $w-1$ advertisements, including advertisement i, are displayed in a slot. Q_1^{min} is defined as the minimum value of the competition part for advertisement i when any w advertisements, including advertisement i, are displayed in a slot.)

$$Q_2^{max}=\max\alpha_{i'}(s_{i'})^{\beta_{i't}},\ Q_2^{min}=\min\alpha_{i'}(s_{i'})^{\beta_{i't}}\ \forall i'\in\{1,2,\cdots,N\},\forall t\in\{1,2,\cdots,T\},$$

(Q_2^{max} is defined as the maximum value of the involvement part for an advertisement. Q_2^{min} is defined as the minimum value of the involvement part for an advertisement.)

$$Q_3^{max}=\max(s_{j_1})^{\delta_{i'j_1}}\cdots(s_{j_{w-2}})^{\delta_{i'j_{w-2}}},\ Q_3^{min}=\min(s_{j_1})^{\delta_{i'j_1}}\cdots(s_{j_{w-1}})^{\delta_{i'j_{w-1}}}\ \forall i'\in\{1,2,\cdots,N\},\forall j_1,j_2\cdots j_{w-1}\in\{1,2,\cdots,N\},j_1\neq j_2\neq\cdots\neq j_{w-1}\neq i'.$$

(Q_3^{max} is defined as the maximum value of the competition part for an advertisement when any $w-1$ advertisements are displayed in a slot. Q_3^{min} is defined as the minimum value of the competition part for an advertisement when any w advertisements are displayed in a slot.)

For each advertisement i and time slot t, let w_{it}' be the smallest integer of w such that $L_{it}^w>R_{it}^w$. Using the value of w_{it}', the value of D_{it} can be calculated as follows:

$$D_{it}=\begin{cases}1 & w_{it}'=2\\ max(s_{j_1})^{\delta_{ij_1}}(s_{j_2})^{\delta_{ij_2}}\cdots(s_{j_{(w_{it}'-2)}})^{\delta_{ij_{(w_{it}'-2)}}}\ \forall i,t & w_{it}'\neq 2\end{cases}\tag{10}$$

If w_{it}' and D_{it} are obtained, then an upper bound (UB) of the optimization model can be calculated. The UB is an optimal objective value of a new formulation. The new formulation is identical to the optimization model except for the objective function. The objective function of the new formulation is $max\sum_{i=1}^{N}\sum_{t=1}^{T}\alpha_i(s_i)^{\beta_{it}}x_{it}*D_{it}$ instead of$max\sum_{i=1}^{N}\sum_{t=1}^{T}\pi_{it}x_{it}$. The new formulation thus represents binary integer programming (BIP).

Proof. The values of L_{it}^w *and* R_{it}^w can be interpreted as described in this paragraph. For an advertisement i and a time slot t, the value of $\sum_{i=1}^{N}\pi_{it}x_{it}$ is less than L_{it}^w when any $w-1$ advertisements including advertisement i are displayed in the time slot t, and the value of $\sum_{i=1}^{N}\pi_{it}x_{it}$ is greater than R_{it}^w when any$w$ advertisements including advertisement i are displayed in the time slot t. So, if $L_{it}^w\leq R_{it}^w$, then the solutions with fewer than w advertisements (including advertisement i) displayed in the time slot$t$ are guaranteed to be not optimal. In other words, if w_{it}' is obtained for advertisement i and time slot t, the solutions with fewer than $w_{it}'-1$ advertisements (including advertisement i) displayed in the time slot t are not optimal. Furthermore, D_{it} represents the value of the competition part that reflects the minimum competition effect when $w_{it}'-1$ (including advertisement i) advertisements are displayed in time slot t. Thus, for each advertisement i and time slot t, the competition effect of the optimal solution is not less than the competition effect indicated by D_{it}. Accordingly, for the new formulation, in which the competition part ($\prod_{\forall j\neq i}\left(1-x_{jt}+s_j x_{jt}\right)^{\delta_{ij}}$) is replaced by D_{it}, the objective value is guaranteed to be an upper bound of the original model. The new formulation is then a BIP as follows: $UB=max\sum_{i=1}^{N}\sum_{t=1}^{T}\alpha_i(s_i)^{\beta_{it}}x_{it}*D_{it}$ subject to (4) – (7). ■

[Lower bound]

The optimization model can be a BIP if no competition effects are present ($\forall\ \delta_{ij}$=0). In Phase 1, the heuristic solves the optimization model without competition effects (BIP). In the next iterations (Phase 2), we assume that the competition part for each advertisement is calculated by using X^* obtained from the previous iteration. We then update the objective function and solve the corresponding BIP. The heuristic to obtain a good lower bound is a modification of the heuristic of Schaal and Hübner (2017). The details of the heuristic are as follows:

Phase 1. Solving the optimization problem without competition effects.
Step 1. Let X^* be the solution of the optimization problem without competition effects.
Step 2. $LB \leftarrow$ the objective function value applying X^* to the original formulation.

Phase 2. Solving the optimization problem with competition effects of the previous iteration.
Step 1. For $\forall\ i,\ t$, compute $\pi_{it}\left(=\alpha_i(s_i)^{\beta_{it}}\prod_{\forall j\neq i}\left(1-x_{jt}+s_j x_{jt}\right)^{\delta_{ij}}\right)$ using X^* and construct the corresponding BIP using π_{it}.
Step 2. Let X^{**} be the solution of the corresponding BIP using π_{it}.
Step 3. If $X^* = X^{**}$, then stop the process; otherwise, go to Step 4.
Step 4. $LB^* \leftarrow$ the objective function value applying X^{**} to the original formulation.
Step 5. If $LB^* > LB$, then $LB \leftarrow LB^*$.
Step 6. $X^* \leftarrow X^{**}$ and repeat Phase 2.

3.2. Hybrid tabu search

This study uses a hybrid tabu search as a supplementary approach to compare the results from large data sets. A tabu search is a meta-heuristic method that can be used to explore a space of possible solutions beyond the local optimality within a reasonable time (Glover & Laguna, 1998). The method is easy to represent a feasible solution as a sequence form in this problem and also to find an improved solution by checking the immediate neighbor sequences of the current solution. Thus, the hybrid tabu search is used as a supplementary approach. The hybrid tabu search generates an initial sequence based on a greedy algorithm, and then the tabu search is repeated with the updated sequence. The sequence represents the assignment of advertisements to time slots. When constraints are satisfied, the sequence can be converted to a feasible solution.

Appendix A presents the details of the greedy algorithm that generates an initial sequence and its information for the hybrid tabu search. After the greedy algorithm, we conduct the hybrid tabu search using this sequence to obtain good feasible solutions. The details of the hybrid tabu search are presented in Appendix B (Algorithm 1 and Algorithm 2). *num*1 is an arbitrary constant that represents a criterion for assigning advertisements, which is introduced to avoid being trapped in local optima. A small example is presented in Appendix C.

4. Computational experiments

In this section, the performances of the model are analyzed in small, large, and standard data sets. The standard data used in this paper is extracted from the specifications provided by the IAB. The integer programming formulation with a non-linear objective function was solved with LINGO version 17.0 for the optimal solution. The heuristic approach used to find valid lower and upper bounds was run with Xpress Mosel version 3.10.0, and the hybrid tabu search was run with JAVA language in Windows 7 on a PC with an Intel(R) Core(TM) i5-4690 CPU 3.5 GHz with 16.00 GB of RAM.

For small data sets, the performances of the solution methodologies were evaluated by comparing the optimal solutions obtained within a reasonable time. The time limit was set to 3600 s. However, for large data sets, because the formulation could not provide good feasible solutions as well as an optimal solution within the time limit, the results obtained by only the heuristic approach and hybrid tabu search were analyzed. The parameters of the hybrid tabu search are as follows: *num*1 was set to 0.05, the number of iterations was $10 \times N$, and the number of tabu search iterations was 5. Table 3 shows the parameter sets used in the formulation. U[a, b] refers to a uniform distribution between a and b.

Table 3
Parameter sets.

Parameter	Value
$(\boldsymbol{N},\ \boldsymbol{T})$	(small): (4,2); (5,2); (5,3); (6,3); (6,4); (7.3); (large): (10,3); (20,5); (30,10); (50,15); (75,20); (100,25)
$\boldsymbol{H}$	400
$\boldsymbol{L_i}$	1 or 2 (small), 1, 2, or 3 (large)
$\boldsymbol{s_i}$	(integer) U[80, 100]
$\boldsymbol{\alpha_i}$	U[5.0, 10.0]
$\boldsymbol{\beta_{it}}$	U[0.10, 0.20]
$\boldsymbol{\delta_{ij}}$	U[−0.020, −0.010]

The parameters N and T in small data sets were designed to solve the problem with LINGO easily and those in big data sets were chosen with the rate of approximately 3 or 4 to 1 (N:T). The slot size and advertisement size were chosen to display an average of 4 or 5 advertisements in a time slot. The ranges for the parameters related to demand function of Corstjens and Doyle (1981), such as α_i, β_{it}, and δ_{ij}, were based on values from shelf-space allocation problems. The results for small and large data sets are presented in Sections 4.1 and 4.2. In Section 4.3, the computational results, using the standard data sets developed by (IAB, 2017), are presented.

4.1. Results for problems with small data sets

Twenty different samples were tested for each problem size. LINGO could find the optimal solutions in all instances of small data sets within the time limit. However, the problems that exceeded the (7, 3) instance set did not provide near-optimal solutions as well as an optimal solution within the time limit. The sample data were also used in the other solution methodologies to compare the results in terms of the effectiveness and efficiency of the heuristic algorithms. Table 4 shows the results for problems with small data sets.

The average computation time of LINGO ranged between 18.8 and 1076 s. In contrast, the average computation time for each of the other two solution methodologies was less than 0.05 s. The heuristic approach to finding the lower and upper bounds found the optimal solutions in 44 of 120 instances, and the hybrid tabu search found the optimal solutions in 58 of 120 instances. Although the optimal solutions were not obtained in more than one-half the instances, the worst optimality gaps for 120 instances were 0.88% and 0.60% for the heuristic approach and the hybrid tabu search, respectively. On average, for the small data sets, the hybrid tabu search performed slightly better and found the solution a little faster than the heuristic approach. In the heuristic approach, the average percentage of $1-(LB/UB)$ was calculated to be 1.74%, and the worst percentage was 3.39%. The average gap between the upper bound and the optimal objective value was calculated to be 1.53%, and the worst gap was 3.16%. The two solution methodologies found comparatively good quality solutions within one second in the experiments with small data sets. The results showed that the two methodologies are appropriate approaches in terms of effectiveness and efficiency.

4.2. Results for problems with large data sets

In this section, the results for problems with large data sets are presented. Because optimal solutions were not able to be intractable to compute within the time limit by using LINGO, the results were analyzed by using the bounds that the heuristic found and the feasible solutions obtained by the hybrid tabu search. In this problem, the number of tabu search iterations was set to T. Each time Algorithm 2 was executed, $5*N$ times were repeated because Algorithm 2 has a random setting in the procedure. The number of tabu sequences was set to be equal to the number of tabu search iterations. Twenty different samples were tested for each problem size. Table 5 shows the results for

Table 4
Results for problems with small data sets.

(**N**, **T**)	Computation time in seconds (avg, max)			% performance gap[a] (avg, max)		
	Optimization	Heuristic for bounds	Hybrid tabu search	Heuristic for bounds[a]	Hybrid tabu search[a]	1-(LB/UB)
(4, 2)	(18.8, 23.0)	(0.025, 0.028)	(0.010, 0.014)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)
(5, 2)	(52.5, 78.0)	(0.027, 0.035)	(0.014, 0.017)	(0.11, 0.46)	(0.02, 0.24)	(1.48, 2.00)
(5, 3)	(109.6, 230)	(0.020, 0.033)	(0.020, 0.026)	(0.14, 0.50)	(0.13, 0.50)	(1.47, 2.16)
(6, 3)	(256.1, 480)	(0.030, 0.056)	(0.030, 0.039)	(0.28, 0.88)	(0.21, 0.60)	(2.32, 3.39)
(6, 4)	(454.9, 766)	(0.030, 0.058)	(0.059, 0.090)	(0.27, 0.72)	(0.20, 0.44)	(2.33, 3.15)
(7, 3)	(1076, 2473)	(0.050, 0.060)	(0.040, 0.050)	(0.40, 0.71)	(0.18, 0.54)	(2.83, 3.36)

[a] $\left(1-\frac{objective\ \ value\ \ of\ \ solution\ \ obtained\ \ by\ \ each\ \ algorithm}{optimal\ \ objective\ \ value}\right)\times 100\%$.

problems with large data sets. The last column of Table 5 lists the optimality gap obtained by LINGO at 3600 s.

The average computation time ranged between 0.05 and 46.47 s in the heuristic approach and between 0.06 and 812.75 s in the hybrid tabu search. As N and T increased, the heuristic approach found better feasible solutions than the hybrid tabu search did. Specifically, in the (100, 25) instance, the heuristic approach found the feasible solution for which the objective value was higher than that of the hybrid tabu search by 15.02%. On average, the heuristic approach performed better and found the solution faster than the hybrid tabu search did for the large data sets. The results showed that the heuristic approach performed better as N and T increased. In the heuristic approach, the average percentage of $1-(LB/UB)$ ranged between 4.51% and 8.97% for the large data sets. It was not always shown that the average percentage of $1-(LB/UB)$ was increased as N and T increased in the instances. A piecewise linearization technique was also used and analyzed to relax the non-linear and non-convex objective function in this problem (Irion, Lu, Al-Khayyal, & Tsao, 2012). The technique showed the same results as the results without competition effects. The average gap from the technique ranged between 15.4% and 17.3% for the large data sets. As in the heuristic approach, the average gap did not always increase as N and T increased in the instances. The last column of Table 5 shows the gap between the best possible objective value and best upper bound (optimality gap) obtained by LINGO at 3600 s. In the (10, 3) and (20, 5) instances, the average optimality gap was calculated to be 26.7% and 48.4%, respectively. LINGO could not obtain any feasible solutions during 3600 s for most of the data between (30, 10) and (100, 25). Although LINGO obtained some feasible solutions during 3600 s for a few data between (30, 10) and (100, 25), the optimality gaps were more than 80%. The reduction of the optimality gap was less than 3% on average, even when the time limit was changed from 3600 s to 10,800 s. Accordingly, the results of the linearization technique and LINGO support the need for alternate algorithms for online banner scheduling to maximize advertising effectiveness in terms of solution quality and computation time. In these experiments, the heuristic approach to finding the upper and lower bounds was better than the hybrid tabu search in terms of effectiveness and efficiency.

4.3. Results for problems with the standard data

In this section, the tests using standard data sets are presented. The standard advertisement specification was developed by (IAB, 2017). Because of the increase in the use of mobile devices and tablet PCs as well as diversification of the types of advertisements, the size specifications for advertisements have consecutively changed recently. Moreover, flexible-sized advertisement units tend to be provided for each type of advertisement. Because this study did not address many other types of advertisements, such as skyscrapers, leaderboards, microbar, and rectangles, this study focused on a vertical banner with advertisement sizes and other parameters from (Deane & Agarwal, 2012a) to deal with the standard data. The advertisement sizes used has been described by the IAB.

For vertical banners, the height of a banner is 900 (width is 120) with advertisement sizes of 120 × 60, 120 × 90, 120 × 150, 120 × 200, 120 × 240, and 120 × 280. The values of α_i, β_{it}, *and* δ_{ij} were the same as those featured in Sections 4.1 and 4.2. The unit for the time slot was 5 min. The unit for the time slot was set larger than that found in previous literature; otherwise, the difference of β_{it} would be too small over time. Twenty different samples were tested for each problem set. Table 6 shows the parameter sets described in Section 4.3 and also presents the results for problems with the standard data sets. The fourth column of Table 6 (number of advertisement sizes) denotes one value of 3, 4, 5, or 6. (3 = {120 × 200, 120 × 240, 120 × 280}, 4 = {120 × 150, 120 × 200, 120 × 240, 120 × 280}, 5 = {120 × 90, 120 × 150, 120 × 200, 120 × 240, 120 × 280}, 6 = {120 × 60, 120 × 90, 120 × 150, 120 × 200, 120 × 240, 120 × 280}). Feasible solutions obtained by the heuristic approach to finding valid lower and upper bounds were greater than or equal to those obtained by the tabu search in almost all the standard data sets. In the 60T_5N data set, the feasible solution gaps between the heuristic approach and hybrid tabu search were approximately 1%, but in the 120T_15N data set, the feasible solution gaps were approximately 10%.

Table 5
Results for problems with large data sets.

(**N**, **T**)	Computation time in seconds (avg, max)		% performance gap (avg, max)				
	Heuristic for bounds	Hybrid tabu search	Heuristic for bounds[a]	Hybrid tabu search[a]	1-(LB/UB)	Piecewise linear	Opt. gap (LINGO)
(10, 3)	(0.05, 0.08)	(0.07, 0.09)	(0.25, 0.96)	(0.04, 0.79)	(4.51, 5.56)	(17.3, 18.5)	(26.7, 31.3)
(20, 5)	(0.14, 0.39)	(0.86, 0.96)	(0.18, 0.83)	(0.16, 0.83)	(8.73, 9.19)	(17.1, 17.6)	(48.4, 85.2)
(30, 10)	(0.64, 1.32)	(7.55, 7.71)	(0.06, 0.31)	(0.56, 2.35)	(8.97, 9.84)	(17.2, 18.5)	–
(50, 15)	(2.55, 5.55)	(54.04, 54.60)	(0.00, 0.00)	(1.42, 2.84)	(8.35, 9.49)	(16.4, 17.2)	–
(75, 20)	(9.93, 18.48)	(251.8, 255.3)	(0.001, 0.02)	(2.29, 3.77)	(7.70, 9.58)	(15.6, 17.2)	–
(100, 25)	(23.71, 46.47)	(799.7, 812.7)	(0.00, 0.00)	(4.57, 15.0)	(7.61, 10.2)	(16.1, 17.9)	–

[a] $\left(1-\frac{objective\ \ value\ \ of\ \ solution\ \ obtained\ \ by\ \ each\ \ algorithm}{\max\{\ LB,\ \ objective\ \ value\ \ obtained\ \ by\ \ hybrid\ \ tabu\ \ search\}}\right)\times 100\%$.

Table 6
Parameter sets with the standard data.

Problem set	Parameter set				Result (average)	
	T	N	# of ad sizes	L_i	1-(LB/UB) (%)	Computation time (s)
60**T**_5**N**_3	60 (5 h)	5	3	35	2.07	0.06
60**T**_5**N**_4	60 (5 h)	5	4	35	2.68	0.06
60**T**_5**N**_5	60 (5 h)	5	5	35	4.05	0.06
60**T**_5**N**_6	60 (5 h)	5	6	35	7.24	0.05
60**T**_10**N**_3	60 (5 h)	10	3	25	5.31	0.82
60**T**_10**N**_4	60 (5 h)	10	4	25	10.54	0.56
60**T**_10**N**_5	60 (5 h)	10	5	25	14.95	0.79
60**T**_10**N**_6	60 (5 h)	10	6	25	18.84	0.42
120**T**_7**N**_3	120 (10 h)	7	3	40	4.32	0.20
120**T**_7**N**_4	120 (10 h)	7	4	40	7.70	0.29
120**T**_7**N**_5	120 (10 h)	7	5	40	11.04	0.24
120**T**_7**N**_6	120 (10 h)	7	6	40	14.64	0.22
120**T**_15**N**_3	120 (10 h)	15	3	30	10.65	1.20
120**T**_15**N**_4	120 (10 h)	15	4	30	15.63	1.11
120**T**_15**N**_5	120 (10 h)	15	5	30	17.45	2.16
120**T**_15**N**_6	120 (10 h)	15	6	30	23.61	1.32

Table 6 provides the results of the heuristic approach.

These problems had complex non-linear objective functions, so LINGO requires quite a bit of time to solve the problem or even to find good quality solutions. In contrast, the heuristic approach, using Xpress Mosel version 3.10.0, was much more efficient at finding good quality solutions. In most cases, the average computation times for the data sets were less than one second. The last four data sets required slightly more than 1 s to obtain lower and upper bounds. Because optimal solutions of the problems could not be obtained within the time limit by using LINGO, the average ratios 1–(LB/optimal objective value) and 1–(optimal objective value/UB) were not calculated.

The average ratio 1–(LB/UB) ranged between 2.07% and 23.61% in this experiment. For example, in the 60T_5N_3 data set, the solution that the lower bound indicated was no worse, on average, than 97.93% for the optimal solution. The average ratio 1–(LB/UB) could be worse (larger) by increasing N, T, and the number of advertisement sizes. In particular, the ratio was influenced most by the number of advertisement sizes. In this experiment, as the number of advertisement sizes was increased, the difference between the biggest and the smallest advertisement sizes for the set increased. When an upper bound of a problem was obtained, the value $w_{it}^{'}$ could be first calculated by comparing Eqs. (8) and (9) for each advertisement i and time slot t. If the difference between the biggest and the smallest advertisement sizes is big in some t, there is a high possibility that the value $w_{it}^{'}$ is not big. This finding means that the possibility of getting a tighter upper bound is low, indicating that more tight bounds and near-optimal solutions can be obtained by using the heuristic approach when the difference in the problem between the biggest and the smallest advertisement sizes is small. Most of the previous literature had only provided MAXSPACE models for which the results were not always able to support advertising effectiveness.

4.4. Managerial insights

The findings of the problem, the heuristic approach, and the analysis have managerial insights for online advertisement publishers. First, it is not always good to assign as many advertisements as possible in a slot. As the number of advertisements displayed in a time slot increases, the advertising effectiveness decreases due to competitive advertising interference. The decrease in the expected CTR caused by the competition might be similar to the value of 1-(LB/UB) in the numerical experiments. Publishers' revenues can be affected along with decreasing CTR value. The gap ranged between about 2% and 23% in the experiments using the standard data sets. The gap was more likely to be larger as the number of advertisements and time slots in a planning horizon increased. It would be more important to assign advertisements in terms of advertising effectiveness as the number of advertisements and time slots increases.

Second, it is important to consider not only the degree to involvement but also the degree to competition. Publishers often try to prioritize and assign the advertisements that have a high degree to involvement in a specific time slot. However, the results showed that they also need to consider the degree to competition between the advertisement in the slot. Thus, it is recommended that publishers choose and schedule advertisements in such a way as to retain the positive effect caused by the degree to involvement while avoiding the negative effect caused by the degree to competition.

Third, unlike in the MAXSPACE problem, the expected CTR values from two groups of advertisements assigned in a slot can be different even when the total lengths of the two groups are the same. Also, the value can be different depending on what slot the group is assigned to. Therefore, it is recommended that publishers analyze the solutions obtained by the heuristic approach, and then identify groups of advertisements that will maximize the degree to involvement and minimize the degree to competition. This approach will help publishers understand the logic of how to assign advertisements and maximize advertising effectiveness.

5. Conclusions

Most of the previous literature had only provided the MAXSPACE models, whose the results were not always able to support advertising effectiveness. For online advertisement publishers, it is important to display advertisements in such a way as to maximize advertising effectiveness, which is directly connected to revenue. The study described in this paper contributes to the literature on online banner advertisement scheduling models by considering critical factors influencing advertising effectiveness. In particular, the degree to competition has not yet been discussed extensively in online banner advertisement scheduling. The expected CTR function was devised to reflect the factors in the model and used as an objective function of the model in terms of measuring advertising effectiveness.

The model devised is an integer programming model with a nonlinear and non-convex objective function. In a large data set, whose optimal solution can be intractable to compute, the problem might not be solved directly using optimization solvers within a reasonable time. The heuristic approach presented provided good quality solutions efficiently, even for large data sets of non-convex and non-linear models through the properties of the objective function. Also, the problem was tested with standardized conditions for advertisements and time slots, and the heuristic approach performed efficiently and effectively in the standard data sets. The model and its heuristic approach are expected to be useful for online advertisement publishers when they choose and schedule advertisements over a planning horizon while considering advertising effectiveness.

Some challenging considerations are relevant for future research. The model proposed and described in this paper featured a variety of parameters related to the effectiveness of advertising. The value of the parameters should be accurately estimated and be dynamically reflected in the model over time. If the model takes additional factors of advertising effectiveness or publisher strategies, such as volume-based pricing discounts, into account, then publishers might find more realistic solutions. Publishers also should consider pricing to maximize their revenue (Ahmed & Kwon, 2012). Scheduling problems can be extended by combining a pricing problem to obtain integrated optimal solutions. Recently, some research has proposed online advertisement allocation based on online resource allocation problems (Legrain & Jaillet, 2016; Mehta, 2013). Requests for advertisement allocation arrive online, and online advertisement publishers immediately need to make decisions whether to display the advertisement or not. The decisions are irrevocable. Online banner advertisement scheduling problem where decisions are online and irrevocable can also be future research. In addition to proposing a realistic model, solution methodologies to obtain good solutions should be considered. Although the proposed heuristic approach described in this paper was efficient, it included binary integer programming, which is an NP-hard problem. Therefore, for future research, other methodologies, such as linearization techniques or meta-heuristics, should be considered to solve the non-linear model efficiently.

CRediT authorship contribution statement

Gwang Kim: Conceptualization, Methodology, Software, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. **Ilkyeong Moon:** Conceptualization, Validation, Writing - review & editing, Supervision.

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Appendix A. Generating an initial sequence of the hybrid tabu search

Phase 1. Calculate R_{it} for each advertisement displayed in each time slot:

$$R_{it} = \alpha_i (s_i)^{\beta_{it}}, \forall\ i \in \{1, 2, \cdots, N\},\ t \in \{1, 2, \cdots, T\}$$

R_{it} is defined as the expected CTR value without competition effects.

Phase 2. Generate an initial sequence S by the N advertisements and T slots in descending order according to R_{it}.

	$R_{i_1t_1} \geq$	$R_{i_2t_2} \geq$	$R_{i_3t_3} \geq$	$\cdots$	$\geq R_{i_{NT}t_{NT}}$
Sequence S	(i_1, t_1) 1st cell	(i_2, t_2) 2nd cell	(i_3, t_3) 3rd cell	$\cdots$ $\cdots$	(i_{NT}, t_{NT}) NT^{th} cell

Appendix B. Procedure of the hybrid tabu search

Algorithm 1 (*Hybrid tabu search***).**

```
Input: S (initial sequence), s_i (height of ad i, ∀ i), H (height of banner),
L_i (lower bound on display frequency of ad i if displayed, ∀ i)
N (number of ads), T (number of slots)
Initialize: CTR ← 0(current objective value); tabuSEQ ← ∅; num1 ← random(0, 1);
(*random(0, 1): an arbitrary constant between 0 and 1)
SEQ_sol ← S; SEQ ← S;
CTR ← Algorithm 2(SEQ);
while time limit is not reached do:
  k ← 1;
  while k < N × T do:
    SEQ_k ← sequence by switching the k^th and (k + 1)^th cells of SEQ;
    if SEQ_k ∈ tabuSEQ then:
      CTR_k ← 0; k ← k + 1;
      continue
    else
      CTR_k ← Algorithm 2(SEQ_k);
      k ← k + 1;
    end
  end
  k ← 1;
  CTR_temp ← 0; temp ← 0;
  while k < N × T do:
    if CTR_temp < CTR_k then:
    temp ← k;
    end
    k ← k + 1;
  end
  if CTR < CTR_temp then:
    CTR ← CTR_temp;
    SEQ_sol ← SEQ_temp;
  end
tabuSEQ ← tabuSEQ ∪ SEQ_temp;
SEQ ← SEQ_temp;
end
Output: SEQ_sol, CTR
```

Algorithm 2 (*Assigning advertisements to time slots using* SEQ).

Input: $SEQ(sequence)$
Initialize: $H_t \leftarrow H \ \forall\ t$; $x_{it} \leftarrow 0 \ \forall\ i, t$; $r_i \leftarrow random(0, 1) \ \forall\ i$
(*$random(0, 1)$: an arbitrary constant between 0 and 1)

\# Assigning advertisements to time slots
$p \leftarrow 1$;
while $p \leq N \times T$ **do:**
  $i(t) \leftarrow i_p(t_p)$ *in the* p^{th} *cell of* SEQ;
  if $r_i < num1$ **then:**
    $p \leftarrow p + 1$;
    continue
  end
  if $s_i \leq H_t$ **then:**
    $x_{it} \leftarrow 1$;
    $H_t \leftarrow H_t - s_i$;
    $p \leftarrow p + 1$;
  end
end

\# Checking the constraint and updating the objective value
$CTR = \sum_{i=1}^{N} \sum_{t=1}^{T} \pi_{it} x_{it}$;
forall i **do:**
  if $L_i \leq \sum_{t=1}^{T} x_{it} \leq T$ *or* $\sum_{t=1}^{T} x_{it} = 0$ **then:**
    continue;
  else
    $CTR = 0$;
  end
end
Output: CTR

Appendix C. A small example of the hybrid tabu search

For a small problem, we will show how a sequence can be translated into a solution. The height of a banner is 10 and the total number of slots is 3. $num1$ is set to 0. We assume to let the size, lower bound on display frequency, and values of R_{it} for each advertisement be as follows:

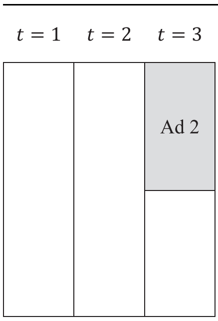
Adi	Size s_i	Lower bound L_i	r_i	R_{i1}	R_{i2}	R_{i3}
1	3	0	0.5	10.2	11.4	12.2
2	5	0	0.5	12.1	12.3	14.4
3	2	0	0.5	8.9	8.7	7.9
4	3	0	0.5	9.6	8.4	10.7
5	4	0	0.5	12.4	11.1	10.4

We generate an initial sequence (S) in descending order according to the values ofR_{it}:

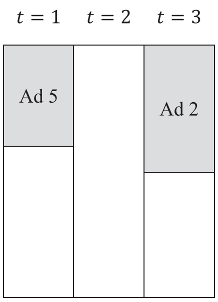
	$R_{23}\geq$	$R_{51}\geq$	$R_{22}\geq$	$R_{13}\geq$	$\cdots$	$R_{42}\geq$	R_{33}
Sequence S	(2, 3)	(5, 1)	(2, 2)	(1, 3)	$\cdots$	(4, 2)	(3, 3)

According to sequence S, we assign advertisements to time slots as follows:

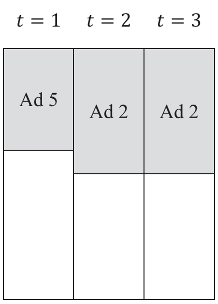
1. The first cell in sequence S is $(i, t) = (2, 3)$ and r_2 is not less than $num1$. Assign ad 2 in slot 3.



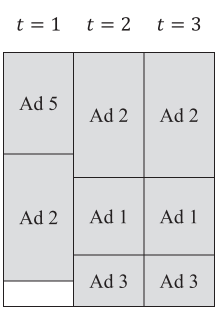
2. The second cell in sequence $S$is $(i, t) = (5, 1)$ and r_5 is not less than $num1$. Assign ad 5 in slot 1.



3. The third cell in sequence S is $(i, t) = (2, 2)$ and r_2 is not less than $num1$. Assign ad 2 in slot 2.



If there is no room to assign advertisement i in slot t or r_i is less than $num1$, we do not assign advertisement i in slot t. According to the sequence, we complete an assignment as follows:



If the assignment satisfies Constraint (2), the assignment can be a feasible solution. During iterations of the tabu search, we generate new sequences by switching two consecutive cells of the current sequence. We generation new assignments using new sequences until the tabu search is terminated.

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