Chapter 11

Coordinated Replenishments at a Single Stocking Point

11.1 Advantages and Disadvantages of Coordination

Advantages of Coordination

1. Savings on unit purchase costs.
2. Savings on unit transportation costs.
3. Savings on ordering costs.
4. Ease of scheduling.

Disadvantages of Coordination

1. A possible increase in the average inventory level.
2. An increase in system control costs.
3. Reduced flexibility.
11.2 The Deterministic Case: Selection of Replenishment Quantities in a Family of Items

11.2.1 Assumptions

(i) The demand rate of each item is constant and deterministic.

(ii) The replenishment quantity of an item need not be an integer.

(iii) The unit variable cost of any of the items does not depend on the quantity.

(iv) The replenishment lead time is of zero duration.

(v) No shortages are allowed.

(vi) The entire order quantity is delivered at the same time.

11.2.2 The Decision Rule

♣ Notation

\( A \) = major setup cost for the family, in dollars

\( a_i \) = minor setup cost for item \( i \), in dollars

\( D_i \) = demand rate of item \( i \), in units/unit time

\( v_i \) = unit variable cost of item \( i \), in $/unit

\( n \) = number of items in the family

\( m_i \) = the integer number of \( T \) intervals that the replenishment quantity of item \( i \) will last (decision variable)

\( T \) = time interval between replenishments of the family (decision variable)

♣ Derivation

\[ Q_i = D_i m_i T \]  

\[ TRC(T, m_1, \ldots, m_n) = A + \frac{\sum_{i=1}^{n} \frac{a_i}{m_i}}{T} + \sum_{i=1}^{n} D_i m_i T v_i r \]
\[
\frac{\partial TRC}{\partial T} = -A + \sum_{i=1}^{n} \frac{a_i}{m_i} + \sum_{i=1}^{n} D_i m_i v_i r = 0
\]

\[
T^*(m_1, \ldots, m_n) = \sqrt{\frac{2(A + \sum_{i=1}^{n} \frac{a_i}{m_i})}{r \sum_{i=1}^{n} m_i D_i v_i}} \quad 11.20
\]

\[
TRC^*(m_1, \ldots, m_n) = \sqrt{2 \left( A + \sum_{i=1}^{n} \frac{a_i}{m_i} \right) r \sum_{i=1}^{n} m_i D_i v_i} \quad 11.21
\]

\[
\text{Minimize}_{m_1, \ldots, m_n} TRC^*(m_1, \ldots, m_n)
\]

\[
\equiv \text{Minimize}_{m_1, \ldots, m_n} F(m_1, \ldots, m_n) = \left( A + \sum_{i=1}^{n} \frac{a_i}{m_i} \right) r \sum_{i=1}^{n} m_i D_i v_i \quad 11.22
\]

If we ignore the integrality of \(m_i\)'s, then

\[
\frac{\partial F(m_1, \ldots, m_n)}{\partial m_j} = -\frac{a_j}{m_j^2} \sum_{i=1}^{n} m_i D_i v_i + D_j v_j \left( A + \sum_{i=1}^{n} \frac{a_i}{m_i} \right) = 0
\]

\[
m_j^2 = \frac{a_j \sum_{i=1}^{n} m_i D_i v_i}{D_j v_j \left( A + \sum_{i=1}^{n} \frac{a_i}{m_i} \right)} \quad \forall j \quad 11.23
\]

\[
\frac{m_j^2}{m_k^2} = \frac{a_j D_k v_k}{D_j v_j} \frac{a_k}{a_k} \quad \rightarrow \quad \frac{m_j}{m_k} = \sqrt{\frac{a_j D_k v_k}{D_j v_j} a_k} \quad j \neq k
\]

\[
\frac{a_j}{D_j v_j} < \frac{D_k v_k}{a_k} \quad \rightarrow \quad m_j \leq m_k
\]

The item \(i\) having the smallest value of \(a_i/D_i v_i\) should have the lowest value of \(m_i\), namely, 1. Without loss of generality, we number the items such that item 1 has the smallest value of \(a_i/D_i v_i\).

\[
m_1 = 1 \quad 11.25
\]

Equation 11.23 \(\rightarrow m_j = \sqrt{\frac{a_j}{D_j v_j}} \sqrt{\frac{\sum_{i=1}^{n} m_i D_i v_i}{\left( A + \sum_{i=1}^{n} \frac{a_i}{m_i} \right)}} \quad j = 2, 3, \ldots, n \quad 11.25\]
\[
\sqrt{\frac{\sum_{i=1}^{n} m_i D_i v_i}{A + \sum_{i=1}^{n} \frac{a_i}{m_i}}} = C
\]

Equation 11.26

\[
m_j = C \sqrt{\frac{a_j}{D_j v_j}} \quad j = 2, 3, \ldots, n
\]

Equation 11.27

\[
\sum_{i=1}^{n} m_i D_i v_i = D_1 v_1 + \sum_{i=2}^{n} C \sqrt{\frac{a_i}{D_i v_i}} D_i v_i = D_1 v_1 + C \sum_{i=2}^{n} \sqrt{a_i D_i v_i}
\]

\[
\sum_{i=1}^{n} \frac{a_i}{m_i} = a_1 + \frac{1}{C} \sum_{i=2}^{n} \sqrt{a_i D_i v_i}
\]

Equation 11.28

Equation 11.29

Substituting equations 11.28 & 11.29 into equation 11.26, we get

\[
\frac{D_1 v_1 + C \sum_{i=2}^{n} \sqrt{a_i D_i v_i}}{A + a_1 + \frac{1}{C} \sum_{i=2}^{n} \sqrt{a_i D_i v_i}} = C^2
\]

\[
C = \sqrt{\frac{D_1 v_1}{A + a_1}}
\]

\[
m_j = \sqrt{\frac{a_j}{D_j v_j} \frac{D_1 v_1}{A + a_1}} \quad j = 2, 3, \ldots, n
\]

Equation 11.30

**Algorithm**

(Step 1) Number the items such that \(a_i/D_i v_i\) is smallest for item 1. Set \(m_1=1\).

(Step 2) Evaluate

\[
m_i = \sqrt{\frac{a_i}{D_i v_i} \frac{D_1 v_1}{A + a_1}}
\]

rounded to the nearest integer.

(Step 3) Evaluate \(T^*\) using the \(m_i\)'s of Step 2.

\[
T^*(m_1, \ldots, m_n) = \sqrt{\frac{2(A + \sum_{i=1}^{n} \frac{a_i}{m_i})}{r \sum_{i=1}^{n} m_i D_i v_i}}
\]

Equation 11.2

(Step 4) Determine

\[
Q_i = D_i m_i T^* \quad \forall i
\]

Equation 11.4

(Numerical Illustration) (p429)
11.2.3 A Bound on the Cost Penalty of the Heuristic Solution

In order to find a lower bound on the minimum cost, we use $m_1 = 1$ and $m_j$'s ($j = 2, \cdots, n$) in equation 11.30 which are not necessarily integer values.

$$TRC_{LB} = \sqrt{2(A + a_1)}D_1 v_1 r + \sum_{j=2}^{n} \sqrt{2a_j D_j v_j r}$$  \hspace{1cm} 11.5

The cost of heuristic is

$$TRC_H = \frac{A + \sum_{i=1}^{n} \frac{a_i}{m_i}}{T} + \sum_{i=1}^{n} \frac{D_i m_i T v_i r}{2}$$  \hspace{1cm} 11.6

(Example) $TRC_{LB} = $2,054.14/year vs. $TRC_H = $2,067.65/year $\rightarrow$ ratio=1.007

11.3 The Deterministic Case with Group Discounts

Unit price or freight rate discounts may be offered on the total dollar value or the total volume of a replenishment made up of several items.

♣ Main Idea of the Algorithm

 Compare the Following Three Possible Solutions

(1) coordinated solution, assuming a quantity discount, leads to total quantities that are always sufficient to achieve the discount $\rightarrow$ the best to use if it is feasible

(2) breakpoint solution

(3) coordinated solution without a quantity discount

Algorithm

(Step 1) Compute the $m_i$’s and $T$ as in Section 11.2 using

$$v_i = v_{oi}(1 - d)$$

Let $Q_{sm}$ be summation of order quantities of all items having $m_i=1$.

If $Q_{sm} \geq Q_b$, use the $m_i$’s, $T$, and $Q_i$’s just found. If not, proceed to Step 2.
(Step 2) Scale up the family cycle time $T$ until the smallest replenishment size equals the quantity breakpoint.

$$T_b = \frac{Q_b}{\sum_{i \in \{i | m_i = 1\}} D_i}$$  \hspace{1cm} 11.8

The associated cost of this breakpoint solution is as follows:

$$TRC(T_b, m_1, \ldots, m_n) = (1 - d) \sum_{i=1}^{n} D_i v_{oi} + \frac{A + \sum_{i=1}^{n} \frac{m_i}{m_i}}{T_b} + \sum_{i=1}^{n} D_i m_i T_b v_{oi} (1 - d) r$$  \hspace{1cm} 11.9

$$TRC(T, m_1, \ldots, m_n) = \sum_{i=1}^{n} D_i v_{oi} + \sqrt{2 \left(A + \sum_{i=1}^{n} \frac{a_i}{m_i}\right) r \sum_{i=1}^{n} m_i D_i v_{oi}}$$  \hspace{1cm} 11.10

(Step 4) Compare the TRC values found in Steps 2 and 3 and use the $m_i$’s, $T$, and $Q_i$’s associated with the lower of these.

(Numerical Illustration) (p432)

11.4 The Case of Probabilistic Demand and No Quantity Discounts

11.4.1 $(S, c, s)$ or Can-Order System

- A special type of continuous review system for controlling coordinated items ← savings in setup costs are of primary concern

- Whenever item $i$’s inventory position drops to or below $s_i$ (must-order point), it triggers a replenishment action that raises item $i$’s level to its order-up-to level $S_i$. At the same time any other item $j$ (within the associated family) with its inventory position at or below its can-order point $c_j$ is included in the replenishment.

Figure 11.2 Behavior of an Item under $(S, c, s)$ Control
11.6 The Production Environment

11.6.1 The Case of Constant Demand and Capacity: The Economic Lot Scheduling Problem (ELSP)

• Find a cycle length, a production sequence, production times, and idle times, so that the production sequence can be completed in the chosen cycle, the cycle can be repeated over time, demand can be fully met, and annual inventory and setup costs can be minimized.

• NP-hard problem \(\rightarrow\) the need to satisfy a production capacity constraint and the need to have only one product in production at a time (synchronization constraint)

\[\text{Notation}\]

\(T\) = common order interval, or time supply, for each product, in units of time

\(p_i\) = production rate of item \(i\), in units/unit time

\(A_i\) = setup cost for item \(i\), in dollars

\(K_i\) = setup time for item \(i\), in dollars

\(D_i\) = demand rate of item \(i\), in units/unit time

\(v_i\) = unit variable cost of item \(i\), in \$/unit

\(n\) = number of items in the family

\[
\text{Minimize } \sum_{i=1}^{n} TRC_i(T) = \left[\frac{A_i}{T} + \frac{rv_iD_i(p_i - D_i)T}{2p_i}\right]
\]

subject to \(\sum_{i=1}^{n} \left( K_i + \frac{D_iT}{p_i} \right) T\)

\[
T^* = \text{Max} \left( \frac{\sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} \frac{rv_iD_i(p_i - D_i)T}{2p_i}}, \frac{\sum_{i=1}^{n} K_i}{1 - \sum_{i=1}^{n} \frac{D_i}{p_i}} \right)
\]

(Numerical Illustration) (p446)
Relevant Literature


Gallego and Moon (1992): In the realm of changing the givens, they examine a multiple product factory that employs a cyclic schedule to minimize holding and setup costs. When setup times can be reduced, at the expense of setup costs, by externalizing setup operations, they show that dramatic savings are possible for highly utilized facilities.


11.7 Shipping Consolidation

Shipment Consolidation Decisions (Higginson and Bookbinder (1994))

(i) Which orders will be consolidated and which will be shipped individually?

(ii) When will orders be released for possible shipping? Immediately, or after some time or quantity-trigger?

(iii) Where will the consolidation take place? At the factory or at an off-site warehouse or terminal?

(iv) Who will consolidate? The manufacturer, customer, or a third party?

Three Possible Policies (Higginson and Bookbinder (1994))

(i) a time policy that ships at a prespecified time

(ii) a quantity policy that ships when a given quantity is achieved

(iii) a time/quantity policy that ships at the earliest of the time and quantity values

The shipper must trade off cost per unit with customer service in deciding on which policy to use.