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## Simultaneous evacuation and entrance planning in complex building based on dynamic network flows

Youngchul Shin<sup>a</sup>, Sungwoo Kim<sup>b</sup>, Ilkyeong Moon<sup>c,\*</sup>

<sup>a</sup> Department of Industrial Engineering, Seoul National University, Seoul, Korea

<sup>b</sup> H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, North Ave NW, Atlanta, GA 30332, USA <sup>c</sup> Department of Industrial Engineering and Institute for Industrial Systems Innovation, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul, Korea

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#### ABSTRACT

This paper presents mathematical models and a heuristic algorithm that address a simultaneous evacuation and entrance planning. For the simultaneous evacuation and entrance planning, four types of mathematical models based on the discrete time dynamic network flow model are developed to provide the optimal routes for evacuees and responders within a critical timeframe. The optimal routes obtained by the mathematical models can minimize the densification of evacuees and responders into specific areas. However, the mathematical model has a weakness in terms of long computation time for the largesize problem. To overcome the limitation, we developed a heuristic algorithm. We also analyzed the characteristics of each model and the heuristic algorithm by conducting case studies. This study pioneers area related to evacuation planning by developing and analyzing four types of mathematical models and a heuristic algorithm which take into account simultaneous evacuation and entrance planning.

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## 1. Introduction

Numerous large cities have confronted various problems related to overpopulation. These municipalities encounter tremendous damage, including casualties, when faced with terrorism, fire, or natural disaster. Recently, global threats and emergencies, such as the terrorist attacks in France in 2015 and Belgium in 2016, have garnered significant attention and highlighted the importance of research on evacuation plans. When emergencies occur in a congested area or inside a building, victims search for the nearest exit [1]. In so looking, they tend to follow the majority of other evacuees such that they move in the same direction [2]. Because of such behavior, numerous evacuees reach exits simultaneously, creating a queue. Consequently, the evacuation time for all victims is extended; that is, the local optimization of each evacuee precludes the global optimization for all the evacuees. Therefore, crowded exits should be efficiently prevented by assigning all evacuees to appropriate exits. Meanwhile, real-time analysis of the location of the emergency and the occupants in the building has become possible because of the advances of modern technology including the internet of things, communications networks, and process methods for big data. Accordingly, people onsite can be apprised of dangerous situations and escape based on the information in real-time. The rapid development in information technology makes possible evacuation planning to use effectively and efficiently in responding to emergencies.

\* Corresponding author.

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E-mail addresses: chulbee@snu.ac.kr (Y. Shin), asdf1358@gmail.com (S. Kim), ikmoon@snu.ac.kr (I. Moon).

From the past, much research related to evacuation planning was conducted in a variety of fields. (e.g., civil engineering, operation research, mass psychology, safety engineering, computer science, etc.) Hamacher and Tjandra [3] proposed the classification for the research on evacuation planning based on macroscopic and microscopic models. Macroscopic models emphasize optimization problems in which evacuees are considered as a homogeneous group. In general, macroscopic models are represented as dynamic network flow models (e.g., maximum dynamic flow model, universal maximum flow model, quickest flow model) [4]. On the basis of these models, solutions provide the optimal egress routes and schedules for evacuees. By contrast, microscopic models are developed as simulation-based models that take into account the individual behavior of evacuees. The models aim to describe interactions between each evacuee during the egress. Because our research focused on the mathematical model from a macroscopic viewpoint, we surveyed the studies relevant to the macroscopic model.

Using the macroscopic model, Francis [5] proposed the uniformity principle, which states that, if evacuation is conducted within the minimum time in the building, then evacuation times of all evacuees are the same. Chalmet et al. [6] developed an evacuation problem as a network model minimizing the evacuation time in the building. Kisko and Francis [7] developed EVACNET+, which is a computer program that provides the routes of evacuation in the building. By describing the building and related information as a network, it provides an optimal evacuation plan. Choi et al. [8] described network flows with side constraints that come from variable arc capacities in evacuation problems. They generated three types of objective functions and proposed greedy algorithms for building evacuation problems. Lin et al. [9] developed a multistage time-varying quickest-flow approach by expanding the dynamic model to a time-varying dynamic model. Chen and Miller-Hooks [10] proposed a mixed-integer program to determine the set of routes that all evacuees can escape to a safety area in minimal time. They also developed a Benders decomposition algorithm to solve the problem more efficiently. Chiu et al. [11] developed a single-destination cell-transmission model based on linear programming for solving a no-notice mass evacuation problem, which is characterized by a large and unexpected incident. Lim et al. [12] proposed a time-expanded network model for finding the optimal path for evacuees by maximizing the number of evacuees entering the safety area within a certain time. They also developed a heuristic algorithm to solve the problem efficiently in terms of computation time. Kang et al. [13] developed an algorithm based on linear and integer programming models to solve building evacuation planning, and they conducted a case study. Kim et al. [14] created a bottleneck relief heuristic to reduce congestion that also minimizes the evacuation time. Bretschneider and Kimms [15] described a dynamic network flow problem with additional variables for the number and direction of used lanes. They also generated an adjustment heuristic to solve the problem more efficiently. Miller-Hooks and Sorrel [16] came up with a mathematical model formulated as dynamic expected flows and a noisy genetic algorithm to determine route instructions while maximizing the number of evacuees within a given time. Li et al. [17] developed a time-extended evacuation network and an algorithm, and they conducted a case study according to the scenarios. Nahum et al. [18] developed the evacuation model considering chance constraints in capacity. They also proposed a genetic algorithm to solve the problem efficiently. Pillac et al. [19] proposed evacuation planning that consists of the sub-problem generating a new evacuation path and a master-problem optimizing the flow of evacuees and the overall evacuation plan. They also conducted a case study to demonstrate the performance of the algorithm. Wang et al. [20] developed the evacuation plan considering side constraints and scenario-based, random travel times. They proposed a relaxation-based heuristic algorithm to find a near-optimal solution for a large-scale problem efficiently. Farahani et al. [21] proposed a mathematical model to maximize the number of evacuees reaching safety areas by selecting the optimal location. They also developed exact algorithms and heuristics and then conducted computational experiments. Osman and Ram [22] presented a two-step approach based on the integer multi-commodity network model to find the optimal routes for evacuees. They also developed a capacity-constrained evacuation-scheduling algorithm and conducted numerical experiments. Osman and Ram [23] also proposed a centralized hybrid approach to find the optimal route of the evacuation path in a network. They developed an algorithm including the mathematical model as the sub-network problem. By conducting computational experiments, they claimed that their method is a novel approach that provides competitive and promising solutions.

By obtaining the optimal route of evacuation according to the mathematical model, one can interpret from the macroscopic viewpoint whether all evacuees can reach safety areas within a timeframe or not. As seen in the literature cited in this section, most studies related to evacuation planning in a building emphasized only the egress for evacuees. For *responders*, such as special weapons and tactics teams and firefighters, *entrance planning* for effectively resolving dangerous situations holds importance equal to evacuation planning [24]. For example, in the case of Brussels in 2016, the terror was raised by multiple and consecutive bombings. In such a case, responders clearly must access several target locations to handle the risk agilely within a period that maximizes the possibility of rescue: a *golden time*. While several definitions of the golden time have been used in various circumstances, we consider the golden time as the maximum time by which all evacuees must be rescued to a safety area after an emergency unfolds. Beyond the golden time, additional critical accidents, such as the collapse of a building or the spread of a large fire, can result in a large number of casualties.

The arbiter makes a decision on the action of the response team, such as the scheduling and moving routes of responders, according to intuition or prior information. This strategy may lead to route congestion for both evacuees and responders and prolong the total evacuation and entrance times. Hence, simultaneous evacuation and entrance planning (SEEP) is required. Accordingly, this paper presents mathematical models and a heuristic algorithm for SEEP by focusing on both the exit of evacuees and the entrance of responders. By establishing SEEP, stakeholders can determine whether such a plan is plausible in an emergency situation within the golden time or not. They can also analyze the situation from the basis of the



Fig. 1. Static network of a simple building layout.

mathematically determined optimal routes for evacuees and responders. In this study, we developed four types of mathematical models according to the discrete time dynamic network flow model. We generated the heuristic algorithm by extending the capacity constrained route planner (CCRP) [25], which accounts for SEEP.

The remainder of the paper is structured as follows: Section 2 presents explanations of the relevant mathematical models. Section 3 explains the heuristic algorithm. Section 4 presents a case study to analyze the mathematical models and a heuristic algorithm. Section 5 summarizes the findings of this research.

#### 2. Mathematical model

The main purpose of this study is to present mathematical models providing the optimal routes for evacuees and responders in an emergency. If evacuation and entrance plans are made separately, evacuees and responders may experience conflict at certain places, which causes congestion and creates a queue. By considering the SEEP, delays in evacuees reaching the safety area and responders entering the target area can be minimized.

To realize the emergency in a building, a static network G = (N, A), with N a set of nodes and A a set of arcs, is introduced. The areas where people are located in the facility, such as rooms or lobbies, are represented as nodes  $i \in N$ . Routes for movement of evacuees or responders between nodes, such as corridors, are represented as arcs  $(i, j) \in A$ . The areas where the route is changed and intersection points are also represented as a node. To apply the number of moving evacuees and responders over time, the dynamic network  $G^T = (N^T, A^T)$  associated with G = (N, A) is defined by the time expansion of *G* over time horizon *T*. Fig. 1 shows an example of converting structure of the building to a network *G* composed of nodes and arcs [3]. Based on network *G*, the movement of the evacuee or responder can be represented in discrete time. That is, by adding a time dimension to a static network flow, the movement of the evacuees or responder can be expressed over time which is the discrete time dynamic network flow  $G^T = (N^T, A^T)$ . Fig. 2 describes the example of the discrete time dynamic network flow.

In emergencies, evacuees are initialized separately in distinct areas. In the network, areas where initialized evacuees are found are called source nodes. To handle these multiple source nodes, a super source node *s* is introduced by connecting all source nodes with zero travel time and infinite arc capacity at time t=0. Infinite arc capacity can be substituted with the number of initialized evacuees in each node. In addition, multiple safety areas will attract evacuees. These areas are defined as sink nodes. Similar to source nodes, these multiple sink nodes are connected with a super sink node *d* with zero travel time and infinite arc capacity can be substituted with the total number of evacuees. In this way, the dynamic network flow problem can be solved as the static network flow problem [3].

Node capacity is dependent on the maximum number of people located in the area while arc capacity corresponds to the maximum number of people passing through the corresponding arc in a discrete time frame. All evacuees and responders are considered members of a homogeneous group in this macroscopic model. The following assumptions are made for this research:

#### Assumptions

- (1) The emergency occurs at t=0 and all evacuees start evacuating at the same time t=0.
- (2) When the emergency occurs, it does not move or change over time.
- (3) Responders enter disaster areas, where the emergency occurs, from safety areas at t = e.
- (4) If a node or an arc reaches maximum capacity, evacuees or responders attempting to pass through the node or the arc will wait until the capacity of the arc or node allows access.
- (5) Arcs are bi-directional except at the node of safety areas  $(i \in D)$  and disaster areas  $(i \in R)$ .
- (6) All evacuees and responders move at the same speed.

This study focuses on the establishment of an evacuation plan as part of the countermeasures undertaken at the beginning of the emergency. Therefore, we made Assumption (1) to incorporate the situation into a plan. In addition, as indicated by Assumption (2), we also considered the emergency as a static situation because accounting for additional disasters goes beyond the scope of this study. Assumption (3) was made to incorporate the required time for responders to arrive after the



Fig. 2. Example of the discrete time dynamic network flow.

emergency is recognized. We made Assumption (4) to show that the overall evacuation time would be delayed in cases for which the evacuees are concentrated in specific areas. We also presumed that evacuees would not head to the disaster area and would escape directly when they reach areas adjacent to safety areas. Accordingly, we made Assumption (5) to prevent unreasonable actions of evacuees. In the case of Assumption (6), although it can be seen as unrealistic, a robust analysis can be performed by considering the traveling time as the minimum value. The purpose of this study is a macroscopic analysis of the possibility to establish the evacuation and entrance planning in a building, which argues that Assumption (6) is not such an unrealistic assumption. The assumptions align with those of previous studies based on a macroscopic model. However, this study differs from previous research in showing two types of flows: one for the evacuee and one for the responder. Because of this unique feature, the evacuee and responder share the capacity of a particular arc or node. Situations that result from this capacity sharing are discussed in Section 3.

#### 2.1. Four types of mathematical models for SEEP

Based on the assumptions, we developed four types of mathematical models for SEEP according to the golden time. We have assumed that the golden time is a given parameter. In other words, the estimation of the golden time is beyond the scope of this research. The primary purpose of this study is developing SEEP according to the degree of golden time. When the golden time is sufficient, all evacuees can reach the safety areas and all responders enter the disaster areas within the timeframe. In this situation, minimizing the overall time of evacuation and entrance time is critical to avoid additional damages. Therefore, two types of mathematical models are developed which minimize the average time of evacuation and entrance, and evacuation time for the last evacuee and the entrance time of the last responder, respectively. When the golden time is adequate for rescuing all people, ensuring the entrance plan with minimizing the average evacuation time of all evacuees within the golden time is critical. Accordingly, a two-step approach model is developed to minimize the average time of evacuation while accommodating the time constraints of the responders reaching the disaster area. When the golden time is insufficient for all evacuees to exit the safety area, all responders must enter the disaster area to ameliorate the disaster and rescue as many evacuees as possible within the golden time. To incorporate the situation, the two-step approach is considered to maximize the number of evacuees reaching the safety area within the golden time while satisfying the time constraints of the responders. Fig. 3 summarizes the four types of mathematical models based on the achievable objectives according to the golden time.



Fig. 3. Four types of mathematical models developed based on the degree of golden time.

The following notations are used to develop the four types of mathematical models: Indices

- *i*, *j*: Node  $(i, j \in N)$
- t: Period (t=0, 1, ..., T)
- s: Super source node
- *d*: Super sink node

#### Parameters

- $\lambda_{ij}$ : Travel time of evacuees and responders from node *i* to node *j*
- *a<sub>i</sub>*: Capacity of node *i*
- $b_{ij}$ : Capacity of arc from node *i* to node *j*
- *c*<sub>*i*</sub>: Minimum number of required responders at disaster area node *i*
- $q_i$ : Number of initialized evacuees in node *i* at t = 0
- *e*: Initial time of responders entering the safety area
- q': Number of arriving responders in safety area node at t = e
- $k_1$ : Weight (importance) of evacuation
- *k*<sub>2</sub>: Weight (importance) of response
- T: Golden time
- ε: Tolerance

Sets

- N: Set of nodes
- S: Set of source nodes
- D: Set of sink nodes
- *R*: Set of disaster area nodes

#### Decision variables

- $x_{ij}(t)$ : Number of evacuees moving from node *i* to node *j* at time *t*
- $z_{ij}(t)$ : Number of responders moving from node *i* to node *j* at time *t*
- $y_i(t)$ : Number of evacuees staying in node *i* at time *t*
- $w_i(t)$ : Number of responders staying in node *i* at time *t*

## 2.2. Model I - minimization of the average evacuation and entrance times

Model I is developed to minimize the sums of average evacuation time of evacuees and entrance time of responders. The average evacuation time and entrance time can be formulated as  $\frac{\sum_{t=0}^{T} \sum_{i \in J} tx_{id}(t)}{\sum_{i \in S} q_i}$  and  $\frac{\sum_{t=0}^{T} \sum_{i \in I} tz_{id}(t)}{q'}$ , respectively. Because

 $\sum_{i \in S} q_i$  and q' are constants, they do not need to be represented in the objective function. By adding these two components and multiplying them by weights  $k_1$  and  $k_2$ , respectively, an objective function represents the sum of the average evacuation and entrance times. As  $k_1$  and  $k_2$  vary in the objective function, the priority between entrance and evacuation can be changed. If the decision maker prioritizes responder entrance over victim evacuation,  $k_1$  is set to be smaller than  $k_2$ . Otherwise,  $k_1$  is larger than  $k_2$ . The objective function and constraints of Model I are as follows:

Objective function

Minimize 
$$k_1 \sum_{t=0}^{T} \sum_{i \in D} tx_{id}(t) + k_2 \sum_{t=0}^{T} \sum_{i \in R} tz_{is}(t)$$
 (1)

Subject to

$$x_{si}(0) = q_i \qquad \forall i \in S \tag{2}$$

$$\sum_{i\in D} z_{di}(e) = q' \tag{3}$$

$$\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t) = \sum_{j \in S} q_j$$
(4)

$$\sum_{t=e}^{T} \sum_{i \in R} z_{is}(t) = q'$$
(5)

$$y_i(t+1) - y_i(t) = \sum_{k \in pred(i)} x_{ki}(t-\lambda_{ki}) - \sum_{j \in succ(i)} x_{ij}(t) \qquad \forall i \in N, \ t = 0, \cdots, T$$
(6)

$$w_i(t+1) - w_i(t) = \sum_{k \in pred(i)} z_{ki}(t-\lambda_{ki}) - \sum_{j \in succ(i)} z_{ij}(t) \qquad \forall i \in N, \ t = e, \cdots, T$$

$$(7)$$

$$y_i(0) = 0 \qquad \forall i \in \mathbb{N}$$
(8)

$$w_i(t) = 0 \qquad \forall i \in N, \ t = 0, \dots, e-1$$
(9)

$$y_i(t) = 0 \qquad \forall i \in D, t = 0, \dots, T$$
(10)

$$w_i(t) = 0 \qquad \forall i \in R, t = 0, \dots, T \tag{11}$$

$$y_i(t) \le a_i \qquad \forall i \in N - D, \ t = 0, \dots, e - 1$$

$$(12)$$

$$y_i(t) + w_i(t) \le a_i \qquad \forall i \in N - D, \ t = e, \dots, T$$
(13)

$$x_{ij}(t) + x_{ji}(\tau) \le b_{ij} \qquad \forall i, j \in \mathbb{N}, t = 0, \dots, e-1; \tau = t - (\lambda_{ij} - 1), \dots, t + (\lambda_{ij} - 1)$$
(14)

$$x_{ij}(t) + x_{ji}(\tau) + z_{ij}(t) + z_{ji}(\tau) \le b_{ij} \quad \forall i, j \in \mathbb{N}, t = e, \dots, T; \tau = t - (\lambda_{ij} - 1), \dots, t + (\lambda_{ij} - 1)$$
(15)

$$\sum_{t=e}^{T} z_{is}(t) \ge c_i \qquad \forall i \in R$$
(16)

$$x_{ij}(t), \ z_{ij}(t), \ y_i(t), \ w_i(t) \in \mathbb{Z}^+ \qquad \forall i, j \in \mathbb{N}, \ t = 0, \dots, T$$
(17)

The objective function (1) minimizes the sum of the average evacuation time of evacuees and entrance time of responders. Constraint (2) indicates the number of evacuees moving from the super source node to each source node at the start of the evacuation. Constraint (3) indicates the number of responders moving from the super sink node to each sink node



Fig. 4. Drawback of using general arc capacity constraint when the travel time is 4 and the arc capacity is 3.

as responders' entrance commences at time t = e. Constraint (4) indicates the number of evacuees moving from a sink node to the super sink node. Constraint (5) indicates the number of responders moving from a disaster area node to the super source node. Constraint (6) indicates the balance equation for the number of remaining and moving evacuees in each node at time *t*. Constraint (7) indicates the balance equation for the number of remaining and moving responders in each node at time *t*. Constraint (8) indicates no remaining evacuees in each node at time t = 0. Constraint (9) indicates no responders remaining in each node at time t = 0, ..., e - 1. Constraint (10) prevents evacuees from being in a sink node. Constraint (11) prevents responders from being in a source node. Constraints (12) and (13) indicate the capacity of node *i* at time *t*. Constraints (14) and (15) indicate the flow capacity of arc (*i*, *j*) at time *t*. With regard to arc capacity, if the travel time between the nodes is 1, the general arc capacity ' $x_{ij}(t) + x_{ij}(t) + z_{ij}(t) \le b_{ij}, t = 0, ..., T$ ;  $\forall i \neq j'$  can be used. However, when the travel time is greater than 1, these constraints are limited because they indicate the capacity at the starting period for each evacuee and responder. Therefore, the arc constraint is formulated as Constraints (14) and (15). Fig. 4 explains the problem of applying a general arc capacity when the travel time is 4 and the arc capacity is 3. Circles made of solid lines represent nodes and nodes made of dotted lines are not actual nodes but represent the middle path of evacues moving by discrete time. Constraint (16) indicates the minimum number of responders who must reach the disaster area. Constraint (17) means that the decision variables  $x_{ij}(t), x_{ij}(t), and <math>w_i(t)$  are non-negative integers.

## 2.3. Model II - minimization of the average evacuation time while satisfying the entrance time constraints

Model II minimizes the average evacuation times while satisfying the time constraints of responders reaching the disaster area. It is based on a two-step approach. The procedure below explains the two-step approach of Model II.

Procedure of Model II

Model II – two-step approach
√ First step Minimize the entrance time of the last responder to the disaster area → Return the least entrance time $e + \sum_{t=e}^{T} \alpha(t)$ √ Second step Multiply tolerance ( $\varepsilon$ ) and the least entrance time determined in Step 1 → $\varepsilon \cdot (e + \sum_{t=e}^{T} \alpha(t))$ Add the $\varepsilon \cdot (e + \sum_{t=e}^{T} \alpha(t))$ as the time constraint for all responders who must reach the disaster area Minimize the average evacuation time of evacuees

In the first step, the least entrance time of the last responder to the disaster area is determined. To conduct this procedure, the integer variable  $\xi(t)$  and binary variable  $\alpha(t)$  are introduced.

$\xi(t)$ :	Cumulative sum of responders reaching the disaster area within time t
$\alpha(t)$ :	Binary variable

If there exist remaining responders who fail to reach the target area within time, 1 Otherwise,  $\boldsymbol{0}$ 

. . .

The value from multiplying the tolerance with the returned least entrance time is added in Constraint (25). The tolerance value can be set by the decision maker in accordance with the situation. If the decision maker gives a sufficient tolerance value, time constraints allow enough time for responders to reach the target areas. Otherwise, the time constraints for the responders are set tightly. The objective function and constraints of the two-step approach are as follows:

Two-step approach

1. Minimize 
$$\sum_{t=e}^{T} \alpha(t)$$
 (18)

Subject to

(2), (3), (5), (7)–(17), and

$$\xi(t) = 0$$
  $t = 0, \dots, e$  (19)

$$\xi(t) = \xi(t-1) + \sum_{i \in \mathbb{R}} z_{is}(t) \qquad t = e+1, \dots, T$$
(20)

$$M_1 - \xi(t) \le M_1 \alpha(t) \qquad t = e, \dots, T \tag{21}$$

 $\alpha(t) \in \{0, 1\}$  t = 0, ..., T (22)

$$\xi(t) \in \mathbb{Z}^+ \qquad t = 0, \dots, T \tag{23}$$

 $M_1$  is a large number

2. Minimize 
$$\sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t)$$
 (24)

Subject to

(2)-(4), (6)-(17), and

$$\sum_{t=e}^{\varepsilon \cdot \left(e + \sum_{i=e}^{T} \alpha(t)\right)} \sum_{i \in \mathbb{R}} z_{is}(t) = q'$$
(25)

In the first step, the objective function (18) minimizes the entrance time of the last responder to the disaster area. Constraint (19) indicates that no responders enter the target area at t=0, ..., e. Constraint (20) expresses that  $\xi(t)$  is the sum of responders entering the target area at t. Constraint (21) indicates  $\alpha(t)=1$  if the remaining responders fail to reach the target area at t. Otherwise,  $\alpha(t)=0$ . Although  $M_1$  is a large number, it can be substituted into q' to make a solution bound tightly. Constraint (22) means that  $\alpha(t)$  is a binary variable. Constraint (23) means that  $\xi(t)$  is a non-negative integer variable. In the second step, the objective function (24) minimizes the average evacuation time of evacuees. Constraint (25) means all responders must reach the disaster area within the timeframe determined by the first step.

## 2.4. Model III - minimization the evacuation time of the last evacuee and the entrance time of the last responder

Model III minimizes the evacuation time of the last evacuee and the entrance time of the last responder. To solve the problem, the integer variable  $\psi(t)$  and the binary variable  $\beta(t)$  are introduced.

ξ(t):	Cumulative sum of evacuees reaching the safety area within time t
$\alpha(t)$ :	Binary variable
	If there are evacuees who do not reach the safety area within time t, 1
	Otherwise, 0

The objective function and constraints are as follows: Objective function

Minimize 
$$k_1 \sum_{t=0}^{T} \beta(t) + k_2 \sum_{t=e}^{T} \alpha(t)$$
 (26)

Subject to (2)–(23), and

$$\Psi(0) = 0$$

(27)

$$\Psi(t) = \Psi(t-1) + \sum_{i \in D} x_{id}(t) \qquad t = 1, \dots, T$$
(28)

$$M_2 - \Psi(t) \le M_2 \beta(t)$$
  $t = 0, ..., T$  (29)

$$\beta(t) \in \{0, 1\} \qquad t = 0, \dots, T \tag{30}$$

$$\Psi(t) \in \mathbb{Z}^+ \qquad t = 0, \dots, T \tag{31}$$

 $M_2$  is a large number

The objective function (26) minimizes the evacuation time of the last evacuee and the entrance time of the last responder. Constraint (27) indicates that no evacuees are at the safety area at t=0. Constraint (28) illustrates that  $\Psi(t)$  is the sum of the number of evacuees at the safety area at t. Constraint (29) means that  $\beta(t)=1$  if some evacuees fail to reach the safety area at t. Otherwise,  $\beta(t)=0$ . Although  $M_2$  is a large number, it can be substituted into  $\sum_{j\in S} q_j$  to tighten the solution bound. Constraint (30) means that  $\beta(t)$  is a binary variable. Constraint (31) means that  $\Psi(t)$  is a non-negative integer variable.

#### 2.5. Model IV - maximization of the number of evacuees within the golden time

Model IV maximizes the number of evacuees who reach the safety area within a certain time while satisfying the time constraints of responders reaching the disaster area. The procedure below explains the two-step approach in this scenario. Procedure of Model IV

Model IV – Two-Step Approach
√ First step
Minimize the entrance time of the last responder to the disaster area
$\rightarrow$ Return the least entrance time $e + \sum_{t=e}^{T} \alpha(t)$
$\checkmark$ Second step
Multiply tolerance ( $\varepsilon$ ) with the entrance time
$\rightarrow \varepsilon \cdot (e + \sum_{t=e}^{T} \alpha(t))$
Add $\varepsilon \cdot (e + \sum_{t=e}^{T} \alpha(t))$ as a time constraint for all responders who must reach the disaster area
Maximize the number of evacuees who reach the safety area.

Similar to Model II, to find the minimum time for responders to reach the disaster area, the binary variable  $\alpha(t)$  and the integer variable  $\xi(t)$  are introduced. After multiplying the tolerance with the entrance time of the last responder, this value is added in Constraint (25). The objective function and the constraints of the two-step approach are as follows: Two-step approach

1. Minimize  $\sum_{t=e}^{T} \alpha(t)$ 

Subject to (2), (3), (5), (7)–(17), (19)–(23) *M*<sub>1</sub> is a large number

2. Maximize  $\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t)$ 

Subject to

(2), (3), (6)-(17), (19)-(23), (25)

By maximizing the objective function  $\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t)$  in the second step, the number of evacuees possible to escape to the safety area within  $t \in \{0,...,T\}$  is maximized. In this model, *T* is required to set smaller than that of other models but is enough for all responders to reach the target area.

#### 3. Heuristic algorithm

Although four types of mathematical models provide the optimal solutions, they show a weakness in terms of the long computation time for solving the large-size problem. Because the mathematical model is based on a time-expanded network, computation time is affected by the upper bound of timeframe *T*. As Fig. 2 shows, at least n(T+1) nodes are needed to solve the problem. Furthermore, the outcomes of decisions based on poorly chosen values for upper bound *T* may be undesirable. An underestimated *T* leads to infeasible solutions and an overestimated *T* causes unnecessary computation time. Therefore, this approach restricts scaling up to the large size of networks because of the long computation time. In addition, unlike the previous evacuation planning based on discrete time network flows, there are two types of flows  $x_{ij}(t)$  and  $z_{ij}(t)$ , representing evacuees and responders moving along with the arcs, respectively. Because the two types of flows share an arc capacity at the same time, the problem solved with a linear program (LP) provides a fractional solution. It means that mathematical models are required to be developed as an integer program (IP). In other words, solutions should be expressed as

an integer value rather than a fractional solution because it represents the number of evacuees or responders. As a result, the complexity of the problem developed as IP increases compared to that of LP.

Because of the extended time for solution generation and the increased complexity, the mathematical models cannot solve the large-size problem within a reasonable time. However, in emergencies, SEEP must be determined rapidly according to real-time information. To overcome the limitation, a heuristic algorithm was developed. The algorithm provides all routes for evacuees and responders from source nodes to sink nodes by the extended capacity constrained route planner (ECCRP). It consists of two sub-network problems. While the mathematical model solves the problem with all input data in one step, the algorithm solves it by iterating two sub-network problems. In seeking the evacuation plan, the mathematical model can be relaxed to LP and provide the solutions as integer values like the general network flow problem does. For entrance planning, because all information on evacuation planning is set as a parameter, the problem is also solved by the static network flow problem as LP [3]. Accordingly, large-size problems can be efficiently solved by the ECCRP.

For the ECCRP consisting of two sub-network problems, routes for the evacuees are provided in the first phase. In the second phase, optimal entrance routes for responders are provided after all the evacuee routes have been added as parameters in the dynamic network flow model. The components and explanations for the ECCRP are as follows:

Components of ECCRP

#### Input:

**Extended Capacity Constrained Route Planner** 

First sub-network problem

Link super source node s to each source node with arc capacity  $= \infty$  and travel time = 0 While any source node  $i \in S$ 

While evacuees still at source node i > 0

obtain paths from source node *i* to super sink node *d* 

 $\text{Minimize} \sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t)$ (30)

Subject to

 $y_i(t) \le a_i, \quad \forall i \in N - D, t = 0, ..., T$  (31)

$$x_{ij}(t) + x_{ji}(\tau) \le b_{ij}, \qquad \forall i, j \in N, t = 0, \dots, T; \tau = t - (\lambda_{ij} - 1), \dots, t + (\lambda_{ij} - 1)$$
(32)

$$x_{sj}(0) = 1, \qquad \forall j \in S \tag{33}$$

$$\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t) = 1$$
(34)

$$y_{i}(t+1) - y_{i}(t) = \sum_{k \in pred(i)} x_{ki}(t-\lambda_{ki}) - \sum_{j \in succ(i)} x_{ij}(t) \qquad \forall i \in N, t = 0, \dots, T$$
(35)

min\_flow = min(number of evacuees remaining at source node  $i \in S$ , node capacity, arc capacity)  $\sqrt{}$  update node capacity

 $y_i(t) \leq a_i - \min_{i} flow, \ \forall i \in N - D, t = 0, \cdots, T$ 

 $\checkmark$  update arc capacity  $x_{ij}(t) + x_{ji}(\tau) \le b_{ij} - \min_{l} flow, \ \forall i, j \in N, t = 0, \dots, T$ ; } } Second sub-network problem Add all of the routes of evacuees as parameters

$$\begin{aligned} \text{Minimize} \sum_{t=0} \sum_{i \in R} t z_{is}(t) \\ \text{Subject to} \\ (3), (5), (7), (9), (11), (13), (15), \text{ and (16)} \\ \textbf{Output evacuation and entrance routes} \end{aligned}$$

Т

The objective function (30) minimizes the time for an evacuee to reach the safety area. Constraint (31) represents node capacity at time *t*. Constraint (32) represents arc capacity at time *t*. Constraint (33) means there is only one evacuee moving from the super source node *s* to the source node  $i \in S$ . Constraint (34) means one evacuee initialized in a source node reaches the safety area. Constraint (35) indicates the balance equation between remaining and moving evacuees in each node at time *t*. By iterating the objective function (30) and Constraints (31)–(35), the algorithm finds the path from source node to sink node and updates the node capacity and arc capacity by subtracting the min\_flow, which accounts for the smallest component among the number of evacuees still at source node  $i \in S$ , node capacities, and arc capacities. When no evacuee remains in source node  $i \in S$ , all movements of evacuees as well as the node and arc capacities are set as parameters in the second step. At this point, by minimizing the objective function (36) with Constraints (3), (5), (7), (9), (11), (13), (15), and (16), the ECCRP provides evacuation and entrance routes.

## 4. Case study

In this research, three types of case studies were conducted. Central City, the huge megamall in Seoul, Korea, was used as the backdrop for the case study. The original layout comes from Kang et al. [13] and Kim et al. [24]. It is modified and enhanced with additional data to create the network. Each facility is set as a node and route of people between nodes is considered an arc. The travel time from a node to an adjacent node is calculated by dividing the running speed by distance. Figs. 5 and 6 illustrate the layout of Central City and the schematic of the network used in this case study.

In Case study I, a Monte Carlo simulation was conducted to verify the need for optimization in evacuation planning. Case study II compared Models I, II, III, and the ECCRP. In Case study III, an experiment for Model IV was conducted according to the golden time *T*. Because Model IV features a short time frame for bound *T* with a value smaller than or equal to the minimum evacuation and entrance times, this case study was carried out separately from the others.

#### 4.1. Data generation

For three types of case studies, it is assumed that all evacuees are initialized at F1-F14 and F16-F19. F42-F45 are considered the safety area. The disaster unfolds at F1-F5, F12, and F13. Accordingly, evacuees escape from F1-F14 and F16-F19



Fig. 5. Floor plan of Central City; the red rectangle depicted is the arc-node network depicted in Fig. 8. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(36)



Fig. 6. Modified network illustrating part of Central City, see red rectangle in Fig. 7. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

<b>Table 1</b> Input data f	or nodes in case study.
N	F1-F45
S	F1-F14 and F16-F19
D	F42-F45
R	F1-F5 and F12-F13

to F42-F45 while responders reach F1-F5 and F12-F13 from F42-F45. F1-F45 corresponds to Nodes 1–45, and Node 0 and 46 are introduced as super source node s and super sink node d, respectively. The number of evacuees for each node is set as a random number between 10 and 50. As a result, the total number of evacuees is 323. The capacity of the nodes and arcs, and the travel time between adjacent nodes are based on data from [13]. We also visited the site where the case study was conducted and calibrated the data through additional surveys. Table 1 shows the set of nodes corresponding to the definitions used for the model.

#### 4.2. Description of Case study I

In Case study I, a Monte Carlo simulation was conducted to justify the use of an optimization model. We conducted the experiments with evacuees choosing a route randomly based on a generated random number between zero and one. We showed the verification of this study by comparing the solutions of the optimal evacuation planning with those of the simulation experiments. To compare evacuation only, all parameters related to the responder in Model I were limited to zero. To avoid unrealistic simulation results, we made assumptions as follows:

- (1) Evacuees do not head to nodes  $i \in S$ , which are disaster areas.
- (2) Evacuees go to safety areas directly when they arrive at nodes  $i \in D$ .
- (3) Evacuees do not move to nodes where movement is blocked, such as F7, F8, F16, and F17.

Evacuees select the path randomly when the capacity of the arc or the adjacent node is available. For the situation in which 100 evacuees are located at node *i*, which is connected to two arcs (i, j) and (i, k), if the generated random numbers [0, 1] for each arc are 0.37 and 0.63, 37 and 63 evacuees move to node *j* and *k*, respectively. In the same manner, when a node is connected with three arcs, the evacuees move to the adjacent three nodes according to the generated three random numbers. For every period  $t \in \{0,...,T\}$ , the probability of selecting arc (i, j) is also generated differently. The solutions with the final fractional values were rounded to obtain the objective value, and the last time of evacuation was set as an integer value.

Optimization model		Objective value 3003	Last time of evacuation 13
Monte Carlo simulation	Mean Min Max Median	4336.32 3998 4829 4342	34.94 28 44 35

Table 2
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Summarization of Case study I.

Table 3				
Summarization	of	Case	study	II.

Method	Range of evacuation time	Range of entrance time	Average time of evacuation	Average time of entrance	Computation time (seconds)
ECCRP	4-13	9–20	9.30	15.36	26.8
Model I	8-13	9–18	11.16	13.06	51.4
Model II	9–13	14–18	10.54	17.73	45.2
Model III	9–13	9–18	11.20	13.67	96.0

#### 4.3. Results of Case study I

Following the above assumptions, experiments of the Monte Carlo simulation were performed 100 times. Results of Case study I are summarized in Table 2. As can be observed in Table 2, the results of the optimization model dominated those of the Monte Carlo simulation. For the objective value, the best result of the simulation showed a relative error of 33.13%. Regarding the last time of evacuation, the relative error was 115.38%. In other words, Case study I addressed the need of optimal evacuation planning. In the next subsections, we will analyze four types of mathematical models and a heuristic algorithm considering the SEEP by conducting Case studies II and III.

#### 4.4. Description of Case study II

The purpose of the Case study II is comparisons among characteristics of Models I, II, III, and ECCRP. We assumed that the total number of responders were 90 and they arrived at the safety area at t=5. Ten responders must reach each disaster node. The weights  $k_1$  and  $k_2$  are each set at 1 and the discrete unit time is set at 5 s. In Model II, tolerance  $\varepsilon$  is set at 1. ECCRP was solved in Java 1.8.071 language with the XPRESS-MP library, and the Models I, II, and III were solved by FICO XPRESS-IVE version 7.9 with an Intel® Core<sup>TM</sup> i5-3470 CPU @ 3.20 GHz.

#### 4.5. Results of Case study II

Table 3 summarizes the results of Case study II. It shows ranges of evacuation and entrance times, average times of evacuation and entrance, and computation times of the ECCRP and Models I, II, and III. As evident from Table 3, the ECCRP outperformed the mathematical models in terms of computation time and quality of the solution with respect to the evacuation time. Furthermore, because it finds the path by iteration, the first evacuation occurred at t=4. However, by solving the evacuation problem first, the performance of the entrance problem suffered. Model I, which minimizes the sum of average times for evacuation and entrance, provided a wide range of solutions in comparison with the other mathematical models. This finding means that all evacuees and responders are distributed to safety and disaster areas more evenly in the timeframe  $t \in \{0,...,T\}$ . Model II, which adds the time constraints of responders in a two-step approach, solves the problem by prioritizing the evacuation over the entrance. Although the average time of entrance was not good, the last responder reached the disaster area at t=18 and the model yields an average evacuation time that outperforms other mathematical models. Model III, which minimizes the evacuation time of the last evacuee and the entrance time of the last responder, showed similar patterns to those of Model I, but the solutions were not as good as those of Model I. Also, the computation time was relatively long. Figs. 7 and 8 illustrate the number of evacuees reaching the safety area and the number of responders entering the disaster area, respectively, over time t.

We observed from Fig. 7 that the ECCRP showed the best performance in terms of the evacuation plan. Model II showed a better performance than Models I and III. However, regarding the entrance plan, the performances of Models I and III were better than the others as shown in Fig. 8. In the case of the ECCRP, the last entry was terminated at t = 20 which was the latest time among the models. Although the last entrance time of Model II was at t = 18, which was the same as Models I and III, the first arrival time was the earliest time compared the other models. In other words, all models developed in this study offered trade-offs between the time of evacuation and entrance. According to the situation, the proper model can be chosen among the models. Table 4 shows the number of evacuees exiting each node in this case study.

From Table 4, one can see that evacuees reached F43–F45 in the ECCRP. In the case of Models I, II, and III, they reached F42–F45. The results differed because the problem addressed by the ECCRP is based on the shortest paths between each node and the safety area, while the mathematical models are searching the global optimality by accounting for an even



Fig. 7. Number of evacuees reaching the safety area over time t.





Table 4
Number of evacuees exiting to each safety
area in Case study II.

	Number of evacuees			
	F42	F43	F44	F45
ECCRP	0	160	73	90
Model I	20	144	119	40
Model II	13	125	140	45
Model III	20	127	136	40

distribution of all evacuees. In Table 5, variances were calculated to see the manner in which evacuees were dispersed in safety areas.

As described in Table 5, Model III has the smallest variance among models. This result addresses that evacuees were distributed to safety areas most evenly in Model III. Because this model emphasizes minimizing the evacuation time of the last evacuee and entrance time of the last responder, it solved the problem by spreading all evacuees and responders evenly. Due to the data property that evacuees were not initialized in all nodes, distributing all evacuees evenly had a negative influence on evacuation performance. Table 6 represents the number of responders entering from each node in this case study.

As indicated in Table 6, responders did not use F42 in the ECCRP or Models I and III. Because F42 is the farthest from the disaster area, responders tended to avoid using it as an entrance route in their effort to minimize travel time. However, in the case of Model II, setting the entrance of responders as constraints affected the routes of the responders. The model does not account for the number of responders using the entrance; rather it only satisfies the time constraints of responders.

#### Table 5

Variance of the number of evacuees reaching each safety area in Case study II.

Variance of the number of evacuees reaching each node
3236.69
2703.69
2834.19
2635.69

#### Table 6

Number of responders entering from each safety area in Case study I.

	Number of responders			
	F42	F43	F44	F45
ECCRP	0	32	40	18
Model I	0	28	42	20
Model II	20	38	18	14
Model III	0	40	30	20

Table 7							
Solutions	of	ECCRP	in	Case	study	I.	

Node	Routes evacuees use (number of evacuees)			
Node 1	1→30→43→46 (30)			
	$1 \rightarrow 1 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (11)			
Node 2	$2 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (10)			
Node 3	$3 \rightarrow 40 \rightarrow 39 \rightarrow 38 \rightarrow 2 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (19)			
Node 4	$4 \rightarrow 39 \rightarrow 38 \rightarrow 2 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (12)			
Node 5	$5 \rightarrow 40 \rightarrow 39 \rightarrow 38 \rightarrow 2 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (1)			
	$5 \rightarrow 40 \rightarrow 39 \rightarrow 38 \rightarrow 37 \rightarrow 25 \rightarrow 20 \rightarrow 44 \rightarrow 46$ (13)			
Node 6	$6 \rightarrow 39 \rightarrow 38 \rightarrow 2 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (8)			
	$6 \rightarrow 39 \rightarrow 38 \rightarrow 37 \rightarrow 25 \rightarrow 20 \rightarrow 44 \rightarrow 46$ (12)			
Node 7	$7 \rightarrow 38 \rightarrow 2 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (18)			
Node 8	$8 \rightarrow 37 \rightarrow 25 \rightarrow 20 \rightarrow 44 \rightarrow 46$ (18)			
Node 9	$9 \rightarrow 29 \rightarrow 30 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (6)			
	$9 \rightarrow 9 \rightarrow 29 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (4)			
Node 10	$10 \rightarrow 29 \rightarrow 29 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (7)			
	$10 \rightarrow 10 \rightarrow 29 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (2)			
	$10 \rightarrow 10 \rightarrow 29 \rightarrow 30 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (2)			
Node 11	$11 \rightarrow 31 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (10)			
Node 12	$12 \rightarrow 11 \rightarrow 31 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (20)			
Node 13	$13 \rightarrow 11 \rightarrow 1 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (20)			
Node 14	$14 \rightarrow 11 \rightarrow 1 \rightarrow 30 \rightarrow 43 \rightarrow 46$ (10)			
	$14 \rightarrow 11 \rightarrow 11 \rightarrow 31 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (10)			
Node 16	$16 \rightarrow 33 \rightarrow 26 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (20)			
Node 17	$17 \rightarrow 34 \rightarrow 33 \rightarrow 26 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (10)			
	$17 \rightarrow 2634 \rightarrow 33 \rightarrow 26 \rightarrow 26 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (10)			
Node 18	$18 \rightarrow 34 \rightarrow 33 \rightarrow 26 \rightarrow 32 \rightarrow 45 \rightarrow 46$ (10)			
	$18 {\rightarrow} 34 {\rightarrow} 35 {\rightarrow} 36 {\rightarrow} 25 {\rightarrow} 20 {\rightarrow} 44 {\rightarrow} 46 (10)$			
Node 19	$19 \rightarrow 36 \rightarrow 25 \rightarrow 20 \rightarrow 44 \rightarrow 46$ (12)			
	$19 \rightarrow 19 \rightarrow 36 \rightarrow 25 \rightarrow 20 \rightarrow 44 \rightarrow 46$ (8)			

Therefore, some responders used F42 as an entering node even though it was the farthest from the disaster area. When F42 is the entering node, other nodes could afford space for evacuees, which leads to a better evacuation solution than offered by the other mathematical models. Table 7 shows the near-optimal routes obtained by the ECCRP in Case study I.

As Table 7 presents, evacuees in F1-F7, F9, F10, F13, and F14 moved to F43; those in F5, F6, F8, F18, and F19 moved to F44; and those in F11, F12, F14, and F16-F18 moved to F45. Evacuees in F5, F6, F14, and F18 moved in separate groups through to different destinations. Although the ECCRP offers an advantage in interpreting the solutions more clearly, it has a limitation that it conducts iterations based on the order defined by the user. In this case study, the order was set as an ascending node number. To figure out whether performance is dependent on the order of iteration or not, four types of experiments were conducted: ascending node number, ascending evacuees' number, descending evacuees' number, and random order number. Table 8 represents the results of these four experiments.

As listed in Table 8, the selection of node order did not affect the evacuation time in this case study. All methods provide solutions in which evacuations finish at t = 13 and entrances are finished at t = 20. The solution gap occurs between the

#### Table 8

Difference in a selection of the node and number of evacuees in ECCRP.

Method	Evacuation time of the last evacuee to the safety area	Entrance time of the last responder to the target area	Computation time (seconds)
Ascending node number	13	20	26.8
Ascending evacuee number	13	20	25.2
Descending evacuee number	13	20	27.2
Random node number	13	20	27.9

Table 3			
Results	of Case	study	III.

Table 0

Golden time T	Number of evacuees entering				
	F42	F43	F44	F45	Sum
8	0	72	38	20	130
9	0	90	38	40	168
10	0	100	38	60	198
11	20	102	48	80	250
12	20	112	78	80	290
13	20	132	88	83	323

ECCRP and mathematical models because the mathematical models solve the entrance problem by accounting for evacuation and entrance simultaneously. By contrast, the ECCRP solves the entrance problem after solving the evacuation problem.

#### 4.6. Description of Case study III

Another case study was conducted with Model IV by varying the golden time *T*. Because the model maximizes the number of evacuees reaching safety areas within the time-bound *T*, we assumed that the time-bound *T* is insufficient for the completing evacuations and entrances. This model is suitable for scenarios in which the golden time is especially urgent and few responders must access certain areas. Accordingly, in this experiment, the number of responders is set to 10 and at least three responders must reach F1 and F2, respectively. The first arrival time to safety the area is set at t=3. The Model IV was solved with FICO XPRESS-IVE version 7.9 with an Intel® Core<sup>TM</sup> i5-3470 CPU @ 3.20 GHz.

## 4.7. Results of Case study III

Table 9 shows the results of Case study III with Model IV. As Table 9 shows, when T is 8 to 10, no one used F42 as a destination for evacuation. Whereas, when T is 11 to 13, 20 evacuees initialized in F19 used F42 as a destination. They used F44 as their destination when T is 8 to 10. Although F42 is farther than F44 from F19, when the golden time is long enough, the optimal evacuation plan is made by spreading some evacuees to the farther area to ensure other evacuees use certain routes. This allows more evacuees to escape within the golden time.

#### 5. Conclusions

The main purpose of this study was to develop the framework for SEEP by developing four types of mathematical models and a heuristic algorithm. Through the mathematical models, evacuees located in each area are provided with the optimal routes to the safety areas. Meanwhile, responders are provided with an optimal path indicating the best places to enter and paths to take to reach the disaster areas. The solutions can support managers who make decisions on evacuation planning and serve as arbiters of the response team. With four types of mathematical models and the heuristic model, the manager can choose a proper entrance and evacuation plan depending on the specific circumstances of the emergency. From the results of the case studies, the recommended mathematical model for a selected possible situation is as follows:

- (1) When the golden time is sufficient and spreading the evacuees evenly is a safe option:
  Model I
- (2) When the golden time is sufficient and minimizing the average evacuation time is critical:Model III
- (3) When the golden time is short, the responder must enter as soon as possible, and most civilians must evacuate:Model IV
- (4) When the entry of the responder should be completed within a certain time, and the evacuation times of evacuees are critical:
  - Model II

Depending on the type of emergency, an appropriate countermeasure is required. In the case of a fire or a gas leak, evacuation of occupants to safety areas is most critical. For a terrorist attack, the entry of responders might be equally or

more critical than evacuations. In the case of a time bomb, the priority level for a responder entering the target area depends on the time available before the explosion. That is, the emergency plan for the building should be planned according to each situation. Appropriate mathematical models should be used according to the four cases presented.

However, the proposed four types of mathematical models based on a dynamic network flow problem showed weakness in terms of long computation times due to the high complexity. The ECCRP did not feature the same weak point as the mathematical model showed. The ECCRP provides complete routes for evacuees and responders and reaches a sub-optimal solution within a reasonable time.

Because of the growth of modern technology, decision makers and planners can analyze real-time emergency situations and related data. Also, the ability to inform evacuees on escape routes and designate appropriate entrance routes for responders in real-time is now possible. To utilize these modern technologies effectively, evacuation and entrance plans must be based on these scientific approaches, and not solely on the intuitive judgment of the response team leader. Accordingly, this study suggests mathematical models and a heuristic algorithm that provide an optimal data-driven evacuation and entrance plan. When an emergency occurs, the integrated planning can provide an optimal path for the evacuee and the responder in real time. Also in non-emergency situations, it is possible to analyze how much time is needed to evacuate to safety area and enter into disaster area through this decision support system. Based on the system, evaluating the appropriateness of the emergency exit location when designing buildings and the acceptable number of people in the building are possible. It can also be used for training of citizens' emergency preparedness and the training of responders for each emergency scenario.

Furthermore, this study pioneers a plan in which both evacuations and emergency responses take place simultaneously. In our knowledge, among the studies related to evacuation planning, this study is the first study considering both of them simultaneously. Therefore, this study is a cornerstone of the related area and it can be expanded to various future studies. However, this study considers only the input parameter as deterministic data. Because of the nature of a disaster situation, many types of uncertainty might manifest. For example, in the case of an emergency in a building, additional collapses of the structure can occur. Therefore, a robust optimization approach to the network flow problem can be used to address the potential collapses of nodes or arcs. In addition, because each evacuee or responder travels at different times, the time value can be defined as an interval, not a nominal value. The direction of occupant movement can also be deemed an uncertainty factor. Furthermore, secondary damage, such as the spread of fire, can be regarded. Accordingly, a stochastic programming or robust optimization approach can be developed to accommodate these uncertain characteristics. For the solution procedure, an exact algorithm can be developed, rather than a heuristic algorithm, to solve the mathematical model. Furthermore, through the application of several different algorithms, comparisons and analyses among algorithms can be conducted.

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