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S.I. : APPLICATIONS OF OR IN DISASTER RELIEF OPERATIONS, PART II

Stochastic facility location model for drones considering uncertain flight distance

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Abstract

This paper developed a stochastic modelling framework to determine the locations and transport capacities of drone facilities for effectively coping with a disaster. The developed model is applicable to emergency planning that incorporates drones into humanitarian logistics while taking into account the uncertain characteristics of drone operating conditions. Because of the importance of speedy decision making in disaster management, a heuristic algorithm was developed using Benders decomposition, which generates time-efficient high-quality solutions. The linear programming rounding method was used to make the algorithm efficient. Computational experiments demonstrated the superiority of the developed algorithm, and a sensitivity analysis was carried out to gain additional insights.

Keywords Humanitarian logistics · Facility location · Stochastic programming · Drone

1 Introduction

E-commerce is experiencing double-digit growth, and one of the e-commerce giants, Amazon, has recently announced the consideration of rapid delivery using drones, also known as *unmanned aerial vehicles* (UAVs) (Ham 2018). With the recent popularization of e-commerce and the beginning of the Fourth Industrial Revolution, the study of logistics has regained momentum, and delivery by drone will be the next big development in the field. Drones are evolving beyond their military origin to become powerful business tools, and incorporating them makes delivery service faster and more convenient.

In the context of disaster management, drones can become very useful as a mode of transportation in humanitarian logistics because they do not need preexisting paths to make deliveries. While trains, boats, and trucks follow restricted pathways, drones can move anywhere and everywhere. Therefore, if a natural disaster strikes and roads are damaged, drones

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can be used as an alternative to serve the destroyed region. However, several concerns are associated with launching drones. Although integrating drones into humanitarian logistics seems efficient and convenient, some features of drones, such as the limited payload or the limited flight time, must be considered. Therefore, studying the operation methods that take into account the uncertain conditions of drone operation is imperative.

For this study, we were concerned with battery capacity, which is a key element of drone operation research. The energy consumption of drones is heavily influenced by payload, weather, and other environmental conditions. Because of the fluctuation in battery consumption, the maximum flight distance of a drone also inevitably varies. Therefore, we proposed a method to utilize drones efficiently in disaster situations while considering this uncertainty.

Various disasters do a great deal of damage and destroy tangible assets such as buildings and equipment. Worse, the catastrophic and fatal aftermath affect the economy and human life. To minimize damage and encourage fast recovery, the appropriate commodities must be delivered to the correct places and to the right people at the perfect time. To deal with this challenge, outstanding response-facility location models, involving the location and selection of distribution centers, warehouses, shelters, medical centers, and other facilities, are essential. In this study, we presented a drone facility location problem that determines the locations, numbers, and transport capacities of drone facilities. A drone facility is an infrastructure where drones can be docked, recharged, and restocked before flying out again for another delivery. Considering the uncertain characteristics of drone operation simultaneously, we developed a stochastic facility location model for a disaster-affected region where drones can be used as a mode of transportation for emergency supplies to demand points.

The remainder of this study was organized as follows: In the next section, we reviewed various important study and underlined research gap. In Sect. 3, we presented the formulation of a drone facility location model (DFLM) with stochastic coverage. We also presented the development of an efficient heuristic algorithm using Benders decomposition in Sect. 4. Numerical experiments were reported in Sect. 5, and conclusions were offered in Sect. 6.

2 Literature review

2.1 Drone operation research

As drones play a growing role in various fields, the study of drones also has increased in practical and academic importance. Much research using drones has already been carried out to understand these trends. Evers et al. (2014) developed the optimization model of information gathering in intelligence, surveillance and reconnaissance missions using drones. They introduced planning based on the orienteering problem to find tours for the drones that are effective in the face of uncertain fuel usage between targets. Burdakov et al. (2017) studied the problem of scheduling drone replacements while maintaining sufficient coverage during a perimeter-guarding task. Replacement strategies were separately introduced for odd and even numbers of drones, and the optimality of each replacement was proven. Kim et al. (2017) proposed a robust approach for finding an optimal flight schedule for drones by considering uncertain battery duration. A box uncertainty set was used to describe the uncertainty of schedule changes, and a numerical example was designed to show the feasibility of the proposed approach.

Interest in research for integrating drones into disaster management has also increased in recent years. Merwaday and Guvenc (2015) introduced a new way of using drones. They explained that when the communications infrastructure has been damaged during natural disasters, drones can be rapidly deployed as unmanned aerial base stations as part of the network architecture for public safety communications. Chowdhury et al. (2017) considered the drone as a mode of transportation to deliver emergency commodities in a disaster-affected region and developed a continuous approximation model that determines the distribution center locations, corresponding service regions, and the order quantities.

These previous studies on the technology and simple applicability of drones are insufficient for practical drone operations. Existing literature on drones has mostly focused on topics other than the planning or optimization of drone operations (Otto et al. 2018). Only a few studies have attempted an optimization approach that extended the traveling salesman problem (TSP). Murray and Chu (2015) and Agatz et al. (2018) published representative studies, and they formulated the TSP with a drone and a flying sidekick TSP. Various optimization approaches to drone operations as well as application of TSP should be extensively studied. A drone facility location problem that we studied in this paper can satisfy this need.

2.2 Facility location problem

Facility location has created challenging problems for many years. Because of the increasing severity of disasters, researchers have paid significant attention to a facility location model in disaster management. Therefore, a rich body of literature features facility location problems. Recently, Boonmee et al. (2017) conducted a survey on the facility location problems related to emergency humanitarian logistics and highlighted the extensive range of these types of problems developed since the 1950s. In addition, Ahmadi-Javid et al. (2017) classified different types of non-emergency and emergency healthcare facility locations in terms of location management, and they reviewed the literature based on location classification.

The model we presented in this paper belongs to a broad class of a set covering location problems that have been used to place a set of facilities in such a way that demand points can be served within some previously defined time or distance. Farahani et al. (2012) presented a literature review on set covering problems in facility location. Because they presented a very comprehensive review by considering publications through 2012, we discussed research that has emerged since 2013.

In many set covering location problems, a demand point is problematically assumed to be covered completely if it lies within the coverage radius of a facility and not covered at all outside the radius (Karatas 2017). Because this unrealistic modeling presumption may lead to potential errors and unjustified solutions, many researchers of recent facility location problem have expanded the model to deal with the uncertainty or to suggest a new framework considering advanced concepts such as cooperative coverage and demand sharing or like ideas.

Various methodologies have been studied to manage the uncertainty. For optimization under uncertainty, two approaches have been mainly used: One is stochastic optimization, in which uncertain parameters are allocated in a probability distribution, and the other is robust optimization. Meng and Shia (2013) formulated a new set covering model based on customer-determined stochastic critical distance. Pereira and Averbakh (2013) studied the robust set covering problem featuring uncertain costs. They developed three exact algorithms to find a robust deviation solution, two of which were based on Benders decomposition. Paul and MacDonald (2016) developed a stochastic optimization model to determine the locations and capacities of distribution centers such that losses were minimized in the event of a disaster. Data from an earthquake-related disaster in the Northridge region of California were used to validate the model. Tayal et al. (2017) formulated a new sustainable stochastic dynamic facility layout problem and optimized the material handling and rearrangement costs using various meta-heuristic techniques. Grass et al. (2018) proposed an accelerated L-shaped method to solve a realistic large-scale two-stage stochastic problem. They developed a realistic large-scale case study for the hurricane-prone south-eastern coast of the United States. Lee and Han (2017), Zhang et al. (2017), Khodaparasti et al. (2018), and Marín et al. (2018) also proposed set covering location models that accounted for uncertainty and developed efficient solution algorithms in recent years.

As can be seen from the literature review, stochastic facility location research has been actively conducted. However, few studies on integrating drones into a facility location problem have been published. Unlike that of previous studies, the developed model presented in this paper was used to consider the location of drone facilities for the efficient operation of drones, which is expected to be a new transportation mode and catalyst for innovation in disaster management and logistics. To the best of our knowledge, no attempts have been made to determine the location of drone facilities for which the stochastic flight distance of the drones was addressed. In addition, the DFLM was formulated as a set covering model in which the allocation of drones and the probability that a drone returns to a facility were considered. These practical constraints narrow the gap between the model and practice.

3 Drone facility location problem with stochastic coverage

As mentioned before, one of the critical factors that should be considered when operating drones is battery capacity, which can translate to traveling time. Weather circumstances, such as typhoons, have a great impact on the fuel consumption of a drone. Therefore, expected flight distance of a drone is not deterministic. In this study, we examined the effects of these uncertainties on disaster management and applied the stochastic optimization.

Uncertainty in travel time was addressed by Goldberg et al. (1990), who analyzed the mean and standard deviation of travel time using data from the Tucson Emergency Medical Services system. Possible traveling time leads to the maximum flight distance. So, the maximum flying duration of each drone is stochastic in this model. Because of the randomness of the maximum flight distance of drones, customer demand covered by a drone facility varies (Fig. 1). This variability of coverage requires more difficult decision making than the previous set-covering location models Therefore, we studied an extended version of the set-covering location model by introducing the stochastic flight distance of drones.

Our objective was to find the optimal locations and transport capacities of facilities according to the minimization of total relevant costs. We assumed that the sets of candidate locations, distance between them, and demands are known. The unit of demand was defined as the quantity of goods that can be transported by one drone delivery. Regardless of delivery distance, a drone can transport supplies only once per period, due to the need for maintenance and battery charging. Therefore, 50 drones are required for delivery to meet 50 demands in a particular location at a specific time. Generally, the facility location problem yields a decision outcome that has long-term effects, so the parameters of the system, such as demand points, operating and distribution costs, and environmental factors, may vary over time. Also, facility additions can be made at different times. In disaster situations, the parameters are particularly subject to variability; that is, decision of the location and the timing for building a facility becomes more critical. Therefore, we decided the operational timing and location for the drone facilities and the process for allocating customers to these drone facilities. The

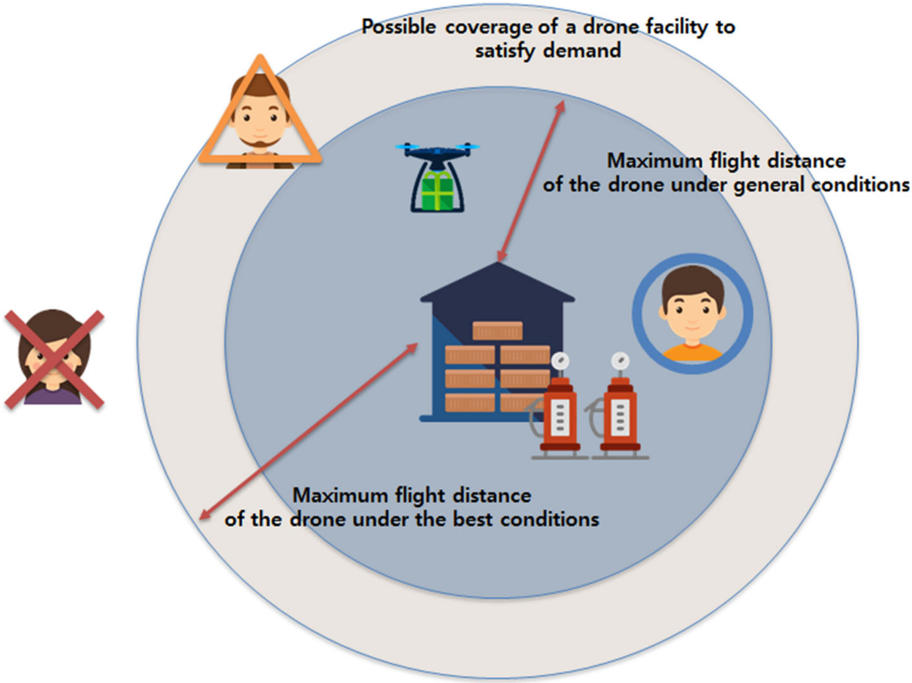

Fig. 1 Variable conditions for drone coverage of customer demand

transport capacity of the drone facility, which is the number of drones available for delivery, is determined simultaneously.

3.1 Notations

In this section, all the used sets, parameters, and decision variables are presented and discussed. The developed drone facility location problem involves three given sets: customers, potential locations of facilities to serve customer demands, and time periods. Let I be the set of facility candidate locations, J be the set of customers, and T be the set of time buckets over the planning horizon. The sets and parameters are defined to develop the mathematical model as follows:

- s_{jt} Demand of customer zone $j \in J$ in time $t \in T$
- S The largest value of all demands (i.e. $\max_{j,t} s_{jt}$)
- d_{ij} Distance between candidate location $i \in I$ and customer zone $j \in J$
- f_i Opening cost of a drone facility at location $i \in I$
- o_i Operating cost of a drone facility at location $i \in I$
- c_i Operation and maintenance cost of a drone at location $i \in I$
- n Maximum number of drones that a drone facility can operate in a unit time
- N Total number of drones that can be operated in a unit time
- α Objective probability that a drone returns to a drone facility
- λ Parameter of the flight distance distribution

Four decision variables are used to construct the developed mathematical model. The decision variables, except integer variable U_{it}, are binary. A description for each of the decision variables follows:

X_{it} 1 if a drone facility is operated at site i in time t, 0 otherwise.
Y_{ijt} 1 if customer j is covered by a drone facility at site i in time t, 0 otherwise.
Z_{it} 1 if a drone facility at site i is opened in time t, 0 otherwise.
U_{it} Number of drones operating from a drone facility at site i in time t.

3.2 Chance constraints

A *chance constraint*, also known as a *probabilistic constraint*, is necessary to manage the stochasticity of the proposed model. The chance constraint, which represents the probability that a drone returns safely to the drone facility is greater than or equal to α, is shown as follows:

$$Prob\big(d \geq 2d_{ij}\big) \geq \alpha \tag{1}$$

where d is a random variable that represents the distance traveled by a drone. In this study, the probability that a drone returns to the drone facility is considered instead of service level. If we had considered service level as a constraint, the drone delivery would be determined from the customer standpoint. So the drone may not return to the drone facility under a service level constraint. This failure to return causes disruption in the next delivery because the number of drones deployed in each drone facility is minimized. Therefore, the probability that a drone returns safely is a more important consideration than service level.

As an application of the chance constraints, the exponential effect of distance was applied. The exponential effect has been widely used in logistics because it is convenient to implement. Beckmann (1999) addressed the effect when the market share of goods or services decreases exponentially as the distance between supplier and customers increases. Berman et al. (2003) suggested the exponential effect as an alternative function for the coverage level. The exponential effect is partially covered when a demand node is between the lower and upper limits of the critical distance within the level of coverage; this position reflects a decreasing function of distance between a node and the closest facility. In our study, the method for dealing with the exponential effect differed from that of other studies. Our model captures the exponential effect by assuming that the flight distance of drones are exponentially distributed. This assumption leads to stochastic coverage of a facility. The probability density function and the cumulative distribution function of d, respectively, are shown as follows:

$$f(d) = \begin{cases} \lambda e^{-\lambda d} & d \geq 0 \\ 0 & d < 0 \end{cases}$$

$$F(d) = \begin{cases} 1 - e^{\lambda d} & d \geq 0 \\ 0 & d < 0 \end{cases}$$

The constraint shown in Eq. (1) can be transformed to a tractable constraint using exponential distribution. Remark 1 demonstrates the way we dealt with chance constraints under the assumption of the flight distance of drones.

Remark 1 (Chance constraints)

$$
\begin{aligned}
&Prob\big(d \geq 2d_{ij}\big) \geq \alpha \\
&\Rightarrow 1 - F\big(2d_{ij}\big) \geq \alpha \\
&\Rightarrow e^{-2\lambda d_{ij}} \geq \alpha \\
&\ Prob\big(d \geq 2d_{ij}\big) \times X_{jt} \geq \alpha \times Y_{ijt} \Rightarrow e^{-2\lambda d_{ij}} \times X_{jt} \geq \alpha \times Y_{ijt}
\end{aligned}
$$

3.3 Mathematical model of DFLM(P)

This subsection describes the model that determines the optimal location, the opening time of drone facilities, and the number of drones deployed from each facility over the planning time horizon.

$$minimize \sum_{i \in I} \sum_{t \in T} (o_i X_{it} + f_i Z_{it} + c_i U_{it}) \tag{2}$$

$$subject\ to$$

$$e^{-2\lambda d_{ij}} X_{it} \geq \alpha Y_{ijt} \quad \forall i \in I, j \in J, t \in T \tag{3}$$

$$\sum_{i \in I} Y_{ijt} \geq \frac{s_{jt}}{S} \quad \forall j \in J, t \in T \tag{4}$$

$$X_{i,t} - X_{i,t-1} \leq Z_{it} \quad \forall i \in I,\ t \in T \tag{5}$$

$$U_{it} \geq \sum_{j \in J} s_{jt} Y_{ijt} \quad \forall i \in I, t \in T \tag{6}$$

$$U_{it} \leq n X_{it} \quad \forall i \in I, t \in T \tag{7}$$

$$\sum_{i \in I} U_{it} \leq N \quad \forall t \in T \tag{8}$$

$$X \in B^{|I| \times |T|}, Y \in B^{|I| \times |J| \times |T|}, Z \in B^{|I| \times |T|}, U \in Z_{+}^{|I| \times |T|} \tag{9}$$

The objective function (2) minimizes the sum of the total relevant costs comprising the costs of opening drone facilities and the operation and maintenance of drones and drone facilities. Constraints (3) guarantee that the probability of drone return to a facility is higher than a target level. Constraints (4) guarantee that demands are covered. Constraints (5) link constraints of the decision for opening a facility. Constraints (6) represent the minimum number of drones needed to satisfy demand. Constraints (7) ensure that drones can be operated only in operating facilities. Constraints (8) indicate that the maximum number of drones can be operated in a unit time. Constraints (9) demonstrate the binary and integer nature of the decision variables.

3.4 Discussion of the DFLM

Several features of the model warrant some discussion either to indicate practical constraints for which the DFLM differs from the related models found in the literature or to point out the flexibility that the DFLM affords. Remarks on the DFLM are in order.

First, it is necessary to analyze how the chance constraints affect the coverage of a drone facility. In the deterministic approach, the coverage of the drone facility is fixed, whereas in the stochastic approach, there is no clear boundary of coverage. From a practical point of view, the chance constraints do not provide a broader feasible solution region than deterministic constraints. Constraints (3) can be replaced by the following Constraints (3–2) under a general deterministic assumption:

$$\frac{1}{\lambda}X_{it} \geq 2d_{ij}Y_{ijt} \quad \forall i \in I, j \in J, t \in T \tag{3–2}$$

Constraints (3–2) mean that a customer's round-trip distance from the operating drone facility should be shorter than the possible flight distance that a drone can cover. Constraints (3) and (3–2) are similar in form, with the difference in the cost of the decision variables. Because λ and d_{ij} are positive values, the following inequality is obtained:

$$\alpha e^{2\lambda d_{ij}} \geq 2\lambda d_{ij} \quad \text{for } \forall \frac{1}{e} \leq \alpha$$

The objective probability that a drone returns to a facility is usually greater than 0.8 or 0.9 considering the practicality. Therefore, the chance constraint was seen to enforce tighter bounds on the coverage of a drone facility in the real application.

Second, it may be questioned whether it is reasonable to consider the operation and maintenance costs of the drones, not the purchase cost. There was a lack of evidence that the purchase cost of the drones would vary depending on the location of the facility. Therefore, the costs associated with the drones are imposed as constants in proportion to the total amount of demand, and taking the purchase cost into account does not cause trade-off with other decision variables.

Third, compared to terrestrial base stations, a drone facility, such as a drone dock, can be easily established to provide on-demand coverage for customers. Therefore, the number of facilities can be easily adjusted as needed. In a typical multi-period facility location model, operating facilities are maintained until all deliveries are made, but in this model, the facility may be abandoned. These changes are undertaken because not only the costs of opening the drone facility but also the operating costs incurred during each period were considered. If the allocated demand area were satisfied by other nearby facilities, then a facility could temporarily suspend operations. This ease of response to change by drone facilities provides flexibility for decision making and enables the DFLM to be used for relatively quick and accurate reactions to disasters or for the relatively more efficient design of supply chains.

4 Solution techniques using Benders decomposition and linear programming relaxation

The very core of decision making in humanitarian logistics is the agility that can save as much time as possible. Most real-life applications of the response facility location problem are extensive and difficult to solve economically. Furthermore, the lag time due to slow decision making takes away precious moments from the golden time during which the majority of causalities are saved. In addition, multiple runs are often required in disaster management because of difficulties in precisely ascertaining future networks, demands and costs. Benders decomposition, a strategy for solving large-scale optimization problems, offers a fix to this situation.

The Benders decomposition algorithm is used to solve a master problem (MP) and a slave problem (SP) iteratively. The SP solution provides information on the assignment of the MP variables in every iteration. Such information, expressed as a Benders cut, restricts assignments that are unacceptable. The Benders cut narrows the search space of the feasible region, which eventually leads to optimality. Benders cuts generated to solve one problem can be valid in a modified version of the same problem, with few revisions or additional effort. (Geoffrion and Graves 1974). Therefore, the Benders decomposition approach offers the possibility of making sequences of related runs in considerably reduced computing times. These useful characteristics enable effective responses to a changing disaster situation.

4.1 Master problem and slave problem

By the use of Benders decomposition, the developed problem, P, is decomposed into the MP that solves X, Y, and Z variables and the slave problem, $\mathrm{SP}(\bar{X}, \bar{Y}, \bar{Z})$, that solves only the U variables by fixing the X, Y, and Z variables to the MP solution and referring to them as $\bar{X}, \bar{Y}, \bar{Z}$. The MP is solved to plan the location and the opening time of a drone facility, and the SP is solved to determine the optimal number of drones that should be deployed from each drone facility. The mathematical models of the MP and SP for the DFLM are as follows:

Master problem (MP)

$$minimize \sum_{i \in I} \sum_{t \in T} (o_i X_{it} + f_i Z_{it}) + \sigma$$

$$subject\ to$$

$$\text{Constraints } (3),\ (4),\ (5) \text{ and } \sigma \geq 0$$

$$X \in B^{|I| \times |T|}, Y \in B^{|I| \times |J| \times |T|}, Z \in B^{|I| \times |T|}$$

Slave problem (SP)

$$minimize \sum_{i \in I} \sum_{t \in T} c_i U_{it}$$

$$subject\ to$$

$$\text{Constraints } (8)$$

$$U_{it} \leq n\bar{X}_{it} \quad \forall i \in I, t \in T \tag{10}$$

$$U_{it} \geq \sum_{j \in J} s_{jt} \bar{Y}_{ijt} \quad \forall i \in I, t \in T \tag{11}$$

$$U \in Z_+^{|I| \times |T|} \tag{12}$$

4.2 Benders cuts using linear programming relaxation of the slave problem

The generation of valid Benders cuts is the basis of Benders decomposition. Good Benders cuts guarantee the convergence of the iterations and determine how fast the algorithm converges (Chu and Xia 2004). However, it is difficult to generate valid Benders cuts for the model in this study because of the duality gap of the integer programming in the SP. One possible way to use Benders cuts is to employ the *no-good cut* method to exclude only current tentative assignment of MP variables that are unacceptable. Such no-good cuts result in an enumerative search and thus slow convergence. However, quick decision making is important in humanitarian logistics. Therefore, this strategy is impractical because it may lead to enumerating all extreme points in the SP.

Linear programming (LP) relaxation can be an alternative to this matter. LP relaxation is a standard technique for designing approximation algorithms. It is done by assuming that decision variables in the SP are real numbers. Then, valid Benders cuts can be generated efficiently according to the strong duality property. Fractional solutions can be rounded off to obtain feasible integer solutions. This approach is usually called an *LP-rounding* method, and it has been successfully used as a strategy for the relaxed Benders algorithm. Shmoys et al. (1997) offered exemplary research by applying LP-rounding strategy to relax the SP into an LP problem. The relaxed SP (RSP) can be formulated with decision variable V which is the relaxation of U. Dual values and variables are very useful in the Benders cut generation. The RSP can be converted to the dual problem as follows:

Dual of slave problem (DSP)

$$maximize \sum_{i\in I}\sum_{t\in T}\left(\sum_{j\in J} s_{jt}\bar{Y}_{ijt}P_{it} - n\bar{X}_{it}Q_{it}\right) - \sum_{t\in T} NR_t$$

$$subject\ to$$

$$P_{it} - Q_{it} - R_t \leq c_i \quad \forall i \in I, t \in T$$

$$P_{it}, Q_{it}, R_t \geq 0 \quad \forall i \in I, t \in T$$

where P_{it}, Q_{it} and R_t are dual variables of the SP. If the DSP is unbounded (such that the primal SP is infeasible), the extreme rays are used to add feasibility cuts to the restricted MP, RMP. If both the primal and dual SP have finite optimal solutions, but the two optimal solutions are different, then a new optimality cut is added to the RMP. Feasibility cuts and optimality cuts are defined in Remark 2.

Remark 2 (Feasibility and optimality cuts) The feasibility cut is

$$\sum_{i\in I}\sum_{t\in T}\left(\sum_{j\in J} s_{jt}\overline{PR}_{it}^{(k)}Y_{ijt} - n\overline{QR}_{it}^{(k)}X_{it}\right) - \sum_{t\in T} N\overline{RR}_t^{(k)} \leq 0$$

where $\overline{PR}_{it}^{(k)}$, $\overline{QR}_{it}^{(k)}$, and $\overline{RR}_t^{(k)}$ are extreme rays of the feasible regions in the SP for iteration k.

The optimality cut is

$$\sigma \geq \sum_{i\in I}\sum_{t\in T}\left(\sum_{j\in J} s_{jt}\bar{P}_{it}^{(k)}Y_{ijt} - n\bar{Q}_{it}^{(k)}X_{it}\right) - \sum_{t\in T} N\bar{R}_{tt}^{(k)}$$

where $\bar{P}_{it}^{(k)}$, $\bar{Q}_{it}^{(k)}$, and $\bar{R}_{it}^{(k)}$ are extreme points for interation k.

4.3 Heuristic algorithm for the DFLM

According to Benders decomposition and LP-rounding, the heuristic algorithm was developed for fast decision making. Algorithm 1 presents a pseudo-code of the developed heuristic approach.

Algorithm 1 Heuristic algorithm for the DFLM

Step 1 *Initialization.* Construct the initial master problem, $MP^{(0)}$. Set the number of iteration $k = 0$ and a tolerance parameter $\varepsilon \geq 0$. Initialize $\bar{\emptyset}_{MP}^{(0)} = 0$, $\bar{\emptyset}_{SP}^{(0)} = 0$, and $UB = \infty$.

Step 2 *Problem solving procedure*

i. Solve $MP^{(k)}$. If it is feasible, obtain the optimal solution $\bar{X}^{(k)}$, $\bar{Y}^{(k)}$, $\bar{Z}^{(k)}$ and the optimal objective value $\bar{\emptyset}_{MP}^{(k)}$. Otherwise, set $\bar{\emptyset}_{MP}^{(k)} = \infty$ and go to *Step 4*.
ii. Construct the slave problem $\mathrm{SP}\big(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}\big)$. Relax $\mathrm{SP}\big(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}\big)$ to the LP problem $\mathrm{RSP}\big(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}\big)$. If $\mathrm{RSP}\big(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}\big)$ is feasible, then obtain the optimal solution $\bar{V}^{(k)}$ and the optimal objective value $\bar{\emptyset}_{SP}^{(k)}$.

Step 3 *Cut generation procedure*. Return the current $\bar{\emptyset}_{MP}^{(k)}, \bar{\emptyset}_{SP}^{(k)}$ as the result value, and update $UB = \sum_{i \in I} \sum_{t \in T} \Big(o_i \bar{\mathrm{X}}^{(k)} + f_i \bar{\mathrm{Z}}^{(k)} \Big) + \bar{\emptyset}_{SP}^{(k)}$. Generate a valid Benders cut and add it to the master problem, $MP^{(k)}$ to construct $MP^{(k+1)}$. If $\mathrm{RSP}\big(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}\big)$ is feasible, add optimality cut; otherwise add feasibility cut.

Step 4 *Termination*. Terminate if $\frac{UB - \bar{\emptyset}_{MP}^{(k)}}{\bar{\emptyset}_{MP}^{(k)}} \leq \varepsilon$. If $\bar{\emptyset}_{MP}^{(k)} = \infty$, then the problem, P, is infeasible. If Algorithm 1 terminates and P is feasible, round up $\bar{V}^{(k)}$ to $\bar{U}^{(k)}$ and return the current $\big(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}, \bar{U}^{(k)}\big)$ as a solution of P. Otherwise, set $k = k+1$ and go back to Step 2.

Finite convergence of Algorithm 1 is assured for any given $\varepsilon \geq 0$. The variables X, Y, and Z are binary, and a feasible region of MP consists of a discrete set. Therefore, Algorithm 1 terminates in a finite number of steps according Theorem 2.4 in Geoffrion (1972).

The process of rounding the relaxed decision variables to the next integer for the number of drones appears in Step 4. Fractional solutions are infeasible because the DFLM is originally an integer programming problem. Hence, this process is necessary and can guarantee the feasibility of the solution. Rounding off to the nearest integer, rather than the higher one, offers a possible solution for a decision maker who prefers a lower cost and is not concerned that the number of returning drones might fall below the target level. In our opinion, satisfying the constraints takes priority over costs in humanitarian logistics. The objective probability for a drone returning to a facility is directly related to a smooth restoration. Therefore, a rounding-up constraints-oriented approach was chosen for the algorithm.

4.4 Discussion of the heuristic algorithm

Some notable characteristics of the algorithm are described in this section. A drone facility can presumably operate a sufficient number of drones because drones incur lower fixed and operation costs than other means of transportation. Condition (13) means the bound that allows for a sufficient number of drones for delivery.

$$\min(n, N) \geq \max_t \sum_{j \in J} s_{jt} \tag{13}$$

It is not difficult to verify that these assumptions make the SPs and RSPs always feasible. If Condition (13) is satisfied, Constraints (7) and (8) in the DFLM become redundant. Then, Constraints (8) and (10) can be removed from the SP without changing the solution, and the

SP can always find a feasible solution. Therefore, in this situation, only the optimality cut is added to MP.

Under the condition that demand s_{jt} is given as an integer, SP is easily solved. Because $\bar{Y}_{ijt}$ is the binary solution of Y_{ijt} satisfying the constraints in MP. $U_{it} = \sum_{j \in J} s_{jt}\bar{Y}_{ijt}$ for $\forall i \in I, t \in T$ is an optimal solution for a feasible SP. For the SP with integer demand, an optimal solution to the respective LP relaxation is the same. The duality gap is zero in this case, and the SP holds strong duality. Therefore, if ε converges to zero, the heuristic algorithm for the DFLM becomes exact algorithm and finds an optimal solution.

5 Computational results

To verify the performance of the DFLM and evaluate the efficiency of the developed algorithm, computational experiments were conducted with an Intel(R) Core(TM) 3.2 GHz processor with 8 GB RAM in the Microsoft Windows 10 operating system. The mathematical model and the heuristic algorithm for the DFLM were solved with FICO Xpress version 7.3 (http://www.fico.com). Examples were generated according to instances on the Euclidean plane of simple location problems from the Benchmarks Library (http://www.math.nsc.ru/AP/benchmarks/english.html). The transportation costs between nodes in the benchmark sets were converted to distances between nodes.

5.1 Description of experiments

According to the DFLM, the size of a problem was determined by the number of facility candidate locations ($|I|$), the number of customers ($|J|$), and the size of the time bucket ($|T|$). The maximum numbers of facility candidate locations and customers were limited to 100 because of the dimension of the matrix in Benchmarks Library. Table 1 shows 30 different problem classes ranging from small to large.

Many parameters have been generated on the basis of work by Shavarani et al. (2018), who studied the San Francisco case. As noted in Shavarani et al. (2018), no accurate data on the costs related to a response facility are available. Therefore, we randomly generated the opening and the operating costs of a drone facility from the uniform distribution. Operation and maintenance cost of a drone was generated by referring to Shavarani et al. (2018). Table 2 shows the range of the costs and demands that had integer values.

Amazon reported that the drones to be used for delivery feature a travel radius of 16 km, which is equivalent to an endurance of 32 km for round trip delivery (Shavarani et al. 2018). Therefore, the parameter for the flight distance distribution of a drone was set to 0.00003125. The default setting for the objective probability that a drone returns to a facility is 0.8. For convenience in the analysis, parameters n and N were each set to 10,000 to prevent an infeasible solution. In the following three subsections, computational results and the analysis were presented.

5.2 Sensitivity analysis on different parameter

Sensitivity analysis can be done by changing some input parameters in the DFLM. First, we examined the probability that a drone returns to a facility, which affects the chance constraints and is related to the success rate of delivery. Table 3 shows a trade-off between costs and target levels. Some obvious managerial insights can be confirmed through the results. The

Table 1 Problem classes generated from Benchmarks Library

Class	Code in Library	$\|I\|$	$\|J\|$	$\|T\|$
C1	111	10	10	3
C2	211	10	20	3
C3	311	10	30	3
C4	411	10	40	3
C5	511	10	50	3
C6	611	10	60	3
C7	711	10	70	3
C8	811	10	80	3
C9	911	10	90	3
C10	1011	10	100	3
C11	1111	100	10	3
C12	1211	100	20	3
C13	1311	100	30	3
C14	1411	100	40	3
C15	1511	100	50	3
C16	1611	100	60	3
C17	1711	100	70	3
C18	1811	100	80	3
C19	1911	100	90	3
C20	2011	100	100	4
C21	2111	10	10	4
C22	2211	20	20	3
C23	2311	30	30	3
C24	2411	40	40	3
C25	2511	50	50	3
C26	2611	60	60	3
C27	2711	70	70	3
C28	2811	80	80	3
C29	2911	90	90	3
C30	3011	100	100	5

Table 2 Range of the parameter values

f_i	o_i	c_i	s_{jt}
U[300,000, 400,000]	U[30,000, 40,000]	U[2, 8]	U[1, 100]

higher the target level, the more facilities need to be built to close the distance to the demand area, which results in higher costs. As can be seen, the facilities were built in more costly locations to achieve higher target levels in some cases.

Analysis were also performed with distributions of flight distance that a drone can fly. The results are summarized in Table 4. Because the smaller λ refers to the longer distance a drone can fly, the results show that the cost decreases as λ decreases. Therefore, fewer drone

Table 3 Sensitivity analysis for different α values

α	Class	Total cost ($)	$\sum_{i\in I}\sum_{t\in T} Z_{it}$	Class	Total cost ($)	$\sum_{i\in I}\sum_{t\in T} Z_{it}$
0.9	C1	2,157,495	5	C11	1,768,044	4
0.8	C1	849,160	2	C11	819,192	2
0.7	C1	436,774	1	C11	407,325	1
0.6	C1	417,470	1	C11	407,325	1
0.5	C1	417,470	1	C11	407,325	1
0.9	C2	Infeasible	–	C12	2,626,039	6
0.8	C2	859,718	2	C12	862,415	2
0.7	C2	429,552	1	C12	415,340	1
0.6	C2	429,545	1	C12	410,709	1
0.5	C2	429,545	1	C12	410,709	1
0.9	C3	Infeasible	–	C13	2,599,149	6
0.8	C3	909,376	2	C13	850,948	2
0.7	C3	846,914	2	C13	422,750	1
0.6	C3	435,048	1	C13	413,673	1
0.5	C3	435,048	1	C13	413,673	1

facilities are necessary when drones can fly longer distances. If the distance that a drone can fly is too short, solving the DFLM is impossible under the given conditions.

5.3 Comparison between deterministic approach and stochastic approach

The business performance of the DFLM and a pure deterministic approach mentioned in Sect. 3.4 were compared. Because the flight distance of a drone is not deterministic, in some circumstances the drones may not return to the drone facility. When a drone does not return, the next delivery by drone is disrupted. More seriously, the delivery that the drone was assigned might have failed to transpire. Satisfying demand during a disaster and the subsequent recovery is critical because it is linked to the safety of the disaster victims.

In this subsection, we verified the need for stochastic modeling in the design of the network considering the drones. The networks were designed by solving the same problem through the deterministic and stochastic model. Afterwards, we looked at how the results changed when the flight distance of a drone follows the normal distribution and the exponential distribution. The mean and variance of the two distributions were $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$, respectively. The evaluation metrics used for comparison were as follows:

- α_N Probability that a drone returns to a facility when flight distance of a drone follows normal distribution
- α_E Probability that a drone returns to a facility when flight distance of a drone follows exponential distribution
- β_N Number of drones that failed to return to the drone facility when flight distance of a drone follows normal distribution
- β_E Number of drones that failed to return to the drone facility when flight distance of a drone follows exponential distribution

Table 4 Sensitivity analysis for different λ values

λ	Class	Total cost ($)	$\sum_{i \in I} \sum_{t \in T} Z_{it}$	Class	Total cost ($)	$\sum_{i \in I} \sum_{t \in T} Z_{it}$
0.00009	C1	2,157,560	5	C11	2,280,740	5
0.00008	C1	2,157,495	5	C11	2,158,056	5
0.00007	C1	2,157,495	5	C11	2,123,846	5
0.00006	C1	2,157,495	5	C11	1,707,678	4
0.00005	C1	1,707,987	4	C11	1,282,127	3
0.00004	C1	852,520	2	C11	831,025	2
0.00003	C1	849,160	2	C11	819,192	2
0.00002	C1	436,774	1	C11	407,325	1
0.00001	C1	417,470	1	C11	407,325	1
0.00009	C2	Infeasible	–	C12	4,687,675	11
0.00008	C2	Infeasible	–	C12	3,422,670	8
0.00007	C2	Infeasible	–	C12	2,798,596	6
0.00006	C2	Infeasible	–	C12	2,137,032	5
0.00005	C2	Infeasible	–	C12	1,700,968	4
0.00004	C2	890,065	2	C12	1,304,549	3
0.00003	C2	839,465	2	C12	862,415	2
0.00002	C2	429,952	1	C12	415,340	1
0.00001	C2	429,545	1	C12	410,709	1
0.00009	C3	Infeasible	–	C13	3,566,052	8
0.00008	C3	Infeasible	–	C13	3,457,633	8
0.00007	C3	Infeasible	–	C13	3,005,912	7
0.00006	C3	Infeasible	–	C13	2,171,188	5
0.00005	C3	Infeasible	–	C13	1,748,157	4
0.00004	C3	Infeasible	–	C13	1,338,633	3
0.00003	C3	909,376	2	C13	834,187	2
0.00002	C3	847,418	2	C13	422,750	1
0.00001	C3	435,048	1	C13	413,673	1

Table 5 Results of the deterministic and stochastic approaches for C1

	Total cost ($)	$\sum_{i \in I} \sum_{t \in T} Z_{it}$	α_N (%)	β_N	α_E (%)	β_E
Deterministic approach	417,470	1	78.3	293.7	81.5	239.5
Stochastic approach	849,160	2	81.5	250.8	90.1	134.2

Problem C1 was used in the experiment, and each evaluation metric value was averaged after 20 repeated experiments. Table 5 summarizes the experimental results to show how each approach copes with uncertainty in terms of evaluation metrics.

Simply looking at network design costs, a pure deterministic approach can lead to overly optimistic judgments about efficiency. However, looking at the evaluation metrics, we can

Table 6 Results of the deterministic and stochastic approaches for C1 when λ = 0.0000625

	Total cost ($)	$\sum_{i \in I} \sum_{t \in T} Z_{it}$	α_N (%)	β_N	α_E (%)	β_E
Deterministic approach	417,470	1	74.5	345.8	74.3	348.5
Stochastic approach	2,157,495	5	83.5	224.3	96.5	47.0

see that the stochastic model copes with uncertainty much better than the deterministic model does. The difference became clear when the flight distance of drones follows an exponential distribution. This is a natural result because the network was designed assuming that the flight distance of drones follows an exponential distribution. If we consider the purchase cost of new drones or the penalty for the delay in subsequent shipping, it is difficult to say which approach dominates in economic terms. To obtain additional insight, we looked at the case of reducing the flight range of drones. Table 6 shows the experimental results when λ was readjusted to 0.0000625.

After doubling the parameters, the differences between the two approaches became clearer. More facilities were established to achieve the target level as the flight range of drones was reduced. As a result, drones returned to the drone facility with a very high probability. It was shown that the distances between the drone facility and customers have a significant impact on the return probability of drones. It was also found that the network was designed to be sustainable overall because even the farthest customer assigned to each drone facility satisfied the target level.

5.4 Comparison between the DFLM and heuristic algorithm

Experiments on various problem classes were performed to confirm the time efficiency of the developed heuristic algorithm. Results of the instances were summarized in Table 7. The gap between the best solution and the best bound was small in the DFLM. However, experimenting with increasing the time limit, it was difficult to find the optimal solution for large size problems through the original model. Some problems could not be solved in the time limit of 3600 s by the DFLM while the heuristic algorithm found a solution. Even if both methods could not solve the problem, the heuristic algorithm yielded a better integer feasible solution within the time limit. As the size of the problem increased, the heuristic algorithm yielded better solutions than the original DFLM. Therefore, it can be said that time efficiency and superiority of the heuristic algorithm were demonstrated.

6 Conclusions

Although drones are unlikely to replace existing technologies completely, they have emerged as alternative options to supplement and complement land- or sea-bound vehicles. Therefore, integrating drones into humanitarian logistics is expected to be efficient and convenient. However, operation methods determined without considering the uncertain conditions of drone operations might produce a negative outcome. In this paper, we provided a stochastic design framework of ways to deploy drones economically to serve a disaster-affected region

Table 7 Comparison between the DFLM and a heuristic algorithm

Class	DFLM			Algorithm 1 ($\varepsilon = 10^{-9}$)		Algorithm 1 ($\varepsilon = 10^{-2}$)	
	Total cost	Best bound	Time (s)	Total cost	Time (s)	Total cost	Time (s)
C1	849,160	849,160	0.59	849,160	0.36	849,160	0.35
C2	859,718	859,718	1.65	859,718	0.69	859,718	0.87
C3	909,376	909,376	1.11	909,376	0.98	909,376	0.84
C4	1,427,428	1,419,362	1199.65	1,423,998	2.06	1,423,998	2.16
C5	1,376,959	1,374,376	3600*	1,376,943	1.69	1,376,943	1.45
C6	1,881,041	1,874,218	3600*	1,880,078	2.52	1,880,078	2.25
C7	1,414,787	1,410,625	3600*	1,414,787	6.11	1,414,787	3.87
C8	Infeasible	–	3.75	Infeasible	1.48	Infeasible	1.82
C9	1,324,900	1,310,827	3600*	1,323,078	7.14	1,323,078	6.24
C10	1,381,471	1,369,414	3600*	1,381,195	6.27	1,381,195	5.08
C11	819,192	819,192	321.75	819,192	21.19	819,192	16.31
C12	862,415	862,415	66.15	862,415	12.56	862,415	14.32
C13	850,948	850,948	500.48	850,948	53.18	851,620	24.78
C14	896,188	895,740	3600*	896,188	189.73	896,359	29.71
C15	1,259,889	1,250,508	3600*	1,255,679	442.15	1,255,679	324.78
C16	1,250,085	1,243,141	3600*	1,261,363	3600*	1,261,363	3600*
C17	955,895	949,376	3600*	955,895	3600*	955,895	3409.63
C18	1,268,390	1,261,861	3600*	1,268,358	3600*	1,268,358	3600*
C19	1,300,603	1,159,146	3600*	1,283,427	3600*	1,283,427	3109.42
C20	2,173,227	1,290,315	3600*	1,406,073	3600*	1,406,073	3600*
C21	899,799	899,799	0.95	899,799	0.44	899,799	0.56
C22	1,262,362	1,262,362	4.11	1,262,362	3.43	1,262,362	3.73

Table 7 continued

Class	DFLM			Algorithm 1 ($\boldsymbol{\varepsilon} = 10^{-9}$)		Algorithm 1 ($\boldsymbol{\varepsilon} = 10^{-2}$)	
	Total cost	Best bound	Time (s)	Total cost	Time (s)	Total cost	Time (s)
C23	1,261,634	1,261,508	3600*	1,261,634	12.76	1,261,634	12.03
C24	976,570	974,590	3600*	976,570	90.27	976,570	25.04
C25	1,281,181	1,279,701	3600*	1,280,919	53.68	1,287,061	51.25
C26	1,294,840	1,294,711	3600*	1,294,840	3063.97	1,296,371	2490.28
C27	924,775	921,781	3600*	924,409	153.05	924,409	120.56
C28	1,280,807	1,271,467	3600*	1,280,553	1328.62	1,280,553	818,52
C29	1,278,449	1,233,711	3600*	1,275,421	3600*	1,275,421	3089.27
C30	1,005,983	977,363	3600*	999,453	3600*	999,453	3600*

*Time limit reached

with uncertain drone flight distance. The developed model is used to determine the optimal locations for the drone facilities and the capacity, which is the number of drones deployed from each facility.

For agile decision making, we developed a heuristic algorithm that produces a high-quality solution. The heuristic algorithm was developed according to Benders decomposition and the LP-rounding technique. The computational results showed that using a heuristic algorithm can reduce delays in decision making. This time efficiency enables effective real-time response in disaster situations. Another meaningful conclusion arising from this study is the remarkable effectiveness of Benders decomposition as a computational strategy for disaster management.

For researchers, we believe that the approach developed is applicable to a range of disaster management operations and opens up a number of future research opportunities. There might be a controversy about the assumption that the flight distance of a drone follows an exponential distribution. Although similar assumptions are used in various fields, some researchers may not accept this assumption. A robust approach, instead of a stochastic optimization, might be applied to this model in the future research. This approach can alleviate complaints about the distributional assumption. We also did not consider costs and times for battery recharging. We assumed that these costs are considered to have a relatively small impact on decision making. However, further consideration of various situations and variables can also yield meaningful conclusions. With consideration of these points, solutions that incorporate drones into disaster management might become more practical and can provide deeper insights.

For practitioners, the approach of this study can give answers to their questions about practicality. Designing constraints by considering the uncertain features of drones and making them tractable provide practitioners with a guideline for the practical use of drones. Furthermore, the DFLM can be generalized to other facility location problems dealing with uncertainty. It can be used not only for disaster management but also for commercial purpose which helps to fully utilize this emerging technology.

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