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Robust empty container repositioning considering foldable containers

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ABSTRACT

Because of the extreme imbalance in intercontinental trade, the repositioning of empty containers creates a significant problem for shipping companies. There are many efforts to reduce the cost of repositioning empty containers, one of which is a foldable container. This paper proposes a robust formulation for the empty container repositioning problem considering foldable containers under demand uncertainty. The robust formulation can be used as a tractable approximation of a multistage stochastic programming formulation which is computationally intractable. Moreover, the robust formulation requires only limited information about the distribution of demand to replicate real-world situations. Computational results show that the proposed formulation performs well in terms of operating costs and there exists a significant cost-saving effect when foldable containers are used in maritime transportation.

1. Introduction

Since the 1970s, the volume of maritime transport has increased sharply because of an increase in worldwide trade. 17.1% of world seaborne trade is transported in the form of containers in 2017 (UNCTAD, 2018). Because of the reusable nature of containerization, the container is returned to the port in an empty state after being used for transporting goods. The time delay between container use and return creates problematic and inconsistent demand and supply for the container. In addition, the extreme imbalance in intercontinental container-shipping volume increases the demand-supply mismatch of empty containers, which contributes to a shortage of empty containers in export dominant ports and a surplus of empty containers in import dominant ports. To satisfy demand for empty containers, it is necessary to reposition empty containers from surplus ports to deficit ports. On average, 20% of total container movements by ocean transportation are empty. Repositioning of empty containers is non-value added transportation with enormous transportation costs and nonprofitable consumption of vessel capacity. Each time an empty container is repositioned, an opportunity cost is incurred for shipping one loaded container, which cuts the shipping company's profitability. Therefore, it is important for the shipping company to reposition containers efficiently and effectively. The empty container repositioning (ECR) problem is to determine an optimal repositioning schedule to satisfy demand for empty containers. The key decision is when and how many empty containers are repositioned to a destined port.

There are many efforts to reduce the cost of repositioning empty containers, one of which is a foldable container. Foldable containers can be transported in a package such that a few folded containers occupy the same volume as one standard container. Despite high purchase costs and the additional folding and unfolding costs of foldable containers, foldable containers have advantages in the reduction of repositioning costs and saving in terms of storage space and vessel capacity. There are two leading companies developing foldable containers: Holland Container Innovations (HCI) of the Netherlands and the Korea Railroad Research Institute. HCI presents a foldable container called 4FOLD and received ISO certification in 2013 (Holland Container Innovations, 2019b). 4FOLD has been used by 15 shipping companies involving APL, the world's third-largest shipping company, with 20% savings of total operation cost on average. HCI shows that folding and unfolding of 4FOLD can be done with standard lifting equipment by a two-person team less than four minutes, which makes the adoption of foldable containers affordable. One successful example of adoption of foldable containers is the Shanghai-Los Angeles-Chicago route served by APL (Holland Container Innovations, 2019a). The exports in Chicago are much less than imports, which incurs huge amount of empty repositioning. APL adopts 4FOLD containers in this route and folded empty container are packed in Chicago and transported to Shanghai via Los Angeles. APL saves approximately 20% of the total operating costs with the reduction coming from hinterland transport and handling costs at the terminal.

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The introduction of foldable containers affects the operational-level planning decisions at shipping companies, which makes the decision process very complex. The flow of loaded containers is a source of empty containers at the destined port, and the vessel capacity is shared by loaded and empty containers. It interconnects two different decisions: the transportation quantity to satisfy demand and the number of empty containers to reposition. When we only consider standard containers to satisfy the demand, the amount of loaded containers is equal to the demand. With the consideration of using foldable containers in operational decisions, the shipping company has to decide which type (standard or foldable) of containers and quantities of containers to satisfy the demand. The quantity and ratio of containers transported in one period affect the number of returned empty containers in the future period. Therefore, existing studies cannot be applied to the integrated model of standard and foldable containers.

Furthermore, in the competitive shipping industry, operational-level planning requires decision making under uncertainty. Although, the shipping company knows the exact demand in the long-term contracts and establishes plan based on demand information, decision makers face many short-term uncertainties, such as weather, port congestion, and demand fluctuation, during implementation. Moreover, uncertainties may prevent planned decisions from being implemented, which may lead to suboptimal decisions. Uncertainties can lead to serious operational failures in the shipping industry. Among the various uncertainties, the uncertainty of customer demand is the most influential; therefore, we consider the uncertainty of demand in this study.

A famous approach for dynamic decision making under uncertainty is multistage stochastic programming in which the uncertainty is characterized by a known probability distribution of parameters. Transportation and repositioning decisions in the ECR problem are considered wait-and-see decisions. Hence, the ECR problem can be modeled with a multistage stochastic programming framework. However, data estimation, such as demand forecasting with historical data, is difficult in practice. It is impossible to achieve complete knowledge about distributions of uncertainties. Moreover, in general, multistage stochastic program is computationally intractable (Shapiro & Nemirovski, 2005); therefore, we utilize a robust optimization framework for which only limited information is required. Adjustable robust optimization, proposed by Ben-Tal, Goryashko, Guslitzer, and Nemirovski (2004), enables dynamic decision making under uncertainty in a robust optimization framework.

In this study, we consider the ECR problem with the adoption of foldable containers. The shipping company decides the type and quantity of empty containers in terms of two different decisions, transporting and repositioning. We focus on the operational planning of ocean transportation between multiple ports over the planning horizon. We also consider demand uncertainty, which leads to the multistage stochastic programming formulation. To tackle the intractability of the multistage formulation, we adopt the concept of adjustable robust optimization. Then, we show that the robust formulation gives a tractable approximation of the stochastic programming formulation.

In summary, contributions of this paper are threefold. First, we propose a mathematical model of the ECR problem considering the use of foldable containers under uncertainty. To the best of our knowledge, it is the first paper to address this topic. Second, we propose the tractable robust formulations with limited information on demand distribution, because it is very difficult to estimate demand distribution precisely with historical data. The robust formulations are used to approximate the multistage stochastic programming formulation. Third, we show the cost-saving and storage-saving effects of using foldable containers through the practical-scaled numerical experiments.

The remainder of this paper is organized as follows. In Section 2, we review literature related to the ECR problem and studies that addressed use of foldable containers. Then, we briefly review the literature related to adjustable robust optimization. In Section 3, we describe the problem definition and assumptions of the problem. In Section 4, we propose model formulations of the ECR problem with foldable containers under demand uncertainty. Then, we explain the main results of this paper, the adjustable robust counterparts using a linear decision rule (LDR) and a restricted linear decision rule (RLDR). In Section 5, we present the results of computational experiments and compare the performance of the proposed model against a predetermined benchmark. In Section 6, we offer a brief discussion and suggest areas for further research.

2. Literature review

The empty container repositioning (ECR) problem has attracted considerable attention in academia. Many researchers have considered various situations and proposed solution methodologies. Wang and Meng (2017) provided a recent review of container liner fleet deployment, and Lee and Song (2017) conducted an extensive review of ocean container transport. Song and Dong (2015) discussed the main causes of empty container repositioning and solutions to the ECR problems. Kuzmicz and Pesch (2019) addressed various aspects and solutions of ECR problems in the context of Eurasian transportation.

Several researchers have considered decision making under uncertainty in ECR problems through stochastic programming with recourse, inventory control-based policies, and robust optimization. Crainic, Gendreau, and Dejax (1993) proposed dynamic deterministic formulations for the empty container allocation problem and extended it to a two-stage stochastic programming formulation under the uncertainties of demand and supply. Cheung and Chen (1998) proposed a two-stage stochastic network formulation of the ECR problem under uncertainties of demand, supply, and capacity. They utilized the stochastic quasi-gradient method and an approximation procedure to obtain solutions. Song (2007) provided an optimal policy for empty container repositioning with uncertain demand that is similar to the optimal policy for inventory control. The structures of the optimal policy were characterized using the Markov decision process. Li, Liu, Leung, and Lai (2004) derived the optimal threshold-type policies called (U, D) policy in a single port case with demand uncertainty and they extended to multi-port case (Li, Leung, Wu, & Liu, 2007). Using the convexity of the cost function, they proposed a heuristic to obtain policies. Lam, Lee, and Tang (2007) considered a dynamic stochastic model of the container allocation problem and proposed an approximate dynamic programming approach. Di Francesco, Crainic, and Zuddas (2009) proposed a stochastic programming model with uncertain data for empty container repositioning and solved the model using multi-scenario optimization. Erera, Morales, and Savelsbergh (2009) modeled the empty repositioning problem using a robust optimization framework. They considered interval uncertainty of forecast values and proposed the concept of a recoverable plan similar to the concept of an adjustable robust counterpart. They showed that the problem modeled using the recoverable plan is polynomially solvable. Long, Lee, and Chew (2012) proposed a two-stage stochastic programming model for empty container repositioning and solved the program with the Sample Average Approximation (SAA). They utilized the scenario aggregation to handle an extremely large number of scenarios. Shu and Song (2014) proposed a two-stage robust optimization model considering both loaded and empty containers. They discussed the complexities of the formulations based on an l_p-norm uncertainty set. von Westarp and Schinas (2016) utilized a fuzzy optimization approach for

Table 1
Comparisons of recent papers related to this research.

Authors (year)	Foldable container	Demand uncertainty	Limited information of distribution	Solution methodology
Konings and Thijs (2001)	✓			-
Song (2007)		✓		Markov decision process based optimal policy
Li et al. (2007)		✓		threshold-type control policy
Lam et al. (2007)		✓		approximate dynamic programming
Di Francesco et al. (2009)		✓		multi-scenario based stochastic programming
Erera et al. (2009)		✓	✓	robust optimization, adjustable robust optimization
Shintani et al. (2010)	✓			mixed integer linear programming
Long et al. (2012)		✓		two-stage stochastic programming, SAA
Moon et al. (2013)	✓			heuristic algorithm
Myung and Moon (2014)	✓			network flow model
Shu and Song (2014)		✓	✓	robust optimization, adjustable robust optimization
Moon and Hong (2016)	✓			LP-based GA and hybrid GA
von Westarp and Schinas (2016)		✓		fuzzy optimization
Myung (2017)	✓			network flow model
Wang and Meng (2017)	✓			network flow model, revised network simplex algorithm
This paper	✓	✓	✓	adjustable robust optimization

the container positioning problem. They considered profit maximization instead of relevant cost minimization and figured out the trade-off between loaded and empty containers. Xie, Liang, Ma, and Yan (2017) captured the coordination issue in empty container repositioning and proposed the inventory sharing game between decision makers at maritime logistics. They studied the equilibrium delivery quantity of empty containers under the decentralized model. Except Erera et al. (2009) and Shu and Song (2014), most of the previous studies assumed the full distribtuional knowledge of uncertainties which is limited in practice. Erera et al. (2009) and Shu and Song (2014) utilized the robust optimization framework to tackle this difficulty, however, their models were limited to two-stage decisions.

The foldable container is a newly commercialized technology and the studies of ECR with the use of foldable containers is recently emerging. Konings and Thijs (2001) analyzed the economic effects of introducing foldable containers into ocean transport systems, and they discussed the technical and logistical conditions for the successful use of foldable containers. Konings (2005) discussed an economical analysis on the adoption of foldable containers considering relevant costs. Shintani, Konings, and Imai (2010) discussed the economic impact of using foldable containers in hinterland repositioning of empty containers. They analyzed several strategies of hinterland transportation in which foldable containers were used. Myung (2017) extended the results of Shintani et al. (2010) by offering efficient solution methods and obtained analytical solutions using a network formulation. Moon, Do Ngoc, and Konings (2013) proposed mathematical models considering foldable and standard containers in maritime transport. They developed heuristic algorithms to solve the proposed models. Then, Myung and Moon (2014) proved that the Moon et al. (2013) model can be reduced to a network flow model that can be solved in polynomial time. Moon and Hong (2016) developed a mathematical model with standard and foldable containers and proposed a linear programming-based genetic algorithm and a hybrid genetic algorithm to solve the model and they obtained near-optimal solutions. Wang, Wang, Zhen, and Qu (2017) considered the ship type decision problem with the use of foldable containers in empty container repositioning. They proposed a network flow model and addressed an exact algorithm based on a revised network simplex algorithm. To the best of our knowledge, there were no studies considering uncertainty into the model of foldable containers.

In this research, we utilize the concept of adjustable robust optimization to obtain a tractable approximation for a multistage stochastic programming formulation with demand uncertainty. Ben-Tal et al. (2004) proposed the concept of adjustable robust counterparts for which decisions can be adjusted dynamically as uncertainty is realized over time. However, they showed that the adjustable robust counterpart is NP-hard, so they proposed the concept of an affinely adjustable robust counterpart (AARC) where adjustable decisions are restricted to affine functions of uncertainty. Then, Ben-Tal, Golany, Nemirovski, and Vial (2005) applied AARC to a supply chain problem named the retailer-supplier flexible commitment problem. Chen and Sim (2009) proposed a tractable deterministic approximation for the goal-driven stochastic optimization model using a piecewise linear decision rule. For this approximation, they developed upper bounds for the expectation of positive parts, which are shown in the objective function of the model. See and Sim (2010) proposed the use of Chen and Sim (2009)' s upper bounds to deal with a multiperiod inventory-management problem. They developed a piecewise linear decision rule named truncated linear decision rule, which extends the result of the linear decision rule.

In this paper, we propose a tractable approximation to the multistage stochastic formulation of the ECR problem considering use of foldable containers. We assume that demand is uncertain and that we have limited information on the distribution of uncertainty such as mean and covariance. We utilize the adjustable robust optimization approach and a linear decision rule for tractability. The summarized characteristics of our research are presented at Table 1.

3. Problem description

In this section, we describe a cycle of container flows and problem definition. Then, we explain the assumptions of the ECR problem we deal with. To understand the ECR problem, container flow must be understood. A consignor sends cargo to the consignee by ocean transport, which is referred to as *demand* in the ECR literature. To meet *demand*, the shipping company sends empty containers to the consignor. The consignor fills the empty containers with cargo and sends these loaded containers to the port. The shipping company transports the containers via an ocean-transport vessel to the destination port where the consignee receives them. The consignee takes the newly arrived cargo out of the containers and sends the emptied containers back to the depot of the shipping company. The empty containers, upon return to the port, are referenced as the *supply* in the ECR problem.

Container flows of this problem are shown in Fig. 1.

1. At the beginning of period t, the customer demand from port i to port j occurs. We aggregate the consignors located near port

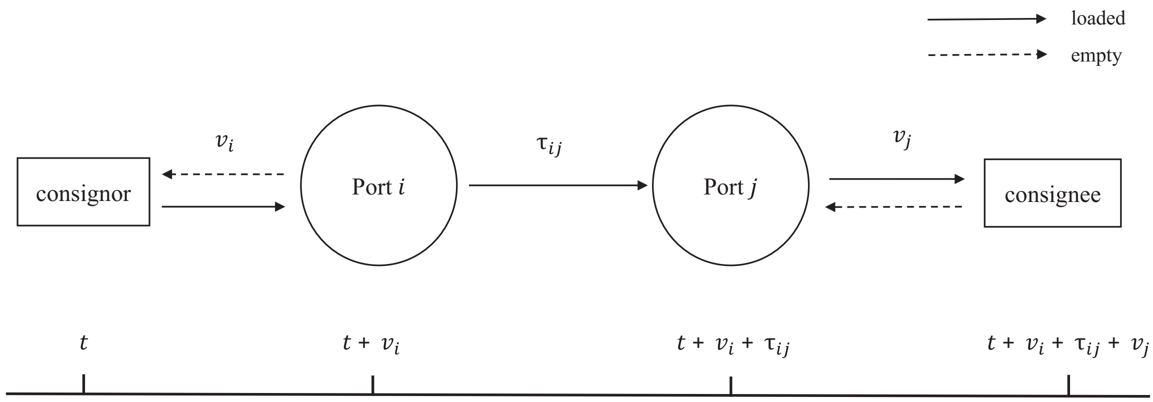

Fig. 1. Cycle of container flows.

i and denote them as demand occurred at port i. We aggregate consignees similarly.

2. The shipping company sends empty containers from the depot at port i to the consignor's site to satisfy the demand. The empty containers are packed at the consignor's site and returned to port i. It takes v_i periods until empty containers are sent to the consignor's site and the loaded containers are returned to port i.
3. The shipping company transports loaded containers to port j via the vessel with given transportation time τ_{ij}.
4. Cargo is delivered to the consignee and emptied at the consigneeâs site. Then, the emptied containers are returned to the depot at port j. It takes v_j periods until the loaded containers are transported to the consignee's site and the emptied containers are returned to the port.

Because of the container flow and reusable property of containers, supplied empty containers are stacked at the import dominant port. To meet the demand at the export dominant port, the shipping company has to reposition empty containers. Empty repositioning consumes the vessel capacity which would be used for loaded containers. Therefore, effective and efficient repositioning plan is crucial to the profit of the shipping company.

In our ECR problem, the shipping company considers both transporting loaded containers and repositioning of empty containers over a planning horizon. Because empty repositioning incurs costs and consumes vessel capacity, foldable containers are used to reduce repositioning costs. Therefore, the shipping company decides the type and quantity of containers to satisfy the demand. According to the transportation decision, the shipping company also decides the type and quantity of repositioned containers to mitigate the trade imbalance. The objective of this problem is to minimize the total operating cost over the planning horizon which consists of transportation, repositioning, holding, penalty, folding, and unfolding costs. One of the challenging decisions is when and how many foldable containers are used, which makes the problem complex.

The assumptions of this problem are as follows:

- The ECR is undertaken at multiple ports on a finite discrete horizon of T periods. The vessel route is not considered, and containers can be repositioned to any port during any period according to given vessel capacity.
- The vessel capacity K_{ijt} from port i to port j in period t, which is shared by loaded and empty containers, is given. The vessel capacity can represent the vessel schedule predetermined by the shipping company.
- The transportation lead time from port i to port j and the inland transportation time (devanning time) at port i are given.
- Demand can be satisfied by both standard and foldable containers. When foldable containers stored at ports are used to satisfy demand, the unfolding operation must be done before transported to the consignor's site.
- Supplied empty containers and repositioned empty containers are two sources of empty containers.
- Unsatisfied demand is satisfied with a short-term lease and incurs a penalty cost.
- Folding and unfolding of foldable containers can be executed only in ports.
- Foldable containers are repositioned in a folded state to occupy less of the vessel capacity than standard containers do.
- Supplied foldable containers from customers are delivered in the unfolded state and used to satisfy demand. Excess foldable containers are stored at a port after being folded.
- The returned container after being emptied is the only source of container supply.

The vessel capacity represents the number of loaded and empty containers in twenty-foot equivalent unit (TEU) that can be transported from port i to port j in period t. The capacity is given by the shipping company in advance according to the fleet schedule. For example, if $K_{ijt} = 0$, no vessel is available vessel in period t to transport containers from port i to port j. Therefore, the vessel capacity can characterize the vessel schedule determined by the shipping company, which leads to the generalization of the first assumption. We assume the containers can be transported to any port during any period, but we can consider the vessel schedule with the vessel capacity.

With the introduction of foldable containers, additional facility and manpower are needed. Because customers may have a negative reaction to additional investments at their sites, folding and unfolding operations are limited to ports. Therefore, customers receive empty foldable containers in an unfolded state, which makes the customers indifferent about the choice of using standard or foldable containers. Although the customer may be concerned about the strength of the foldable container, for this study, the strength of the foldable container is assumed to be the same as a standard one. Hence, the demand can be satisfied by either standard or foldable containers.

According to the assumptions, we propose model formulations of the ECR problem considered. In the next section, we present a deterministic and a multistage stochastic programming formulation. Then, we introduce a robust formulation using a linear decision rule and show that the robust formulation is a tractable approximation of the multistage stochastic formulation of the ECR problem.

4. Model formulation

In this section, we propose a deterministic linear programming formulation with deterministic demand. Then, we regard demand as a random variable that represents demand uncertainty. We propose a multistage stochastic programming formulation with non-anticipativity constraints. However, in general, a multistage stochastic programming formulation is computationally intractable; therefore, we utilize the concept of adjustable robust optimization for tractable approximation.

The notations for the parameters are as follows:

- P ports
- T periods
- C^S_{ij} unit transportation cost of a standard container from port i to port j
- C^F_{ij} unit transportation cost of a foldable container from port i to port j
- R^S_{ij} unit repositioning cost of a standard container from port i to port j
- R^F_{ij} unit repositioning cost of a foldable container from port i to port j
- H^S_i holding cost of a standard container per unit per period at port i
- H^F_i holding cost of a foldable container per unit per period at port i
- P^S_i penalty cost of a standard container per unit per period at port i
- P^F_i penalty cost of a foldable container per unit per period at port i
- K_{ijt} vessel capacity (TEUs) from port i to port j in period t
- N number of foldable containers used to build one folded pack
- FC_i unit folding cost of a foldable container at port i
- UC_i unit unfolding cost of a foldable container at port i
- τ_{ij} transportation time from port i to port j
- ν_i inland transportation time (or devanning time) at port i
- D_{ijt} demand for transporting containers from port i to port j in period t

Decision variables used in this model are as follows:

- r^S_{ijt} repositioning quantity of standard containers from port i to port j in period t
- r^F_{ijt} repositioning quantity of foldable containers from port i to port j in period t
- x^S_{ijt} number of standard containers used to satisfy demand from port i to port j in period t
- x^F_{ijt} number of foldable containers used to satisfy demand from port i to port j in period t
- z^S_{it} inventory level of standard containers at port i at the end of period t
- z^F_{it} inventory level of foldable containers at port i at the end of period t

The total operating cost consists of consists of transportation, repositioning, holding, penalty, folding, and unfolding costs. The transportation costs consist of expenses incurred during container flow. In other words, costs which are directly involved to the delivery between consignors and consignees: empty container movement to the consignors, ocean transportation by the vessel, and delivery and return from consignees. Recent information technology enables the shipping company to valuate the exact transportation cost per unit delivery. The repositioning cost is the handling cost of transporting empty containers to mitigate the trade imbalance. The holding cost is incurred when empty containers are stored at the depot in ports. The penalty cost is related to the short-term leasing cost incurred to meet unsatisfied demand. Folding and unfolding costs are incurred when folding and unfolding operations are executed at ports.

Fig. 2 represents inflows and outflows at port i in period t in a time-space expanded network of the ECR model. According to the container flow and repositioning operation, two different sources of empty containers, $x_{ji,t-\nu_j-\tau_{ji}-\nu_i}$ and $r_{ji,t-\tau_{ji}}$, are transported from port j.

Using the above notation, we explain the balance equation of port i in period t. A balance equation is presented in Fig. 3. Three types of inflows were considered: repositioning quantities from other ports, number of supplied containers after use, and inventory from the last period. It takes τ_{ji} periods to reposition empty containers from port j to port i. It takes $\nu_j+\tau_{ji}+\nu_i$ periods to finish one cycle of container flows such that supplied containers are returned after one cycle. Three types of outflows were considered: repositioning quantities to other ports, number of empty containers used to satisfy customer demand, and inventory amount. Fig. 3 presents the balance equation of standard containers, which is the same for foldable containers.

4.1. Deterministic formulation

First, we consider the deterministic demand. The model we developed is similar to that of Moon et al. (2013) and Tsang and Mak (2015). An explanation of this deterministic formulation helps in understanding the stochastic model. Let RC_{DET} denote total repositioning and transportation costs and HC_{DET} denote total holding and penalty costs and FC_{DET} denote total folding and unfolding costs.

$$\text{RC}_{\text{DET}} = \sum_{t=1}^{T}\sum_{i\in P}\sum_{j\in P}\left(R^S_{ij}r^S_{ijt} + C^S_{ij}x^S_{ijt} + R^F_{ij}r^F_{ijt} + C^F_{ij}x^F_{ijt}\right)$$

$$\text{HC}_{\text{DET}} = \sum_{t=1}^{T}\sum_{i\in P}\left(H^S_i(z^S_{it})^+ + H^F_i(z^F_{it})^+ + P^S_i(z^S_{it})^- + P^F_i(z^F_{it})^-\right)$$

$$\text{FC}_{\text{DET}} = \sum_{t=1}^{T}\sum_{i\in P}\left(FC_i\left(\sum_{j\in P}(x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}} - x^F_{ijt})\right)^+ + UC_i\left(\sum_{j\in P}(x^F_{ijt} - x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}})\right)^+\right)$$

The deterministic formulation is as follows:

$$\text{TC}_{\text{DET}} = \tag{1}$$

$$\min \quad \text{RC}_{\text{DET}} + \text{HC}_{\text{DET}} + \text{FC}_{\text{DET}} \tag{2}$$

$$\text{s.t.} \quad z^S_{it} = z^S_{i,t-1} + \sum_{j\in P} r^S_{ji,t-\tau_{ji}} - \sum_{j\in P} r^S_{ijt} + \sum_{j\in P} x^S_{ji,t-\nu_i-\nu_j-\tau_{ji}} - \sum_{j\in P} x^S_{ijt}, \quad \forall i\in P, t=1,\dots,T \tag{3}$$

$$z^F_{it} = z^F_{i,t-1} + \sum_{j\in P} r^F_{ji,t-\tau_{ji}} - \sum_{j\in P} r^F_{ijt} + \sum_{j\in P} x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}} - \sum_{j\in P} x^F_{ijt}, \quad \forall i\in P, t=1,\dots,T \tag{4}$$

$$x^S_{ijt} + x^F_{ijt} = D_{ijt}, \qquad \forall i,j\in P, t=1,\dots,T \tag{5}$$

$$r^S_{ijt} + \frac{1}{N}r^F_{ijt} + x^S_{ij,t-\nu_i} + x^F_{ij,t-\nu_i} \le K_{ijt}, \quad \forall i,j\in P, t=1,\dots,T \tag{6}$$

$$r^S_{ijt}, r^F_{ijt} \ge 0, x^S_{ijt}, x^F_{ijt} \ge 0, \qquad \forall i,j\in P, t=1,\dots,T \tag{7}$$

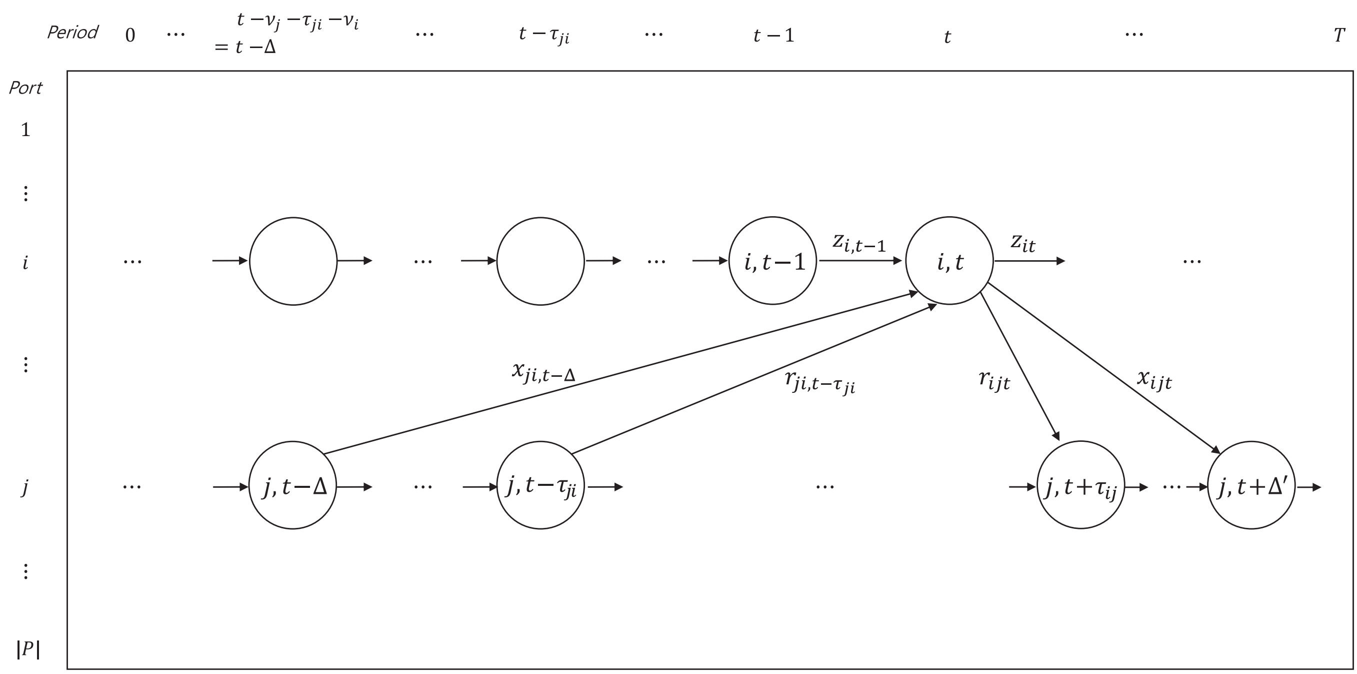

Fig. 2. Inflows and ouflows at port i in period t in a time-space expanded network of the ECR model.

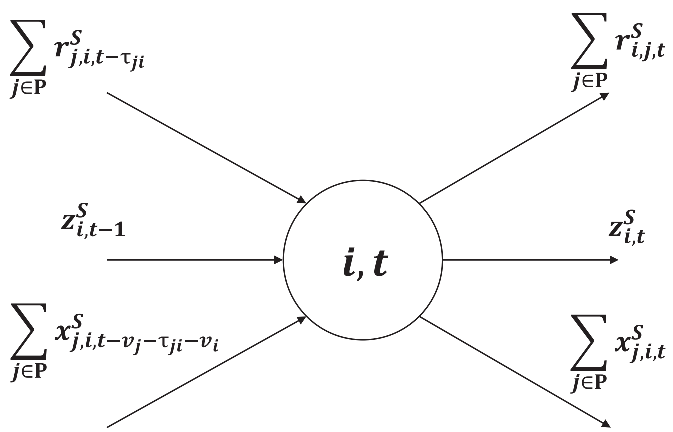

Fig. 3. Balance equation of standard containers.

The objective function represents total operating cost, including repositioning, transportation, inventory holding, penalty, and folding/unfolding costs. Constraints (3) and (4) represent balance equations for standard and foldable containers, respectively. Constraint (5) represents that the demand is satisfied with standard and foldable containers from the depot at the port. Constraint (6) is a capacity constraint for the vessel. It shows that when repositioning foldable containers, $1/N$ unit of capacity is used. Empty foldable containers consume less capacity, which leads to more available capacity for loaded containers that is a value-added activity for the shipping company. Constraint (7) is a non-negativity constraint.

Although repositioning and transportation decisions are based on the number of containers, the above formulation is a linear program. Because hundreds or thousands of containers are usually used, solutions that are rounded up are very close to the optimal solution. Moreover, the formulation does not have any binary variables. In many cases, rounding binary variables makes the optimal solution of a linear program highly suboptimal. Fortunately, the ECR formulation does not contain any binary variables and the quantity of containers is over hundreds, which makes the linear program formulation of the ECR problem reasonable.

4.2. Multistage stochastic programming formulation

We regard demand as a random variable to incorporate uncertainties into the model. Multistage stochastic programming formulation can be proposed with random demand. We denote stochastic demand as $\tilde{d}_{ijt}$ and assume that stochastic demand, $\tilde{d}_{ijt}$, is realized dynamically over the planning horizon. At the beginning of period, $\tilde{d}_{ijt}$ is realized, and then the shipping company makes transportation and repositioning decisions based on the demand realization and past information. Let RC_{STOC} denote total expected repositioning and transportation costs and HC_{STOC} denote total expected holding and penalty costs and FC_{STOC} denote total expected folding and unfolding costs.

$$\text{RC}_{\text{STOC}} = \mathbb{E}\Bigg[\sum_{t=1}^{T}\sum_{i\in P}\sum_{j\in P}\Big(R^S_{ij}r^S_{ijt}(\omega) + C^S_{ij}x^S_{ijt}(\omega) + R^F_{ij}r^F_{ijt}(\omega) + C^F_{ij}x^F_{ijt}(\omega)\Big)\Bigg]$$

$$\text{HC}_{\text{STOC}} = \mathbb{E}\Bigg[\sum_{t=1}^{T}\sum_{i\in P}\Big(H^S_i(z^S_{it}(\omega))^+ + H^F_i(z^F_{it}(\omega))^+ + P^S_i(z^S_{it}(\omega))^- + P^F_i(z^F_{it}(\omega))^-\Big)\Bigg]$$

$$\text{FC}_{\text{STOC}} = \mathbb{E}\Bigg[\sum_{t=1}^{T}\sum_{i\in P}\Bigg(FC_i\Big(\sum_{j\in P}(x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}(\omega) - x^F_{ijt}(\omega))\Big)^+ + UC_i\Bigg(\sum_{j\in P}(x^F_{ijt}(\omega) - x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}(\omega))\Bigg)^+\Bigg)\Bigg]$$

The multistage stochastic programming formulation is as follows:

$$\text{TC}_{\text{STOC}} = \tag{8}$$

$$\min \quad \text{RC}_{\text{STOC}} + \text{HC}_{\text{STOC}} + \text{FC}_{\text{STOC}} \tag{9}$$

$$\text{s.t.}\quad z^S_{it}(\omega) = z^S_{i,t-1}(\omega) + \sum_{j\in P} r^S_{ji,t-\tau_{ji}}(\omega) - \sum_{j\in P} r^S_{ijt}(\omega) + \sum_{j\in P} x^S_{ji,t-\nu_i-\nu_j-\tau_{ji}}(\omega) - \sum_{j\in P} x^S_{ijt}(\omega), \qquad \forall i\in P, t=1,\ldots,T \tag{10}$$

$$z^F_{it}(\omega) = z^F_{i,t-1}(\omega) + \sum_{j\in P} r^F_{ji,t-\tau_{ji}}(\omega) - \sum_{j\in P} r^F_{ijt}(\omega) + \sum_{j\in P} x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}(\omega) - \sum_{j\in P} x^F_{ijt}(\omega), \qquad \forall i\in P, t=1,\ldots,T \tag{11}$$

$$x^S_{ijt}(\omega) + x^F_{ijt}(\omega) = D_{ijt}(\omega), \qquad \forall i,j\in P, t=1,\ldots,T \tag{12}$$

$$r^S_{ijt}(\omega) + \frac{1}{N} r^F_{ijt}(\omega) + x^S_{ij,t-\nu_i}(\omega) + x^F_{ij,t-\nu_i}(\omega) \le K_{ijt}, \qquad \forall i,j\in P, t=1,\ldots,T \tag{13}$$

$$r^S_{ijt}(\omega) = r^S_{ijt}(\xi), \quad r^F_{ijt}(\omega) = r^F_{ijt}(\xi), \qquad \forall \xi\in\Omega^t(\omega), t=1,\ldots,T, \omega\in\Omega \tag{14}$$

$$x^S_{ijt}(\omega) = x^S_{ijt}(\xi), \quad x^F_{ijt}(\omega) = x^F_{ijt}(\xi), \qquad \forall \xi\in\Omega^t(\omega), t=1,\ldots,T, \omega\in\Omega \tag{15}$$

$$r^S_{ijt}(\omega), r^F_{ijt}(\omega) \ge 0, x^S_{ijt}(\omega), x^F_{ijt}(\omega) \ge 0, \quad \forall i,j\in P, t=1,\ldots,T \tag{16}$$

$\omega\in\Omega$ represents a possible realization or scenario of random demand over T periods where Ω represents a set of all possible realizations or scenarios. The objective function contains expectations over all possible realizations, which reflects the risk-neutral decision making of the shipping company. Decision variables depend on the possible realization of demand. Constraints (14) and (15) represent non-anticipativity constraints where $\Omega^t(\omega)$ denotes a set of scenarios whose demand is the same as ω until t period. A non-anticipativity constraint means that the decisions only depend on the past realizations and do not depend on the future. Therefore, if any two different demand scenarios have the same demand history until period t, the decision on the subsequent period must be the same for both demand scenarios. For example, suppose that ω_1 and ω_2 have the same demand history until period t, i.e., $d_{ijk}(\omega_1) = d_{ijk}(\omega_2)$ for $k=1,\ldots,t$. Then, the decisions on period t should be the same, that is, $x_{ijt}(\omega_1) = x_{ijt}(\omega_2)$ and $r_{ijt}(\omega_1) = r_{ijt}(\omega_2)$ for both standard and foldable cases.

The presented formulation accounts for decision making under demand uncertainty; however, in general the optimal solution is difficult to obtain (Shapiro & Nemirovski, 2005) because the evaluation of $\mathbb{E}[(\cdot)^+]$ in the multistage setting is extremely difficult. Moreover, the formulation requires complete knowledge of the distribution of demand uncertainty, which is needed for the evaluation of expectation in the objective function. In practice, it is difficult to estimate the distribution precisely from historical data. Therefore, in many practical circumstances, the first and second moments are estimated based on past data to reach the best possible forecast of future demand. With estimations of the first and second moments, we need a tractable and distributionally robust approach to handle the expectation of positive parts $\mathbb{E}[(\cdot)^+]$. To incorporate those practical conditions, we adopt an adjustable robust optimization technique similar to that of Ben-Tal et al. (2005); it requires only limited information on distributions and is computationally tractable when using a linear decision rule.

4.3. Adjustable robust counterpart

In this research, we consider multistage decision making under uncertainty, which means that decisions are made after observing past data realization. Decisions can represent wait-and-see decisions which depend on a portion of uncertain data. This adjustability can be represented by adjustable robust counterpart which was proposed by Ben-Tal et al. (2004). Therefore, we adopt the adjustable robust optimization technique and the concept of a linear decision rule. For this direction, we need to make two preparations before proposing an adjustable robust counterpart. First, we will introduce the factor-based demand model which represents affine parameterizations of uncertainty based on uncertain factors. Second, we adopt upper bounds to the expectations of the positive parts which appear at the objective function of the multistage formulation. The following contents are extended from the results of See and Sim (2010).

4.3.1. Factor-based demand model

For utilizing the concept of the adjustable robust counterpart, we utilize a factor-based demand model similar to the model of See and Sim (2010). A factor-based demand model represents the uncertain demand which is affinely dependent on uncertain factors. We need a specific assumption of uncertain factors for tractability.

Assumption 1. The uncertain factors $\tilde{z} = \left\{\tilde{z}_{ijt}\right\}_{(i,j\in P,t=1,\ldots,T)}$ are zero mean random variables with a covariance matrix Σ. Uncertain factors $\tilde{z}$ are distributed in the conic quadratic representable support set, $\mathcal{W}$.

The support set $\mathcal{W}$ is conic quadratic representable if $\mathcal{W}$ is represented by a quadratic cone or a second-order cone, e.g., $\mathcal{W} = \left\{z\in\mathbb{R}^n \mid z_1 \ge \sqrt{z_2^2+\cdots+z_n^2}\right\}$. $\mathcal{W}$ would be intervals, polyhedrons, or ellipsoids. This assumption is essential for the tractability of the formulation. Without this assumption, the robust counterpart over $\mathcal{W}$ would be intractable.

Under Assumption 1, we can express the factor-based demand as follows:

$$d_{ijt}(\tilde{z}) = d^0_{ijt} + \sum_{i'\in P}\sum_{j'\in P}\sum_{k=1}^{T} d^k_{i'j'ijt}\tilde{z}_{i'j'k} \qquad \forall i,j\in P, t=1,\ldots,T \tag{17}$$

$$d^k_{i'j'ijt} = 0 \qquad \forall i,j\in P, t=1,\ldots,T, k\ge t+1 \tag{18}$$

For example, consider demand for two ports and two periods. Then, $d_{121}(\tilde{z}) = d^0_{1,2,1} + d^1_{1,2,1,2,1}\tilde{z}_{1,2,1} + d^1_{2,1,1,2,1}\tilde{z}_{2,1,1}$ and $d_{122}(\tilde{z}) = d^0_{1,2,2} + d^1_{1,2,1,2,2}\tilde{z}_{1,2,1} + d^2_{1,2,1,2,2}\tilde{z}_{1,2,2} + d^1_{2,1,1,2,2}\tilde{z}_{2,1,1} + d^2_{2,1,1,2,2}\tilde{z}_{2,1,2}$ which are affine functions of $\tilde{z}$. $d_{211}(\tilde{z})$ and $d_{212}(\tilde{z})$ can be represented similarly.

Eq. (17) shows that the uncertain demand is affinely dependent on uncertain factors $\tilde{z}_{ijt}$. As uncertain factors are realized dynamically, Eq. (18) shows that the uncertain demand is depend only on the realized uncertain factors. See and Sim (2010) showed that many demand models, such as the independently distributed demand, ARMA(p, q) demand process, and any other demand models characterized by random factors, can be expressed as a factor-based model.

4.3.2. Bound on expectations of positive parts

One of the most difficult things in the multistage stochastic programming formulation is the evaluation of the expectation in the objective function. It requires the complete knowledge of distribution, which is restricted in practice. Even if the distribution is known precisely, the evaluation of the expectation is computationally intractable in multistage case. Therefore, we use upper bounds similar to those of Scarf (1958) with assumption of limited information about the distribution of the uncertain factors. The upper bounds provide tight bounds on expectations of positive parts $\mathbb{E}[(\cdot)^+]$ with distributionally robust properties. Expectations of positive parts $\mathbb{E}[(\cdot)^+]$ appear in the objective function of the multistage stochastic programming formulation, for example, holding and penalty costs $\mathbb{E}[H_i^S(z_{it}^S)^+ + P_i^S(z_{it}^S)^-]$. Hence, we need the bound on $\mathbb{E}[(\cdot)^+]$ with distributionally robust and tractable properties. Chen and Sim (2009) proposed the bounds in the form of affine functions of random factors. Therefore, we adopt the results of Chen and Sim (2009).

Theorem 1 (Chen & Sim, 2009)**.** *Under Assumption 1 on uncertain factors, the following functions, $\Pi^i(y_o, \boldsymbol{y})$, $i \in \{1, 2, 3\}$ are the upper bounds of $\mathbb{E}\big[(y_o + \boldsymbol{y}'\tilde{\boldsymbol{z}})^+\big]$ where $x^+ = \max\{x, 0\}$:*

1. $\Pi^1(y_0, \boldsymbol{y}) := \big(y_0 + \max_{\tilde{\boldsymbol{z}} \in \mathcal{W}} \tilde{\boldsymbol{z}}'\boldsymbol{y}\big)^+$
2. $\Pi^2(y_0, \boldsymbol{y}) := y_0 + \big(-y_0 + \max_{\tilde{\boldsymbol{z}} \in \mathcal{W}} \tilde{\boldsymbol{z}}'(-\boldsymbol{y})\big)^+$
3. $\Pi^3(y_0, \boldsymbol{y}) := \frac{1}{2}y_0 + \frac{1}{2}\sqrt{y_0^2 + \boldsymbol{y}'\boldsymbol{\Sigma}\boldsymbol{y}}$

Proof. We refer the reader to Chen and Sim (2009) for the proof. □

Remark 1. Chen and Sim (2009) The first bound in Theorem 1 is derived from the positive part of support of the uncertain factors. The second bound is derived from the negative part of support of the uncertain factors. The third bound is derived from the covariance of the uncertain factors.

Remark 2. Chen and Sim (2009) proposed five upper bounds of expectation of positive parts. However, the last two bounds require the estimation of forward and backward deviations defined by Chen, Sim, and Sun (2007), which may reflect a new concept for practitioners. Therefore, for the simplicity and applicability in the shipping industry, we omit the last two bounds.

Theorem 1 shows that the three bounds are upper bounds on the expectations of positive parts, respectively. Then, Chen and Sim (2009) integrated these bounds for better bound.

Theorem 2 (Chen & Sim, 2009)**.** *Let*

$$\Pi(y_0, \boldsymbol{y}) := \min_{y_{i0}, \boldsymbol{y_i}} \sum_{i=1}^{3} \Pi^i(y_{i0}, \boldsymbol{y_i})$$
$$\text{s.t.} \sum_{i=1}^{3} y_{i0} = y_0,$$
$$\sum_{i=1}^{3} \boldsymbol{y_i} = \boldsymbol{y}.$$

$\Pi(y_0, \boldsymbol{y})$ is a better upper bound of the expectation of positive parts than the three bounds from Theorem 1, that is,

$$\mathbb{E}\big[(y_o + \boldsymbol{y}'\tilde{\boldsymbol{z}})^+\big] \le \Pi(y_0, \boldsymbol{y}) \le \min_{i=1,2,3} \Pi^i(y_0, \boldsymbol{y})$$

Proof. We refer the reader to Chen and Sim (2009) for the proof. □

The epigraph form of the bound in Theorem 2, $\Pi(y_0, \boldsymbol{y}) \le M$, is

$$\begin{aligned}
& \exists\ y_{i0} \in \mathbb{R}, \boldsymbol{y_i} \in \mathbb{R}^N, r_i \in \mathbb{R}, i = 1, 2, 3 \\
\text{s.t.}\ & r_1 + r_2 + r_3 \le M \\
& y_{10} + \max_{\tilde{\boldsymbol{z}} \in \mathcal{W}} \tilde{\boldsymbol{z}}'\boldsymbol{y_1} \le r_1 \\
& 0 \le r_1 \\
& \max_{\tilde{\boldsymbol{z}} \in \mathcal{W}} \tilde{\boldsymbol{z}}'(-\boldsymbol{y_2}) \le r_2 \\
& y_{20} \le r_2 \\
& \frac{1}{2}y_{30} + \frac{1}{2}\sqrt{y_{30}^2 + \boldsymbol{y_3}'\boldsymbol{\Sigma}\boldsymbol{y_3}} \le r_3 \\
& y_{10} + y_{20} + y_{30} = y_0 \\
& \boldsymbol{y_1} + \boldsymbol{y_2} + \boldsymbol{y_3} = \boldsymbol{y}
\end{aligned}$$

where N is the dimension of uncertain factors $\tilde{z}$.

Remark 3. Under Assumption 1, the bound in Theorem 2 is a second-order cone program (SOCP), which is computationally tractable and solved efficiently with a commercial solver. With this bound, we can approximate the objective function of the multistage stochastic programming formulation.

Theorem 2 shows that the integration of the three bounds from Theorem 1 generates a better bound than the three upper bounds provided separately. Hence, we adopt the bound from Theorem 2 to propose the adjustable robust counterparts.

4.3.3. Linear decision rule (LDR) formulation

In this section, we explain the LDR and propose a robust formulation based on the LDR. We show that the LDR formulation is a second-order cone program which is computationally tractable and can be solved using commercial solvers. Then, we show that the LDR formulation is a tractable approximation of a multistage stochastic programming formulation for the ECR problem.

Ben-Tal et al. (2004) showed that the adjustable robust counterpart is NP-hard, so they proposed an affinely adjustable robust counterpart (AARC) for tractability. The idea of AARC is to restrict decisions to affine functions of uncertainties. An LDR is based on the same concept of an AARC, which means that the repositioning decisions are restricted to affine functions of random factors.

$$r_{ijt}^S(\tilde{z}) = r_{ijt}^{S,0} + \sum_{i',j' \in P} \sum_{k=1}^{t} r_{i'j'ijt}^{S,k} \tilde{z}_{i'j'k} \qquad \forall i, j \in P,\ t = 1, \ldots, T \tag{19}$$

$$r_{ijt}^F(\tilde{z}) = r_{ijt}^{F,0} + \sum_{i',j' \in P} \sum_{k=1}^{t} r_{i'j'ijt}^{F,k} \tilde{z}_{i'j'k} \qquad \forall i, j \in P,\ t = 1, \ldots, T \tag{20}$$

$$r_{i'j'ijt}^{S,k} = 0 \qquad \forall i, j \in P,\ t = 1, \ldots, T,\ k = t+1, \ldots, T \tag{21}$$

$$r_{i'j'ijt}^{F,k} = 0 \qquad \forall i, j \in P,\ t = 1, \ldots, T,\ k = t+1, \ldots, T \tag{22}$$

For example, consider repositioning decisions of standard containers for two ports and two periods. Then, $r_{121}^S(\tilde{z}) = r_{1,2,1}^{S,0} + r_{1,2,1,2,1}^{S,1}\tilde{z}_{1,2,1} + r_{2,1,1,2,1}^{S,1}\tilde{z}_{2,1,1}$ and $r_{122}^S(\tilde{z}) = r_{1,2,2}^{S,0} + r_{1,2,1,2,2}^{S,1}\tilde{z}_{1,2,1} + r_{1,2,1,2,2}^{S,2}\tilde{z}_{1,2,2} + r_{2,1,1,2,2}^{S,1}\tilde{z}_{2,1,1} + r_{2,1,1,2,2}^{S,2}\tilde{z}_{2,1,2}$ which are affine functions of $\tilde{z}$. $r_{211}^S(\tilde{z})$ and $r_{212}^S(\tilde{z})$ can be represented similarly.

Eqs. (19) and (20) show that the repositioning decisions for standard and foldable containers are restricted to affine functions of $\tilde{z}$, respectively. Eqs. (21) and (22) show the non-anticipativity of the repositioning decisions. Other decision variables, x_{ijt}, are also affinely dependent on random factors $\tilde{z}$. We omit non-anticipativity constraints for brevity.

$$x_{ijt}^S(\tilde{z}) = x_{ijt}^{S,0} + \sum_{i',j' \in P} \sum_{k=1}^{t} x_{i'j'ijt}^{S,k} \tilde{z}_{i'j'k} \qquad \forall i, j \in P,\ t = 1, \ldots, T \tag{23}$$

$$x_{ijt}^F(\tilde{z}) = x_{ijt}^{F,0} + \sum_{i',j' \in P} \sum_{k=1}^{t} x_{i'j'ijt}^{F,k} \tilde{z}_{i'j'k} \qquad \forall i, j \in P,\ t = 1, \ldots, T \tag{24}$$

We can approximate the expected repositioning and transporting costs by using the LDR for repositioning and transporting decisions. For example, consider repositioning and transporting costs of the standard container case. The foldable container case can be expressed similarly.

$$\begin{aligned}
&\mathbb{E}[R^S_{ij} r^S_{ijt}(\tilde{z}) + C^S_{ij} x^S_{ijt}(\tilde{z})] \\
&\leq \mathbb{E}\left[R^S_{ij}\left(r^{S,0}_{ijt} + \sum_{i',j' \in P}\sum_{k=1}^{t} r^{S,k}_{i'j'ijt}\tilde{z}_{i'j'k}\right) + C^S_{ij}\left(x^{S,0}_{ijt} + \sum_{i',j' \in P}\sum_{k=1}^{t} x^{S,k}_{i'j'ijt}\tilde{z}_{i'j'k}\right)\right] \\
&= R^S_{ij} r^{S,0}_{ijt} + C^S_{ij} x^{S,0}_{ijt}
\end{aligned}$$

The first inequality holds by the LDR and the second equality holds by the zero-mean assumption of uncertain factors. We can approximate the expected holding and penalty costs, and the folding and unfolding costs by the LDR and the bound from Theorem 2. For example, consider the holding and penalty costs of the foldable container case.

$$\begin{aligned}
&\mathbb{E}[H^S_i (z^S_{it}(\tilde{z}))^+ + P^S_i (z^S_{it}(\tilde{z}))^-] \\
&\leq \mathbb{E}\left[H^S_i\left(z^{S,0}_{it} + \sum_{i',j' \in P}\sum_{k=1}^{t} z^{S,k}_{i'j'it}\tilde{z}_{i'j'k}\right)^+ + P^S_i\left(z^{S,0}_{it} + \sum_{i',j' \in P}\sum_{k=1}^{t} z^{S,k}_{i'j'it}\tilde{z}_{i'j'k}\right)^-\right] \\
&\leq H^S_i \Pi(z^{S,0}_{it}, \boldsymbol{z}^S_{i,t}) + P^S_i \Pi(-z^{S,0}_{it}, -\boldsymbol{z}^S_{i,t})
\end{aligned}$$

The first inequality holds by the LDR and the second inequality holds by the bound from Theorem 2. Expected folding and unfolding costs can be approximated similarly.

Using the LDR and bounds from Theorem 2, we propose the LDR formulation for the ECR problem. Let RC_{LDR} denote total repositioning and transportation costs using the LDR and HC_{LDR} denote total holding and penalty costs using the LDR and Theorem 2. Let FC_{LDR} denote total folding and unfolding costs using the LDR and Theorem 2.

$$\text{RC}_{\text{LDR}} = \sum_{t=1}^{T}\sum_{i \in P}\sum_{j \in P}\left(R^S_{ij} r^{S,0}_{ijt} + C^S_{ij} x^{S,0}_{ijt} + R^F_{ij} r^{F,0}_{ijt} + C^F_{ij} x^{F,0}_{ijt}\right)$$

$$\begin{aligned}
\text{HC}_{\text{LDR}} = \sum_{t=1}^{T}\sum_{i \in P}\Big(&H^S_i \Pi(z^{S,0}_{it}, \boldsymbol{z}^S_{i,t}) + H^F_i \Pi(z^{F,0}_{it}, \boldsymbol{z}^F_{i,t}) \\
&+ P^S_i \Pi(-z^{S,0}_{it}, -\boldsymbol{z}^S_{i,t}) + P^F_i \Pi(-z^{F,0}_{it}, -\boldsymbol{z}^F_{i,t})\Big)
\end{aligned}$$

$$\begin{aligned}
\text{FC}_{\text{LDR}} = \sum_{t=1}^{T}\sum_{i \in P}\Bigg(&FC_i \Pi\Bigg(\sum_{j \in P}(x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}} - x^{F,0}_{ijt}), \\
&\sum_{j \in P}(\boldsymbol{x}^F_{ji,t-\nu_i-\nu_j-\tau_{ji}} - \boldsymbol{x}^F_{ijt})\Bigg) \\
&+ UC_i \Pi\Bigg(\sum_{j \in P}(x^{F,0}_{ijt} - x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}), \sum_{j \in P}(\boldsymbol{x}^F_{ijt} - \boldsymbol{x}^F_{ji,t-\nu_i-\nu_j-\tau_{ji}})\Bigg)\Bigg)
\end{aligned}$$

The LDR formulation is as follows:

$$\text{TC}_{\text{LDR}} = \tag{25}$$

$$\begin{aligned}
\min \quad & \text{RC}_{\text{LDR}} + \text{HC}_{\text{LDR}} + \text{FC}_{\text{LDR}} \\
\text{s.t.} \quad & z^{S,0}_{it} = z^{S,0}_{i,t-1} + \sum_{j \in P} r^{S,0}_{ji,t-\tau_{ji}} - \sum_{j \in P} r^{S,0}_{ijt} + \sum_{j \in P} x^{S,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}} - \sum_{j \in P} x^{S,0}_{ijt}, \\
& \forall i \in P, t = 1, \ldots, T
\end{aligned} \tag{26}$$

$$\begin{aligned}
z^{S,k}_{i'j'it} = z^{S,k}_{i'j'i,t-1} &+ \sum_{j \in P} r^{S,k}_{i'j'ji,t-\tau_{ji}} - \sum_{j \in P} r^{S,k}_{i'j'ijt} + \sum_{j \in P} x^{S,k}_{i'j'ji,t-\nu_i-\nu_j-\tau_{ji}} \\
&- \sum_{j \in P} x^{S,k}_{i'j'ijt}, \qquad \forall i', j', i \in P, t = 1, \ldots, T, k \leq t
\end{aligned} \tag{27}$$

$$z^{F,0}_{it} = z^{F,0}_{i,t-1} + \sum_{j \in P} r^{F,0}_{ji,t-\tau_{ji}} - \sum_{j \in P} r^{F,0}_{ijt} + \sum_{j \in P} x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}} - \sum_{j \in P} x^{F,0}_{ijt}, \quad \forall i \in P, t = 1, \ldots, T \tag{28}$$

$$\begin{aligned}
z^{F,k}_{i'j'it} = z^{F,k}_{i'j'i,t-1} &+ \sum_{j \in P} r^{F,k}_{i'j'ji,t-\tau_{ji}} - \sum_{j \in P} r^{F,k}_{i'j'ijt} + \sum_{j \in P} x^{F,k}_{i'j'ji,t-\nu_i-\nu_j-\tau_{ji}} \\
&- \sum_{j \in P} x^{F,k}_{i'j'ijt}, \qquad \forall i', j', i \in P, t = 1, \ldots, T, k \leq t
\end{aligned} \tag{29}$$

$$x^{S,0}_{ijt} + x^{F,0}_{ijt} = d^0_{ijt}, \qquad \forall i, j \in P, t = 1, \ldots, T \tag{30}$$

$$x^{S,k}_{i'j'ijt} + x^{F,k}_{i'j'ijt} = d^k_{i'j'ijt}, \quad \forall i', j', i, j \in P, t = 1, \ldots, T, k \leq t \tag{31}$$

$$\begin{aligned}
&r^{S,0}_{ijt} + \frac{1}{N} r^{F,0}_{ijt} + x^{S,0}_{ij,t-\nu_i} + x^{F,0}_{ij,t-\nu_i} \\
&+ \sum_{i',j' \in P}\sum_{k=1}^{t}\left(r^{S,k}_{i'j'ijt} + \frac{1}{N} r^{F,k}_{i'j'ijt} + x^{S,k}_{i'j'ij,t-\nu_i} + x^{F,k}_{i'j'ij,t-\nu_i}\right)\tilde{z}_{ijk} \leq K_{ijt}, \\
&\forall i, j \in P, t = 1, \ldots, T, \tilde{z} \in \mathcal{W}
\end{aligned} \tag{32}$$

The objective function includes the upper bounds of the expectations of positive parts. Because the bounds are second-order cones, the objective function is a second-order cone. Moreover, the bounds are derived only with mean, support, and covariance of uncertainties. Hence, the above formulation does not need any distributional assumptions. All constraints, except Constraint (32), are linear and Constraint (32) can be transformed to a robust counterpart under Assumption 1. If the uncertain factors have interval or ellipsoidal uncertainty, then the transformed robust counterpart is computationally tractable. Hence, if we assume that Constraint (32) can be transformed to be tractable, then the LDR formulation is computationally tractable. From this, we can obtain the following result.

Theorem 3. $\text{TC}_{\text{STOC}} \leq \text{TC}_{\text{LDR}}$, *where* TC_{STOC} *is the optimal expected cost of the multistage stochastic programming formulation, and* TC_{LDR} *is the optimal expected cost under the linear decision rule.*

Proof. The proof is similar to that of See and Sim (2010). We refer the reader to the electronic companion of See and Sim (2010).

First, we will show that the inventory levels of standard and foldable containers are expressed as affine functions of random factors. We only show the proof of the standard container case, because the foldable container case is the same. Note that constraint (10) is $z^S_{it}(\tilde{z}) = z^S_{i,t-1}(\tilde{z}) + \sum_{j \in P} r^S_{ji,t-\tau_{ji}}(\tilde{z}) - \sum_{j \in P} r^S_{ijt}(\tilde{z}) + \sum_{j \in P} x^S_{ji,t-\nu_i-\nu_j\tau_{ji}}(\tilde{z}) - \sum_{j \in P} x^S_{ijt}(\tilde{z})$. By summation over period t and using the linear decision rule, we can obtain

$$\begin{aligned}
z^S_{it}(\tilde{z}) &= z^{S,0}_{i0} + \sum_{\tau=1}^{t}\sum_{j \in P} r^{S,0}_{ji,\tau-\tau_{ji}} - \sum_{\tau=1}^{t}\sum_{j \in P} r^{S,0}_{ij,\tau} \\
&\quad + \sum_{\tau=1}^{t}\sum_{j \in P} x^{S,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}} - \sum_{\tau=1}^{t}\sum_{j \in P} x^{S,0}_{ij,\tau} \\
&\quad + \sum_{i',j',j \in P}\sum_{\tau=1}^{t}\sum_{k=1}^{t} r^{S,k}_{i'j'ji,\tau-\tau_{ji}}\tilde{z}_{i'j'k} - \sum_{i',j',j \in P}\sum_{\tau=1}^{t}\sum_{k=1}^{t} r^{S,k}_{i'j'ij,\tau}\tilde{z}_{i'j'k} \\
&\quad + \sum_{i',j',j \in P}\sum_{\tau=1}^{t}\sum_{k=1}^{t} x^{S,k}_{i'j'ji,t-\nu_i-\nu_j-\tau_{ji}}\tilde{z}_{i'j'k} - \sum_{i',j',j \in P}\sum_{\tau=1}^{t}\sum_{k=1}^{t} x^{S,k}_{i'j'ij\tau}\tilde{z}_{i'j'k} \\
&= z^{S,0}_{it} + \sum_{i',j' \in P} z^{S,k}_{i'j'it}\tilde{z}_{i'j'k}
\end{aligned}$$

Hence, $z^{S,0}_{it}$ is also an affine function of random factors and constraints (26) and (27) are derived. The linear decision rule solution is a feasible solution of the multistage stochastic programming formulation. Under the linear decision rule and by Theorem 2, we obtain

$$\mathbb{E}\Bigg(\sum_{i,j\in P}\left(R^S_{ij}r^S_{ijt}(\tilde{z})+C^S_{ij}x^S_{ijt}(\tilde{z})+R^F_{ij}r^F_{ijt}(\tilde{z})+C^F_{ij}x^F_{ijt}(\tilde{z})\right)\Bigg)$$
$$+\mathbb{E}\Bigg(\sum_{j\in P}H^S_i\left(z^S_{it}(\tilde{z})\right)^+ + H^F_i\left(z^F_{it}(\tilde{z})\right)^+ + P^S_i\left(z^S_{it}(\tilde{z})\right)^- + P^F_i\left(z^F_{it}(\tilde{z})\right)^-\Bigg)$$
$$+\mathbb{E}\Bigg(\sum_{i\in P}FC_i\Big(\sum_{j\in P}(x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}(\omega)-x^F_{ijt}(\omega))^+\Big)\Bigg)$$
$$+\mathbb{E}\Bigg(\sum_{i\in P}FC_i\Big(\sum_{j\in P}(x^F_{ijt}(\omega)-x^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}(\omega))^+\Big)\Bigg)$$
$$\leq \sum_{i,j\in P}\left(R^S_{ij}r^{S,0}_{ijt}+C^S_{ij}x^{S,0}_{ijt}+R^F_{ij}r^{F,0}_{ijt}+C^F_{ij}x^{F,0}_{ijt}\right)$$
$$+\sum_{i\in P}\left(H^S_i\Pi(z^{S,0}_{it},\boldsymbol{z}^S_{i,t})+H^F_i\Pi(z^{F,0}_{it},\boldsymbol{z}^F_{i,t})\right)$$
$$+\sum_{i\in P}\left(P^S_i\Pi(-z^{S,0}_{it},-\boldsymbol{z}^S_{i,t})+P^F_i\Pi(-z^{F,0}_{it},-\boldsymbol{z}^F_{i,t})\right)$$
$$+\sum_{i\in P}\Bigg(FC_i\Pi\Big(\sum_{j\in P}(x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}-x^{F,0}_{ijt}),\sum_{j\in P}(\boldsymbol{x}^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}-\boldsymbol{x}^F_{ijt})\Big)$$
$$+UC_i\Pi\Bigg(\sum_{j\in P}(x^{F,0}_{ijt}-x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}),\sum_{j\in P}(\boldsymbol{x}^F_{ijt}-\boldsymbol{x}^F_{ji,t-\nu_i-\nu_j-\tau_{ji}})\Bigg)\Bigg).$$

Therefore, we conclude $TC_{STOC} \leq TC_{LDR}$. □

Theorem 3 shows that the LDR formulation is a tractable approximation of the multistage stochastic programming formulation. The optimal solution of the multistage stochastic programming formulation is difficult to obtain; however, the optimal solution of the LDR formulation can be used. We show the analysis of LDR formulation performance and a comparison against a benchmark in the numerical experiments section.

4.3.4. Restricted linear decision rule (RLDR) formulation

Although the LDR formulation is computationally tractable, it has a lot of decision variables because each decision is dependent on all possible uncertainties. For example, each repositioning decision, $\boldsymbol{r}_{ijt}$, has $|P|\times|P|\times|T|+1$ variables in the LDR formulation. For a simpler formulation, we propose a restricted linear decision rule (RLDR) similar to the idea of Ang, Lim, and Sim (2012). A restricted linear decision rule means that decision rules are restricted to affine functions of the uncertain factors $\tilde{\boldsymbol{z}}_{ij}$ rather than all possible realizations of the uncertain factors. For example, the repositioning decision from port i to port j would not be affected by random factors $\{\tilde{z}_{k,l}\}_{k,l\neq i,j}$. Therefore, the RLDR may perform well in practice despite the additional assumptions required. The RLDR is as follows:

$$r^S_{ijt}(\tilde{z})=r^{S,0}_{ijt}+\sum_{k=1}^{t}r^{S,k}_{ijt}\tilde{z}_{ijk}\qquad \forall i,j\in P,\ t=1,\ldots,T$$
$$r^F_{ijt}(\tilde{z})=r^{F,0}_{ijt}+\sum_{k=1}^{t}r^{F,k}_{ijt}\tilde{z}_{ijk}\qquad \forall i,j\in P,\ t=1,\ldots,T$$
$$x^S_{ijt}(\tilde{z})=x^{S,0}_{ijt}+\sum_{k=1}^{t}x^{S,k}_{ijt}\tilde{z}_{ijk}\qquad \forall i,j\in P,\ t=1,\ldots,T$$
$$x^F_{ijt}(\tilde{z})=x^{F,0}_{ijt}+\sum_{k=1}^{t}x^{F,k}_{ijt}\tilde{z}_{ijk}\qquad \forall i,j\in P,\ t=1,\ldots,T$$

For example, consider repositioning decisions of standard containers for two ports and three periods. Then, $r^S_{121}(\tilde{z})=r^{S,0}_{1,2,1}+r^{S,1}_{1,2,1}\tilde{z}_{1,2,1}$, $r^S_{122}(\tilde{z})=r^{S,0}_{1,2,2}+r^{S,1}_{1,2,2}\tilde{z}_{1,2,1}+r^{S,2}_{1,2,2}\tilde{z}_{1,2,2}$, and $r^S_{123}(\tilde{z})=r^{S,0}_{1,2,3}+r^{S,1}_{1,2,3}\tilde{z}_{1,2,1}+r^{S,2}_{1,2,3}\tilde{z}_{1,2,2}+r^{S,3}_{1,2,3}\tilde{z}_{1,2,3}$ which are affine functions of $\tilde{z}_{12}$. In the opposite direction, $r^S_{211}(\tilde{z})=r^{S,0}_{2,1,1}+r^{S,1}_{2,1,1}\tilde{z}_{2,1,1}$, $r^S_{212}(\tilde{z})=r^{S,0}_{2,1,2}+r^{S,1}_{2,1,2}\tilde{z}_{2,1,1}+r^{S,2}_{2,1,2}\tilde{z}_{2,1,2}$, and $r^S_{213}(\tilde{z})=r^{S,0}_{2,1,3}+r^{S,1}_{2,1,3}\tilde{z}_{2,1,1}+r^{S,2}_{2,1,3}\tilde{z}_{2,1,2}+r^{S,3}_{2,1,3}\tilde{z}_{2,1,3}$ which are affine functions of $\tilde{z}_{21}$.

To utilize the RLDR, an additional assumption on demand is needed. Random demand $d_{ijt}(\tilde{z})$ should be a function of $\tilde{\boldsymbol{z}}_{ij}$ rather than of $\tilde{\boldsymbol{z}}$. In other words, $d_{ijt}(\tilde{z})$ depends only on the random factors related to the (i, j) pair. Therefore, $d_{ijt}(\tilde{z})$ can be represented as follows:

$$d_{ijt}(\tilde{z})=d^0_{ijt}+\sum_{k=1}^{t}d^k_{ijt}\tilde{z}_{ijk}.$$

Without this assumption, the RLDR would be infeasible. Therefore, we assume the above assumption in the RLDR formulation.

Using the RLDR and bounds from Theorem 2, we propose the RLDR formulation of the ECR problem. Let RC_{RLDR} denote total repositioning and transportation costs using the RLDR and HC_{RLDR} denote total holding and penalty costs using the RLDR and Theorem 2. Let FC_{RLDR} denote total folding and unfolding costs using the RLDR and Theorem 2.

$$RC_{RLDR}=\sum_{t=1}^{T}\sum_{i\in P}\sum_{j\in P}\left(R^S_{ij}r^{S,0}_{ijt}+C^S_{ij}x^{S,0}_{ijt}+R^F_{ij}r^{F,0}_{ijt}+C^F_{ij}x^{F,0}_{ijt}\right)$$
$$HC_{RLDR}=\sum_{t=1}^{T}\sum_{i\in P}\Big(H^S_i\Pi(z^{S,0}_{it},\boldsymbol{z}^S_{i,t})+H^F_i\Pi(z^{F,0}_{it},\boldsymbol{z}^F_{i,t})$$
$$+P^S_i\Pi(-z^{S,0}_{it},-\boldsymbol{z}^S_{i,t})+P^F_i\Pi(-z^{F,0}_{it},-\boldsymbol{z}^F_{i,t})\Big)$$
$$FC_{RLDR}=\sum_{t=1}^{T}\sum_{i\in P}\Bigg(FC_i\Pi\Bigg(\sum_{j\in P}(x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}-x^{F,0}_{ijt}),$$
$$\sum_{j\in P}(\boldsymbol{x}^F_{ji,t-\nu_i-\nu_j-\tau_{ji}}-\boldsymbol{x}^F_{ijt})\Bigg)$$
$$+UC_i\Pi\Bigg(\sum_{j\in P}(x^{F,0}_{ijt}-x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}),\sum_{j\in P}(\boldsymbol{x}^F_{ijt}-\boldsymbol{x}^F_{ji,t-\nu_i-\nu_j-\tau_{ji}})\Bigg)$$

The RLDR formulation is as follows:

$TC_{RLDR}=$

$$\min\quad RC_{RLDR}+HC_{RLDR}+FC_{RLDR}\tag{33}$$

$$\text{s.t.}\quad z^{S,0}_{it}=z^{S,0}_{i,t-1}+\sum_{j\in P}r^{S,0}_{ji,t-\tau_{ji}}-\sum_{j\in P}r^{S,0}_{ijt}+\sum_{j\in P}x^{S,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}-\sum_{j\in P}x^{S,0}_{ijt},\qquad \forall i\in P,t=1,\ldots,T\tag{34}$$

$$z^{S,k}_{j,i,i,t}=z^{S,k}_{j,i,i,t-1}+r^{S,k}_{j,i,t-\tau_{ji}}+x^{S,k}_{j,i,t-\nu_i-\nu_j-\tau_{ji}},\qquad \forall i,j\in P,t=1,\ldots,T,k\leq t\tag{35}$$

$$z^{S,k}_{i,j,i,t}=z^{S,k}_{i,j,i,t-1}-r^{S,k}_{i,j,t}-x^{S,k}_{i,j,t},\qquad \forall i,j\in P,t=1,\ldots,T,k\leq t\tag{36}$$

$$z^{F,0}_{it}=z^{F,0}_{i,t-1}+\sum_{j\in P}r^{F,0}_{ji,t-\tau_{ji}}-\sum_{j\in P}r^{F,0}_{ijt}+\sum_{j\in P}x^{F,0}_{ji,t-\nu_i-\nu_j-\tau_{ji}}-\sum_{j\in P}x^{F,0}_{ijt},\qquad \forall i\in P,t=1,\ldots,T\tag{37}$$

$$z^{F,k}_{j,i,i,t}=z^{F,k}_{j,i,i,t-1}+r^{F,k}_{j,i,t-\tau_{ji}}+x^{F,k}_{j,i,t-\nu_i-\nu_j-\tau_{ji}},\qquad \forall i,j\in P,t=1,\ldots,T,k\leq t\tag{38}$$

$$z^{F,k}_{i,j,i,t}=z^{F,k}_{i,j,i,t-1}-r^{F,k}_{i,j,t}-x^{F,k}_{i,j,t},\qquad \forall i,j\in P,t=1,\ldots,T,k\leq t\tag{39}$$

$$x_{ijt}^{S,0} + x_{ijt}^{F,0} = d_{ijt}^{0}, \qquad \forall i, j \in P, t = 1, \ldots, T \tag{40}$$

$$x_{ijt}^{S,k} + x_{ijt}^{F,k} = d_{ijt}^{k}, \qquad \forall i, j \in P, t = 1, \ldots, T, k \le t \tag{41}$$

$$r_{ijt}^{S,0} + \frac{1}{N} r_{ijt}^{F,0} + x_{ij,t-\nu_i}^{S,0} + x_{ij,t-\nu_i}^{F,0} + \sum_{k=1}^{t} \left(r_{ijt}^{S,k} + \frac{1}{N} r_{ijt}^{F,k} + x_{ij,t-\nu_i}^{S,k} + x_{ij,t-\nu_i}^{F,k} \right) \tilde{z}_{ijk} \le K_{ijt}, \quad \forall i, j \in P, t = 1, \ldots, T, \tilde{z} \in W \tag{42}$$

Fewer decision variables of the RLDR formulation are used than in the LDR formulation. Hence, we can obtain the result shown in Theorem 4.

Theorem 4. $\mathrm{TC}_{\mathrm{STOC}} \le \mathrm{TC}_{\mathrm{LDR}} \le \mathrm{TC}_{\mathrm{RLDR}}$, *where* $\mathrm{TC}_{\mathrm{RLDR}}$ *is the optimal expected cost under the RLDR.*

Proof. The first inequality comes from Theorem 3. The second inequality is true because the RLDR formulation is a formulation that adds constraints to the LDR formulation, and the objective function of the RLDR formulation is larger than that of the LDR formulation. □

Theorem 4 shows that the RLDR formulation is also a tractable approximation. However, the expected cost of the RLDR formulation is worse than that of the LDR formulation. Despite the worse RLDR performance, the size of the RLDR formulation is much smaller than that of the LDR formulation. Therefore, the RLDR formulation is easy to handle and has competitive advantages in practice. We analyzed the performance and computation time for the validity of the RLDR formulation and compare the RLDR against a benchmark.

5. Computational experiments

In this section, we present the numerical analysis of the proposed formulations based on the expected operating costs, computation time, and optimality gap against the benchmark. We define expected value given perfect information (EV|PI) for the benchmark against the proposed models. Then, we conduct simulations for further analysis such as cost-saving effects of foldable containers, cost ratio of total operating costs, and container storage at depots of ports. The following results were solved using Xpress software version 7.9 on a PC with an Intel(R) Core(TM) i5-6600 CPU 3.30 GHz with 32.00GB of RAM.

5.1. Expected value given perfect information (EV|PI)

For validating the performance of the proposed model, we need a benchmark for comparison. However, it is difficult to obtain an optimal solution of a multistage stochastic programming formulation, so we utilize an alternative of $\mathrm{TC}_{\mathrm{STOC}}$ which is possible to calculate. Therefore, we define expected value given perfect information (EV|PI) as follows:

$$\mathrm{EV|PI} = \mathbb{E}_D[\mathrm{TC}_{\mathrm{DET}}|D] \approx \frac{1}{K} \sum_{k=1}^{K} \left(\mathrm{TC}_{\mathrm{DET}}|D_k \right)$$

EV|PI represents the expected value of the total operating costs given the information of demand. We generate K samples of demand scenarios and calculate $\mathrm{TC}_{\mathrm{DET}}$ for each demand scenario. Then, we calculate EV|PI as an expectation over K samples. EV|PI can be an alternative of $\mathrm{TC}_{\mathrm{STOC}}$, because it is similar to multistage scenario generation approach. EV|PI would be less than the objective function value, because it is calculated based on the complete knowledge about future demand. Therefore, it can be used as a benchmark for comparing the performances of the robust formulations.

Table 2
Mean of demand process, μ_{ij}^{0}.

From-to	NB	SH	BS	VC	LA
NB	–	50	50	300	400
SH	50	–	50	300	400
BS	100	100	–	200	300
VC	150	150	100	–	100
LA	200	200	150	100	–

5.2. Demand process

In this experiment, we utilize the demand process proposed by Graves (1999), which can be represented as the factor-based demand model. To compare performances of the LDR and RLDR formulations, we assume that the demand only depends on random factors $\tilde{\boldsymbol{z}}_{ij}$. Then, the demand is expressed as follows:

$$d_{ijt}(\tilde{z}) = d_{ijt}^{0} + \sum_{k=1}^{t} d_{ijt}^{k} \tilde{z}_{ijk} = \mu_{ijt}^{0} + \sum_{k=1}^{t-1} \alpha \tilde{z}_{ijk} + \tilde{z}_{ijt} \qquad \forall i, j \in P, \ t = 1, \ldots, T$$

In this demand process, $d_{ijt}^{k} = \alpha$ for $k = 1, \ldots, t-1$ and $d_{ijt}^{t} = 1$. We assume that $\tilde{z}_{ijt}$ are independent uniformly distributed random variables in $[-\bar{z}_{ij}, \bar{z}_{ij}]$. Supports of uncertain factors, $\bar{z}_{ij}$, are listed in Appendix A (Table A.19). When $\alpha = 1$, the demand process is a random walk, and when $\alpha = 0$, the demand process is a stationary i.i.d. process. In this experiment, we use two different values of $\alpha \in \{0, 0.25\}$.

5.3. Computational results

We consider a numerical example of five ports and 20 planning periods. The five ports represent Ningbo (NB), Shanghai (SH), Busan (BS), Vancouver (VC), and Los Angeles (LA), respectively, which are five major ports in the North America (NA)-Asia shipping network. We consider the NA-Asia instance, because the trade imbalance between NA and Asia is highly significant. We set a base period of 4 days to set the transportation time between Asian ports as 1 period and the transportation time to cross the Pacific Ocean as 4 periods or 16 days. We refer to the vessel schedule announced by Maersk, the world's largest shipping company. We assume that the inland transportation time ν is 1 base period and $N = 4$, which means that four folded foldable containers are used to build one pack such as 4FOLD of HCI. The demand parameters shown in Table 2 are determined by referring to the monthly cargo volume data for each port. To reflect the trade imbalance, we set the mean of demand from the export dominant ports to the import dominant ports to be double for the return direction. We assume that the number of supplied containers determined before the beginning of the planning horizon and the initial inventory of containers are given. We also assume that the initial inventory of foldable containers are one over ten of that of standard containers. For simplicity, we assume that the parameters are the same over the planning horizon, for example, $\mu_{ijt}^{0} = \mu_{ij}^{0}$ for all t. We let $H_i^S = 0.2$, $H_i^F = 0.1$, $P_i^S = 2$, $P_i^F = 4$, $FC_i = 0.1$, and $UC_i = 0.1$ for all five ports. The unit holding cost of foldable containers is the half of that of standard containers, because foldable containers are stored with folded state. The penalty cost of foldable containers is twofold that of standard containers, because the purchase cost and leasing cost of foldable containers are much expensive. The transportation time between ports, other cost parameters, capacity, and supports of uncertain factors are listed in Appendix A. $[-\bar{z}_{ij}, \bar{z}_{ij}]$ is the interval of uniform distribution of $\tilde{z}_{ijt}$, so we can calculate the standard deviation with $\bar{z}_{ij}$. We use these parameters as a baseline, and vary

Table 3
Computational results with different holding costs

α	HC	Expected Cost			Time (s)		Gap (%)	
		EV|PI	LDR	RLDR	LDR	RLDR	LDR	RLDR
0	0.2	193,141	193,252	193,451	11418.9	1815.9	0.057	0.161
	0.1	161,704	161,832	162,109	14423.8	2713.2	0.079	0.250
	0.04	142,437	142,552	142,779	11216.9	2173.1	0.081	0.240
0.25	0.2	193,146	193,696	194,362	14344.5	2795.5	0.285	0.629
	0.1	161,715	162,614	163,217	12368.6	2361.5	0.556	0.929
	0.04	142,446	143,236	143,761	12268.0	2532.0	0.554	0.923

Table 4
Computational results using standard containers only with different holding costs.

α	HC	Expected Cost			Time (s)		Gap (%)	
		EV|PI	LDR	RLDR	LDR	RLDR	LDR	RLDR
0	0.2	218,071	218,164	218,280	746.2	200.0	0.043	0.096
	0.1	174,037	174,128	174,243	673.9	231.4	0.053	0.119
	0.04	147,616	147,698	147,803	605.1	227.5	0.056	0.127
0.25	0.2	218,077	218,700	219,069	777.1	193.2	0.286	0.455
	0.1	174,045	174,697	175,070	751.8	162.8	0.375	0.589
	0.04	147,626	148,256	148,616	605.6	217.2	0.427	0.671

Table 5
Comparisons between using standard containers only and using standard and foldable containers both.

α	HC	Standard & Foldable			Standard			Gap (%)		
		EV|PI	LDR	RLDR	EV|PI	LDR	RLDR	EV|PI	LDR	RLDR
0	0.2	193,141	193,252	193,451	218,071	218,164	218,280	11.43	11.42	11.37
	0.1	161,704	161,832	162,109	174,037	174,128	174,243	7.09	7.06	6.96
	0.04	142,437	142,552	142,779	147,616	147,698	147,803	3.51	3.48	3.40
0.25	0.2	193,146	193,696	194,362	218,077	218,700	219,069	11.43	11.43	11.28
	0.1	161,715	162,614	163,217	174,045	174,697	175,070	7.08	6.92	6.77
	0.04	142,446	143,236	143,761	147,626	148,256	148,616	3.51	3.39	3.27

the cost parameters, such as holding, transportation, and repositioning costs. Finally, we generate 10,000 samples for calculating the benchmark EV|PI.

The computational results with various holding costs and the values for α are summarized in Table 3. We set the holding cost H_i^S as {0.2, 0.1, 0.04} for a resulting P_i^S/H_i^S ratio of {10, 20, 50}. The structure of the optimal solutions of inventory models often depends on the ratio P_i^S/H_i^S. The ECR model is similar to inventory models, which makes the experiments with varying holding costs meaningful. The performance gap of the LDR formulation presented in Table 3 is calculated by $(\text{TC}_{\text{LDR}} - \text{EV|PI})/\text{EV|PI} \times 100$. The gap of the RLDR formulation is calculated similarly.

The expected costs show that the result of Theorem 4 holds, and the performance gap shows that the expected costs of both the LDR and RLDR formulations are very close to the EV|PI. Bertsimas, Iancu, and Parrilo (2010) showed that the LDR can be optimal in specific conditions such as convex objective functions and the box uncertainty set. There are a few studies about an optimality and a performance guarantee of the LDR in the multistage setting (Bertsimas & Goyal, 2012; Bertsimas, Goyal, & Lu, 2015; El Housni & Goyal, 2017). The tight performance gap from Table 3 would be justified by the theoretical results, even though the LDR and RLDR formulations are based on the multistage setting and bounds from Theorem 2. The performance gap can be interpreted as the price of robustness, which means that additional cost is incurred to obtain distributionally robust properties. The computation time of the LDR formulation is over 10,000 seconds, which might seem unreasonable. However, the length of time between the decisions on the first period and the next uncertainty realization is one base period or 4 days, which is sufficient for updating the data and solving the LDR formulation. Therefore, the LDR formulation can be used in the rolling horizon manner, that is, the formulation over the entire planning horizon is solved and only the first decision is implemented. Then, the realized uncertainty data is updated and the formulation is solved with the updated data. The computation time of the RLDR formulation is much less than that of the LDR formulation, because the number of variables are quite small. The smaller computation time of the RLDR formulation offers a competitive advantage in practice.

To analyze the cost-saving effect for the use of foldable containers, we compare the computational results against the results of using only standard containers. The computational results for using only standard containers at various holding costs are summarized in Table 4. We compared two cases and show the findings in Table 5. The gap shows that in our experiments, at most 11.43% cost savings is realized when foldable containers are used in maritime transport. The cost-saving effect decreases as the holding cost H_i decreases because the considerable cost saving using foldable containers occurs in the holding cost part. However, the cost-saving effect might be overestimated because the expected total cost represents the expected operating costs over the planning horizon. The operating costs do not include fixed or purchase costs of the foldable containers, which may be very costly. Nevertheless, a considerable cost saving may be realized for using commercialized foldable containers.

The shipping industry is highly affected by crude oil prices, because transportation and repositioning costs are proportional to crude oil prices. The computational results with various repositioning and transportation costs (TC) are summarized in Table 6. We denote three different parameters as {1,2,4} such that transportation and repositioning costs are once, twice, and four times the costs in Appendix A, respectively. The computational results for

Table 6
Computational results with different transport/repositioning costs.

α	TC	Expected Cost			Time (s)		Gap (%)	
		EV|PI	LDR	RLDR	LDR	RLDR	LDR	RLDR
0	1	193,141	193,252	193,451	11536.8	1767.9	0.057	0.161
	2	321,818	321,953	322,311	10777.9	2361.2	0.042	0.153
	4	577,061	577,234	577,530	10460.1	2364.2	0.030	0.081
0.25	1	193,146	193,696	194,362	14163.4	2795.6	0.285	0.629
	2	321,837	322,947	323,685	13390.6	2443.9	0.345	0.574
	4	577,101	578,322	578,915	14216.3	2548.8	0.212	0.314

Table 7
Computational results using standard containers only with different transport/repositioning costs.

α	TC	Expected Cost			Time (s)		Gap (%)	
		EV|PI	LDR	RLDR	LDR	RLDR	LDR	RLDR
0	1	218,071	218,164	218,280	808.1	193.8	0.043	0.096
	2	347,883	347,956	348,031	696.7	221.0	0.021	0.042
	4	603,707	603,893	603,931	896.3	230.6	0.031	0.037
0.25	1	218,077	218,700	219,069	767.7	193.3	0.286	0.455
	2	347,901	348,544	348,898	613.0	204.2	0.185	0.287
	4	603,828	604,801	604,936	985.9	192.7	0.161	0.183

Table 8
Comparisons between using standard containers only and using standard and foldable containers both.

α	TC	Standard & Foldable			Standard			Gap (%)		
		EV|PI	LDR	RLDR	EV|PI	LDR	RLDR	EV|PI	LDR	RLDR
0	1	193,141	193,252	193,451	218,071	218,164	218,280	11.43	11.42	11.37
	2	321,818	321,953	322,311	347,883	347,956	348,031	7.49	7.47	7.39
	4	577,061	577,234	577,530	603,707	603,893	603,931	4.41	4.41	4.37
0.25	1	193,146	193,696	194,362	218,077	218,700	219,069	11.43	11.43	11.28
	2	321,837	322,947	323,685	347,901	348,544	348,898	7.49	7.34	7.23
	4	577,101	578,322	578,915	603,828	604,801	604,936	4.43	4.38	4.30

Table 9
Computational results with different folding/unfolding costs.

α	FC	Expected Cost			Time (s)		Gap (%)	
		EV|PI	LDR	RLDR	LDR	RLDR	LDR	RLDR
0	0.05	160,909	161,021	161,301	12616.2	2637.4	0.070	0.244
	0.1	161,704	161,832	162,109	14423.8	2713.2	0.079	0.250
	0.2	163,294	163,436	163,681	10641.1	2128.0	0.087	0.237
	0.4	166,273	166,394	166,598	12961.6	2633.5	0.073	0.196
0.25	0.05	160,918	161,760	162,368	12722.9	2818.0	0.523	0.901
	0.1	161,715	162,614	163,217	12368.6	2361.5	0.556	0.929
	0.2	163,305	164,268	164,813	14135.0	2730.1	0.590	0.923
	0.4	166,283	167,175	167,676	18427.4	2927.7	0.536	0.838

using both standard and foldable containers and using only standard containers are summarized in Tables 6 and 7. Comparisons between the two cases are represented in Table 8. With varying transportation and repositioning costs, we observe a tight performance gap and a significant cost-saving effect using foldable containers.

The adoption of foldable containers leads to a new type of cost, folding and unfolding costs. The folding and unfolding operations need additional labor, which makes the introduction of folding containers difficult in areas where labor costs are high. The computational results with various folding and unfolding costs (FC) are summarized in Tables 9 and 10. We use four different parameters for FC as {0.05, 0.1, 0.2, 0.4} with holding cost HC=0.1. The lower folding cost represents the case when the additional labor cost of folding/unfolding operations is small. The changes in total costs are relatively small compared to the changes in the HC or TC cases, because most of the total operating costs are transportation costs, and the portion of folding and unfolding costs is small. However, the cost-saving effects decrease as folding and unfolding costs increase, because the utilization of foldable containers may decrease when folding and unfolding costs are high. We note in Table 10 that the total costs of the standard container case have not changed, because the total costs are not affected by folding and unfolding costs. For further analysis, we compare the transportation and repositioning quantities of foldable containers with various folding and unfolding costs in Section 5.4.

In summary, we observed that the LDR and RLDR formulations perform closely to that of the EV|PI, which reflects the applicability in practice. The performance gap is tight and endurable for robustness and tractability. In addition, the computation time of the RLDR formulation is shorter than that of the LDR formulation, which reflects a competitive advantage in practice. Finally, we

Table 10
Comparisons between using standard containers only and using standard and foldable containers both.

α	FC	Standard & Foldable			Standard			Gap (%)		
		EV|PI	LDR	RLDR	EV|PI	LDR	RLDR	EV|PI	LDR	RLDR
0	0.05	160,909	161,021	161,301	174,037	174,128	174,243	8.16	8.14	8.02
	0.1	161,704	161,832	162,109	174,037	174,128	174,243	7.63	7.60	7.49
	0.2	163,294	163,436	163,681	174,037	174,128	174,243	6.58	6.54	6.45
	0.4	166,273	166,394	166,598	174,037	174,128	174,243	4.67	4.65	4.59
0.25	0.05	160,918	161,760	162,368	174,037	174,128	174,243	8.15	7.65	7.31
	0.1	161,715	162,614	163,217	174,037	174,128	174,243	7.62	7.08	6.76
	0.2	163,305	164,268	164,813	174,037	174,128	174,243	6.57	6.00	5.72
	0.4	166,283	167,175	167,676	174,037	174,128	174,243	4.66	4.16	3.92

Table 11
Comparisons of average total operating cost over simulations.

α	HC	Standard and foldable			Standard			Gap (%)		
		EV|PI	LDR	RLDR	EV|PI	LDR	RLDR	EV|PI	LDR	RLDR
0	0.2	193141.0	193779.8	193869.4	218071.0	218144.4	218218.7	11.43	11.17	11.16
	0.1	161704.0	162186.2	162366.5	174036.6	174111.7	174185.3	7.09	6.85	6.79
	0.04	142436.6	143012.4	143035.5	147616.0	147681.8	147744.3	3.51	3.16	3.19
0.25	0.2	193146.3	193961.9	194371.9	218076.6	218454.4	218622.0	11.43	11.21	11.09
	0.1	161714.6	162591.6	163030.8	174045.0	174446.0	174620.9	7.08	6.80	6.64
	0.04	142446.4	143311.4	143569.8	147626.0	147994.8	148163.4	3.51	3.16	3.10

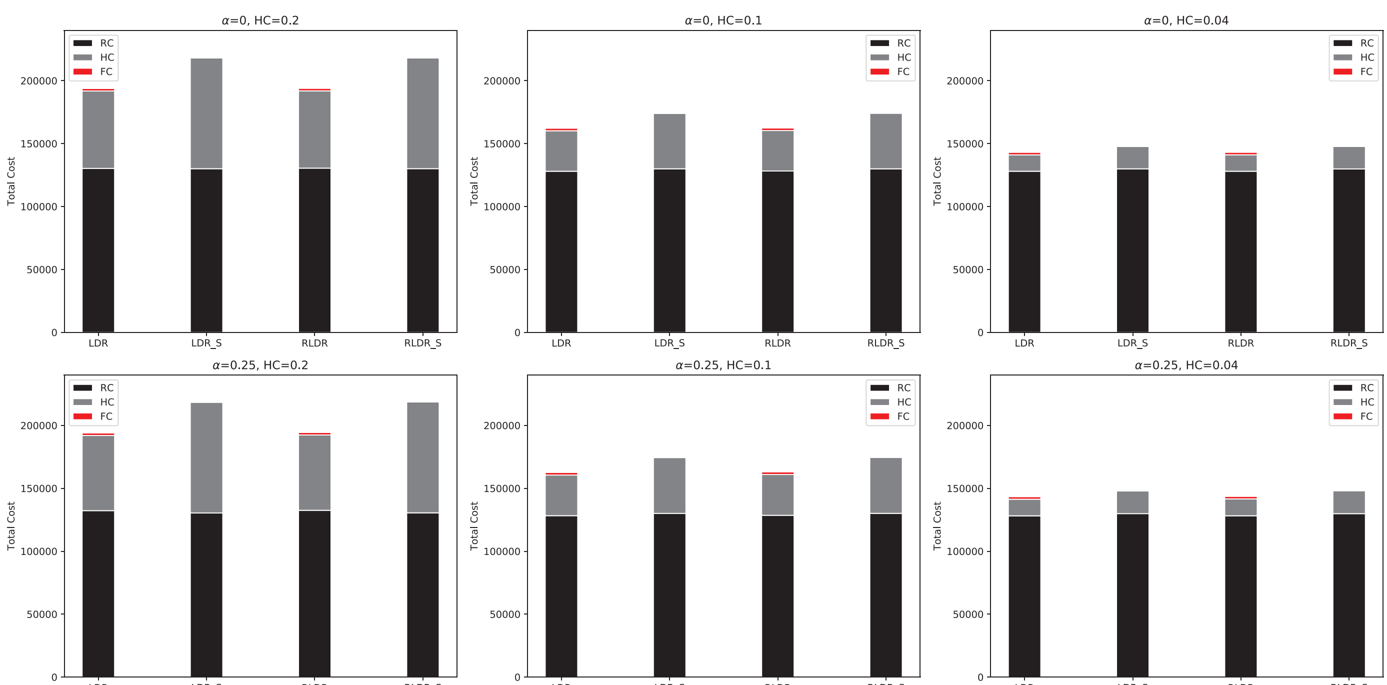

Fig. 4. Average cost ratio over total operating costs.

observed that operating costs can be reduced significantly by using foldable containers.

5.4. Simulation results

The optimal solutions of the LDR and RLDR formulations are optimal affine policies, that is, we obtain optimal parameters of affine policies. To see the obtained optimal policies perform well on actual uncertainty realizations, it is necessary to implement the policies with uncertainty realizations and analyze the results. Based on the affine policies, the actual decisions such as repositioning and transportation quantities are calculated when the uncertainties are realized. Therefore, we implement the affine policies from the LDR and RLDR formulations with actual uncertainty realizations and compare the results with EV|PI. To evaluate the total operating costs of implementing optimal policies, we conduct simulations with scenarios of uncertainty realizations. We use the same samples for calculating EV|PI with varying holding costs and values of α. We calculate the transporting and repositioning decision based on the optimal policies and uncertainty realization. Since the decisions of the LDR and RLDR formulations are the number of containers, we round transporting and repositioning quantities to be integers. The average of total operating costs over 10,000 samples and gap using foldable containers are summarized in Table 11. The gap shows the savings in operating costs using foldable containers when implementing optimal policies.

The cost ratio over total operating costs are presented in Fig. 4. RC represents average repositioning and transportation costs over

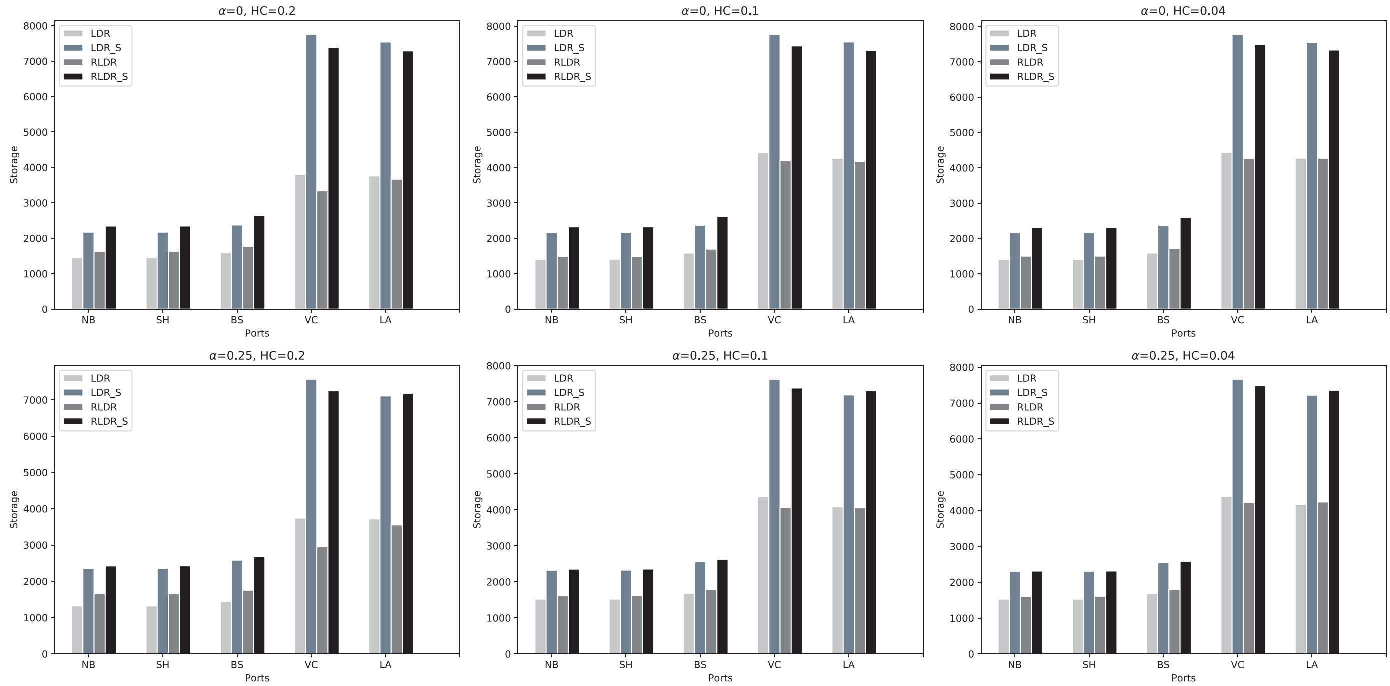

Fig. 5. Average storage of empty containers at ports.

Table 12
Total transportation and repositioning quantities of foldable containers between NA-Asia network.

		LDR				RLDR			
		NA to Asia		Asia to NA		NA to Asia		Asia to NA	
α	FC	TQ	RQ	TQ	RQ	TQ	RQ	TQ	RQ
0	0.05	47.1	2535.3	10900.8	0.0	0.0	2544.5	10950.8	0.0
	0.1	366.6	2531.0	10897.6	0.0	164.0	2356.5	10764.6	0.0
	0.2	392.5	2503.0	10873.7	0.0	99.0	2128.3	10523.4	0.0
	0.4	0.0	293.4	8706.1	0.0	0.0	0.0	8464.5	0.0
0.25	0.05	27.3	2832.5	11292.7	0.0	0.0	2817.5	11290.1	0.0
	0.1	331.8	2823.3	11252.4	0.0	183.0	2501.9	10921.4	0.0
	0.2	339.7	2632.1	11028.4	0.0	161.0	1976.7	10355.7	0.0
	0.4	0.0	2.3	8402.6	0.0	0.0	0.0	8439.1	0.0

samples and HC represents average holding and penalty costs over samples. FC represents folding and unfolding costs over samples. LDR and LDR_S represent the total operating cost and the cost ratio of the LDR formulation when using standard and foldable containers and using only standard containers, respectively. RLDR and $RLDR_S$ are defined similarly. Fig. 4 shows that even though the folding and unfolding costs are added, the significant savings of holding and penalty costs lead to reduction in the total operating costs. As unit holding cost decreases, the proportion of holding costs in total operating costs decreases and the cost-saving effect diminishes. One of the most influential advantages of using foldable containers is the saving in holding costs.

Another important advantage is the saving of storage space at the port. Empty standard containers occupy substantial space at the port, which causes port congestion and operations delays. Foldable containers are stored in the folded state at the port, which leads to considerable storage saving. The average storage at each port is presented in Fig. 5. Although the number of empty containers is not meaningfully reduced, the space taken for storage at the port diminishes substantially. The decline in import dominant ports such as Vancouver and Los Angeles is particularly notable, because supplied empty containers are typically stacked in import dominant ports. The saving in port storage leads to mitigation of port congestion and unnecessary operations that can not be captured by the cost-saving effect.

The utilization of foldable containers is highly influenced by the additional costs of using foldable containers, such as folding and unfolding costs. When using foldable containers, folding and unfolding operations are required with additional labor. Therefore, decisions about operations of foldable containers can change a lot depending on the folding and unfolding costs. We conduct simulations over various folding and unfolding costs and summarize transportation and repositioning quantities of foldable containers between NA and Asia in Table 12. TQ (RQ) represents total transportation (repositioning) quantities of foldable containers over the planning horizon, respectively. We compare two cases, NA to Asia (from VC, LA to NB, SH, BS) and Asia to NA (from NB, SH, BS to VC, LA), for analyzing decisions between imbalanced ports. Since the ports on the NA side are import-oriented ports, empty containers are stocked at NA ports and foldable containers are repositioned from NA to Asia. However, as the folding and unfolding costs increase, the repositioning of foldable containers decreases drastically. Because of insufficiencies of empty containers at Asian ports, repositioning of foldable containers does not occur and foldable containers are used to transport goods from Asia to NA. In this case, the utilization of foldable containers decreases as folding and unfolding costs increase. Therefore, it is clear that decisions about foldable containers are influenced by folding/unfolding costs, which may affect the adoption and active utilization of foldable containers.

6. Conclusions

In this paper, we consider the ECR problem with foldable containers under demand uncertainty. For incorporating demand uncertainty into the decision-making process, we propose a multistage stochastic programming formulation. However, in general, a multistage stochastic formulation is computationally intractable; therefore, we adopt the adjustable robust optimization technique and propose a tractable formulation with the LDR and RLDR. The two robust formulations are tractable approximations of a multistage stochastic programming formulation and have distributionally robust properties. Hence, we evaluate the performances of proposed formulations and compare the results with EV|PI benchmark. Furthermore, we validate the advantages of using foldable containers by showing the cost-saving and storage-saving effects through simulations with scenarios. We expect our model to serve as a bridge for analyzing the advantages of foldable containers under uncertainties.

For further research, we intend to extend our study to account for the pooling effects of foldable containers. In inventory management models, a centralized system with one large distribution center leads to lower variability than a decentralized system with many distribution centers. Likewise, we can view empty containers as inventory and repositioning decisions of empty containers as ordering decisions. There may be risk pooling effects similar to those in inventory management models, that is, the variability of operating costs may decrease because multiple folded containers act like one standard container. On the other hand, because of the tremendous operating costs of hinterland transport, the use of foldable containers in hinterland transport proves to be another interesting topic. In addition, port congestion and empty container movement with trucks are important topics in maritime logistics. Therefore, the use of foldable containers in hinterland transport will be influential to shipping companies and container terminals.

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Appendix A. Parameters

We generated instances for numerical experiments by aggregating several vessel schedules of Maersk, the world's largest shipping company. We considered the NA-Asia shipping network provided by Maersk (2019). The transportation time between Ningbo and Shanghai is 1–3 days, and the transportation time between Busan and Shanghai or Ningbo is 2–3 days according to the announced schedule. The transportation time to cross the Pacific Ocean is 10–20 days, depending on the ports and schedule. Therefore, we set a base period of 4 days to set the transportation time between Asian ports as 1 period and the transportation time to cross the Pacific Ocean as 4 periods (Table A.13). It is worth mentioning that actual travel time is not linearly dependent on the distance.

Likewise, the transportation cost also is not linearly dependent on the travel time or travel distance. There is a lot of demand between NA-Asia compared to the demand between Asian ports. The shipping company can enjoy the advantages of economies of scale in transportation between NA-Asia. Moreover, transportation costs include inland transportation costs and loading and unloading costs which must occur when the container is transported. These costs represent a considerable portion of transportation costs. For the above reasons, the unit transportation cost between ports does not depend on the actual travel time or travel distance. Therefore, we set unit transportation costs between Asian ports as 1 and between Asia and NA as 2 (Tables A.14 and A.15).

Table A.13
Transportation time between ports, τ_{ij}.

From-to	NB	SH	BS	VC	LA
NB	–	1	1	4	4
SH	1	–	1	4	4
BS	1	1	–	4	4
VC	4	4	4	–	2
LA	4	4	4	2	–

Table A.14
Unit transportation cost of standard containers, C^S_{ij}.

From-to	NB	SH	BS	VC	LA
NB	–	1	1	2	2
SH	1	–	1	2	2
BS	1	1	–	2	2
VC	2	2	2	–	1
LA	2	2	2	1	–

Table A.15
Unit transportation cost of foldable containers, C^F_{ij}.

From-to	NB	SH	BS	VC	LA
NB	–	1	1	2	2
SH	1	–	1	2	2
BS	1	1	–	2	2
VC	2	2	2	–	1
LA	2	2	2	1	–

Table A.16
Unit repositioning cost of standard containers, R^S_{ij}.

From-to	NB	SH	BS	VC	LA
NB	–	0.8	0.8	1.6	1.6
SH	0.8	–	0.8	1.6	1.6
BS	0.8	0.8	–	1.6	1.6
VC	1.6	1.6	1.6	–	0.8
LA	1.6	1.6	1.6	0.8	–

Table A.17
Unit repositioning cost of foldable containers, R^F_{ij}.

From-to	NB	SH	BS	VC	LA
NB	–	0.4	0.4	0.8	0.8
SH	0.4	–	0.4	0.8	0.8
BS	0.4	0.4	–	0.8	0.8
VC	0.8	0.8	0.8	–	0.4
LA	0.8	0.8	0.8	0.4	–

Table A.18
Capacity, K_{ij}.

From-to	NB	SH	BS	VC	LA
NB	–	300	300	1200	1600
SH	300	–	300	1200	1600
BS	400	400	–	800	1200
VC	600	600	400	–	400
LA	800	800	600	400	–

Table A.19
Support of uncertain factors, $\bar{z}_{ij}$.

From-to	NB	SH	BS	VC	LA
NB	–	5	5	10	20
SH	5	–	5	10	20
BS	8	8	–	8	10
VC	8	8	8	–	8
LA	10	10	8	8	–

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